Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE 2008

MARKING SCHEME

MATHEMATICS

HIGHER LEVEL

PAPER 2

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips- numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5·50 may be written as €5,50.

QUESTION 1

Part (a)	**15 marks**	**Att 5**
Part (b)	**15 (5,5,5) marks**	**Att 6(2,2,2)**
Part (c)	**20 (5,10,5) marks**	**Att 7(2,3,2)**

Part (a) **15 marks** **Att 5**

The height and the diameter of a solid cylinder are both 8 cm in length.

✍ Find the curved surface area of the cylinder correct to the nearest whole number.

15 marks **Att 5**

CSA of cylinder $2\pi rh = 2\,(3{\cdot}142)\,4{\cdot}8 = 201{\cdot}088\ \text{cm}^2$

$201\ \text{cm}^2$ to nearest whole number

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect substitution into correct formula
B3 Incorrect *r*
B4 Incorrect relevant area formula
B5 Using a value of π which affects accuracy of answer

Slips (-1)

S1 Arithmetic slips to a maximum of (-*3*)
S2 Not rounding to nearest whole number

Attempts (5 marks)

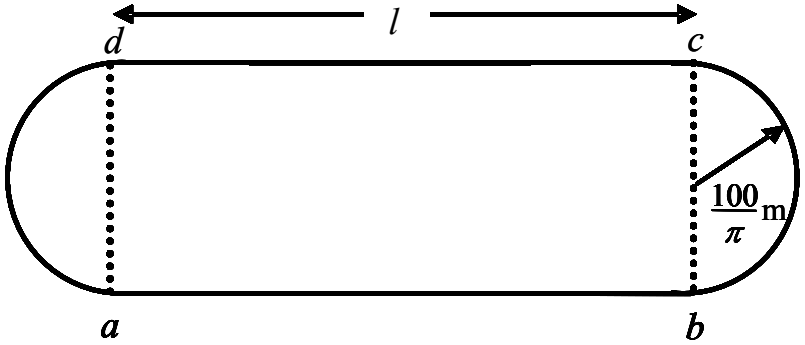
A1 Correct formula with some substitution
A2 Correct *r* indicated

Worthless (0)

W1 Volume of cylinder

Part (b) **15 (5,5,5) marks** **Att 6 (2,2,2)**

(b) The diagram shows the perimeter of a running track, consisting of two straight sections of length l, and two semi-circular sections, at each end, of radius $\frac{100}{\pi}$ m, as shown.



(i) ✍ Given that the perimeter of the track measures 400 m, find l.

(ii) A 1500 m race starts at the point a and goes in the direction $abcd$.
✍ At what point does the race finish?

(iii) An athlete completes this distance in 3 mins 26 sec.
✍ Find his average speed in m/s, correct to one decimal place.

(b)(i) **5 marks** **Att 2**

$2l + 2\pi r = 400$

$l + \pi r = 200$

$l + \pi(\frac{100}{\pi}) = 200 \quad \Rightarrow l + 100 = 200 \quad \Rightarrow l = 100\text{m}$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect substitution into correct perimeter formula
B3 Incorrect relevant perimeter formula
B4 Using incorrect r
B5 Using a value of π which affects accuracy of answer
B6 Early rounding off which affects accuracy of answer

Slips (-1)

S1 Arithmetic slips to a maximum of (-3)

Attempts (2 marks)

A1 Correct perimeter formula

Worthless (0)

W1 Area of rectangle and/or disc

(b)(ii) **5 marks** **Att 2**

$1500 = 3(400) + 300$ or $\frac{1500}{400} = 3{\cdot}75$ laps

i.e three perimeters and 300m more from *a* Starting at *a* the race finishes at *d* as it lies 0·75 laps or 300 m from *a*

$|ab| = |bc| = |cd| = 100$

Race finishes at *d*

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 $\frac{400}{1500}$

B3 Early rounding off of answer from (b) (i) which affects accuracy of answer

Slips (-1)

S1 Arithmetic slips to a maximum of (-*3*)

Misreadings (-1)

M1 Race in opposite direction i.e. *adcb*

Attempts (2 marks)

A1 Finds number of complete perimeters

Worthless (0)

W1 $\frac{400}{1500}$ and stops

(b)(iii) **5 marks** **Att 2**

3mins 26 sec = 206 sec

Average speed = $\frac{1500}{206}$ =7·28 m/sec = 7·3m/sec to one decimal place

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 3 mins ≠180 secs

B3 Speed = $\frac{206}{1500}$ m/sec

B4 Speed expressed in metres per min

Slips (-1)

S1 Arithmetic slips to a maximum of (-*3*)

Attempts (2 marks)

A1 Converting minutes to seconds

Worthless (0)

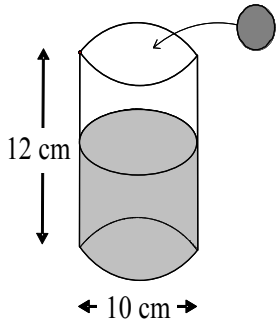
W1 Av Speed = product of distance by time

Part (c) **20 (5,10,5) marks** **Att 7(2,3,2)**

(c) A spherical golf ball has a diameter of 4 cm.

(i) ✍ Find the volume of the golf ball in terms of π.

A cylindrical hole on a golf course is 10 cm in diameter and 12 cm deep. The hole is half full of water.



(ii) ✍ Calculate the volume of water in the hole, in terms of π.

The golf ball is dropped into the hole.

(iii) ✍ Find the rise in the level of the water, correct to two decimal places.

(c)(i) **5 marks** **Att 2**

Volume of golf ball (sphere) = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi 2^3$ or $\frac{32}{3}\pi$ cm^3 or $10{\cdot}67\,\pi$ cm^3

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect substitution into correct formula
B3 Incorrect relevant volume formula

Slips (-1)

S1 Arithmetic slips to maximum *(-3)*
S2 Answer not in terms of π

Attempts (2 marks)

A1 Indicates radius = half length of diameter

Worthless (0)

W1 Surface area of sphere

(c)(ii) **10 marks** **Att 3**

Volume of cylinder where $r = 5$ and $h = 6$

$\pi r^2 h = \pi 5^2 \times 6$ or $150\,\pi$ cm^3

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect r and /or incorrect h
B3 Incorrect relevant volume formula

Slips (-1)
S1 Arithmetic slips to maximum *(-3*)
S2 Answer not in terms of π

Attempts (2 marks)
A1 Indicates radius half length of diameter
A2 Some indication of relevant height

