Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2021

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Contents

Paper 1: Marking Scheme

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D
No of categories	3	4	5
5-mark scale	0, 2, 5	0, 2, 3, 5	
10-mark scale		0, 3, 7, 10	0, 3, 5, 8, 10
15-mark scale			0, 4, 8, 12, 15
20-mark scale			0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

A rounding penalty is applied only once in each section (a), (b), (c) etc. A penalty for an omitted unit is applied only once in each section (a), (b), (c) etc. There is no penalty for omitted units if the question specifies the unit to be used in the answer, and there is generally no penalty for an omitted euro symbol in questions involving money.

In general, accept a candidate’s work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Section A (120)		**Section B (100)**	
Question 1 (30) (a) 10C (b) 10D (c) 10D **Question 2 (30)** (a) 10D (b) 20D **Question 3 (30)** (a) 10D (b)(i) 10C (b)(ii) 10D	**Question 4 (30)** (a) 15D (b)(i) 5B (b)(ii) 10C **Question 5 (30)** (a)(i) 10D (a)(ii) 10D (b) 10D **Question 6 (30)** (a) 10D (b) 10C (c) 10D	**Question 7 (50)** (a)(i) 5C (a)(ii) 5C (a)(iii) 5C (a)(iv) 10C (b)(i) 10C (b)(ii) 10C (b)(iii) 5C **Question 8 (50)** (a)(i) 5C (a)(ii) 15D (b)(i) 5C (b)(ii) 10C (b)(iii) 5C (c) 10D	**Question 9 (50)** (a)(i) 5C (a)(ii) 5B (a)(iii) 10C (b) 10D (c) 10D (d) 10D **Question 10 (50)** (a)(i) 5C (a)(ii) 10C (a)(iii) 5B (a)(iv) 5B (b)(i) 5B (b)(ii) 10C (b)(iii) 5C (b)(iv) 5C

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✘	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
$\boxed{*}$	Star	Rounding / Unit / Arithmetic error Misreading	
	Horizontal wavy	Error	
✔l	Tick L		The work presented in the body of the script merits low partial credit
✔m	Tick M		The work presented in the body of the script merits mid partial credit (or partial credit)
✔h	Tick H		The work presented in the body of the script merits high partial credit
$\boxed{F*}$	F star		The work presented in the body of the script merits Full Credit (– 1)
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page (portion of the page)	
$\boxed{o}$	Oversimplify	The candidate has oversimplified the work	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

In a **C scale** where * and and appear in the body of the work, then ✔l should be placed in the right margin.

In the case of a **D scale** with the same annotations, ✔m should be placed in the right margin.

A ✔ in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is then indicated in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	**Model Solution – 30 Marks**	**Marking Notes**
(a)	$\frac{(4-2i)}{(2+4i)} = \frac{4-2i}{2+4i} \times \frac{2-4i}{2-4i}$ $= \frac{(8-16i-4i-8)}{2^2+4^2}$ $= \frac{-20i}{20}$ $= 0 - 1i$ $\therefore k = -1$ **OR** $\frac{(4-2i)}{(2+4i)} = \frac{-i(2+4i)}{2+4i}$ $= 0 - 1i$ $\therefore k = -1$ **OR** $4 - 2i = ki(2 + 4i)$ **Re**: $4 = -4k$ $\therefore k = -1$ **or Im**: $-2i = 2ki$ $\therefore k = -1$	**Scale 10C(0, 3, 7, 10)** *Note:* Accept $0 - 1i$ *Low Partial Credit:* • $2 - 4i$ • $4 - 2i = ki(2 + 4i)$ *High Partial Credit:* • $\frac{4-2i}{2+4i} \times \frac{2-4i}{2-4i}$ • $4 - 2i = -4k + 2ki$ • Sets Re = Re **or** Im = Im *Full Credit –1:* • $0 - i$ or $-1i$ as solution, with k not identified.

Q1	Model Solution – 30 Marks	Marking Notes
(b)	$-5+12i = (a+bi)^2$ $a^2+2abi-b^2$ Re: $a^2-b^2=-5$ Im: $2ab=12 \ \therefore \ b=\frac{6}{a}$ $a^2-\left(\frac{6}{a}\right)^2 = -5$ $a^4+5a^2-36 = 0$ $(a^2+9)(a^2-4) = 0$ $\therefore \ a=\pm 2$ and $b=\pm 3$ Answer: $2+3i, \ -2-3i$ **OR** $r=\sqrt{5^2+12^2}=13$ $\tan\theta=-\frac{12}{5}$ so $\cos\theta=-\frac{5}{13}$ $(-5+12i)^{\frac{1}{2}}$ $=[13(\cos(\theta+2n\pi)+i\sin(\theta+2n\pi))]^{\frac{1}{2}}$ $=\sqrt{13}\left(\cos\left(\frac{\theta}{2}+n\pi\right)+i\sin\left(\frac{\theta}{2}+n\pi\right)\right)$ $2\sin^2\left(\frac{\theta}{2}\right)=1-\cos\theta=1+\frac{5}{13}$ So $\sin\left(\frac{\theta}{2}\right)=\frac{3}{\sqrt{13}}$ and so $\cos\left(\frac{\theta}{2}\right)=\frac{2}{\sqrt{13}}$ $n=0$: $\sqrt{13}\left(\cos\left(\frac{\theta}{2}\right)+i\sin\left(\frac{\theta}{2}\right)\right)$ $=\sqrt{13}\left(\frac{2}{\sqrt{13}}+i\frac{3}{\sqrt{13}}\right)=2+3i$ $n=1$: $\sqrt{13}\left(\cos\left(\frac{\theta}{2}+\pi\right)+i\sin\left(\frac{\theta}{2}+\pi\right)\right)$ $=\sqrt{13}\left(-\frac{2}{\sqrt{13}}-i\frac{3}{\sqrt{13}}\right)=-2-3i$	**Scale 10D(0, 3, 5, 8, 10)** *Note*: Accept $2+3i$ for *Full Credit* *Low Partial Credit:* • $(a+bi)^2=-5+12i$ • $a+bi=(-5+12i)^{\frac{1}{2}}$ • r or θ found • $-5+12i$ plotted on Argand diagram. • Shows some knowledge of De Moivre's theorem *Mid Partial Credit:* • Relevant equation in a single variable • Writes $-5+12i$ in polar form *High Partial Credit:* • Finds $a=2$ or $b=3$ • $-2-3i$ found • Correct solution in polar form (accept with mishandling of $2n\pi$)

Q1	Model Solution – 30 Marks	Marking Notes
(c)	$z^3 = r(\cos\theta + i\sin\theta)$ $z = \left(r(\cos\theta + i\sin\theta)\right)^{\frac{1}{3}}$ $= 2\left(\cos\frac{\pi+2n\pi}{3} + i\sin\frac{\pi+2n\pi}{3}\right)$ $n = 0$: $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 1 + \sqrt{3}i$ $n = 1$: $z = 2\left(\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right) = -2$ $n = 2$: $z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 1 - \sqrt{3}i$	**Scale 10D (0, 3, 5, 8, 10)** *Note*: if $((r(\cos\theta + i\sin\theta))^3$ is used, award *Low Partial Credit* at most. *Note*: polar form must be used to achieve any credit *Low Partial Credit:* • $z = \left(r(\cos\theta + i\sin\theta)\right)^{\frac{1}{3}}$ • r found • θ found • $-8 + 0i$ plotted on an Argand diagram • Shows some knowledge of De Moivre's theorem *Mid Partial Credit:* • $z = 8^{\frac{1}{3}}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ • $8^{\frac{1}{3}}\left(\cos\frac{\pi+2n\pi}{3} + i\sin\frac{\pi+2n\pi}{3}\right)$ *High Partial Credit:* • One root evaluated in the form $a + bi$ from De Moivre's expression • Three solutions in polar form

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$\lvert -3+p\rvert = 5$ [LPC] $9-6p+p^2 = 25$ [MPC] $p^2-6p-16 = 0$ $(p+2)(p-8) = 0$ [HPC] $p=-2$ or $p=8$ **OR** $\lvert -3+p\rvert = 5$ $-3+p=5$ **or** $-3+p=-5$ $p=8$ **or** $p=-2$	**Scale 10D(0, 3, 5, 8, 10)** **If solving as a quadratic equation:** *Low Partial Credit:* • $x=-3$ substituted into equation *Mid Partial Credit:* • relevant quadratic in p found *High Partial Credit:* • quadratic factorised • one missing or incorrect term in quadratic, but finishes correctly. **If solving as two linear equations:** *Low Partial Credit:* • 1 linear equation • $x=-3$ substituted into equation *Mid Partial Credit:* • 1 value of p found *High Partial Credit:* • Both linear equations in p given

Q2	Model Solution – 30 Marks	Marking Notes
(b)	$f(-4) = 0$ $(-4)^3 + q(-4)^2 - 22(-4) + 56 = 0$ $-64 + 16q + 88 + 56 = 0$ $16q + 80 = 0$ $16q = -80$ $q = -5$ $x^3 - 5x^2 - 22x + 56 = 0$ $x^2(x+4) - 9x(x+4) + 14(x+4) = 0$ $(x^2 - 9x + 14)(x+4) = 0$ $(x-2)(x-7)(x+4) = 0$ Roots $= (-4,\ 2,\ 7)$ **OR** $$\begin{array}{r\|l} & x^2 - 9x + 14 \\ \hline x+4 & x^3 - 5x^2 - 22x + 56 \\ & \underline{x^3 + 4x^2} \\ & \quad -9x^2 - 22x + 56 \\ & \quad \underline{-9x^2 - 36x} \\ & \qquad 14x + 56 \\ & \qquad \underline{14x + 56} \\ & \qquad\qquad 0 \end{array}$$ Remainder $= 0$, $\therefore q = -5$ Roots $= (-4,\ 2,\ 7)$	**Scale 20D (0, 5, 10, 15, 20)** *Note:* If q set equal to 5 and $x + 4$ divided into $f(x)$, needs conclusion in order to be accepted as showing that $q = -5$. *Low Partial Credit:* • States $f(-4) = 0$ • Any correct division • Sets up long division correctly *Mid Partial Credit:* • Shows that $q = -5$ • Correct quotient in quadratic form found (accept in terms of q) *High Partial Credit:* • Find $x = 2$ and $x = 7$ • Correct quotient in quadratic form found **and** shows that $q = -5$ Full Credit –1: • Apply a * for -4 not listed as a root

<table>
<tr><th>Q3</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>
$xz = 2\sqrt{2}$
$yz = 8\sqrt{6}$
$xy = 4\sqrt{3}$
$\Rightarrow x^2y^2z^2 = (2\sqrt{2})(8\sqrt{6})(\,4\sqrt{3})$
$\Rightarrow x^2y^2z^2 = 384$
$\Rightarrow xyz = \sqrt{384} = 8\sqrt{6}$ [cm³]

OR

$y = \frac{4\sqrt{3}}{x}$ and $z = \frac{2\sqrt{2}}{x}$
$\Rightarrow \left(\frac{4\sqrt{3}}{x}\right)\left(\frac{2\sqrt{2}}{x}\right) = 8\sqrt{6}$
$\Rightarrow x = 1$ cm
$\Rightarrow y = 4\sqrt{3}$ cm
$\Rightarrow z = 2\sqrt{2}$ cm
$\therefore xyz = 8\sqrt{6}$ [cm³]
</td><td>
Scale 10D (0, 3, 5, 8, 10)

Note: $(2\sqrt{2})(8\sqrt{6})(\,4\sqrt{3})$ on its own is not awarded any credit.

