Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2010

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 25 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.
A *Formulae and Tables* booklet may be obtained from the Superintendent.
Take the value of g to be 9.8 ms^{-2}.
Marks may be lost if necessary work is not clearly shown.

1. **(a)** A car is travelling at a uniform speed of 14 ms^{-1} when the driver notices a traffic light turning red 98 m ahead.

Find the minimum constant deceleration required to stop the car at the traffic light,

(i) if the driver immediately applies the brake

(ii) if the driver hesitates for 1 second before applying the brake.

(b) A particle passes P with speed 20 ms^{-1} and moves in a straight line to Q with uniform acceleration.

In the first second of its motion after passing P it travels 25 m.

In the last 3 seconds of its motion before reaching Q it travels $\frac{13}{20}$ of $|PQ|$.

Find the distance from P to Q.

2. **(a)** Two particles, A and B, start initially from points with position vectors $6\vec{i} - 14\vec{j}$ and $3\vec{i} - 2\vec{j}$ respectively. The velocities of A and B are constant and equal to $4\vec{i} - 3\vec{j}$ and $5\vec{i} - 7\vec{j}$ respectively.

(i) Find the velocity of B relative to A.

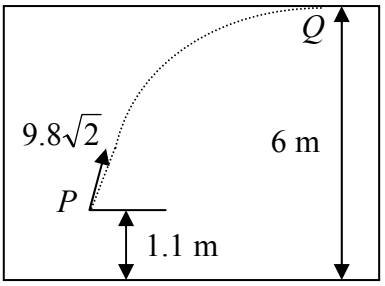
(ii) Show that the particles collide.

(b) When a motor-cyclist travels along a straight road from South to North at a constant speed of 12.5 ms^{-1} the wind appears to her to come from a direction North $45°$ East.

When she returns along the same road at the same constant speed, the wind appears to come from a direction South $45°$ East.

Find the magnitude and direction of the velocity of the wind.

3. **(a)** In a room of height 6 m, a ball is projected from a point P.



P is 1.1 m above the floor.

The velocity of projection is $9.8\sqrt{2}$ ms^{-1} at an angle of 45° to the horizontal.

The ball strikes the ceiling at Q without first striking a wall.
Find the length of the straight line PQ.

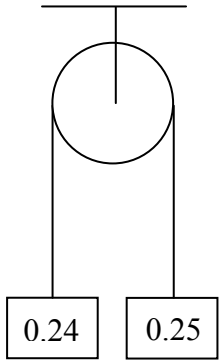
(b) A particle is projected up an inclined plane with initial speed 80 ms^{-1}.
The line of projection makes an angle of $30°$ with the inclined plane and the plane is inclined at an angle θ to the horizontal.
The plane of projection is vertical and contains the line of greatest slope.

The particle strikes the plane at an angle of $\tan^{-1}\dfrac{2}{\sqrt{3}}$.

Find **(i)** the value of θ

(ii) the speed with which the particle strikes the plane.

4. **(a)** Two particles of masses 0.24 kg and 0.25 kg are connected by a light inextensible string passing over a small, smooth, fixed pulley.

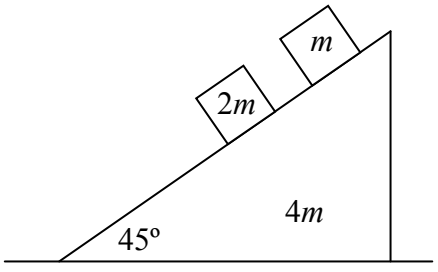


The system is released from rest.

Find **(i)** the tension in the string

(ii) the speed of the two masses when the 0.25 kg mass has descended 1.6 m.

(b) A smooth wedge of mass $4m$ and slope 45º rests on a smooth horizontal surface.



Particles of mass $2m$ and m are placed on the smooth inclined face of the wedge.

The system is released from rest.

(i) Show, on separate diagrams, the forces acting on the wedge and on the particles.

(ii) Find the acceleration of the wedge.

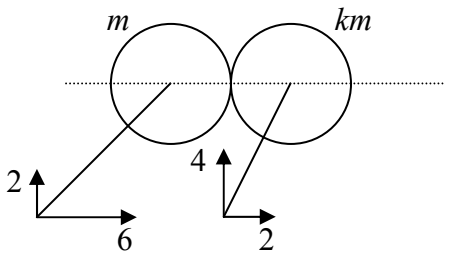
5. **(a)** A sphere, of mass m and speed u, impinges directly on a stationary sphere of mass $3m$.

The coefficient of restitution between the spheres is e.

(i) Find, in terms of u and e, the speed of each sphere after the collision.

(ii) If $e = \frac{1}{4}$, find the percentage loss in kinetic energy due to the collision.