Worthless (0)
W1 Surface area formula for cylinder

(c)(iii) **5 marks** **Att 2**

Let rise in cylinder $= h$

$$\pi 5^2 h = \frac{4}{3}\pi 2^3$$

$$25\pi h = \frac{32}{3}\pi$$

$$25h = \frac{32}{3}$$

$$h = \frac{32}{75} = 0{\cdot}42666$$

$h = 0{\cdot}43$cm to 2 dec. places

or Vol of water in cyl. + vol of sphere

$$\pi 5^2 \times 6 + \frac{4}{3}\pi 2^3 = 150\pi + \frac{32}{3}\pi = \frac{482}{3}\pi$$

Let height in cylinder = H

$$\pi 5^2 \mathrm{H} = \frac{482}{3}\pi$$

$$25\mathrm{H} = \frac{482}{3} \quad \mathrm{H} = 6{\cdot}4266$$

rise = 6·4266-6 = 0·4266
= 0·43cm to 2 dec. places

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect squaring and /or cubing
B3 Transposition error
B4 Using a value of π which affects accuracy of answer
B5 Incorrect substitution into correct formula
B6 Incorrect *r*
B7 Incorrect relevant volume formula
B8 Early rounding off which affects accuracy of answer

Slips (-1)
S1 Not rounding off to 2 dec places
S2 Arithmetic slips to a maximum of (*-3*)
S3 Leaving answer as 6 ·4266 or equivalent

Attempts (2 marks)
A1 Volume of either sphere or cylinder carried forward from (c)(i) or (c)(ii)
A2 Addition of volumes

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5,5,5,5) marks**	**Att 8 (2,2,2,2)**
Part (c)	**20 (5,5,5,5) marks**	**Att 8(2,2,2,2)**

Part (a) **10 marks** **Att 3**

(a) a (3, 6) and b (-1, 3) are two points.

✍ Find $|ab|$.

(a) **10 marks** **Att 3**

Formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|ab| = \sqrt{(3+1)^2 + (6-3)^2} = \sqrt{4^2 + 3^2} = \sqrt{25}$ or 5

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula and continues
B3 Switches both x and y in substitution

Slips (-1)
S1 Arithmetic errors

Attempts(3 marks)
A1 Correct formula with or without some substitution
A2 Subtracts the x's and /or y's

Part (b) **20 (5,5,5,5) marks** **Att 8(2,2,2,2)**

The line L: $3x - 5y + 15 = 0$ and the line M: $3x + 4y - 12 = 0$ cut the x-axis at the points c and d respectively.

(i) ✍ Find the coordinates of c and d.

(ii) ✍ Find e, the point of intersection of L and M.

(iii) ✍ Show the lines L and M on a coordinate diagram on graph paper.

(iv) ✍ Find the area of Δcde.

(b)(i) **5 marks** **Att 2**

L: $3x - 5y + 15 = 0$	M: $3x + 4y - 12 = 0$
$y = 0$	$y = 0$
$3x - 0 + 15 = 0$	$3x + 0 - 12 = 0$
$3x = -15$	$3x = 12$
$x = -5$	$x = 4$
c (-5,0)	d (4,0)

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Transposition error.
B3 Finds point where L cuts the x- axis only
B4 Finds point where M cuts x-axis only

Slips (-1)
S1 Finds where L(or M) cuts both axes but does not identify c (or d) or incorrectly identifies c(or d)
S2 Arithmetic slips to a maximum of (-3)

Misreadings (-1)
M1 Finds where both L and M intersect Yaxis

Attempts (2 marks)
A1 Some attempt at substitution of 0

(b)(ii) **5 marks** **Att 2**

M:	$3x + 4y - 12 = 0$		
L:	$3x - 5y + 15 = 0$		
	$9y - 27 = 0$	$3x + 4(3) - 12 = 0$	
	$9y = 27$	$3x + 12 - 12 = 0$	
	$y = 3$	$3x = 0$	
		$x = 0$	e(0,3)

* Note: Accept $(0,3) \in L$ and $(0,3) \in M$ shown in each case

Blunders (-3)

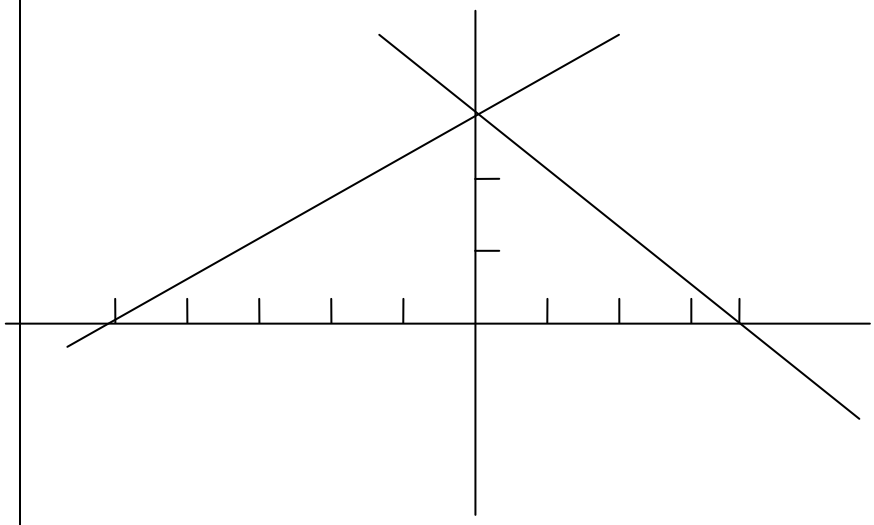
B1 Correct answer without work shown (✍)
B2 Transposition error

Slips (-1)
S1 Arithmetic slips to maximum (-3)
S2 Not finding second co-ordinate

Attempts (2marks)
A1 Any correct step and stops
A2 Graphical solution correct

Worthless(0)
W1 Graphical solution incorrect

(b)(iii) **5 marks** **Att 2**



$c(-5,0)$, $d(4,0)$, $e(0,3)$

* Accept candidate's perpendicular axes

Blunders(-3)

B1 Incorrect scale

B2 One line only sketched

Slips (-1)

Attempts (2 marks)

A1 One point only plotted

A2 Axes only drawn

(b)(iv) **5 marks** **Att 2**

Area = $\frac{1}{2} \cdot 9{\cdot}3 = \frac{27}{2}$ or $13{\cdot}5$

* Accept any valid method

Blunders (-3)

B1 Correct answer without work shown (✍)

B2 Incorrect relevant area formula

B3 Sum of areas of two smaller triangles not equal to area of required triangle

Slips (-1)

S1 Arithmetic errors to a maximum of (*-3*)

S2 Sum of areas of smaller triangles not found

Attempts (2 marks)

A1 Relevant area formula with some substitution

Part (c) **20(5,5,5,5) marks** **Att 8 (2,2,2,2)**

p is the point (2, -3) and q is the point (-2, 1).