If solving without isolating variables:
Low Partial Credit:
• $xz = 2\sqrt{2}$, or similar
• xy, xz, and yz stated
• Volume $= xyz$
• $V = x(8\sqrt{6})$ or similar

Mid Partial Credit:
• $xz = 2\sqrt{2}$ and $yz = 8\sqrt{6}$ and $xy = 4\sqrt{3}$

High Partial Credit:
• $x^2y^2z^2 = (2\sqrt{2})(8\sqrt{6})(\,4\sqrt{3})$

If solving by isolating variables:
Low Partial Credit:
• $xz = 2\sqrt{2}$, or similar

Mid Partial Credit:
• $y = \frac{4\sqrt{3}}{x}$ and $z = \frac{2\sqrt{2}}{x}$, or similar

High Partial Credit:
• 1 dimension found
</td></tr>
<tr><td>(b)
(i)</td><td>
$3x^2 + 8x - 35 = 0$
$(3x - 7)(x + 5) = 0$
$x = \frac{7}{3} \quad x = -5$

OR

Roots $= \frac{-8 \pm \sqrt{8^2 - 4(3)(-35)}}{2(3)}$

$x = \frac{7}{3} \quad x = -5$
</td><td>
Scale 10C(0, 3, 7, 10)

Low Partial Credit:
• Effort at factorisation
• Quadratic formula with some substitution

High Partial Credit:
• Factors found
• Formula fully substituted
</td></tr>
</table>

Q3	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$3.(3^m)^2 + 8(3^m) - 35 = 0$ [*as* $3^m > 0$*, then* $3^m \neq -5$*, so:*] $$3^m = \frac{7}{3}$$ $$\log_3 3^m = \log_3\left(\frac{7}{3}\right)$$ $$m = \log_3\left(\frac{7}{3}\right)$$ $$= \log_3 7 - \log_3 3$$ $$= \log_3 7 - 1$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Some work in writing given equation in the form of that in **(b)(i)** • −5 explicitly excluded • $x = 3^m$ *Mid Partial Credit:* • $3^m = \frac{7}{3}$ *High Partial Credit:* • m isolated correctly, for example, $m = \log_3\left(\frac{7}{3}\right)$ *Full Credit –1:* • $3^m = -5$ written down, and not explicitly excluded

<table>
<tr><th>Q4</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$\boldsymbol{P(1)}$: $2^{3(1)-1}+3=7$, which is div. by 7

$\boldsymbol{P(k)}$: Assume $2^{3k-1}+3$ is div. by 7

$$2^{3k-1}+3=7M$$
$$2^{3k-1}=7M-3$$

$\boldsymbol{P(k+1)}$: $2^{3k+2}+3$

$=2^3(2^{3k-1})+3$

$=8(7M-3)+3$

$=56M-21$

$P(k+1)$ is divisible by 7

True for $n=1$ and, if true for $n=k$, then true for $n=k+1$. Therefore, true for all $n\geq 1$.

OR

$\boldsymbol{P(k+1)}$: $2^{3k+2}+3$

$=2^3(2^{3k-1})+3$

$=(7+1)(2^{3k-1})+3$

$=(7.2^{3k-1})+(2^{3k-1}+3)$

Both divisible by 7

True for $n=1$ and, if true for $n=k$, then true for $n=k+1$. Therefore, true for all $n\geq 1$.

</td><td>

Scale 15D (0, 4, 8, 12, 15)

Accept Step $P(1)$, Step $P(k)$, Step $P(k+1)$ in any order

Low Partial Credit:
- Any **one** of Step $P(1)$, Step $P(k)$, or Step $P(k+1)$

Mid Partial Credit:
- Any **two** of Step $P(1)$, Step $P(k)$, or Step $P(k+1)$

High Partial Credit:
- Some valid work in using Step $P(k)$ to prove Step $P(k+1)$

Full Credit –1:
- Omits conclusion but otherwise correct

</td></tr>
<tr><td>(b)
(i)</td><td>

$$T_n=p+(n-1)(7)$$
$$T_n=p+7n-7$$

</td><td>

Scale 5B (0, 2, 5)

Note: Accept $p+(n-1)(7)$.

Partial Credit:
- $a=p$ or $d=7$
- T_n formula with some substitution

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</table>

Q4	Model Solution – 30 Marks	Marking Notes
(b) **(ii)**	$$p + 7n - 7 = 2021$$ $$p + 7n = 2028$$ $$7n = 2028 - p$$ so $2028 - p$ is the nearest multiple of 7 that is less than 2028. This is 2023. So: $$2028 - p = 2023$$ $$p = 5$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $p + 7n - 7 = 2021$ *High Partial Credit:* • 2023 • 289

Q5	Model Solution – 30 Marks	Marking Notes
(a) (i)	$f'(x) = 6x^2 + 12x - 12$ $= 6(x^2 + 2x - 2)$ $= 6(x^2 + 2x + 1 - 1 - 2)$ $= 6[(x+1)^2 - 3]$ $= 6(x+1)^2 - 18$ $\therefore a = 6, \quad b = 1, \quad c = -18$ **OR** $6x^2 + 12x - 12 = ax^2 + 2abx + ab^2 + c$ $\Rightarrow a = 6$ $\therefore\ 2(6)(b) = 12$ $\Rightarrow b = 1$ $\therefore 6(1)^2 + c = -12$ $\Rightarrow c = -18$	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit:* • Any correct differentiation *Mid Partial Credit:* • Differentiation fully correct *High Partial Credit:* • any 2 of a or b or c correctly identified • $6[(x+1)^2 - 3]$ *Full Credit –1* • $6(x+1)^2 - 18$
(a) (ii)	$f'(x) > g'(x)$ $6x^2 + 12x - 12 > 36$ $6x^2 + 12x - 48 > 0$ $x^2 + 2x - 8 > 0$ $x < -4$ or $x > 2$.	**Scale 10D(0, 3, 5, 8, 10)** *Note:* Accept inclusion of -4 and 2. *Low Partial Credit:* • Any correct differentiation for $g(x)$ *Mid Partial Credit:* • Inequality correctly formulated *High Partial Credit:* • roots of quadratic found *Full Credit –1:* • $-4 > x > 2$

Q5	Model Solution – 30 Marks	Marking Notes
(b)	$h'(x) = 4\cos 2x$ $h'\left(\frac{\pi}{6}\right) = 4\cos 2\left(\frac{\pi}{6}\right) = 4\cos\left(\frac{\pi}{3}\right)$ $= 2 = m_T$ $h\left(\frac{\pi}{6}\right) = 2\sin 2\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right)$ $= \sqrt{3}$ $\left(\frac{\pi}{6}, \sqrt{3}\right) \quad m = 2$ $y - \sqrt{3} = 2\left(x - \frac{\pi}{6}\right)$ $x = 0 \Rightarrow y - \sqrt{3} = 2\left(-\frac{\pi}{6}\right)$ $y = \sqrt{3} - \frac{\pi}{3} = 0{\cdot}6848\ldots$ $\therefore k = 0{\cdot}68$ [2 D.P.] **OR** *Finds* $\left(\frac{\pi}{6}, \sqrt{3}\right)$ *and slope* $= 2$*, then:* $y = mx + c$ implies $\sqrt{3} = 2\left(\frac{\pi}{6}\right) + k$ i.e. $k = \sqrt{3} - \frac{\pi}{3}$ $= 0{\cdot}6848\ldots = 0{\cdot}68$ [2 D.P.]	**Scale 10D(0, 3, 5, 8, 10)** *Note:* Consider solution as involving 4 steps: Step 1: differentiate $2\sin 2x$ Step 2: find slope at $x = \frac{\pi}{6}$ is 2 Step 3: find y-value at $x = \frac{\pi}{6}$ Step 4: find k (for example, using slope or equation of line) *Low Partial Credit:* • Work of merit, for example, some correct differentiation; substitutes $\frac{\pi}{6}$ in to $h(x)$; formula for slope or equation of a line with some substitution *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct

<table>
<tr><th>Q6</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$h'(x) = a(x+1)(x-3)$
$= a(x^2 - 2x - 3)$
$(0, 6) \in h'(x)$
$\therefore 6 = a(0 + 0 - 3)$
$\Rightarrow a = -2$
$h'(x) = -2(x^2 - 2x - 3)$
$h'(x) = -2x^2 + 4x + 6$
OR
$h'(x) = -2x^2 + 4x + 6$
y-intercept $= h'(0) = 6$
$h'(x) = -2(x+1)(x-3)$
$\therefore$ Roots = –1 and 3
OR
$h'(x) = ax^2 + bx + c$
$h'(0) = c = 6$
So $h'(-1) = a - b + 6 = 0$
and $h'(3) = 9a + 3b + 6 = 0$
$3 \times h'(1) = 3a - 3b + 18 = 0$
So $12a + 24 = 0$
$\therefore a = -2$ and $b = 4$
i.e. $h'(x) = -2x^2 + 4x + 6$

</td><td>

Scale 10D (0, 3, 5, 8, 10)
Note: Accept 3 points from graph verified as belonging to given equation of $h'(x)$
Note: three points identified from graph is *Low Partial Credit*.

