

Coimisiún na Scrúduithe Stáit State Examinations Commission

Leaving Certificate 2024

Marking Scheme

Applied Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

In considering this marking scheme for the written examination, the following points should be noted.

- 1. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods. The descriptions, methods and definitions in the scheme are not exhaustive and alternative valid answers are acceptable.
- 2. The detail required in any answer is determined by the context and manner in which the question is asked, and also by the number of marks assigned to the answer in the examination paper. Therefore, in any instance, it may vary from year to year.
- **3.** A solidus (/) indicates different valid attempts.
- **4.** A number of different types of penalties are applied to candidates' work, including:
 - mathematical error ("blunders")

-3

mathematical/numerical slip

-1

misreading (if not serious or leading to oversimplification)

- -1
- **5.** A misreading or slip or omission which oversimplifies the question may be regarded as equivalent to a mathematical error and is marked accordingly.
- 6. In cases where a question item is marked using a marking scale, the scale is provided in **bold**. For a 30 mark item marked using a marking scale:
 - 27 marks are awarded where candidate work shows one systemic error.
 - 24 marks are awarded where candidate work shows two systemic errors.
 - 16 marks are awarded where candidate work shows more than two systemic errors but there is evidence of correct application of method or algorithm
 - 8 marks are awarded where a valid attempt is presented which cannot be awarded higher marks.

For a 20 mark item marked using a marking scale:

- 17 marks are awarded where candidate work shows one systemic error.
- 14 marks are awarded where candidate work shows two systemic errors.
- 8 marks are awarded where a valid attempt is presented which cannot be awarded higher marks.

For a 15 mark item marked using a marking scale:

- 12 marks are awarded where candidate work shows one systemic error.
- 6 marks are awarded where candidate work shows two systemic errors or where a valid attempt is presented which cannot be awarded higher marks.

For a 10 mark item marked using a marking scale:

- 7 marks are awarded where candidate work shows one systemic error.
- 4 marks are awarded where candidate work shows two systemic errors or where a valid attempt is presented which cannot be awarded higher marks.

- **7.** A zero should only be recorded when the candidate has attempted the question item but does not merit marks. If a candidate does not attempt a question item examiners should record NR.
- **8.** Examiners are expected to annotate parts of the responses as directed at the marking conference. (See below.)