(i) ✍ Find r, the midpoint of $[pq]$.

K is the line through r, perpendicular to $[pq]$.

(ii) ✍ Find the equation of K.

(iii) ✍ Show that s (3, 2) is on the line K.

(iv) ✍ Prove that the triangle Δpqs is isosceles.

(c)(i) **5 marks** **Att 2**

r: $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}) = (\frac{2-2}{2}, \frac{-3+1}{2}) = (\frac{0}{2}, \frac{-2}{2})$ or (0,-1)

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant midpoint formula and continues
B3 Mixes both x and y in substitution
B4 Finds one co-ordinate only

Slips (-1)
S1 Arithmetic errors

Attempts (2 marks)
A1 Writes midpoint formula with or without substitution

(c)(ii) **5 marks** **Att 2**

Slope $pq = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-1}{2--2} = \frac{-4}{4} = -1$

Slope $K = 1$ Equation K : $y - y_1 = m(x - x_1)$

$y - - 1 = 1(x-0)$

$y + 1 = x$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula and continues
B3 Switches both x and y in substitution

Slips (-1)

S1 Arithmetic errors
S2 Incorrect perpendicular slope
S3 Taking p or q instead of r for point on K

Attempts (2marks)

A1 Correct slope formula and/or line formula with or without some substitution
A2 Indicates product of perpendicular slopes equals -1

(c)(iii) **5 marks** **Att 2**

s (3, 2) on line K

$y + 1 = x \quad \Rightarrow \quad$ LHS: 2 + 1 = 3 = RHS

Blunders(-3)

B1 Mixes x and y in substitution
B2 Transposition error

Slips(-1)

S1 Arithmetic errors to maximum (*-3)*

Attempts(2 marks)

A1 Graphical solution correct

Worthless(0)

W1 Graphical solution incorrect

(c)(iv) **5 marks** **Att 2**

Formula : $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|sq| = \sqrt{(-2-3)^2 + (1-2)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1}$ or $\sqrt{26}$

$|sp| = \sqrt{(-2-3)^2 + (3-2)^2} = \sqrt{(-5)^2 + (1)^2} = \sqrt{25+1}$ or $\sqrt{26}$

Triangle $\Delta\, pqs$ is isosceles

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect relevant formula and continues
B3 Switches both *x* and *y* in substitution
B4 Substitutes correctly for *x* and *y* in each case but does not simplify
B5 $(-1)^2 \neq 1$

Slips (-1)

S1 $|sq| \neq |sp|$ without a conclusion
S2 Arithmetic errors to maximum (*-3*)

Attempts(2 marks)

A1 Correct formula with or without some substitution
A2 Incorrect relevant formula with some correct substitution

QUESTION 3

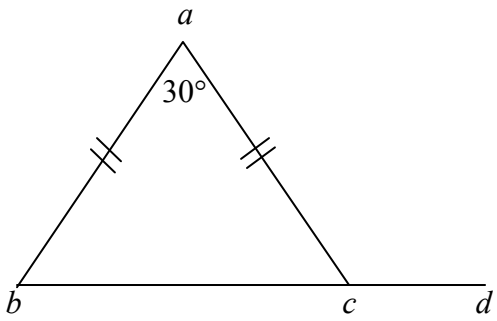
Part (a)	**15 marks**	**Att 5**
Part (b)	**25 (20,5) marks**	**Att 9(7,2)**
Part (c)	**10 (3,3,3,1) marks**	**Att 3(1,1,1)**

Part (a) **15 marks** **Att 5**

(a) abc is an isosceles triangle, with $|ab| = |ac|$

and $|\angle bac| = 30°$.

✍ Find $|\angle acd|$.



(a) **15 marks** **Att 5**

$|\angle acd| = |\angle bac| + |\angle abc|$ (exterior angle = sum of interior opposites)

$|\angle abc| = |\angle acb|$ (isosceles triangle)

$= \frac{1}{2}(180° - 30°) = 75°$

$|\angle acd| = 30° + 75° = 105°$ or $|\angle acb| = 75°$

$|\angle acd| = 180° - 75° = 105°$

* Note: Some or all steps may be indicated on diagram drawn by candidate

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Sum of angles in triangle ≠ 180
B3 $|\angle acb| + |\angle acd| \neq 180$

Slips (-1)

S1 Arithmetic slips to a maximum of (-*3*)

Attempts (5 marks)

A1 Diagram from examination paper drawn and equal angles indicated

Worthless (0)

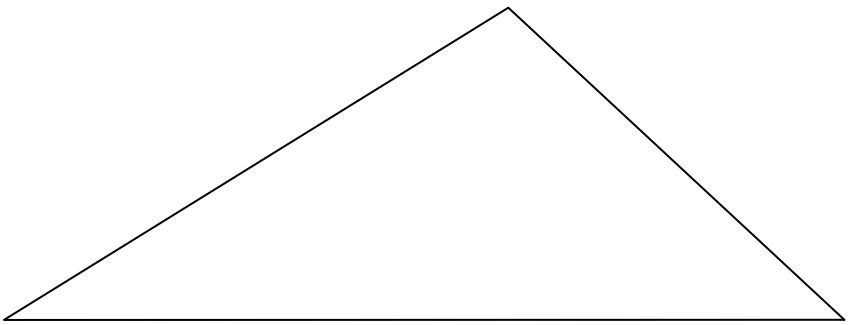
W1 Diagram from examination paper either partially or fully drawn

Part (b) **25 (20,5) marks** **Att 9(7,2)**

(b) (i) ✍ Construct a triangle of sides 11 cm, 8 cm and 6 cm.

(b) (ii) ✍ Prove that the measures of the three angles of a triangle sum to 180°.

(b)(i) **20 marks** **Att 7**



* Accept constructions with a tolerance of 2 mm

Blunders(-3)

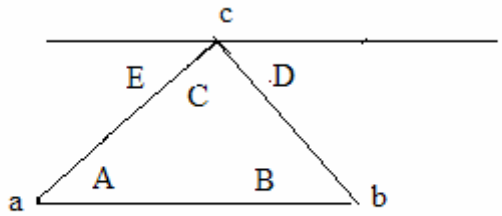
B1 Each incorrect side

B2 Constructing right angle between two sides

Attempts(7 marks)

A1 Any one correct side drawn

(b)(ii) **5 marks** **Att 2**



Given: Δ*abc*
To prove : $|\angle A| + |\angle B| + |\angle C| = 180^0$
Construction: Through *c* draw a line parallel to [*ab*]
Mark the angles E and D step 1
Proof: $|\angle A| = |\angle E|$ alternates
$|\angle B| = |\angle D|$ alternates step 2
$|\angle C| + |\angle A| + |\angle B| = |\angle C| + |\angle E| + |\angle D|$
But $|\angle C| + |\angle E| + |\angle D| = 180^0$ step 3
$\Rightarrow |\angle A| + |\angle B| + |\angle C| = 180^0$