Low Partial Credit:
• Identifies or uses a relevant value from the graph,
for example, $(x + 1)$ or 3
• A factorisation of given $h'(x)$,
for example $2(-x^2 + 2x + 3)$

Mid Partial Credit:
• Generates $(x + 1)(x - 3)$ from graph
• Correctly verifies one point from the graph into the given $h'(x)$
• Full factorisation of given $h'(x)$
• Using simultaneous equations, finds one value (a, b, or c)

High Partial Credit:
• Generates $x^2 - 2x - 3$ from graph
• From given $h'(x)$, finds two roots
• Correctly verifies two points from the graph into the given $h'(x)$
• From given $h'(x)$, shows that y-intercept is 6 **and** fully factorises
• Using simultaneous equations, finds two values (from a, b, and c)

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</table>

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$h''(x) = -4x + 4 = 0$ at max/min of $h'(x)$ $\therefore x = 1$ $h'''(x) = -4 < 0$, i.e. max $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** [*Quadratic with negative x^2, so max occurs halfway between the roots:*] $x = \frac{-1+3}{2} = 1$ $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** $h'(x) = -2(x^2 - 2x - 3)$ $= -2(x^2 - 2x + 1 - 1 - 3)$ $= -2((x-1)^2 - 4)$ $= -2(x-1)^2 + 8$ $\therefore$ max positive slope$= 8$	**Scale 10C(0, 3, 7, 10)** *Note*: It is possible to accept for *Full Credit* without $h'''(x) < 0$ *Low Partial Credit:* • Some correct differentiation of $h'(x)$ • Finds $h''(x)$ • $h'(x) = -2(x^2 - 2x - 3)$ • Indicates axis of symmetry on graph *High Partial Credit:* • $x = 1$ • $h'(x) = -2((x-1)^2 - 4)$
(c)	$h(x) = \int h'(x)\,dx$ $h(x) = -\frac{2x^3}{3} + \frac{4x^2}{2} + 6x + C$ $(0, -2) \in h(x)$: $-2 = -0 + 0 + 0 + C$ $\Rightarrow C = -2$ $\therefore h(x) = -\frac{2x^3}{3} + 2x^2 + 6x - 2$	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept correct answer without work. *Low Partial Credit:* • Any indication of integration *Mid Partial Credit:* • Integration of 3 terms fully correct *High Partial Credit:* • Relevant equation in C (with substitution)

<table>
<tr><th>Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>
<table>
<tr><th>Swing</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td>Length of Arc (cm)</td><td>45</td><td>$\mathbf{\frac{81}{2}}$</td><td>$\frac{729}{20}$</td><td>$\mathbf{\frac{6561}{200}}$</td><td>$\mathbf{\frac{59049}{2000}}$</td></tr>
</table>
OR
Answers as decimals:
$40{\cdot}5$, $32{\cdot}805$, and $29{\cdot}5245$
</td><td>
Scale 5C(0, 2, 3, 5)
Note: Accept decimals rounded to one decimal place.

Low Partial Credit:
• 1 correct table entry

High Partial Credit:
• 2 correct table entries
</td></tr>
<tr><td>(a)
(ii)</td><td>
$T_n = 45(0{\cdot}9)^{n-1}$
$T_{25} = 45(0{\cdot}9)^{24}$
$= 3{\cdot}5894 \ldots$
$= 3{\cdot}6$ cm [1 D.P.]
</td><td>
Scale 5C (0, 2, 3, 5)

Low Partial Credit:
• T_n formula with some substitution
• Identifies a or r

High Partial Credit:
• Formula fully substituted

Full Credit –1:
• Correct answer, no or incorrect unit
</td></tr>
<tr><td>(a)
(iii)</td><td>
$S_{40} = \dfrac{45(1 - 0{\cdot}9^{40})}{1 - 0{\cdot}9}$
$S_{40} = 443{\cdot}3486 \ldots$
$= 443$ [cm] [$\in \mathbb{N}$]
</td><td>
Scale 5C (0, 2, 3, 5)

Low Partial Credit:
• Sum of two or more relevant arc-lengths
• S_{40} formula with some substitution
• Identifies a or r

High Partial Credit:
• Formula fully substituted
</td></tr>
</table>

Q7	Model Solution – 50 Marks	Marking Notes
(a) (iv)	$45(0{\cdot}9)^{n-1} = 2$ $(0{\cdot}9)^{n-1} = \frac{2}{45}$ $n - 1 = \log_{0\cdot9}\left(\frac{2}{45}\right)$ $n - 1 = 29{\cdot}5510\ldots$ $n = 30{\cdot}5510\ldots$ $\therefore p = 31$	**Scale 10C (0, 3, 7, 10)** *Note:* If solving by trial and improvement, T_{30} and T_{31} must both be evaluated for *Full Credit*. *Low Partial Credit:* • Two or more different values substituted and evaluated for n, other than 30 and 31 • $45(0{\cdot}9)^{n-1} = 2$ *High Partial Credit:* • log equation without indices • Evaluates T_{30} and T_{31} but no conclusion • $p = 31$ with T_{31} evaluated
(b) (i)	$l = 2\pi r\left(\frac{\theta}{360°}\right)$ [*degrees*] $2\pi(100)\left(\frac{\theta}{360°}\right) = 45$ $\theta = \frac{45 \times 360°}{200\pi} = 25{\cdot}7831\ldots°$ $= 26$ [°] [$\in \mathbb{N}$] **OR** $l = r\theta$ [*radians*] $100\theta = 45$ $\theta = 0{\cdot}45$ radians $\theta = 0{\cdot}45\left(\frac{180}{\pi}\right)° = 25{\cdot}7831\ldots°$ $= 26$ [°] [$\in \mathbb{N}$]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Formula for length of arc with some substitution • Circumference found *High Partial Credit:* • θ found in radians • Arc-length formula for θ in degrees, fully substituted • Verifies that $\theta = 26$ gives arc-length of 45 cm, to the nearest cm

Q7	Model Solution – 50 Marks	Marking Notes
(b) (ii)	$26 + 26(0{\cdot}9) + 26(0{\cdot}9)^2 + 26(0{\cdot}9)^3 + \cdots$ $$S_\infty = \frac{26}{1 - 0{\cdot}9} = 260\ [°]$$	**Scale 10C (0, 3, 7, 10)** *Note:* Accept solution based on more accurate value for θ. *Low Partial Credit:* • a or r identified • First line in model solution, or similar • S_∞ formula with some substitution *High Partial Credit:* • S_∞ formula fully substituted • S_n evaluated for a large enough value of n that gives the correct answer, when rounded to the nearest degree.
(b) (iii)	[*Half total distance and half accumulated angle occur at same point*]: Distance: $S_\infty = \frac{45}{1-0{\cdot}9} = 450$ cm Half = 225 [cm] **OR** $$\frac{26(1 - 0{\cdot}9^n)}{1 - 0{\cdot}9} = \frac{260°}{2}$$ $$1 - 0{\cdot}9^n = \frac{1}{2}$$ $$-0{\cdot}9^n = -\frac{1}{2}$$ $$n = 6{\cdot}5788 \ldots$$ $$\frac{45(1 - 0{\cdot}9^{6{\cdot}5788})}{1 - 0{\cdot}9}$$ $$= 224{\cdot}9996 \ldots$$ $$= 225\ [\text{cm}]\ [\in \mathbb{N}]$$ **OR** Half accumulated angle $= \frac{260°}{2} = 130°$ So distance $= 2\pi(100)\left(\frac{130°}{360°}\right)$ $= 226{\cdot}89 \ldots = 227$ [cm] $[\in \mathbb{N}]$	**Scale 5C (0, 2, 3, 5)** *Note*: Accept $n = 6{\cdot}5788 \ldots$ rounded to $n = 7$ and finished ($234{\cdot}766 \ldots = 235$ [cm] $[\in \mathbb{N}]$) *Low Partial Credit:* • Finds half accumulated angle • S_n formula with some substitution • S_∞ distance formula with some substitution *High Partial Credit:* • n found • S_∞ found for distance

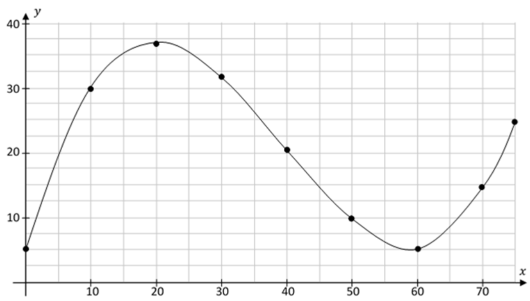
<table>
<tr><th>Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$h(10)$
$= 0{\cdot}001(10)^3 - 0{\cdot}12(10)^2 + p(10) + 5$
$= 30$

$10p = 36$
$p = 3{\cdot}6$</td><td>Scale 5C(0, 2, 3, 5)

Low Partial Credit:
• $h(10)$ with some relevant substitution

High Partial Credit:
• Equation in p</td></tr>
<tr><td>(a)
(ii)</td><td>
<table>
<tr><th>x</th><th>0</th><th>10</th><th>20</th><th>30</th><th>40</th></tr>
<tr><td>$h(x)$</td><td>5</td><td>30</td><td>37</td><td>32</td><td>21</td></tr>
</table>

<table>
<tr><th>x</th><th>50</th><th>60</th><th>70</th><th>75</th></tr>
<tr><td>$h(x)$</td><td>10</td><td>5</td><td>12</td><td>21·875</td></tr>
</table>



</td><td>Scale 15D (0, 4, 8, 12, 15)
14 items are required: 5 table entries and 9 plots (which also need to be joined appropriately for Full Credit)

Low Partial Credit:
• Any one item correct

Mid Partial Credit:
• any 7 items correct

High Partial Credit:
• any 11 items correct

Full Credit –1:
• All items correct but points not joined or joined incorrectly
• All items but one correct, and points appropriately joined</td></tr>
<tr><td>(b)
(i)</td><td>$h'(x) = 0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$</td><td>Scale 5C (0, 2, 3, 5)

Low Partial Credit:
• correct differentiation of 1 term

High Partial Credit:
• correct differentiation of 2 terms</td></tr>
</table>

Q8	Model Solution – 50 Marks	Marking Notes
(b) (ii)	$h'(x) = 0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$ $h'(20) \;= 0{\cdot}003(20)^2 - 0{\cdot}24(20) + 3{\cdot}6$ $= 0$, so [local] max/min at $x = 20$ $h''(20) \;= 0{\cdot}006(20) - 0{\cdot}24 < 0$, so local max Also $h(20) > h(0)$ and $h(20) > h(75)$ **OR** $h'(x) = 0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$ $h'(20) \;= 0{\cdot}003(20)^2 - 0{\cdot}24(20) + 3{\cdot}6$ $= 0$, so [local] max/min at $x = 20$ From graph, turning point at $x = 20$ is a [local] max, and it is above the two endpoints [0 and 75]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$ • States $h'(x) = 0$, or similar *High Partial Credit:* • Shows that $h'(20) = 0$, but no further justification that it is the max in the range $[0, 75]$.
(b) (iii)	$h''(x) = 0{\cdot}006x - 0{\cdot}24 = 0$ $0{\cdot}006x = 0{\cdot}24$ $x = 40$ $h(40) = 0{\cdot}001(40)^3 - 0{\cdot}12(40)^2$ $+ 3{\cdot}6(40) + 5$ $= 21$ [m]	**Scale 5C (0, 2, 3, 5)** *Note:* work presented in (b)(iii) must involve calculus, or be based on calculus from (b)(ii), to be awarded any credit. *Low Partial Credit:* • Some correct differentiation of $h'(x)$ • $h''(x)$ indicated *High Partial Credit:* • $x = 40$