Symbol	Name	Use
×	Cross	Incorrect element
✓	Tick	Correct element
S	Slip	Deduct one mark
2	Box 2	Partially correct element – award 2 marks
Λ	۸	Missing element
~~~	Horizontal wavy line	To be noticed
{	Vertical wavy line	Additional page

9. Bonus marks at the rate of 5% of the marks obtained will be given to a candidate who answers the written examination paper entirely through Irish and who obtains 75% or less of the total mark available (i.e. 300 marks or less). In calculating the bonus to be applied decimals are always rounded down, not up ¬ e.g., 4.5 becomes 4; 4.9 becomes 4, etc. See below for when a candidate is awarded more than 300 marks in the written examination paper.

#### Marcanna Breise as ucht freagairt trí Ghaeilge

Léiríonn an tábla thíos an méid marcanna breise ba chóir a bhronnadh ar iarrthóirí a ghnóthaíonn níos mó ná 75% d'iomlán na marcanna.

N.B. Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don scrúdú. Ba chóir freisin an marc bónais sin **a shlánú síos**.

#### Tábla 400 @ 5%

Bain úsáid as an tábla seo i gcás na n-ábhar a bhfuil 400 marc san iomlán ag gabháil leo agus inarb é 5% gnáthráta an bhónais.

Bain úsáid as an ngnáthráta i gcás 300 marc agus faoina bhun sin. Os cionn an mharc sin, féach an tábla thíos.

Bunmharc	Marc Bónais
301 - 306	14
307 - 313	13
314 - 320	12
321 - 326	11
327 - 333	10
334 - 340	9
341 - 346	8
347 - 353	7

Bunmharc	Marc Bónais
354 - 360	6
361 - 366	5
367 - 373	4
374 - 380	3
381 - 386	2
387 - 393	1
394 - 400	0

$$AB = \begin{pmatrix} -1 & -1 & 2 \\ 13 & 2 & -3 \\ -6 & 11 & 6 \end{pmatrix}$$

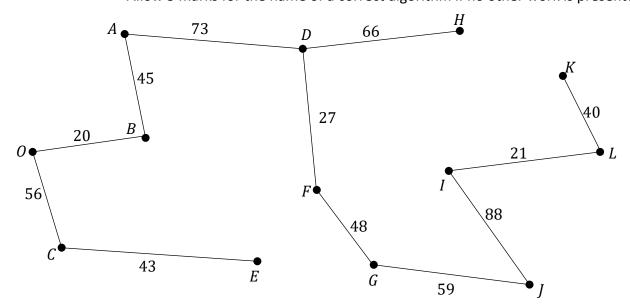
10 [**0/4/7**]

# 1(b) (i)

<u>Kruskal's algorithm</u>	Prim's algorithm
OB  = 20	Choose node $0$ , say.
IL  = 21	OB  = 20
DF  = 27	AB  = 45
KL  = 40	0C  = 56
CE  = 43	CE  = 43
AB  = 45	AD  = 73
FG  = 48	DF  = 27
OC  = 56	FG  = 48
GJ  = 59	GJ  = 59
OA  = 62	DH  = 66
DH  = 66	IJ  = 88
BE  = 71	IL  = 21
AD  = 73	KL  = 40
CB  = 82	
IJ  = 88	

15 [**0/6/9/12**]

Deduct 3 marks if the algorithm used is not correctly named. Allow 3 marks for the name of a correct algorithm if no other work is presented.



$$2(56+43) + 20 + 45 + 73 + 2(66) + 27 + 48 + 59 + 88 + 21 + 40 = 751 \text{ m}$$

**1(c)** 

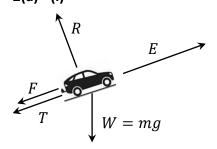
$$s_{J} = 10t + t^{2}$$

$$s_{K} = 4(t-1) + 2(t-1)^{2}$$

$$s_{J} = s_{K} + 21 + 4$$

$$t = 5 \pm \sqrt{2} s$$
5
5

5

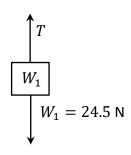


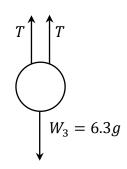
10

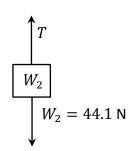
# 2(a) (ii)

$$4400 = 2725g \sin \theta + 1525 + 375$$
 5 5  $\theta = 5.37^{\circ}$  5

# 2(b) (i)







5, 5, 5

# 2(b) (ii)

$$24.5 - T = 2.5a$$
 5
 $44.1 - T = 4.5b$  and  $2T - 6.3g = 6.3\left(\frac{a+b}{2}\right)$  5
 $T = 31.18 \text{ N}$  5

# 3(a)

# 3(b) (i)

 $t = \frac{\pi}{2}$  so  $v = \frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4} = \frac{\pi^2}{8} - \frac{1}{2} \approx 0.73$ 