Blunders (-3)
B1 Any step incorrect

Attempts (2 marks)
A1 Triangle with vertices or angles indicated

Worthless (0)
W1 Wrong theorem

Part (c) **10 (3,3,3,1) marks** **Att 3(1,1,1)**

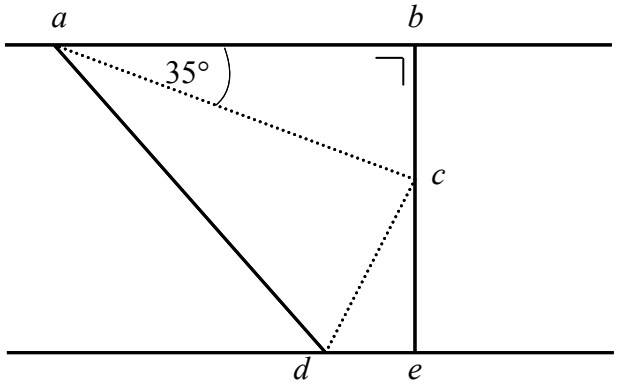
(c) *ab* is parallel to *de*,

ac bisects $\angle bad$,

dc bisects $\angle ade$,

be is perpendicular to *ab* and

$|\angle bac| = 35°$.



(i) ✍ Find $|\angle ade|$.

(ii) ✍ Find $|\angle acd|$.

(iii) ✍ Prove that the triangles *adc*, *abc* and *cde* are equiangular.

(iv) ✍ Given that $|ab| = 5$ and $|bc| = 3{\cdot}5$, write $|de| : |ec|$ in the form $m : n$,

where $m, n \in \mathrm{N}$.

(c)(i) **3 marks** **Att 1**

$|\angle bad| = 35° + 35° = 70°$

$|\angle ade| = 180° - 70° = 110°$

or taking quadrilateral *abed*

$|\angle bad| + |\angle ade| + |\angle abe| + |\angle bed| = 360°$

$70° + |\angle ade| + 90° + 90° = 360°$

$|\angle ade| = 110°$

* Note: Any blunder results in an attempt mark of 1.

*Blunders (see * above)*

B1 Correct answer without work shown (✍)

B2 Sum of angles on straight line ≠ 180°

B3 Sum of angles in quadrilateral ≠ 360°

Slips (-1)

S1 Arithmetic slip(Max 2)

Attempts (1 mark)

A1 Measure of any correct relevant angle indicated

Worthless (0)

W1 Diagram from examination paper either partially or fully reproduced

(c)(ii) **3 marks** **Att 1**

$|\angle adc| = |\angle cde| = \frac{1}{2}(110°) = 55°$

$|\angle acd| = 180° - (35° + 55°) = 180° - 90° = 90°$

* Note: Any blunder results in an attempt mark of 1.

* Some or all steps may be indicated on candidate's diagram

*Blunders (See 1st * above)*

B1 Correct answer without work shown (✍)

B2 Sum of angles in $\Delta\, adc \neq 180°$

B3 Sum of angles on line $\neq 180°$

Slips (-1)

S1 Arithmetic slip (Max 2)

Worthless (0)

W1 Diagram from examination paper either partially or fully reproduced

(c)(iii) **3 marks** **Att 1**

Measures of angles in $\Delta\, adc$ are 90°, 55°, 35°

Measures of angles in $\Delta\, abc$ are 90°, 35°, with remaining angle 55° (sum =180°)

Measures of angles in $\Delta\, dce$ are 90° (given), $|\angle cde| = 55° \Rightarrow |\angle dce| = 35°$

$\Rightarrow$ triangles equiangular

* Note: Any blunder results in an attempt mark of 1.

* Some steps may be indicated on candidate's diagram

*Blunders (See 1st * above)*

B1 Sum of angles in any triangle $\neq 180$

Slips (-1)

S1 Arithmetic slip(Max 2)

S2 Showing a pair of the triangles are equiangular.

(c)(iv) **1 mark** **hit or miss**

de side opposite 35°

ce side opposite 55°

$|bc| : |ab| = |de| : |ec| = 3{\cdot}5 : 5 = 7:10$

QUESTION 4

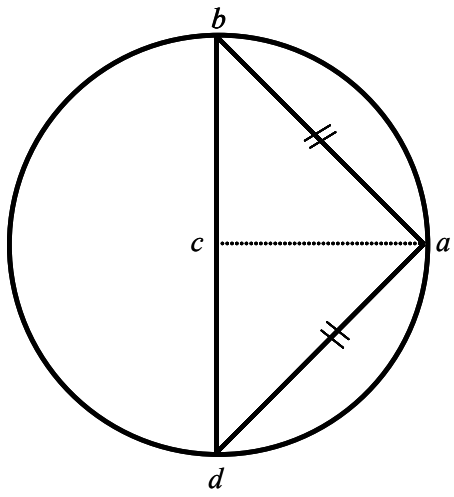
Part (a)	**20 (15,5) marks**	**Att 7(5,2)**
Part (b)	**20 (15,5) marks**	**Att 7(5,2)**
Part (c)	**10 (5,5) marks**	**Att 4(2,2)**

Part (a) **20(15,5) marks** **Att 7(5,2)**

(a) [*bd*] is the diameter of the circle, *c* is the centre of the circle and $|ba| = |ad|$.

Find (i) ✍ $|\angle adb|$,

(ii) ✍ $|\angle dac|$.



(a)(i) **15 marks** **Att 5**

$|\angle bad| = 90°$ (angle in semi-circle)
$|\angle abd| = |\angle adb|$ (isosceles triangle)
$|\angle abd| + |\angle adb| = 90°$
$|\angle adb| = 45°$

* Some or all steps may be indicated on candidate's diagram

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Sum of measure of angles in triangle ≠ 180°

Slips (-1)

S1 Arithmetic slips to maximum of (-*3*)

Attempts (5 marks)

A1 Angle at arc in semi-circle indicated as right angle
A2 Correct angles indicated in isosceles triangle but value not found

Worthless (0)

W1 Diagram from examination paper reproduced either partially or fully

(a)(ii) **5 marks** **Att 2**

$|ac| = |cd|$ (radii)
$|\angle cda| = |\angle dac|$
$|\angle dac| = 45°$ since $|\angle cda| = 45°$

* Some or all steps may be indicated on candidate's diagram

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Sum of angles in a triangle ≠ 180°

Slips (-1)
S1 Arithmetic slips to maximum of (-*3*)

Attempts (2 marks)
A1 $|ac| = |cd|$ indicated

Worthless (0)
W1 Diagram from examination paper reproduced either partially or fully

Part (b) **20 (15,5) marks** **Att 7(5,2)**

(b) (i) ✍ Prove that a line through the centre

of a circle perpendicular to a chord

bisects the chord.

