Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2006

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

THURSDAY, 8 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

1. **(a)** Find the real number a such that for all $x \neq 9$,

$$\frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a.$$

(b) $f(x) = 3x^3 + mx^2 - 17x + n$, where m and n are constants.
Given that $x-3$ and $x+2$ are factors of $f(x)$, find the value of m and the value of n.

(c) $x^2 - t$ is a factor of $x^3 - px^2 - qx + r$.

(i) Show that $pq = r$.

(ii) Express the roots of $x^3 - px^2 - qx + r = 0$ in terms of p and q.

2. **(a)** Solve the simultaneous equations

$$y = 2x - 5$$
$$x^2 + xy = 2.$$

(b) **(i)** Find the range of values of $t \in \mathbf{R}$ for which the quadratic equation $(2t-1)x^2 + 5tx + 2t = 0$ has real roots.

(ii) Explain why the roots are real when t is an integer.

(c) $f(x) = 1 - b^{2x}$ and $g(x) = b^{1+2x}$, where b is a positive real number.

Find, in terms of b, the value of x for which $f(x) = g(x)$.

3. **(a)** Given that $z = 2 + i$, where $i^2 = -1$, find the real number d such that $z + \frac{d}{z}$ is real.

(b) **(i)** Use matrix methods to solve the simultaneous equations

$$4x - 2y = 5$$
$$8x + 3y = -4$$

(ii) Find the two values of k which satisfy the matrix equation

$$\begin{pmatrix} 1 & k \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = 11.$$

(c) **(i)** Express $-8 - 8\sqrt{3}\,i$ in the form $r(\cos\theta + i\sin\theta)$.

(ii) Hence find $\left(-8 - 8\sqrt{3}\,i\right)^3$.

(iii) Find the four complex numbers z such that

$$z^4 = -8 - 8\sqrt{3}\,i.$$

Give your answers in the form $a + bi$, with a and b fully evaluated.

4. **(a)** $-2 + 2 + 6 + \ldots + (4n - 6)$ are the first n terms of an arithmetic series. S_n, the sum of these n terms, is 160. Find the value of n.

(b) The sum to infinity of a geometric series is $\frac{9}{2}$.

The second term of the series is -2.
Find the value of r, the common ratio of the series.

(c) The sequence $u_1, u_2, u_3, \ldots$, defined by $u_1 = 3$ and $u_{n+1} = 2u_n + 3$, is as follows:
3, 9, 21, 45, 93……

(i) Find u_6, and verify that it is equal to the sum of the first six terms of a geometric series with first term 3 and common ratio 2.

(ii) Given that, for all k, u_k is the sum of the first k terms of a geometric series with first term 3 and common ratio 2, find $\sum_{k=1}^{n} u_k$.

5. **(a)** Find the value of the middle term of the binomial expansion of

$$\left(\frac{x}{y}-\frac{y}{x}\right)^8.$$

(b) **(i)** Express $\dfrac{2}{(r+1)(r+3)}$ in the form $\dfrac{A}{r+1}+\dfrac{B}{r+3}$.

(ii) Hence find $\displaystyle\sum_{r=1}^{n}\frac{2}{(r+1)(r+3)}$.

(iii) Hence evaluate $\displaystyle\sum_{r=1}^{\infty}\frac{2}{(r+1)(r+3)}$.

(c) **(i)** Given two real numbers a and b, where $a>1$ and $b>1$, prove that

$$\frac{1}{\log_b a}+\frac{1}{\log_a b}\geq 2.$$

(ii) Under what condition is $\dfrac{1}{\log_b a}+\dfrac{1}{\log_a b}=2$.

6. **(a)** Differentiate $\sqrt{x}(x+2)$ with respect to x.

(b) The equation of a curve is $y=3x^4-2x^3-9x^2+8$.

(i) Show that the curve has a local maximum at the point (0, 8).

(ii) Find the coordinates of the two local minimum points on the curve.

(iii) Draw a sketch of the curve.

(c) Prove by induction that $\dfrac{d}{dx}\left(x^n\right)=nx^{n-1},\ n\geq 1,\ n\in\mathbf{N}$.

7. **(a)** Taking $x_1 = 2$ as the first approximation to the real root of the equation

$$x^3 + x - 9 = 0,$$

use the Newton-Raphson method to find x_2, the second approximation.

(b) The parametric equations of a curve are:

$$x = 3\cos\theta - \cos^3\theta$$

$$y = 3\sin\theta - \sin^3\theta, \text{ where } 0 < \theta < \frac{\pi}{2}.$$

(i) Find $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$.

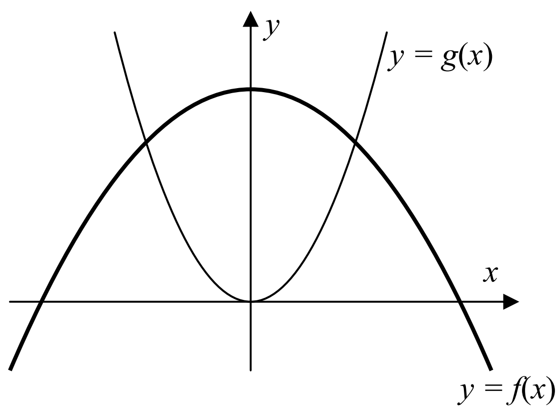
(ii) Hence show that $\frac{dy}{dx} = \frac{-1}{\tan^3\theta}$.

(c) Given $y = \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right)$, find $\frac{dy}{dx}$ and express it in the form $\frac{a}{b-x^n}$.

8. **(a)** Find **(i)** $\int \sqrt{x}\,dx$ **(ii)** $\int e^{-2x}\,dx.$

(b) Evaluate **(i)** $\int_1^2 x(1+x^2)^3\,dx$ **(ii)** $\int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta \, d\theta.$

(c) The diagram shows the graphs of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = 12 - 3x^2$ and $g(x) = 9x^2$.



(i) Calculate the area of the region enclosed by the curve $y = f(x)$ and the x-axis.

(ii) Show that the region enclosed by the curves $y = f(x)$ and $y = g(x)$ has half that area.

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