before impact (m s⁻¹) after impact (m s⁻¹)  $A \quad u \cos \alpha \, \vec{\imath} + u \sin \alpha \, \vec{\jmath} \qquad v_1 \vec{\imath} + u \sin \alpha \, \vec{\jmath} \\ B \quad -u \cos \alpha \, \vec{\imath} + u \sin \alpha \, \vec{\jmath} \qquad v_2 \vec{\imath} + u \sin \alpha \, \vec{\jmath} \\ \text{PCM} \quad m(u \cos \alpha) + m(-u \cos \alpha) = m(v_1) + m(v_2) \\ v_1 + v_2 = 0 \\ \text{NEL} \quad v_1 - v_2 = -2eu \cos \alpha \qquad \qquad 5$ 

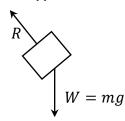
NEL 
$$v_1 - v_2 = -2eu \cos \alpha$$
 5  
 $v_1 = -eu \cos \alpha$   $v_2 = eu \cos \alpha$  5

# 3(b) (ii)

$$\frac{\tan^2 \alpha}{e^2} = 1 \text{ so } e = \tan \alpha \text{ as required}$$

5

4 (i)



5

4 (ii)

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 + mg(6.7 - 6.7\cos\alpha) \text{ and } R - mg\cos\alpha = \frac{mv^2}{6.7}$$

$$R = 3mg\cos\alpha - 2mg + \frac{mv_1^2}{6.7}$$

4 (iii)

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + 6.7mg$$

$$v_2 = \sqrt{v_1^2 - 13.4g}$$
5

4 (iv)  $D \downarrow W = mg$ 

5

4 (v)

$$ma = -W - D = -mg - mkv$$
  
 $a = \frac{dv}{dt} = -g - kv = -(g + kv)$  as required 5

4 (vi)

$$\int \frac{dv}{g+kv} = -\int dt$$

$$= -t + c 5$$

let 
$$u = g + kv$$
, so  $du = kdv$   $\therefore \int \frac{dv}{g + kv} = \frac{1}{k} \int \frac{du}{u} = \frac{1}{k} \ln|u| = \frac{1}{k} \ln(g + kv)$  5

$$v = v_2 = \sqrt{{v_1}^2 - 13.4g}$$
 when  $t = 0$  so  $c = \frac{1}{k} \ln(g + k\sqrt{{v_1}^2 - 13.4g})$ 

$$\ln \frac{g+kv}{g+k\sqrt{v_1^2-13.4g}} = -kt \text{ so } \frac{g+kv}{g+k\sqrt{v_1^2-13.4g}} = e^{-kt}, \text{ i.e. } v = \frac{\left(g+k\sqrt{v_1^2-13.4g}\right)e^{-kt}-g}{k} \text{ m s}^{-1}$$

PCM 
$$2m(2u) + 3m(-u) = 2mv_1 + 3mv_2$$
 i.e.  $u = 2v_1 + 3v_2$ 

NEL 
$$v_1 - v_2 = -e(2u + u) = -3eu$$
 5

$$v_1 = \frac{u(1-9e)}{5}$$
 and  $v_2 = \frac{u(1+6e)}{5}$ 

# 5(a) (ii)

 $v_2 > 0$  for all possible values of e

$$v_1 > 0 \text{ for } 0 \le e < \frac{1}{9}$$

# 5(b) (i)

$$v = at_1$$
 so  $a = \frac{v}{t_1}$ 

$$s_1 = \frac{1}{2}at_1^2 = \frac{1}{2}vt_1 = \frac{vt_2}{4}$$

$$s_2 = vt_2 5$$

$$0 = v - bt_3 \text{ so } b = \frac{v}{t_3}$$

$$s_3 = vt_3 - \frac{1}{2}bt_3^2 = \frac{1}{2}vt_3$$

$$T = t_1 + t_2 + t_3$$

$$= \frac{3t_2}{2} + t_3 \text{ so } t_3 = \frac{2T - 3t_2}{2}$$

$$s_3 = \frac{v(2T - 3t_2)}{4}$$

$$d = s_1 + s_2 + s_3 = \frac{vt_2}{4} + vt_2 + \frac{2T - 3t_2}{2} = \frac{v(T + t_2)}{2}$$

so 
$$T = \frac{2d - t_2 v}{v}$$
 as required

# 5(b) (ii)

$$\frac{d}{T} = \frac{2v}{3} \text{ so } vT = \frac{4vT}{3} - t_2 v$$

$$3t_2 = T$$
 so

so 
$$T = 6t_1$$
 as required

$$P_{n+1} = 1.012P_n - x 5$$

# 6(a) (ii)

$$P_n = a^n P_0 + b \left( \frac{1 - a^n}{1 - a} \right) \text{ or } P_n = C a^n + D$$

$$P_n = 1.012^n \left(13500 - \frac{x}{0.012}\right) + \frac{x}{0.012}$$