(b) A smooth sphere, of mass m, moving with velocity $6\vec{i} + 2\vec{j}$ collides with a smooth sphere, of mass km, moving with velocity $2\vec{i} + 4\vec{j}$ on a smooth horizontal table.



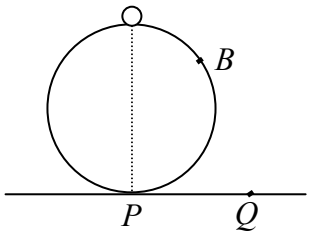
After the collision the spheres move in parallel directions.

The coefficient of restitution between the spheres is e.

(i) Find e in terms of k.

(ii) Prove that $k \geq \frac{1}{3}$.

6. **(a)** A particle of mass m kg lies on the top of a smooth sphere of radius 2 m.
The sphere is fixed on a horizontal table at P.



The particle is slightly displaced and slides down the sphere. The particle leaves the sphere at B and strikes the table at Q.

Find **(i)** the speed of the particle at B

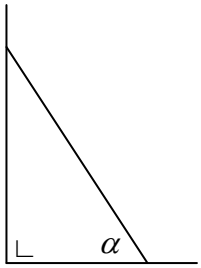
(ii) the speed of the particle on striking the table at Q.

(b) A particle moves with simple harmonic motion of amplitude 0.75 m.
The period of the motion is 4 s.

Find **(i)** the maximum speed of the particle

(ii) the time taken by the particle to move from the position of maximum speed to a position at which its speed is half its maximum value.

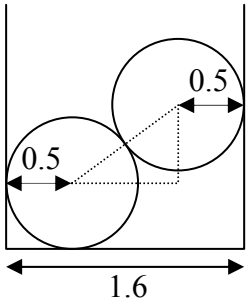
7. **(a)** One end of a uniform ladder, of weight W, rests against a smooth vertical wall, and the other end rests on rough horizontal ground. The coefficient of friction between the ladder and the ground is μ.
The ladder makes an angle α with the horizontal and is in a vertical plane which is perpendicular to the wall.



Show that a person of weight $3W$ can safely climb to the top of the ladder if

$$\mu > \frac{7}{8\tan\alpha}.$$

(b) Two uniform smooth spheres each of weight W and radius 0.5 m, rest inside a hollow cylinder of diameter 1.6 m.



The cylinder is fixed with its base horizontal.

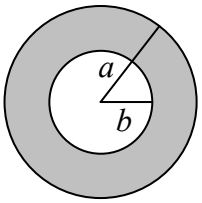
(i) Show on separate diagrams the forces acting on each sphere.

(ii) Find, in terms of W, the reaction between the two spheres.

(iii) Find, in terms of W, the reaction between the lower sphere and the base of the cylinder.

8. **(a)** Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

(b) An annulus is created when a central hole of radius b is removed from a uniform circular disc of radius a.



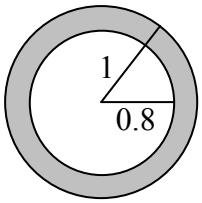
The mass of the annulus (shaded area) is M.

(i) Show that the moment of inertia of the annulus about an axis through its centre and perpendicular to its plane is $\frac{M\left(a^2+b^2\right)}{2}$.

(ii) The annulus rolls, from rest, down an incline of 30°. Find its angular velocity, in terms of g, a and b, when it has rolled a distance $\frac{a}{2}$.

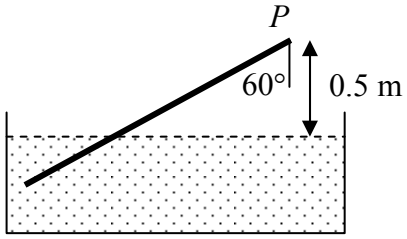
9. **(a)** State the Principle of Archimedes.

A buoy in the form of a hollow spherical shell of external radius 1 m and internal radius 0.8 m floats in water with 61% of its volume immersed.



Find the density of the material of the shell.

(b) A uniform rod, of length 1.5 m and weight W, is freely hinged at a point P.



The rod is free to move about a horizontal axis through P.
The other end of the rod is immersed in water.

The point P is 0.5 m above the surface of the water.

The rod is in equilibrium and is inclined at an angle of $60°$ to the vertical.

Find **(i)** the relative density of the rod

(ii) the reaction at the hinge in terms of W.

10. **(a)** Solve the differential equation

$$y\frac{dy}{dx} = x + xy^2$$

given that $y = 0$ when $x = 0$.

(b) The acceleration of a cyclist freewheeling down a slight hill is

$$0.12 - 0.0006v^2 \ \text{ms}^{-2}$$

where the velocity v is in metres per second.

The cyclist starts from rest at the top of the hill.

Find **(i)** the speed of the cyclist after travelling 120 m down the hill

(ii) the time taken by the cyclist to travel the 120 m if his average speed is $2.65 \ \text{ms}^{-1}$.

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