Q8	Model Solution – 50 Marks	Marking Notes
(c)	$\frac{1}{75}\int_0^{75} h(x)\ \mathrm{d}x \quad = \frac{1}{75}\left[\frac{0\cdot001x^4}{4} - \frac{0\cdot12x^3}{3} + \frac{3\cdot6x^2}{2} + 5x + C\right]$ $= \frac{1}{75}\left[\frac{0\cdot001(75)^4}{4} - \frac{0\cdot12(75)^3}{3} + \frac{3\cdot6(75)^2}{2} + 5(75) + C - (0 + C)\right]$ $= \frac{1}{75}(1535\cdot15625)$ $= 20\cdot46875$ m $= 20\cdot47$ [m] [2 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Integration is required in order to be awarded any credit *Low Partial Credit:* • Integration indicated *Mid Partial Credit:* • Integration of terms fully correct (accept without C) *High Partial Credit:* • limits substituted correctly (and $\frac{1}{75}$ present)

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$95 = Ae^{(-0\cdot081)(0)} + 20$ $75 = Ae^{0}$ $75 = A$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some substitution into function, including $A = 75$ *High Partial Credit:* • Equation in A • Substitutes $A = 75$ and $t = 0$
(a) (ii)	*Any valid description, for example*: "The lowest temperature the coffee will cool to" **OR** "The lower limit of the coffee's temperature" **OR** "Room temperature" *etc.*	**Scale 5B (0, 2, 5)** *Partial Credit:* • reference to temperature **or** limit
(a) (iii)	$T(10) = 75e^{(-0\cdot081)(10)} + 20$ $T(10) = 53\cdot364$ ∴ Decrease $95 - 53\cdot364 = 41\cdot636$ $= 42°$ C	**Scale 10C (0, 3, 7, 10)** *Note:* Award *High Partial Credit* for finding $T'(10)$, i.e. the rate of decrease at $t = 10$. *Low Partial Credit:* • Some substitution into function • Finds $T'(t)$ *High Partial Credit:* • $T(10)$ evaluated • $T'(10)$ evaluated ($-2\cdot7025$ ...) *Full Credit –1:* • Correct answer with no or incorrect unit

Q9	Model Solution – 50 Marks	Marking Notes
(b)	$82 = 75e^{(-0{\cdot}081)(t)} + 20$ $62 = 75e^{-0.{\cdot}081t}$ $\frac{62}{75} = e^{-0{\cdot}081t}$ $\ln\left(\frac{62}{75}\right) = -0{\cdot}081t$ $t = 2{\cdot}3500 \ldots$ mins $t = 2$ mins 21 secs [nearest sec]	**Scale 10D (0, 3, 5, 8, 10)** *Note*: Accept $2{\cdot}35$ mins or 141 seconds *Low Partial Credit:* • Some substitution into equation *Mid Partial Credit* • Fully substituted equation *High Partial Credit:* • Equation in t with no indices (logs handled correctly)
(c)	$T(t) = 75e^{-0.081t} + 20$ $T'(t) = -6{\cdot}075e^{-0{\cdot}081t}$ $-6{\cdot}075e^{-0{\cdot}081t} = -4{\cdot}05$ $e^{-0{\cdot}081t} = \frac{2}{3}$ $-0{\cdot}081t = \ln\left(\frac{2}{3}\right)$ $t = 5{\cdot}0057 \ldots$ $T(5 \cdot 0057 \ldots) = 75e^{-0{\cdot}081(5{\cdot}0057\ldots)} + 20$ $= 70$ [°C]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* differentiation must be used in order to be awarded any credit. *Note*: Chain rule must be applied in order to be awarded more than *Low Partial Credit* *Low Partial Credit:* • Some correct differentiation *Mid Partial Credit:* • Equation in t *High Partial Credit:* • $t = 5{\cdot}0057 \ldots$ or $t = 5$ • One error in finding t and finishes correctly

<table>
<tr><th>Q9</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(d)</td><td>
$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{20}$
$V = \frac{1}{64} = x^3$ so $x = \frac{1}{4}$
and $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}x}$
$= -\frac{1}{20} \div 3x^2 = -\frac{1}{60x^2}$

At $x = \frac{1}{4}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{60(0{\cdot}25)^2}$
$= -\frac{4}{15}$ cm/sec
</td><td>
Scale 10D (0, 3, 5, 8, 10)
Note: Accept if the rates of change are treated as positive.

Low Partial Credit:
• States a relevant derivative, for example, $\frac{\mathrm{d}V}{\mathrm{d}t}$
• $x^3 = \frac{1}{64}$

Mid Partial Credit:
• Any two of the following:
○ $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{20}$
○ $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$
○ $x = \frac{1}{4}$
○ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}x}$, or similar

High Partial Credit:
• $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}x}$, and any two others from the MPC list above

Full Credit –1:
• Unit incorrect or omitted
</td></tr>
</table>

Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$V(t) = 60 + 41t - 3t^2 = 0$ $(-t + 15)(3t + 4) = 0$ $t = 15$ days **OR** $t = \frac{-41 \pm \sqrt{41^2 - 4(-3)(60)}}{2(-3)}$ $= \frac{-41 \pm \sqrt{2401}}{-6}$ $= 15$, as $t > 0$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $V(t) = 0$ • Quadratic formula with some substitution *High Partial Credit:* • $V(t)$ fully factorised • Formula fully substituted *Full Credit –1:* • Gives 15 and $-\frac{4}{3}$
(a) (ii)	$V'(t) = 41 - 6t$ $V'(5) = 41 - 6(5) = 11$ litres / day	**Scale 10C (0, 3, 7, 10)** *Note:* differentiation must be used in order to be awarded any credit. *Low Partial Credit:* • Any relevant differentiation *High Partial Credit:* • $V'(5)$ substituted *Full Credit –1* • Correct answer but no or incorrect unit
(a) (iii)	$V'(t) = 0$ $41 - 6t = 0$ $41 = 6t$ $t = \frac{41}{6}$ or $6{\cdot}8333\ldots$ [$V''(t) = -6 < 0$, so max]	**Scale 5B (0, 2, 5)** *Partial Credit:* • $V'(t) = 0$ • $41 - 6t$ • $V''(t) = -6$
(a) (iv)	$V\left(\frac{41}{6}\right) = 60 + 41\left(\frac{41}{6}\right) - 3\left(\frac{41}{6}\right)^2$ $= 200{\cdot}0833\ldots$ $= 200$ [litres] [nearest litre]	**Scale 5B (0, 2, 5)** *Partial Credit:* • $V'(t) = 0$ • $41 - 6t$ • $V''(t) = -6$ • Some relevant substitution into $V(t)$

Q10	Model Solution – 50 Marks	Marking Notes
(b) (i)	$I(t) = 1{\cdot}5 + \sin\frac{\pi t}{5}$ $\sin A \geq -1$ So $I(t) \geq 0{\cdot}5$ Radius increases every year, as $I(t) > 0$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Some relevant work, for example, substitutes value of $t > 0$ into $I(t)$; states $I(t) > 0$; states $\sin\frac{\pi t}{5} > -1$
(b) (ii)	**Show**: $I(6) = 1{\cdot}5 + \sin\frac{6\pi}{5} = 0{\cdot}9122\ldots$ $I(5) = 1{\cdot}5 + \sin\frac{5\pi}{5} = 1{\cdot}5$ $I(6) < I(5)$ **Explanation**: It grew less in year 6 than in year 5, *or similar*	**Scale 10C (0, 3, 7, 10)** Note: Accept without the conclusion (i.e. that $I(6) < I(5)$), as long as $I(5)$ and $I(6)$ evaluated, and valid explanation given. *Low Partial Credit:* • $I(5)$ or $I(6)$ with some substitution • Valid explanation *High Partial Credit:* • $I(5)$ and $I(6)$ with full substitution • Valid explanation **and** some substitution into $I(5)$ or $I(6)$
(b) (iii)	$r(2) = r(1) + I(2)$ $r(2) = r(0) + I(1) + I(2)$ $r(2) = 10 + 1{\cdot}5 + \sin\frac{\pi}{5} + 1{\cdot}5 + \sin\frac{2\pi}{5}$ $r(2) = 13 + \sin\frac{\pi}{5} + \sin\frac{2\pi}{5}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $r(2) = r(1) + I(2)$ • $r(3) = r(2) + I(3)$ • Some relevant substitution into $r(1)$ *High Partial Credit:* • $r(2) = r(0) + I(1) + I(2)$ • $r(1) = 10 + 1 \cdot 5 + \sin\frac{\pi}{5}$ *Full Credit –1:* • $r(2) = 10 + 1{\cdot}5 + \sin\frac{\pi}{5} + 1{\cdot}5 + \sin\frac{2\pi}{5}$

Q10	Model Solution – 50 Marks	Marking Notes
(b) (iv)	$r(10) = 10 + 10(1{\cdot}5) + \sin\frac{\pi}{5} + \sin\frac{2\pi}{5} + \cdots + \sin\frac{10\pi}{5}$ $= 10 + 15 + 0$ $= 25$ cm $V_2 = kV_1$ $\pi 25^2 h = k\pi 10^2 h$ $625 = 100k$ $k = 6{\cdot}25$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $r(10)$ with some substitution • Formula for volume of a cylinder with some substitution *High Partial Credit:* • $r(10)$ fully substituted

Paper 2: Marking Scheme

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D
No of categories	3	4	5
5-mark scale	0, 2, 5	0, 2, 3, 5	
10-mark scale		0, 3, 7, 10	0, 3, 5, 8, 10
15-mark scale			0, 4, 8, 12, 15
20-mark scale			0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

A rounding penalty is applied only once in each section (a), (b), (c) etc. It is explicitly indicated in the scheme where penalties for incorrect or omitted units are to be applied. There is no penalty for omitted units if the question specifies the unit to be used in the answer, and there is generally no penalty for an omitted euro symbol in questions involving money.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Section A (120)		**Section B (100)**	
Question 1 (30) (a) 15D (b) 10C (c) 5C **Question 2 (30)** (a) 10C (b) 10D (c)(i) 5C (c)(ii) 5C **Question 3 (30)** (a) 15D (b)(i) 10D (b)(ii) 5B	**Question 4 (30)** (a)(i) 10C (a)(ii) 10D (b) 10D **Question 5 (30)** (a)(i) 15D (a)(ii) 5B (b) 10C **Question 6 (30)** (a) 20D (b) 10D	**Question 7 (50)** (a) 5C (b) 15D (c) 10D (d) 5C (e) 10C (f) 5C **Question 8 (50)** (a)(i) 10D (a)(ii) 10D (b)(i) 5C (b)(ii) 10D (c)(i) 10D (c)(ii) 5C	**Question 9 (50)** (a)(i) 10D (a)(ii) 10D (b)(i) 5C (b)(ii) 10C (b)(iii) 5C (b)(iv) 5C (b)(v) 5C **Question 10 (50)** (a)(i) 10D (a)(ii) 10C (a)(iii) 10D (b) 10C (c) 10D

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✘	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
$\boxed{*}$	Star	Rounding / Unit / Arithmetic error Misreading	
〰	Horizontal wavy	Error	
✔l	Tick L		The work presented in the body of the script merits low partial credit
✔m	Tick M		The work presented in the body of the script merits mid partial credit (or partial credit)
✔h	Tick H		The work presented in the body of the script merits high partial credit
$\boxed{F*}$	F star		The work presented in the body of the script merits Full Credit (– 1)
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
⦚	Vertical wavy	No work on this page (portion of the page)	
$\boxed{o}$	Oversimplify	The candidate has oversimplified the work	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

In a **C scale** where * and 〰 and 〰 appear in the body of the work, then ✔l should be placed in the right margin.