# 6(a) (iii)

$$P_1 = 1.012P_0 - x = 13622 - x = 13514$$
 so  $x = 148$  people 5

# 6(a) (iv)

$$P_7 = 1.012^7 \left(13500 - \frac{148}{0.012}\right) + \frac{148}{0.012} = 13602 \text{ people}$$

# 6(b) (i)

$$\int ve^{-\frac{v^2}{4}}dv = \int ds$$

$$= s + c$$
 5

let 
$$u = -\frac{v^2}{4}$$
, so  $du = -\frac{v}{2}dv : \int ve^{-\frac{v^2}{4}}dv = -2\int e^u du = -2e^u = -2e^{-\frac{v^2}{4}}$ 

v = 0 when s = 0 so c = -2

$$-2e^{-\frac{v^2}{4}} = s - 2$$
, i.e.  $v = 2\sqrt{\ln\frac{2}{2-s}} \,\mathrm{m}\,\mathrm{s}^{-1}$ 

# 6(b) (ii)

$$s = 0.3 \text{ so } v = 0.81 \text{ m s}^{-1}$$

$$v_H = v \cos \alpha : t = \frac{35}{20 \cos \alpha}$$

7(a) (ii)

$$2.5 = 20\sin\alpha \, t - 4.9t^2$$

$$2.5 = 35 \tan \alpha - 4.9 \left(\frac{35 \sec \alpha}{20}\right)^2$$

 $15.00625 \tan^2 \alpha - 35 \tan \alpha + 17.50625 = 0$ 

$$\alpha = 58.09^{\circ} \text{ or } \alpha = 35.996^{\circ}$$

7(b) (i)



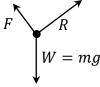
5

7(b) (ii)

$$R\cos\beta = mg$$
 so  $R = \frac{5mg}{3}$  and  $F_c = \frac{mv^2}{r} = R\sin\beta$ 

$$\frac{mv^2}{8} = \frac{4mg}{3}$$
 i.e.  $v = 10.2$  m s⁻¹

7(b) (iii)



7(b) (iv)

$$R\cos\beta + \mu R\sin\beta = mg$$
 so  $R = \frac{5mg}{3+4\mu}$  and  $F_c = \frac{mv^2}{r} = R\sin\beta - \mu R\cos\beta$  5

$$\frac{m(7.5)^2}{8} = \left(\frac{5mg}{3+4\mu}\right) \left(\frac{4-3\mu}{5}\right) \text{ so } \frac{4-3\mu}{3+4\mu} = 0.717 \text{ i.e. } \mu = 0.315$$

8(a) (i)

Stage	State	Action	Destination	Value
Year 4	G	GY	Y	6*
	Н	HY	Y	5 *
	I	IY	Y	7 *
Year 3	D	DG	G	-5 + 6 = 1
		DH	Н	-3 + 5 = 2
		DI	I	3 + 7 = 10 *
	E	EG	G	-2 + 6 = 4
		EH	Н	5 + 5 = 10
		EI	I	6 + 7 = 13 *
	F	FG	G	4+6=10*
		FH	Н	3 + 5 = 8
		FI	I	2 + 7 = 9
Year 2	A	AD	D	5 + 10 = 15 *
		AE	E	-2 + 13 = 11
		AF	F	-5 + 10 = 5
	В	BD	D	7 + 10 = 17 *
		BE	E	-3 + 13 = 10
		BF	F	4 + 10 = 14
	С	CD	D	5+10 = 15 *
		CE	E	-4 + 13 = 9
		CF	F	-1 + 10 = 9
Year 1	X	XA	A	-11 + 15 = 4
		XB	В	-13 + 17 = 4
		XC	С	-9 + 15 = 6 *

plan for maximum profit =  $X \rightarrow C \rightarrow D \rightarrow I \rightarrow Y$ 

30 **[0/8/16/24/27**]

# 8(a) (ii)

e.g. Bellman's principle can be used to find the shortest or longest path, Dijkstra's algorithm can only be used to find the shortest path.

# 8(b)

replacement of physical quantities with correct units and simplification to m

15 [**0/6/9/12**]

#### 9 (i)

$$M_2 = M_1 + 0.75M_0 = 245 + 150 = 395 \text{ kg}$$
  
 $M_3 = M_2 + 0.75M_1 = 395 + 183.75 = 578.75 \text{ kg}$ 

#### 9 (ii)

$$\begin{split} M_{n+2} &= M_{n+1} + 0.75 M_n \text{ i.e. } 4 M_{n+2} - 4 M_{n+1} - 3 M_n = 0 \\ 4 x^2 - 4 x - 3 &= 0 \text{ i.e. } (2 x - 3) (2 x + 1) = 0 \text{ i.e. } x = 1.5 \text{ or } x = -0.5 \\ M_n &= \alpha (1.5)^n + \beta (-0.5)^n \\ M_0 &= 200 \text{ so } \alpha + \beta = 200 \text{ and } M_1 = 245 \text{ so } 3\alpha - \beta = 490 \\ \alpha &= 172.5 \text{ and } \beta = 27.5, \text{ i.e. } M_n = 172.5 (1.5)^n + 27.5 (-0.5)^n \end{split}$$