(b) (ii)

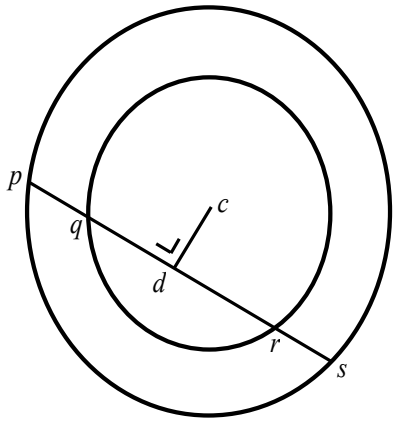
c is the centre of both circles.

[*ps*] is a chord of the larger circle.

[*ps*] intersects the smaller circle

at *q* and *r*.
cd is perpendicular to *ps*.

✍ Prove |*pq*| = |*rs*|.



(b)(i) **15 marks** **Att 5**

Given: Circle C, centre c on D, with chord $ab \perp$ D, and $ab \cap$ D = {p}

Construction: Join ca and cb step 1

To Prove : $|ap| = |bp|$

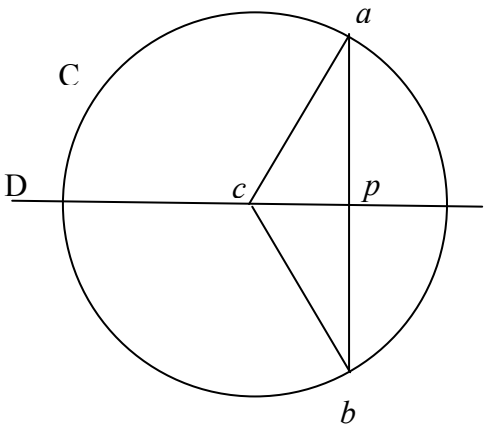
Proof: $|ca| = |cb|$ (radii) step 2

$|\angle cpa| = |\angle cpb|$ (right angles) step3

$|cp| = |cp|$

$\Rightarrow$RHS $\Rightarrow \Delta cap$ and Δcpb congruent step 4

$\Rightarrow |ap| = |bp|$ step5



or $|ca| = |cb|$ (radii)

$\Rightarrow |\angle cap| = |\angle cbp|$ (isosceles triangle) step2

$|\angle cpa| = |\angle cpb|$ (right angles)

$\Rightarrow |\angle acp| = |\angle$bcp$|$ step 3

$\Rightarrow$ASA $\Rightarrow \Delta cap$ and Δcpb congruent step 4

$\Rightarrow |ap| = |bp|$ step5

or $|ca| = |cb|$ (radii)

$\Rightarrow |\angle cap| = |\angle cbp|$ (isosceles triangle) step 2

$|\angle cpa| = |\angle cpb|$ (right angles)

$\Rightarrow |\angle acp| = |\angle bcp|$ (sum of angles in triangle =180) step3

$\Rightarrow |cp| = |cp|$

$\Rightarrow$ SAS $\Rightarrow \Delta cap$ and Δcpb congruent step 4

$\Rightarrow |ap| = |bp|$ step5

* Some steps may be indicated on diagram

* Accept any other valid proofs

Blunders(-3)

B1 Each step incorrect or omitted

B2 Each step incomplete

Attempts(5marks)

A1 Diagram with circle drawn, and diameter or chord indicated

Worthless(0)

W1 Wrong Theorem

W2 Circle and nothing else

(b)(ii) **5 marks** **Att 2**

$|dp| = |ds|$ since cd perpendicular bisector of [ps]
$|dp| = |dq| + |pq|$ and $|ds| = |dr| + |rs|$
But $|dq| = |dr|$ (theorem)
$|pq| = |rs|$

Blunders (-3)
B1 $|dp| \neq |ds|$
B2 $|dq| \neq |dr|$
B3 $|qd|$ equals radius or equivalent

Slips (-1)
S1 Arithmetic slips to maximum of (-*3*)

Attempts (2 marks)
A1 $|dp| = |ds|$ indicated
A2 $|dq| = |dr|$ indicated
A3 Steps towards showing Δcpd and Δcsd congruent
A4 q as midpoint of [pd] or equivalent

Worthless (0)
W1 Diagram from examination paper reproduced either partially or fully

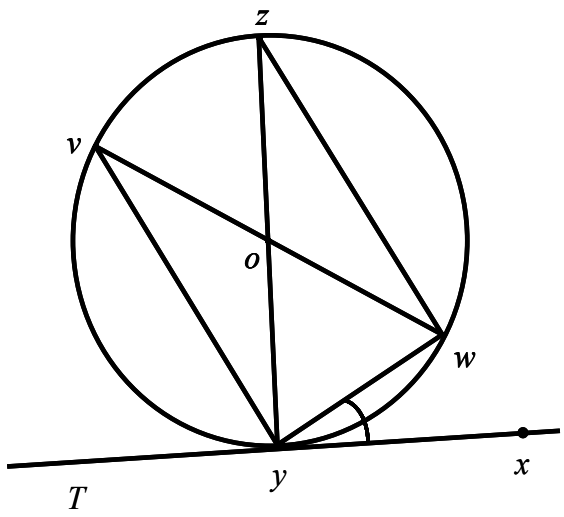
Part (c) **10 (5,5) marks** **Att 4(2,2)**

(c) T is a tangent to the circle and o is the centre of the circle.

$|\angle xyw| = 40°$.

(i) ✍ Find $|\angle wvy|$.

(ii) ✍ Using congruent triangles or otherwise, prove $|zw| = |vy|$



(c)(i) **5 marks** **Att 2**

$|\angle zyw| = 50°$ since diameter zy perpendicular to T
$|\angle ywv| = 50°$ since $|yo| = |wo|$ (radii)
$|\angle yow| = 180° - (50° + 50°) = 80°$
But $|\angle yow| = 2|\angle wvy|$
$|\angle wvy| = 40°$

or taking Δyvw
$|\angle wvy| = 180° - |\angle vyw| + |\angle ywv|$
$= 180° - (90° + 50°)$
$= 40°$

* Some or all steps may be indicated on candidate's diagram.

