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LEAVING CERTIFICATE EXAMINATION, 2001

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 22 JUNE – AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2.
Marks may be lost if necessary work is not clearly shown or you do not indicate where a calculator has been used.

1. **(a)** Points p and q lie in a straight line, where $|pq| = 1200$ metres.
Starting from rest at p, a train accelerates at 1 m/s^2 until it reaches the speed limit of 20 m/s. It continues at this speed of 20 m/s and then decelerates at 2 m/s^2, coming to rest at q.

Find the time it takes the train to go from p to q.

Find the shortest time it takes the train to go from rest at p to rest at q if there is no speed limit, assuming that the acceleration and deceleration remain unchanged at 1 m/s^2 and 2 m/s^2 , respectively.

(b) A particle is projected vertically upwards with an initial velocity of u m/s and another particle is projected vertically upwards from the same point and with the same initial velocity T seconds later.

Show that the particles

(i) will meet $\left(\frac{T}{2}+\frac{u}{g}\right)$ seconds from the instant of projection of the first particle

(ii) will meet at a height of $\frac{4u^2 - g^2T^2}{8g}$ metres.

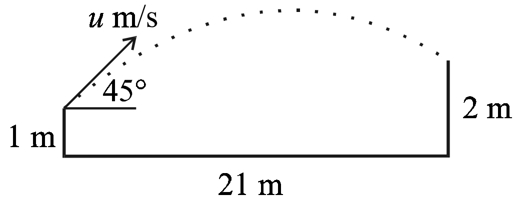
2. **(a)** Ship B is travelling at $5\sqrt{34}$ km/hr in the direction $\tan^{-1}\frac{6}{7}$ north of east and ship C is travelling at $5\sqrt{5}$ km/hr in the direction $\tan^{-1}7$ north of west.

Show that the speed and direction of ship B relative to ship C is 25 km/hr at $\tan^{-1}\frac{1}{3}$ north of east.

(b) The speed of an aeroplane in still air is 160 km/hr. It flies in a straight line from p to q and back again. Point q is due north of point p. Throughout the journey there is a wind blowing from the south-west at 32 km/hr. The time for the whole journey is 5 hours.

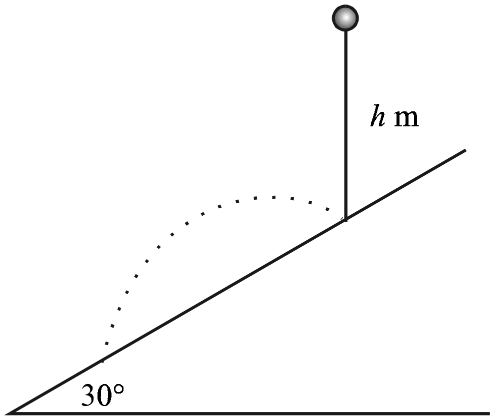
Find the distance from p to q. Give your answer to the nearest km.

3. **(a)** A player hits a ball with an initial speed of u m/s from a height of 1 m at an angle of 45° to the horizontal ground.
A member of the opposing team, 21 m away, catches the ball at a height of 2 m above the ground.



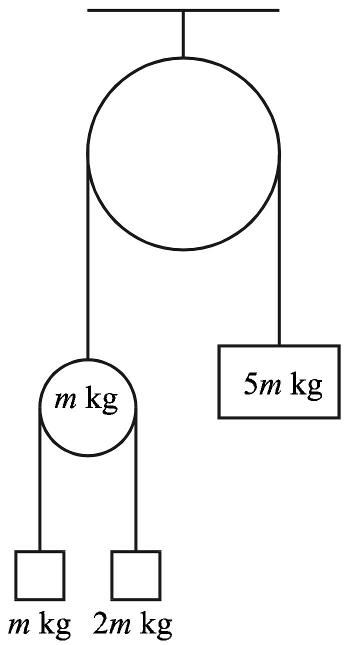
Find the value of u.

(b) A ball is dropped from a height of h m onto a smooth inclined plane. The ball strikes the plane at p and rebounds.
The plane is inclined at an angle of 30° to the horizontal and the coefficient of restitution between the ball and the plane is $\frac{1}{2}$.



Find how far down the plane from p is the ball's next point of impact.
Express your answer in terms of h.

4. A smooth pulley, of mass m kg, is connected by a light inextensible string passing over a smooth light fixed pulley to a particle of mass $5m$ kg.
Two particles of masses m kg and $2m$ kg are connected by a light inextensible string passing over the smooth pulley of mass m kg.



The system is released from rest.

(i) Draw a diagram showing all the forces acting on each particle and on the smooth pulley of mass m kg.

(ii) Find the acceleration of each particle, in terms of g.

(iii) When the particle of mass $2m$ kg has moved down 1 metre relative to the fixed pulley, find how far the particle of mass $5m$ kg has moved relative to the fixed pulley.

5. **(a)** A uniform smooth sphere of mass 2 kg and moving with speed u m/s collides with another uniform smooth sphere of mass 3 kg which is at rest. The velocity of the sphere of mass 2 kg before impact makes an angle of 45° with the line of centres at impact. The coefficient of restitution between the spheres is e.

(i) Find, in terms of e and u, the speed of each sphere after the collision.