# 9 (iii)

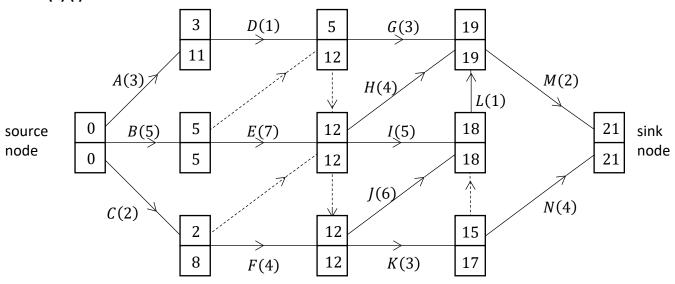
$$M_0 = 200, M_1 = 245, M_2 = 395, M_3 = 578.75, M_4 = 875, M_5 = 1309.0625$$
 total mass =  $3602.8125 \, \mathrm{kg}$ 

# 9 (iv)

$$\begin{split} P_{n+2} &= P_{n+1} + 0.75P_n - 2^{n+2} \text{ has a particular solution of the form } f(n) = a + b(2^n) \\ f(n+2) &= f(n+1) + 0.75f(n) - 2^{n+2} \\ a + b(2^{n+2}) &= a + b(2^{n+1}) + 0.75a + 0.75b(2^n) - 2^{n+2} \text{ i.e. } 3a - 2^{n+4} = 5b(2^n) \text{ for all } n \\ a &= 0 \text{ and } b = -3.2 \\ P_n &= \alpha(1.5)^n + \beta(-0.5)^n - 3.2(2^n) \\ P_0 &= 199 \text{ so } \alpha + \beta - 3.2 = 199 \\ P_1 &= 243 \text{ so } 3\alpha - \beta - 12.8 = 486 \\ \alpha &= 175.25 \text{ and } \beta = 26.95, \text{ i.e. } P_n = 175.25(1.5)^n + 26.95(-0.5)^n - 3.2(2^n) \end{split}$$

#### 9 (v)

$$P_0 = 199, P_1 = 243, P_2 = 388.25, P_3 = 562.5, P_4 = 837.6875, P_5 = 1227.5625$$
 total mass = 3458 kg



20 [0/8/14/17]

10(a) (ii)

# 10(a) (iii)

one where a delay in completing an activity will lead to a delay in completing the overall project 5

# 10(a) (iv)

no effect on the time to complete the project, but B, E, K, N will be a new critical path 5

10(b)

$$\frac{dx}{dt} = -kx$$

so 
$$\int \frac{dx}{x} = -\int kdt$$
 so  $\ln x = -kt + c$ 

x = H when t = 0 so  $c = \ln H$  i.e.  $kt = \ln \frac{H}{x}$ 

$$x = \frac{H}{3}$$
 when  $t = 45$  so  $k = \frac{\ln 3}{45}$  so when  $x = \frac{H}{8}$ ,  $t = 45 \left(\frac{\ln 8}{\ln 3}\right) = 85.2$  s

Section	Indicative Content	Marks
Introduction and Research	Background research on brief  And the modelled  And the modelled  And the modelled  Background research on brief	20
	Research specific problem(s)	
	Identify relevant variables	
	Present relevant data	
	<ul> <li>Provide citations and references</li> </ul>	
The Modelling Process	Explain and justify model and assumptions	20
	Compute solutions	
	<ul> <li>Present solutions using appropriate mathematical and graphical representations</li> </ul>	
	<ul> <li>Analysis of solution(s) – sensitivity to changes in assumptions; comparison with other solutions or real-world data</li> </ul>	
	<ul> <li>Iterative process</li> </ul>	
Interpretation of Results	Interpretation of solution(s) in real-world context	15
	Conclusions and reflections	
Communication and Innovation	This is not a distinct section of the report.	15
	<ul> <li>Innovative and creative approaches</li> </ul>	
	Overall coherence	

# To be noted by examiner:

- Before commencing marking read the entire reporting booklet to familiarise yourself with the content presented for marking.
- Be careful not to penalise skilful brevity, nor to reward unwarranted length.
- Mark descriptors should be interpreted in the context of the challenges and demands of the specific problem which the candidate has chosen.