In the case of a **D scale** with the same annotations, ✔m should be placed in the right margin.

A ✔ in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is them indicated in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	$11 \times 0{\cdot}15^1 \times 0{\cdot}85^{10} = 0 \cdot 3248 \ldots$ $= 0 \cdot 325$ [3 D.P.] **OR** $11 \times \frac{3}{20} \times \left(\frac{17}{20}\right)^{10} = 0{\cdot}3248 \ldots$ $= 0 \cdot 325$ [3 D.P.]	**Scale 15D (0, 4, 8, 12, 15)** *Note:* multiplication between relevant terms is necessary to be awarded above *Low Partial Credit* *Low Partial Credit:* • $0{\cdot}15$ or $0{\cdot}85$ or $\frac{3}{20}$ or $\frac{17}{20}$ or 11 • x^{10} where $0 < x < 1$ *Mid Partial Credit:* • Three terms multiplied, two correct, from 11, $(0{\cdot}15^1)$, and $(0{\cdot}85^{10})$ • $(0{\cdot}15^1) \times (0{\cdot}85^{10})$ *High Partial Credit:* • $11 \times 0{\cdot}15^1 \times 0{\cdot}85^{10}$ or equivalent • $(0{\cdot}15^1) \times (0{\cdot}85^{10})$ evaluated $[0 \cdot 0295 \ldots]$
(b)	$P(0 \text{ or } 1 \text{ or } 2 \text{ left-footed})$ $= 0{\cdot}85^{11} + \binom{11}{1} \times 0{\cdot}15^1 \times 0{\cdot}85^{10}$ $+ \binom{11}{2} \times 0{\cdot}15^2 \times 0{\cdot}85^9$ $= 0{\cdot}1673 \ldots + 0{\cdot}3248 \ldots + 0{\cdot}2866 \ldots$ $= 0{\cdot}7787 \ldots$ $= 0{\cdot}78$ [2 D.P.]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • First line of solution • $0{\cdot}15$ or $0{\cdot}85$ or $\frac{3}{20}$ or $\frac{17}{20}$ or $\binom{11}{1}$ *High Partial Credit:* • Two out of $P(0)$, $P(1)$, $P(2)$ fully substituted

Q1	Model Solution – 30 Marks	Marking Notes
(c)	From 10, $P(0 \text{ or } 1 \text{ or } 2 \text{ left-footed})$ $= 0{\cdot}85^{10} + \binom{10}{9} \times 0{\cdot}15^{1} 0{\cdot}85^{9}$ $+ \binom{10}{8} \times 0{\cdot}15^{2} 0{\cdot}85^{8}$ $= 0{\cdot}1968 \ldots + 0{\cdot}3474 \ldots + 0{\cdot}2758 \ldots$ $= 0{\cdot}82019 \ldots$ $= 0{\cdot}82$ [2 D.P.]	**Scale 5C (0, 2, 3, 5)** *Note:* Accept $82{\cdot}02\%$ *Low Partial Credit:* • First line of solution • $0{\cdot}15$ or $0{\cdot}85$ or $\frac{3}{20}$ or $\frac{17}{20}$ or $\binom{10}{1}$ *High Partial Credit:* • Two out of $P(0)$, $P(1)$, $P(2)$ fully substituted

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$3k-6\left(\frac{2k+2}{3}\right)+2=0$ $\Rightarrow 3k-4k-4+2=0$ $\Rightarrow k=-2$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Some substitution into equation of line *High Partial Credit:* • Equation of line fully substituted
(b)	$s-2t-8=0$ so $s=2t+8$ $\frac{\lvert 4s+3t+6\rvert}{\sqrt{4^2+3^2}}=1$ $\frac{\lvert 8t+32+3t+6\rvert}{5}=1$ $\lvert 11t+38\rvert=5$ $11t+38=5$ **or** $11t+38=-5$ $\therefore\ t=-3$ **or** $\therefore\ t=-\frac{43}{11}$ $\therefore\ s=2$ **or** $\therefore\ s=\frac{2}{11}$ **OR** $\frac{\lvert 4s+3t+6\rvert}{\sqrt{4^2+3^2}}=1$ $\lvert 4s+3t+6\rvert=5$ $4s+3t=-1$ **or** $4s+3t=-11$ Intersection of either with $s-2t=8$: $s=2,\ t=-3$ **or** $s=\frac{2}{11},\ t=-\frac{43}{11}$	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Only one pair of s and t required. *Low Partial Credit:* • Some substitution of (s,t) into equation of line • Some substitution into distance of point to line formula • Finds one co-ordinate of point of intersection of two given lines • Correct answer without work *Mid Partial Credit:* • Full substitution of s and t into both (1) the equation of the line **and** (2) the equation of the distance of a point to a line formula • $4s+3t=-1$ **or** $4s+3t=-11$ *High Partial Credit:* • $\frac{\lvert 8t+32+3t+6\rvert}{5}=1$, or similar • ($4s+3t=-1$ **or** $4s+3t=-11$) **and** $s-2t=8$

<table>
<tr><th>Q2</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(c)
(i)</td><td>

$|AC| = \sqrt{12^2 + 9^2} = 15$

$|AD| = \frac{2}{3}(15) = 10$

OR

$$\left(\frac{2 \times 16 + 1 \times 4}{2+1}, \ \frac{2 \times 11 + 1 \times 2}{2+1}\right) = D(12, 8)$$

$$|AD| = \sqrt{(12-8)^2 + (8-2)^2} = 10$$

OR

Let D be (x, y). Then

$\left(\frac{3x - 1 \times 4}{3-1}, \ \frac{3y - 1 \times 2}{3-1}\right) = (16, 11)$

So $D = (x, \ y) = (12, 8)$

$|AD| = \sqrt{8^2 + 6^2} = 10$

</td><td>

Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- Some substitution into formula for $|AC|$
- Some substitution into formula for D
- Identifies relevant translation

High Partial Credit:

- $|AC| = 15$ found
- D found **and** $|AD|$ fully substituted

</td></tr>
<tr><td>(c)
(ii)</td><td>

$|AB| = 33 \ \Rightarrow B \ is \ (37, 2).$

The translation $\overrightarrow{CB}$: x increases by 21

$x_E = 16 + \frac{1}{3}(21) = 23$

The translation $\overrightarrow{CB}$: y decreases by 9

$y_E = 11 - \frac{1}{3}(9) = 8$

So $E = (23, 8)$.

OR

$$\left(\frac{2 \times 16 + 1 \times 37}{2+1}, \frac{2 \times 11 + 1 \times 2}{2+1}\right) = E(23, 8)$$

OR

DE and AB parallel

∴ equation DE: $y = 8$

Then x coordinate by translation or ratio or equation $BC \cap DE = 23$

</td><td>

Scale 5C (0, 2, 3, 5)

Note: finding slant distances alone is not given credit

Low Partial Credit:

- One co-ordinate of *B* **or** *E* found
- $|DE| = 11$

High Partial Credit:

- B found **and** some work towards finding E
- E found

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Q3	Model Solution – 30 Marks	Marking Notes
(a)	$\lvert DB\rvert = \frac{1}{2}\left(4\sqrt{3}\right) = 2\sqrt{3}$ $\lvert CD\rvert = \sqrt{4^2 + 2^2} = \sqrt{20}$ or $2\sqrt{5}$ Radius $= \lvert CB\rvert$ $= \sqrt{\left(2\sqrt{5}\right)^2 + \left(2\sqrt{3}\right)^2} = 4\sqrt{2}$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • $\lvert DB\rvert$ (or $\lvert DA\rvert$) found • Pythagoras' Theorem with some relevant substitution • Right angle indicated at D; $[AC]$ or $[BC]$ drawn, and length r indicated *Mid Partial Credit:* • $\lvert CD\rvert$ found • $\lvert DB\rvert$ found **and** Pythagoras' Theorem with some relevant substitution *High Partial Credit:* • $\lvert DB\rvert$ **and** $\lvert CD\rvert$ found
(b) (i)	$x^2 + y^2 + 4x - 2y - 95 = 0$: Centre $(-2, 1)$. Radius $= 10$ $(x-7)^2 + (y-13)^2 = 25$: Centre $(7, 13)$. Radius $= 5$ $r_1 + r_2 = 15$ Distance between the two centres: $\sqrt{12^2 + 9^2} = \sqrt{225} = 15$ Touch Externally since $r_1 + r_2 =$ distance between two centres.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Either centre **or** either radius found *Mid Partial Credit:* • Finds two from the following four: two centres and two radii *High Partial Credit:* • Both centres **and** both radii found *Full Credit –1:* • Both centres **and** both radii **and** distance between the centres found but no or incorrect conclusion

Q3	Model Solution – 30 Marks	Marking Notes
(b) (ii)	*Accept any point on the line* $y = \frac{4x+11}{3}$ *for which* $x > 4$. Slope of the two centres: $\frac{13-1}{7+2} = \frac{4}{3}$ For example: $\left(7+1,\ 13+\frac{4}{3}\right) = \left(8,\ 14\frac{1}{3}\right)$ **OR** $(-2, 1) \rightarrow (7, 13)$ given by x: $+9$, y: $+12$ So $(7, 13) \rightarrow (7+9, 13+12) = (16, 25)$ **OR** Any circle that touches c at that point must be on l (i.e. line through the two centres). Slope of the two centres: $\frac{13-1}{7+2} = \frac{4}{3}$ $y - 1 = \frac{4}{3}(x+2)$ l: $4x - 3y + 11 = 0$ now sub $x = 10$ $\Rightarrow\ \ y = 17$ One centre $= (10, 17)$.	**Scale 5B (0, 2, 5)** *Note:* no credit is awarded for just identifying either/both centres. *Partial Credit:* • Work of merit, for example: identifies that the point must be on the line containing the two centres; clearly recognises that $x > 4$ or $y > 9$; finds the point $(4, 9)$, or either ordinate; finds slope of line containing two centres *Full Credit –1:* • Answer as another point on the line joining the centres, but with $x < 4$ (i.e. touches internally)