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 $|\angle zyx| \neq 90°$
B3 Sum of angles in a triangle $\neq 180°$
B4 $|\angle zyw| \neq |\angle ywv|$
B5 $|\angle yow| \neq 2|\angle wvy|$

Slips (-1)

S1 Arithmetic slips to a maximum of *(-3)*

Attempts (2 marks)

A1 $|\angle zyw| = 50°$
A2 $|yo| = |wo|$ or equivalent

Worthless (0)

W1 Diagram from examination paper reproduced either partially or fully
W2 Angles at centre of circle indicated as right angles

(c)(ii) **5 marks** **Att 2**

Congruent triangles:

Taking Δywv *and* Δywz or in Δvoy and Δzow

$|\angle wyv| = |\angle ywz|$ (both right angles) $|ov| = |oz|$ (radii) step 1

$[wy]$ common to both triangles $|oy| = |ow|$

[wv] and [yz] hypothenuse in each case $|\angle voy| = |\angle zow|$ (vertically opposite) step 2

$\Rightarrow$ RHS $\Rightarrow$ congruent triangles $\Rightarrow$ SAS $\Rightarrow$ congruent triangles

$\Rightarrow |zw| = |vy|$ $\Rightarrow |zw| = ||vy|$ step 3

* Note: Also possible to show Δvoy and Δzow congruent by ASA

Otherwise: Taking Δywv and Δyzw

$$|wv|^2 = |wy|^2 + |vy|^2$$

$$|zy|^2 = |wy|^2 + |zw|^2 \quad \text{step1}$$

but $|wv|^2 = |zy|^2$ since both diameters step 2

$$\Rightarrow |wy|^2 + |vy|^2 = |wy|^2 + |zw|^2$$

$$\Rightarrow |vy|^2 = |zw|^2 \Rightarrow |vy| = |zw| \quad \text{step3}$$

Blunders (-3)

B1 Any step incorrect or omitted

B2 Incorrect identification of hypotenuse

Slips (-1)

Attempts (2 marks)

A1 Indicates pair of sides or pair of angles relevant to proving congruence

A2 Sum of angles in a triangle = 180°

Worthless (0)

W1 Numerical values given to $|vy|$ and $|zw|$ from measurement on examination paper

QUESTION 5

Part (a)	**5 marks**	**Att 2**
Part (b)	**35 (15,20) marks**	**Att 12(5,7)**
Part (c)	**10 (3,3,3,1) marks**	**Att 3(1,1,1)**

Part (a) **5 marks** **Att 2**

✍ Given that $\tan A = 4$, write $\cos A$ in the form $\frac{1}{\sqrt{x}}$, $x \in \mathrm{N}$.

(a) **5 marks** **Att 2**

$$\text{Tan } A = 4 = \frac{4}{1} = \frac{opp}{adj}$$

Let *hypotenuse* = h

$$h^2 = 4^2 + 1^2 = 17 \Rightarrow h = \sqrt{17}$$

$$\text{Cos } A = \frac{adj}{hyp} = \frac{1}{\sqrt{17}}$$

Blunders(-3)

B1 Correct answer without work shown (✍)
B2 Incorrect ratio for *Tan* function
B3 Pythagoras incorrect
B4 Incorrect squaring
B5 Incorrect ratio for *Cos* function

Slips(-1)
S1 Arithmetic slips

Attempts (2 marks)
A1 *Tan* function or *Cos* function ratio correct
A2 Pythagoras indicated

Part (b) **35 (15,20) marks** **Att 12(5,7)**

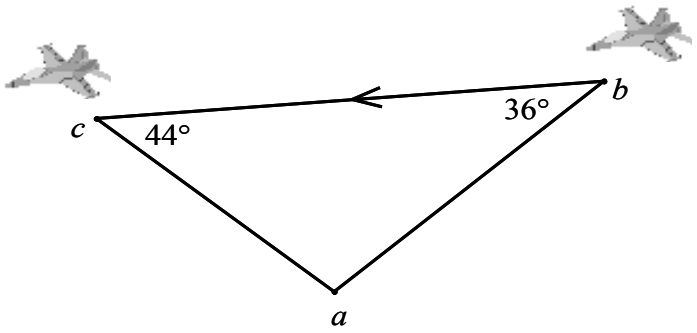
b and *c* are two airports as shown.

When airport *b* is viewed from *a*,

$|\angle abc| = 36°$.

When airport *c* is viewed from *a*,

$|\angle acb| = 44°$.



It takes a plane 25 minutes travelling at a speed of 384 km/h to go from airport *b* to airport *c*.

Find **(i)** ✍ the distance between both airports, i.e. |*bc*|,

(ii) ✍ the distance airport *c* is from point *a*, i.e. |*ac*|, correct to the nearest km.

(b)(i) **15 marks** **Att 5**

$$\frac{25}{60}(384) = \frac{5}{12}(384) = 160 \text{ km}$$

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Early rounding off which affects accuracy of answer

Slips (-1)

S1 Arithmetic errors to a maximum of (-*3*)
S2 Mishandles converting 25 minutes in terms of hours

Attempts (5 marks)

A1 Use of 25 and 60

(b)(ii) **20 marks** **Att 7**

$|\angle cab| = 180° - (36° + 44°) = 100°$

$$\frac{Sin100^0}{160} = \frac{Sin36^0}{|ac|} \Rightarrow \frac{160Sin36^0}{Sin100^0} = \frac{160(0.587785)}{0.9848} = 95{\cdot}497$$

95 km, to nearest km.

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Incorrect ratio in use of Sine Rule
B3 Error in cross multiplication
B4 Reads wrong page of tables or uses calculator in incorrect mode
B5 Early rounding off which affects the answer
B6 Sum of angles in triangle ≠ 180°

Slips (-1)

S1 Arithmetic slips to maximum (*-3*)
S2 Answer not to nearest km.

Misreadings (-1)

M1 Calculates $|ab|$

Attempts (7 marks)

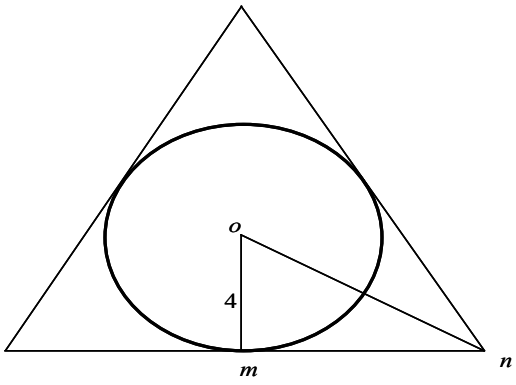
A1 Sine Rule with some substitution
A2 Uses $|\angle cab| = 90°$ and continues

Worthless (0)

W1 Treats triangle as right angled
W2 $\frac{100}{160} = \frac{36}{|ac|}$ or equivalent

Part (c) **10 (3,3,3,1) marks** **Att 3(1,1,1)**

(c) The diagram shows an equilateral triangle and the incircle of the triangle with centre *o*.



(i) ✍ Given that $|om| = 4$, find $|mn|$, giving your answer in surd form.

(ii) ✍ Find $|on|$.