Marking Scheme

Higher Level Applied Mathematics Mathematical Modelling Project – Marking Scheme

1 Introduction & Research	Very thorough 16 – 20	Thorough 11 – 15	Basic 6 – 10	Very basic 0 – 5
(20 marks)	Problem and variables identified, research presented and cited, data presented where relevant.	Work of a good standard, but some issues with, for example, identification of variables or citation of research.	Basic statement of problem with some evidence of research undertaken.	Statement of problem with no evidence of research.
<b>2a</b> The Modelling Process –	Very thorough 12 – 15	Thorough 8 – 11	Basic 4 – 7	Very basic 0 – 3
Explain & Justify (15 marks)	Model fully explained, including further iterations, assumptions identified and justified.	Model well explained, assumptions clearly described.	Model and assumptions poorly explained.	Model outlined with no explanation of assumption.
<b>2b</b> The Modelling Process –	Very thorough 16 – 20	Thorough 11 – 15	Basic 6 – 10	Very basic 0 – 5
Compute & Iterate (20 marks)	Computation of mathematical solution(s) fully explained. Full iterative process.	Partial explanation of mathematical solution(s). Iteration(s) presented.	Solution(s) computed without explanation. Some evidence of iterative process	Solution(s) computed without explanation.

2c	Very thorough	Thorough	Basic	Very basic
The Modelling Process –	12 – 15	8 - 11	4-7	0-3
Present & Analyse	Solution(s) presented using	Solution(s) presented and	Solution(s) presented.	Solution(s) presented.
(15 marks)	appropriate	analysed, but with some	Some evidence of analysis.	
	mathematical/graphical	issues with, for example,		
	representations.	mathematical/graphical		
	Solution(s) analysed with	representations.		
	reference to model's			
	assumptions or other			
	solutions or real-world			
	data.			
က	Very thorough	Thorough	Basic	Very basic
Interpretation of Results	12 – 15	8 - 11	4-7	0 – 3
(15 marks)	Excellent interpretation of	Good interpretation of	Some interpretation of	Results interpreted poorly
	results in a real-world	results in a real-world	results presented.	or conclusion(s) drawn
	context.	context.	Limited conclusion(s)	poorly or project reflected
	Conclusion(s) drawn and	Conclusion(s) drawn <i>or</i>	drawn <i>or</i> limited reflection	on poorly.
	project reflected on.	project reflected on.	on project.	
4	Very thorough	Thorough	Basic	Very basic
Communication &	12 - 15	8 – 11	4-7	0 – 3
Innovation	Project approached and/or	Project approached and/or	Project approached or	Little or no evidence of
(15 marks)	presented in a highly	presented with	presented with limited	innovation/creativity.
	innovative and/or creative	innovation/creativity.	innovation/creativity.	Poor overall coherence.
	way.	Good overall coherence.	Fair overall coherence.	
	Excellent overall			
	coherence.			