Q4	Model Solution – 30 Marks	Marking Notes
(a) (i)	$\cos(A+B) = \cos A \,.\, \cos B - \sin A \,.\, \sin B$ $\cos(A+A) = \cos A \,.\, \cos A - \sin A \,.\, \sin A$ $\cos(2A) = \cos^2 A - \sin^2 A$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $\cos(A+B)$ formula with some substitution • Tested with one or more values of A *High Partial Credit:* • $\cos(A+A)$ $= \cos A \,.\, \cos A - \sin A \,.\, \sin A$
(a) (ii)	$\cos 2A = \cos^2 A - \sin^2 A$ $\Rightarrow \cos\theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$ using a right-angled triangle $\cos\frac{\theta}{2} = \frac{2}{\sqrt{5}}$ $\cos\theta = \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$ **OR** $\cos\theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$ $= \left(1 - \sin^2 \frac{\theta}{2}\right) - \sin^2 \frac{\theta}{2}$ $= 1 - 2\sin^2 \frac{\theta}{2}$ $= 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $\cos\theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$ • Valid work to find $\cos\frac{\theta}{2}$ from $\sin\frac{\theta}{2}$ • Some relevant substitution *Mid Partial Credit* • Finds $\cos\frac{\theta}{2} = \frac{2}{\sqrt{5}}$. Also accept $\cos\frac{\theta}{2} = \cos\left(\sin^{-1}\frac{1}{\sqrt{5}}\right) = 0{\cdot}8944\ldots$ • $\cos\theta = \left(1 - \sin^2 \frac{\theta}{2}\right) - \sin^2 \frac{\theta}{2}$ *High Partial Credit:* • $\cos\theta$ formula fully substituted

Q4	Model Solution – 30 Marks	Marking Notes
(b)	$\tan(\text{angle}) = -\sqrt{3}$, so reference angle $= 60°$ $\mathbf{150° \leq B + 150° \leq 510°}$ In Quad's 2 or 4, so angles are $\mathbf{300°}$ or $\mathbf{480°}$ $B + 150 = 300$ **or** $B + 150 = 480$ So $B = 150°$ **or** $B = 330°$ **OR** $\tan(B + 150) = \frac{\tan B + \tan 150}{1 - \tan B \tan 150}$ $= \frac{\tan B - \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)\tan B} = -\sqrt{3}$ So $\tan B - \frac{1}{\sqrt{3}} = -\sqrt{3} - \tan B$ So $2\tan B = -\frac{2}{\sqrt{3}}$, i.e. $\tan B = -\frac{1}{\sqrt{3}}$ Reference angle $= 30°$ In Quad's 2 or 4, so $B = 150°$ **or** $B = 330°$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Reference angle $= 60°$ • Correct range for $B + 150°$ • Quadrants 2 and 4 identified • $\tan(A + B)$ formula with some substitution *Mid Partial Credit:* • $300°$ or $480°$ • Solution to equation outside of required range (for e.g., $B = -30°$ or $-210°$) • $\frac{\tan B + \tan 150}{1 - \tan B \tan 150} = -\sqrt{3}$ *High Partial Credit:* • $B = 150°$ or $B = 330°$ • $300°$ and $480°$

Q5	Model Solution – 30 Marks	Marking Notes
(a) (i)	$V_{sphere} = \frac{4}{3}\pi r^3$ $V_{cone} = \frac{1}{3}\pi r^2 h$ $V_{space} = \frac{4}{3}\pi r^3 - 2.\frac{1}{3}\pi r^2 h$ $= \frac{4}{3}\pi r^3 - \frac{2}{3}\pi r^2 r$ $= \frac{2}{3}\pi r^3$ $= \frac{1}{2} \times V_{sphere}$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • 2 volume formulas given (sphere and cone) • Volume formula given with some relevant manipulation / substitution *Mid Partial Credit:* • Volume of space in terms of r and h *High Partial Credit:* • Volume of space in terms of 1 variable *Full Credit -1:* • Incorrect or no conclusion, otherwise correct
(a) (ii)	$V_c = \frac{2}{3}\pi r^3 = \frac{686}{3}\pi$ $2r^3 = 686$ $r^3 = 343$ $r = 7$ cm	**Scale 5B (0, 2, 5)** *Partial Credit:* • $2.\frac{1}{3}\pi r^2 h = \frac{686}{3}\pi$ or equivalent, for example, $\frac{1}{3}\pi r^3 = \frac{686}{6}\pi$ *Full Credit –1* • Answer correct but incorrect or no units

<table>
<tr><th>Q5</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>

$60 \times 1{\cdot}75 = 105$ km travelled by van until $10{:}45$.

$105 + 60t = 95t$

So $t = 3$ hours

$10{:}45 + 3$ hr $= 13{:}45$

OR

$95 \times (t - 1{\cdot}75) = 60t$
$\Rightarrow\ t = 4\frac{3}{4}$ hours

$9{:}00 + 4\frac{3}{4}$ hr $= 13{:}45$

OR

Let distance travelled = x km

$$\frac{x}{60} - \frac{x}{95} = \frac{7}{4}$$

$x = 285$ km

$\Rightarrow\ t = \frac{285}{95} = 3$ hours

$10{:}45 + 3$ hr $= 13{:}45$

OR

$\frac{1{\cdot}75 \times 60}{95 - 60} = 3$ hours.
$10{:}45 + 3 = 13{:}45$

OR

<table>
<tr><th>Time</th><th>Dist A (km)</th><th>Dist B (km)</th></tr>
<tr><td>$10{:}45$</td><td>$1{\cdot}75 \times 60$
$= 105$</td><td>0</td></tr>
<tr><td>$11{:}45$</td><td>$105 + 60$
$= 165$</td><td>95</td></tr>
<tr><td>$12{:}45$</td><td>$165 + 60$
$= 225$</td><td>$95 + 95$
$= 190$</td></tr>
<tr><td>$13{:}45$</td><td>$225 + 60$
$= 285$</td><td>$190 + 95$
$= 285$</td></tr>
</table>

Ans: $13{:}45$

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Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Work of merit, for example:
$1{\cdot}75$ or $\frac{7}{4}$ or 105 km;
or $(t - 1{\cdot}75)$ or $(t + 1{\cdot}75)$;
or $\frac{x}{60}$ or $\frac{x}{95}$

High Partial Credit:

- equation in t or in x, from which correct time / distance can be directly found
- correct table / diagram showing destination with all required distances / times indicated, but no conclusion
- $\frac{1{\cdot}75 \times 60}{95 - 60}$

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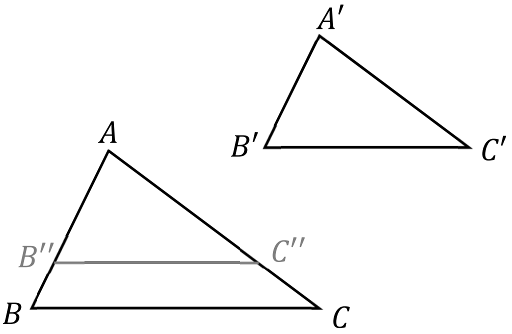
</table>

Q6	Model Solution – 30 Marks	Marking Notes
(a)	**Step A (Given / Diagram)**  *Given*: ABC and $A'B'C'$ [similar triangles] [*To Prove*: $\frac{\lvert AB\rvert}{\lvert A'B'\rvert} = \frac{\lvert BC\rvert}{\lvert B'C'\rvert} = \frac{\lvert CA\rvert}{\lvert C'A'\rvert}$.] **Step B (Construction / Diagram):** *Construction*: Mark B'' on $[AB]$ such that $\lvert AB''\rvert = \lvert A'B'\rvert$. Mark C'' on $[AC]$ such that $\lvert AC''\rvert = \lvert A'C'\rvert$. [Join B'' to C''.] **Step C:** *Proof*: $\Delta\ AB''C''$ is congruent to $\Delta\ A'B'C'$ **Reason:** SAS **Step D:** $\therefore\ B''C'' \parallel BC$ **Reason:** corresponding angles, $\lvert\angle AB''C''\rvert = \lvert\angle ABC\rvert$ **Step E:** $\therefore\ \frac{\lvert AB\rvert}{\lvert AB''\rvert} = \frac{\lvert AC\rvert}{\lvert AC''\rvert}$ $\therefore\ \frac{\lvert AB\rvert}{\lvert A'B'\rvert} = \frac{\lvert AC\rvert}{\lvert A'C'\rvert}$. Similarly, $\frac{\lvert AB\rvert}{\lvert A'B'\rvert} = \frac{\lvert BC\rvert}{\lvert B'C'\rvert}$. Hence, $\frac{\lvert AB\rvert}{\lvert A'B'\rvert} = \frac{\lvert BC\rvert}{\lvert B'C'\rvert} = \frac{\lvert CA\rvert}{\lvert C'A'\rvert}$.	**Scale 20D (0, 5, 10, 15, 20)** Consider the proof as requiring five steps, equivalent to those outlined in the model solution. Accept steps without reasons for up to *High Partial Credit*, but not for *Full Credit* Accept without last line if *To Prove* is filled in correctly. *Low Partial Credit:* • Work of merit, for example, relevant diagram(s) drawn, or effort at 'Given' *Mid Partial Credit:* • Any 2 steps *High Partial Credit:* • 4 steps presented

Q6	Model Solution – 30 Marks	Marking Notes
(b)	**S1.** $\lvert\angle HBQ\rvert = \lvert\angle HAP\rvert$ .... alternate **S2.** $\lvert\angle QHB\rvert = \lvert\angle PHA\rvert$ .... vertically opposite **S3.** So triangles are similar **S4.** So $\dfrac{\lvert AH\rvert}{\lvert HB\rvert} = \dfrac{\lvert AP\rvert}{\lvert QB\rvert}$ **S5.** So $\lvert AH\rvert \times \lvert QB\rvert = \lvert AP\rvert \times \lvert HB\rvert$	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Step 5 is not considered done unless steps 1 to 4 are all present *Low Partial Credit:* • 1 relevant step listed or shown on diagram (no justification) • Mentions the relevant justifications *Mid Partial Credit:* • 3 relevant steps listed or shown on diagram (no justification) *High Partial Credit:* • All valid steps included but with no justification • 4 steps correct with at least one justification