(iii) Write down the height of the equilateral triangle.

(iv) ✍ Calculate the area of the equilateral triangle, giving your answer in surd form.

(c)(i) **3 marks** **Att 1**

$$Tan\ 30° = \frac{|om|}{|mn|} = \frac{4}{|mn|} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{|mn|} \Rightarrow |mn| = 4\sqrt{3} \quad \text{or} \quad \frac{\sqrt{3}}{3} = \frac{4}{|mn|} \Rightarrow |mn| = \frac{12}{\sqrt{3}}$$

* Note: Any blunder results in an attempt mark of 1.

*Blunders (See * above)*

B1 Correct answer without work shown (✍)
B2 Incorrect ratio for *Tan* function
B3 Error in cross multiplication
B4 Reads from page in tables not relevant to *Tan* function or uses calculator in incorrect mode

Slips (-1)

S1 Arithmetic slip (Max 2)
S2 Answer not in surd form

Attempts (1 mark)

A1 Indicates use of 4 in a relevant ratio

(c)(ii) **3 marks** **Att 1**

$|on|^2 = 4^2 + (4\sqrt{3})^2 = 16 + 48 = 64 \Rightarrow |on| = 8$ or $Sin30° = \frac{4}{|on|} \Rightarrow \frac{1}{2} = \frac{4}{|on|}$

$\Rightarrow \quad |on| = 8$

* Note: Any blunder results in an attempt mark of 1.
* Accept candidate's value from (c)(i)

*Blunders (See 1st * above)*

B1 Correct answer without work shown (✍)
B2 Pythagoras incorrect
B3 Incorrect squaring
B4 $|on|^2 = 64$ and stops
B5 Incorrect ratio for *Sine* function
B6 Reads wrong page of tables or uses calculator in incorrect mode
B7 Error in cross multiplication

Slips (-1)
S1 Arithmetic slip(Max 2).

Attempts (1 mark)
A1 Pythagoras indicated
A2 Sine Rule with some substitution

(c)(iii) **3 marks** **Att 1**

12 (i.e. 8+4).

* Note: Any blunder results in an attempt mark of 1.
* Accept candidate's value for $|on|$

*Blunders (See 1st * above)*
B1 Shows incorrect operator e.g. $|on| - 4$ instead of $|on| + 4$

Attempts (1 mark)
A2 Indicates some use of 4 or 8

(c)(iv) **1 mark** **hit or miss**

Area = ½ base × perpendicular height = $4\sqrt{3} \times 12 = 48\sqrt{3}$
or area = 6 × (Area Δ *mon*) = 6 × (½ × $4\sqrt{3}$ × 4) = $48\sqrt{3}$

QUESTION 6

Part (a)	**15 marks**	**Att 5**
Part (b)	**15 (5,10)marks**	**Att 5(2,3)**
Part (c)	**20 (5,5,5,5) marks**	**Att 8(2,2,2,2)**

Part (a) **15 marks** **Att 5**

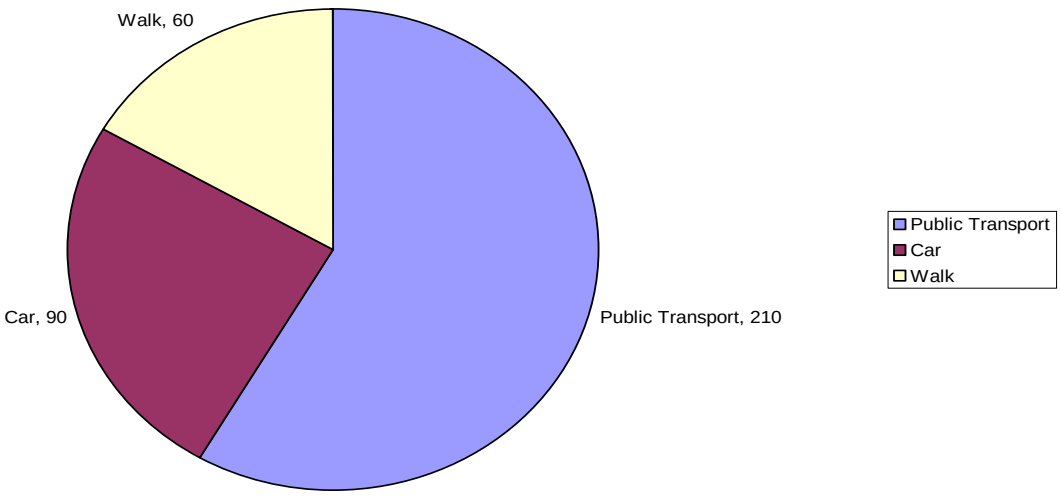
60 people were asked how they travelled to work. The following table is a summary of the results:

Type of transport	Public Transport	Car	Walk
No. of people	35	15	10

✍ Draw a pie chart to illustrate the above information.

(a) **15 marks** **Att 5**

	Public Transport	Car	Walk
	35	15	10
Pie Chart Angles	210°	90°	60°



Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Sum of angles ≠ 360°
B3 Divisor other than 60
B4 Incorrect plotting

Slips (-1)
S1 Arithmetic slips to maximum of (*-3*)

Atempts (5 marks)
A1 Use of 360 indicated or implied
A2 Circle drawn

Worthless (0)
W1 Bar Chart

Part (b) **15 (5,10) marks** **Att 5(2,3)**

A professional golfer plays 50 rounds of golf over a season. The following were the number of shots taken in each round:

69	66	70	70	71	70	68	71	76	72
69	74	75	73	77	70	73	74	66	74
69	74	74	70	75	73	69	76	80	72
73	69	79	72	69	74	79	73	77	72
69	67	70	69	68	70	70	71	68	66

(i) ✍ Complete the following frequency table.

No. shots per round	66 – 69	69 – 72	72 – 75	75 – 81
Number of rounds				

[Note: 66 – 69 means 66 or more but less than 69, etc.]

(ii) ✍ Using mid interval values, calculate the mean number of shots per round, giving your answer correct to the nearest whole number.