(ii) If the sphere of mass 2 kg makes an angle $\tan^{-1} 10$ with the line of centres after impact, find e.

(b) Two identical smooth spheres, each of mass m and moving in the same direction collide directly. The coefficient of restitution between the spheres is e.
If u is the magnitude of the relative velocity between the spheres before impact, show that

(i) each sphere receives an impulse (change in momentum) of magnitude $\frac{1}{2}mu(1+e)$

(ii) the loss in the total kinetic energy of the two spheres due to the impact is $\frac{1}{4}mu^2(1-e^2)$.

6. **(a)** A particle moving with simple harmonic motion has speeds of 5 cm/s and 2 cm/s when it is at points 3 cm and 4 cm, respectively, from the centre of the motion.

(i) Find the amplitude and the period of the motion.

(ii) Find the maximum speed of the particle.

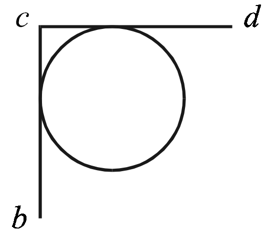
(b) A particle of mass m kg is suspended from a fixed point p by a light elastic string of natural length l and elastic constant $\frac{4mg}{l}$.

(i) Find the distance of the equilibrium position from the point p, in terms of l.

(ii) The particle is pulled down until it is at a distance $\frac{7l}{4}$ vertically below p and is then released from rest.

Find the time taken, in terms of l, for the string to go slack.

7. **(a)** Two identical uniform rods, $[bc]$ and $[cd]$, each of weight W and each of length $4l$, are rigidly connected at c so that $|\angle bcd| = 90°$. The rods rest in limiting equilibrium (that is, just on the point of slipping) in contact with a fixed rough circular peg of radius a with the rod $[cd]$ horizontal and the rod $[bc]$ vertical and where $l < a < 2l$. The coefficient of friction between each rod and the peg is μ where $\mu < 1$.



(i) Draw a diagram showing all the forces acting on the rods.

(ii) Show that $a = \dfrac{l(1+\mu^2)}{(1-\mu)}$.

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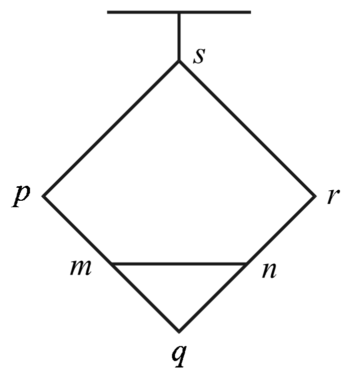
(b) Four identical uniform rods, each of weight W, are freely jointed at their ends to form a square $pqrs$. The square is suspended from s and is held in the form of a square by a light rod, $[mn]$, joining the midpoints of the rods $[pq]$ and $[qr]$.

Calculate, in terms of W, the force in the light rod $[mn]$.



8. **(a)** Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

(b) State the Parallel and Perpendicular Axes Theorems.

Hence, or otherwise, find the moment of inertia of a uniform circular disc of mass m and radius r about an axis tangential to its circumference and lying in the plane of the disc.

The disc may rotate smoothly about a fixed horizontal axis tangential to its circumference and lying in the plane of the disc. The disc is held in the horizontal plane and then released from rest.

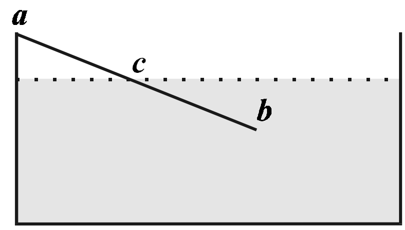
Find the angular speed of the disc when it has rotated through an angle ϑ, in terms of r and ϑ.

Find, in terms of r, the maximum angular speed in the subsequent motion.

9. **(a)** A compound object made up of two bodies of relative densities 1.5 and 2.3 weighs 9.408 N in water and 4.704 N in a liquid of relative density 1.3.

Find the volume of each body in the compound object.

(b) A uniform rod, [*ab*], is of length $2l$. The end *a* of the rod is hinged smoothly at the edge of a tank. The rod is in an inclined position with part [*cb*] immersed in the uniform liquid in the tank. The density of the rod is ρ and the density of the liquid is σ. The rod is at rest.



Show that the length of the immersed part, [*cb*], of the rod is

$$2l\left(1-\sqrt{1-\frac{\rho}{\sigma}}\right).$$

10. **(a)** Find $\frac{d}{dx}\left(\frac{y}{x}\right)$.

Hence, or otherwise, solve the differential equation

$$\frac{1}{x}\frac{dy}{dx}-\frac{y}{x^2}=\frac{1}{x}$$

given that $y = 1$ when $x = 1$.

(b) A car of mass m kg is travelling along a level road. The resistance to motion is mkv^2 N, where v m/s is the speed. When the car is travelling at 14 m/s, the engine cuts out. Ten seconds after the engine cuts out, the speed of the car is 7 m/s.

(i) Show that $k=\frac{1}{140}$.

(ii) The car travels a distance of s metres in the first T seconds after the engine cuts out.
Show that

$$s=140\ln\left(1+\frac{T}{10}\right).$$