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$25 \times 1{\cdot}2 = 30$ $30 + 28 + 4 = 62$ [km]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $1{\cdot}2$ or $\frac{72}{60}$ *High Partial Credit:* • 30 km coming from $25 \times 1{\cdot}2$
(b)	$4{\cdot}8 - 1{\cdot}2 = 3{\cdot}6$ hours Let x be the speed at swimming, so: $T_{run} + T_{swim} = \frac{28}{5{\cdot}6x} + \frac{4}{x} = 3{\cdot}6$ $\frac{28+4(5{\cdot}6)}{5{\cdot}6x} = 3{\cdot}6$ $50{\cdot}4 = 20{\cdot}16x$ $x = 2{\cdot}5$ [km/h] **OR** $4{\cdot}8 - 1{\cdot}2 = 3{\cdot}6$ hours $T_{run} = \frac{28}{4} \times \frac{T_{swim}}{5{\cdot}6}$, so $T_{run} + T_{swim} = \left(\frac{28}{4} \times \frac{T_{swim}}{5{\cdot}6}\right) + T_{swim}$ $= \frac{9}{4} \times T_{swim} = 3{\cdot}6$ So $T_{swim} = \frac{8}{5}$ i.e. $\text{Speed}_{swim} = 4 \div \frac{8}{5} = 2{\cdot}5$ [km/h]	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • Work of merit, for example: $4{\cdot}8 - 1{\cdot}2$ or $5{\cdot}6x$ or $\frac{28}{4}$ or $\frac{T_{swim}}{5{\cdot}6}$ *Mid Partial Credit:* • $\frac{4}{x}$ or $\frac{28}{5{\cdot}6x}$ or $\frac{5}{x}$ • $\frac{28}{4} \times \frac{T_{swim}}{5{\cdot}6}$ *High Partial Credit:* • An equation in one variable that can be solved to give a relevant time or speed
(c)	$30^2 = 28^2 + 4^2 - 2(28)(4)\cos C$ $\cos C = \frac{28^2+4^2-30^2}{2(28)(4)}$ $\cos C = -\frac{100}{224}$ $C = 116{\cdot}51 \ldots = 116{\cdot}5$ [1 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • cosine rule formulated with some substitution *Mid Partial Credit:* • cosine rule formulated with full substitution *High Partial Credit:* • $\cos C = \frac{28^2+4^2-30^2}{2(28)(4)}$ or equivalent • $28^2 + 4^2 - 2(28)(4)\cos 116{\cdot}5°$ fully evaluated

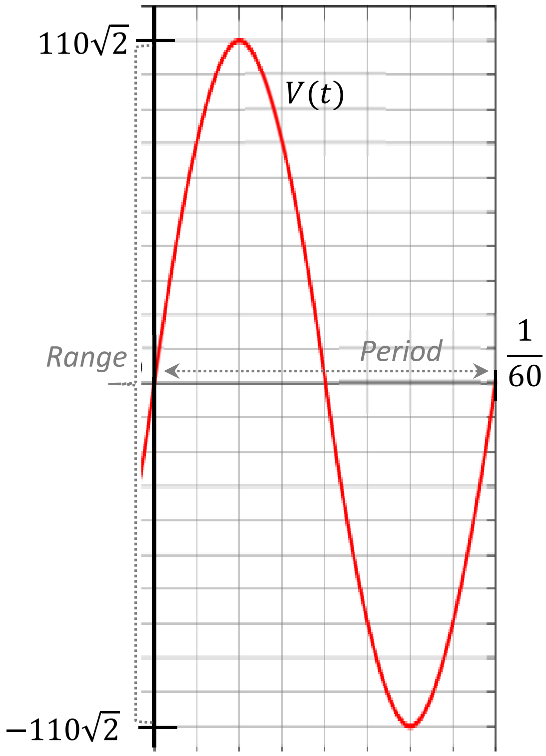
Q7	Model Solution – 50 Marks	Marking Notes
(d)	$\text{Area} = \frac{1}{2} A\ B \sin C$ $\text{Area} = \frac{1}{2}(28)(4)\sin 116{\cdot}5°$ $= 50{\cdot}11 \ldots = 50{\cdot}1$ [km^2] [1 D.P.] **Or** $\frac{30}{\sin 116{\cdot}5} = \frac{28}{\sin B}$ $\sin B = 0 \cdot 83527 \ldots$ $B = 56 \cdot 33^0$ $\text{Area} = \frac{1}{2}(30)(4)\sin 56{\cdot}33$ $= 50{\cdot}11 \ldots = 50{\cdot}1$ [km^2] [1 D.P.]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Area formula with some substitution *High Partial Credit:* • Area formula with full substitution
(e)	Area $= \frac{1}{2}$ base $\times\ d$ $50{\cdot}1 = \frac{1}{2}(30)d$ $d = \frac{50{\cdot}1}{15} = 3{\cdot}34 = 3{\cdot}3$ [km] [1 D.P.]	**Scale 10C (0, 3, 7, 10)** *Note:* divides $116{\cdot}5°$ by 2 and continues: award *Low Partial Credit* at most *Low Partial Credit:* • Indicates shortest distance with right angle (no credit awarded for this in (d)) • Equation with some substitution *High Partial Credit:* • Equation with full substitution
(f)	$\tan(0{\cdot}05) = \frac{x}{30}$ $30 \times \tan(0{\cdot}05) = x$ $x = 0{\cdot}02617 \ldots$ km $= 26$ [m] [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Relevant work on the diagram, for example, joins T to B and indicates angle of $0 \cdot 05°$ • Tan formula with some substitution *High Partial Credit:* • Tan formula with full substitution

Q8	Model Solution – 50 Marks	Marking Notes
(a) (i)	Top 10% means 90% below. $P(z < 1 \cdot 28) = 0{\cdot}8997.$ $\frac{x-176}{36} = 1{\cdot}28$ $\Rightarrow x = 222{\cdot}08$ Minimum mark of 223	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept use of $P(z < 1{\cdot}29)$, to give $x = 222{\cdot}44$ *Note:* Accept answer rounded to 222 instead of 223 *Low Partial Credit:* • Mean or standard deviation indicated • z-formula with some substitution *Mid Partial Credit:* • z-score found ($1{\cdot}28$ or $1{\cdot}29$) • z-formula fully substituted $\left(\frac{x-176}{36}\right)$ *High Partial Credit:* • $\frac{x-176}{36} = 1{\cdot}28$
(a) (ii)	$P(165 < x < 210)$ $P\left(\frac{165-176}{36} < z < \frac{210-176}{36}\right)$ $= P(-0{\cdot}31 < z < 0{\cdot}94)$ [2 D.P.] $= P(z < 0{\cdot}94) - P(z > -0{\cdot}31)$ $P(z < 0{\cdot}94) = 0{\cdot}8264$ $P(z < -0{\cdot}31) = 1 - P(z < 0{\cdot}31)$ $= 1 - 0{\cdot}6217 = 0{\cdot}3783$ So answer $= 0{\cdot}8264 - 0{\cdot}3783 = 0{\cdot}4481$ $= 44{\cdot}81\%$ of 1st years got the Distinction.	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Also accept use of $-0{\cdot}30$ instead of $-0{\cdot}31$, and/or use of $0{\cdot}95$ instead of $0{\cdot}94$. *Low Partial Credit:* • Mean or standard deviation indicated • *z* formula with some substitution • $-0{\cdot}305$ … or $0{\cdot}94$ … *Mid Partial Credit:* • One relevant probability found directly from tables ($0{\cdot}6217$ or $0{\cdot}8264$) *High Partial Credit:* • $0{\cdot}3783$ found • $0{\cdot}8264$ and $0{\cdot}6217$ found *Full Credit –1:* • Uses $P(z > -0{\cdot}35)$ and finishes correctly

Q8	Model Solution – 50 Marks	Marking Notes
(b) (i)	$T = \frac{19{\cdot}8-21}{\left(\frac{5{\cdot}2}{\sqrt{60}}\right)} = -1{\cdot}787\ \ldots$	**Scale 5C (0, 2, 3, 5)** *Note*: Accept $1{\cdot}787\ \ldots$ *Note*: $\frac{s}{\sqrt{n}}$ must be used in order to be awarded above *Low Partial Credit* *Low Partial Credit:* • Mean or standard deviation indicated • Relevant formula with some substitution *High Partial Credit:* • Formula fully substituted
(b) (ii)	p-value: $p = 2[1 - P(z < 1{\cdot}79)]$ $= 2(1 - 0{\cdot}9633)$ $= 0{\cdot}0734$ Conclusion: There is **not** enough evidence to say that the claim in the news report is incorrect [as $0{\cdot}0734 > 0{\cdot}05$]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept $P(z < 1{\cdot}78)$, so p-value of $2(1 - 0{\cdot}9625) = 0{\cdot}075$. *Note:* Accept conclusion based on z-score rather than p-value. *Low Partial Credit:* • $P(z < 1{\cdot}79)$ • $0{\cdot}9633$ *Mid Partial Credit:* • $2[1 - P(z < 1{\cdot}79)]$ • Work of merit in finding p-value and correct conclusion based on this *High Partial Credit:* • p-value found but no or incorrect conclusion

Q8	Model Solution – 50 Marks	Marking Notes
(c) (i)	**Assuming no replacement:** $\frac{18}{23}\times\frac{17}{22}\times\frac{16}{21}\times\frac{5}{20}$ $= 0{\cdot}11518\ldots = 0{\cdot}1152$ [4 D.P.] **OR** **Assuming replacement:** $\left(\frac{18}{23}\right)^3\times\frac{5}{23}$ $= 0{\cdot}10420\ldots = 0{\cdot}1042$ [4 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* multiplication between relevant terms is necessary to be awarded above *Low Partial Credit* *Low Partial Credit:* • One relevant fraction *Mid Partial Credit:* • Product of four fractions, two of them correct *High Partial Credit:* • $\frac{18}{23}\times\frac{17}{22}\times\frac{16}{21}\times\frac{5}{20}$ • $\left(\frac{18}{23}\right)^3\times\frac{5}{23}$
(c) (ii)	$\left(\frac{12}{23}\times\frac{6}{22}\times\frac{5}{21}\right)3! = \frac{360}{1771}$ $= 0\cdot 20327\ldots\ = 0{\cdot}2033$ [4 D.P.] **OR** $\left(\frac{12}{23}\times\frac{6}{22}\times\frac{5}{21}\right)+\left(\frac{12}{23}\times\frac{5}{22}\times\frac{6}{21}\right)+$ $\left(\frac{6}{23}\times\frac{12}{22}\times\frac{5}{21}\right)+\left(\frac{6}{23}\times\frac{5}{22}\times\frac{12}{21}\right)+$ $\left(\frac{5}{23}\times\frac{6}{22}\times\frac{12}{21}\right)+\left(\frac{5}{23}\times\frac{12}{22}\times\frac{6}{21}\right)$ $= 0\cdot 20327\ldots\ = 0{\cdot}2033$ [4 D.P.] **OR** $\frac{\binom{12}{1}\times\binom{6}{1}\times\binom{5}{1}}{\binom{23}{3}} = 0{\cdot}2033$ [4 D.P.]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • One relevant fraction, for example, $\frac{12}{23}$ or $\frac{6}{22}$ or $\frac{5}{23}$ • Counts / lists different possible arrangements, for example, 3! or 6 *High Partial Credit:* • $\frac{12}{23}\times\frac{6}{22}\times\frac{5}{21}$ or any other relevant triple • Assumes keys are replaced and finishes