(b)(i) **5 marks** **Att 2**

No. shots per round	66 – 69	69 – 72	72 – 75	75 – 81
Number of rounds	7	19	15	9

Blunders (-3)
B1 Omits any number (frequencies do not sum to 50)
B2 Cumulative frequencies

Slips (-1)
S1 Arithmetic errors

Attempts (2 marks)
A1 Any one value filled in correctly into table

Worthless (0)
W1 Copies table and stops without making any further entries

(b)(ii) **10 marks** **Att 3**

$$\text{Mean} = \frac{7(67{\cdot}5) + 19(70{\cdot}5) + 15(73{\cdot}5) + 9(78)}{50}$$

$$= \frac{472{\cdot}5 + 1339{\cdot}5 + 1102{\cdot}5 + 702}{50}$$

$$= \frac{3616{\cdot}5}{50}$$

$$= 72{\cdot}33$$

= 72 to nearest whole number

* Accept candidates work from (b)(i)
Blunders (-3)
B1 Correct answer without work shown (✍)
B2 Consistent incorrect mid interval value
B3 Division by 4
B4 Division by sum of mid intervals
B5 Consistently adds interval value to frequency instead of multiplying

Slips (-1)
S1 Arithmetic slips to maximum (-3)

Attempts (3 marks)
A1 One correct multiplication in numerator
A2 Indicates division by 50
A3 One correct midinterval
Worthless (0)
W1 Sum of frequencies divided by 4

Part (c) **20 (5,5,5,5) marks** **Att 8(2,2,2,2)**

(c) At a Garda checkpoint, the speed of 100 vehicles passing was recorded. The following were the results:

Speed in km/h	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of cars	8	24	40	18	10

[Note: 20 – 40 means 20 or more but less than 40, etc.]

(i) Construct the cumulative frequency table.

(ii) On graph paper construct the ogive.

(iii) ✍ Use your graph to estimate the median.

(iv) ✍ Use your graph to estimate the number of vehicles with a speed of at least 70 km/h.

(c)(i) **5 marks** **Att 2**

Speed in km/h	<20	<40	<60	<80	<100
No. of cars	8	32	72	90	100

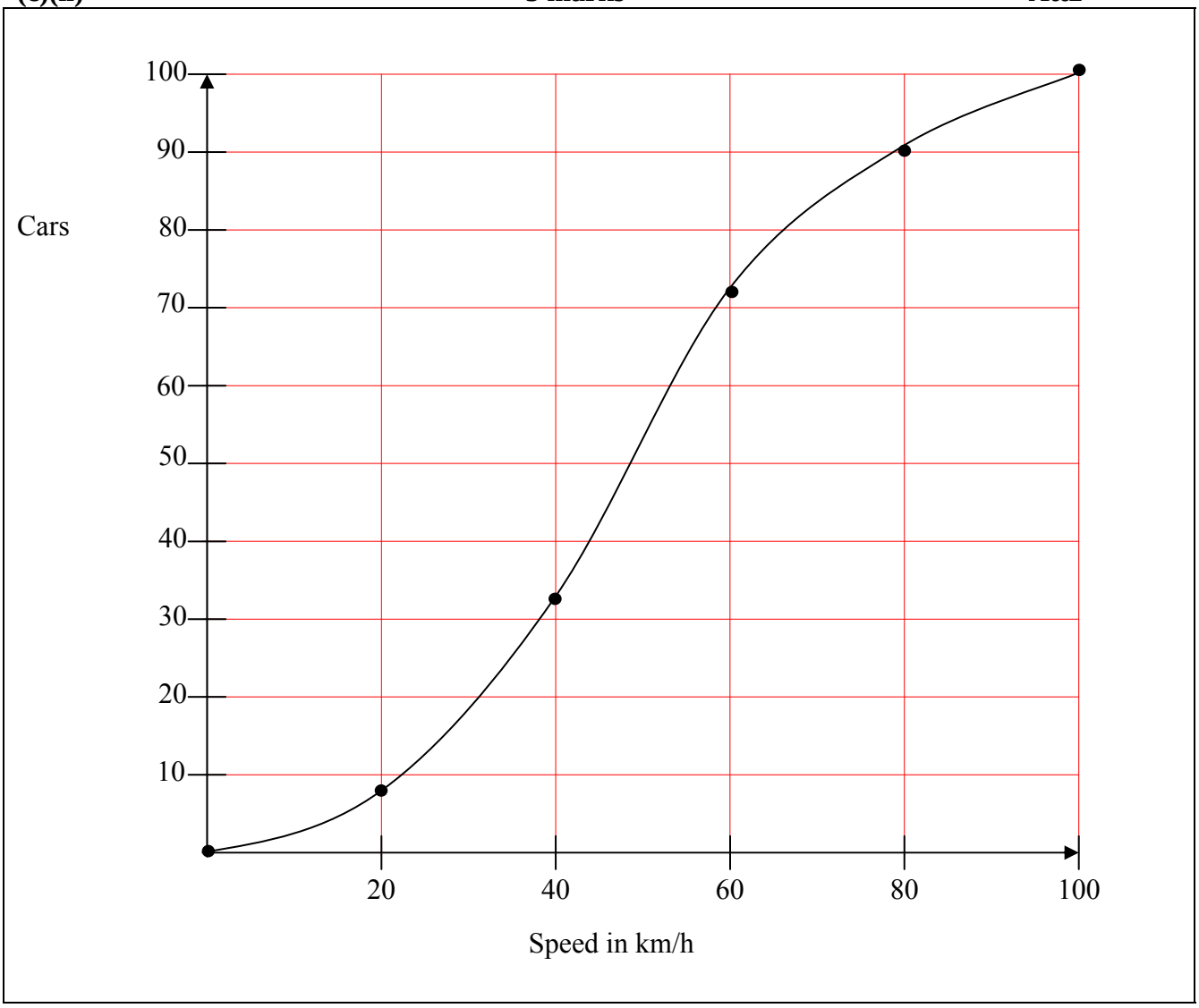
Blunders (-3)
B1 Omits any number (sum ≠100)

Slips (-1)
S1 Arithmetic slips to maximum (*-3*)

Attempts (2 marks)
A1 Any one value filled in correctly into table
A2 Any indication of addition of frequencies

Worthless (0)
W1 Copies table and stops

(c)(ii) **5 marks** **Att2**



Blunders (-3)
B1 Incorrect scales
B2 Plots points but not joined
B3 Draws a 'cumulative' histogram
B4 Draws a 'cumulative' cumulative ogive

Slips (-1)
S1 Each incorrect plot
S2 Each point omitted

Attempts (2 marks)
A1 Draws scaled axes and stops

(c)(iii) **5 marks** **Att 2**

Median = 49

* Accept median consistent with candidate's work

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Takes 'median' from horizontal axis
B3 Line drawn from incorrect starting point of correct axis for median
B4 Work for median correct but not clearly marked

Attempts (2marks)
A1 Draws line from 50^{th} frequency to ogive
A2 Indicates use of 50

(c)(iv) **5 marks** **Att 2**

No of vehicles with a speed of less than 70 km/hr = 82 (using graph) No. of vehicles with speed greater than 70 km/hr = 100 – 82= 18

* Accept answer consistent with candidate's work

Blunders (-3)

B1 Correct answer without work shown (✍)
B2 Number of vehicles with speed of less than 70 km/hr

Attempts (2marks)
A1 Graphical indication of use of 70 km/hr