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$\lvert\angle ABC\rvert = 180 - 20 = 160°$ $\frac{1450}{\sin 160} = \frac{x}{\sin 8{\cdot}57}$ $x = \frac{1450 \times \sin 8{\cdot}57}{\sin 160}$ $x = 631{\cdot}7626 \ldots$ Time $= \frac{631{\cdot}7626\ldots}{420} = 1{\cdot}504$ … hours $= 90$ mins **or** $1{\cdot}5$ hours **or** 1 hour 30 mins **OR** $\lvert AB\rvert = 2 \times 420 = 840$ km $\lvert BC\rvert^2 = 1450^2 + 840^2 - 2(1450)(840)\cos 8 \cdot 57$ $\lvert BC\rvert^2 = 399299{\cdot}05 \ldots$ $\lvert BC\rvert = 631{\cdot}90 \ldots$ $\frac{631{\cdot}90\ldots}{420} = 1{\cdot}50$ … hours etc	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • sine rule or cosine rule stated with some substitution • finds $\lvert AB\rvert$ or $\lvert\angle ABC\rvert$ *Mid Partial Credit:* • sine rule or cosine rule with full substitution *High Partial Credit:* • $\lvert BC\rvert$ (that is, x) found
(a) (ii)	$\text{Time}_{[AC]} = \frac{1450}{420} = 3{\cdot}4523$ … hours Total time $= 2 + 1{\cdot}5 + 3{\cdot}4523 \ldots$ $= 6{\cdot}9523$ … hours $= 25\,028{\cdot}57$ … seconds Max possible flight time $= \frac{100\,000}{3{\cdot}8}$ $= 26\,315{\cdot}7$ … seconds, which is greater than $25\,028{\cdot}57$ … sec **OR** *Finds* $25\,028{\cdot}57$ … seconds Litres required $= 3{\cdot}8 \times 25\,028{\cdot}57 \ldots$ $= 95\,108 \cdot 57$ … litres, which is less than $100\,000$ litres.	**Scale 10D (0, 3, 5, 8, 10)** Consider solution as requiring four steps, equivalent to: **1.** Finds time in hours to travel $[AC]$ **2.** Finds total time in hours (3 sides) **3.** Converts total time to seconds **4.** Converts time (sec) to litres required *Low Partial Credit:* • One relevant calculation, for example, $\frac{1450}{420}$ or $2 + 1{\cdot}5$ or $\frac{100\,000}{3{\cdot}8}$ *Mid Partial Credit:* • Two steps *High Partial Credit:* • Three steps

Q9	Model Solution – 50 Marks	Marking Notes
(b) (i)	Range: $\left[-110\sqrt{2},\ 110\sqrt{2}\right]$ Period: $\frac{2\pi}{120\pi}$ or $\frac{1}{60}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, some indication of the period of a sine function; some mention of $110\sqrt{2}$ *High Partial Credit* • Period or range correct *Full Credit –1:* • Apply a * for period and range swapped
(b) (ii)		**Scale 10C (0, 3, 7, 10)** *Note:* Consider solution as requiring 3 aspects: **1.** sine curve of at least one period, including $(0, 0)$ **2.** range indicated **3.** period indicated *Low Partial Credit:* • Period or range from **(b)(i)** indicated on axes, but no or incorrect graph • Graph a recognisable portion of a sine curve, or similar *High Partial Credit:* • Two of the aspects above present on the graph
(b) (iii)	$V(6{\cdot}67) = 110\sqrt{2}\sin 120\pi 6{\cdot}67$ $= 147{\cdot}949\ldots = 147{\cdot}95$ [Volts] [2 D.P.]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Formula with some substitution *High Partial Credit:* • Formula with full substitution *Full Credit –1:* • Calculator in incorrect mode

Q9	Model Solution – 50 Marks	Marking Notes
(b) (iv)	*Accept any value of t satisfying* $t = \frac{1+8n}{480}$ *or* $t = \frac{3+8n}{480}$*, as long as in the correct form.* $V(t) = 110\sqrt{2}\,\sin 120\pi t = 110$ So $\sin 120\pi t = \frac{1}{\sqrt{2}}$ So $120\pi t = \frac{\pi}{4}$, i.e. $t = \frac{1}{480}$ [seconds] **or** $120\pi t = \frac{3\pi}{4}$ => $t = \frac{3}{480}$ [seconds]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Equation with some substitution *High Partial Credit:* • Equation with full substitution *Full Credit –1:* • Calculator in incorrect mode
(b) (v)	$V(t) = 110\sqrt{2}\,\sin(120\pi t)$ $V'(t) = 120\pi \times 110\sqrt{2}\,\cos(120\pi t)$ $V'(2) = 120\pi \times 110\sqrt{2}\,\cos(120\pi \times 2)$ $= 58646{\cdot}0 = 58646$ Volts /sec [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** *Note:* $V'(t)$ must be correct in order to be awarded more than *Low Partial Credit*. *Low Partial Credit:* • any correct differentiation *High Partial Credit:* • Expression for $V'(t)$ *Full Credit –1* • Calculator in incorrect mode • Correct answer with no or incorrect units

Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$P(2 \text{ from the 1st 9 are O}^-) \times P(10\text{th is O}^-)$ $= \binom{9}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^7 \times \frac{8}{100}$ $= 0{\cdot}01028 \ldots = 0{\cdot}0103$ [4 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $\frac{8}{100}$ or $\frac{92}{100}$ or $\binom{9}{2}$ • First line of solution indicated (accept with "and" instead of ×) *Mid Partial Credit:* • $\binom{9}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^7$ • $\binom{10}{3}\left(\frac{8}{100}\right)^3\left(\frac{92}{100}\right)^7$ evaluated *High Partial Credit:* • $\binom{9}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^7\frac{8}{100}$ • $\binom{10}{3}\left(\frac{8}{100}\right)^3\left(\frac{92}{100}\right)^7\frac{8}{100}$ evaluated
(a) (ii)	$1 - P(\text{none are O}^-)$ $= 1 - \left(\frac{92}{100}\right)^5$ $= 0{\cdot}34091 \ldots = 0{\cdot}3409$ [4 D.P.] **OR** $P(1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5 \text{ are O}^-)$ $= \binom{5}{1}\left(\frac{8}{100}\right)^1\left(\frac{92}{100}\right)^4 + \binom{5}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^3$ $+\binom{5}{3}\left(\frac{8}{100}\right)^3\left(\frac{92}{100}\right)^2 + \binom{5}{4}\left(\frac{8}{100}\right)^4\left(\frac{92}{100}\right)^1$ $+\left(\frac{8}{100}\right)^5$ $= 0{\cdot}34091 \ldots = 0{\cdot}3409$ [4 D.P.]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $\left(\frac{92}{100}\right)^a$ where $0 < a < 5$ • First line of either solution *High Partial Credit:* • $\left(\frac{92}{100}\right)^5$ • Three terms in second solution
(a) (iii)	$1 - 0{\cdot}92^k > 0{\cdot}97$ so $0{\cdot}92^k < 0{\cdot}03$ Find where $0{\cdot}92^k = 0{\cdot}03$ i.e. $k(\ln(0{\cdot}92)) = \ln(0{\cdot}03)$ so $k = \frac{\ln(0{\cdot}03)}{\ln(0{\cdot}92)} = 42{\cdot}05 \ldots$ so least $k = 43$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $0{\cdot}92^k$ • $0{\cdot}03$ *Mid Partial Credit:* • $1 - 0{\cdot}92^k > 0{\cdot}97$ or $= 0{\cdot}97$ • $k(\ln(0{\cdot}92))$ *High Partial Credit:* • Equation in k without indices (logs handled correctly)

Q10	Model Solution – 50 Marks	Marking Notes
(b)	**Interpretation 1:** **initial €70 charged regardless** $0{\cdot}8(70) + 0{\cdot}2(70 + 150 + 80)$ $= 0{\cdot}8(70) + 0{\cdot}2(300) = €116$ **Interpretation 2:** **initial €70 not charged if not successful** $0{\cdot}8(70) + 0{\cdot}2(150 + 80)$ $= 0{\cdot}8(70) + 0{\cdot}2(230) = €102$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • A correct calculation, for example, $0{\cdot}8(70)$ or 300 or 230 *High Partial Credit:* • $0{\cdot}8(70)$ and $0{\cdot}2(300)$ **or** $0{\cdot}8(70)$ and $0{\cdot}2(230)$
(c)	Average pay-out per customer: $120\,000(0{\cdot}0001) + 40\,000(0{\cdot}002)$ $= €92$ Target profit per customer: $\frac{900\,000}{18\,000} = €50$ Required premium: $50 + 92 = €142$ **OR** Average pay-out per customer: $120\,000(0{\cdot}0001) + 40\,000(0{\cdot}002)$ $= €92$ Total expected payout: $€92 \times 18\,000 = €1\,656\,000$ Total revenue required: $€1\,656\,000 + €900\,000 = €2\,556\,000$ Required premium: $€2\,556\,000 \div 18\,000 = €142$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • One relevant calculation, for example, $120\,000(0{\cdot}0001)$ or $\frac{900\,000}{18\,000}$ *Mid Partial Credit:* • Finds €92 (average payout per person) *High Partial Credit:* • Finds €2 556 000 (total revenue) • Finds €92 and €50

Marcanna Breise as ucht freagairt trí Ghaeilge

Léiríonn an tábla thíos an méid marcanna breise ba chóir a bhronnadh ar iarrthóirí a ghnóthaíonn níos mó ná 75% d'iomlán na marcanna.

N.B. Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don scrúdú. Ba chóir freisin an marc bónais sin **a shlánú síos**.

Tábla 220 @ 5%
Bain úsáid as an tábla seo i gcás na n-ábhar a bhfuil 220 marc san iomlán ag gabháil leo agus inarb é 5% gnáthráta an bhónais.

Bain úsáid as an ngnáthráta i gcás 165 marc agus faoina bhun sin. Os cionn an mharc sin, féach an tábla thíos.

Bunmharc	Marc Bónais
166	8
167 - 173	7
174 - 180	6
181 - 186	5
187 - 193	4

Bunmharc	Marc Bónais
194 - 200	3
201 - 206	2
207 - 213	1
214 - 220	0