Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION 2020

APPLIED MATHEMATICS – HIGHER LEVEL

2 HOURS 30 MINUTES

Six questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be $9{\cdot}8$ m s^{-2}.

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

Diagrams are generally not drawn to scale.

1. **(a)** A car is travelling on a straight level road at a uniform speed of 26 m s^{-1} when the driver notices a tractor 91·2 m ahead.
The tractor is travelling at a uniform speed of 6 m s^{-1} in the same direction as the car.
The driver of the car hesitates for t seconds before applying the brake.
The maximum deceleration of the car is 5 m s^{-2}.

Find the maximum value of t which would avoid a collision between the car and the tractor.

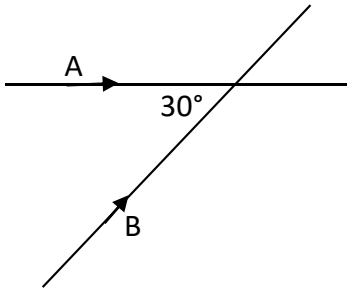
(b) A 60 gram mass is projected vertically upwards with an initial speed of 15 m s^{-1} and half a second later a 40 gram mass is projected vertically upwards from the same point with an initial speed of 22·65 m s^{-1}.

(i) Calculate the height at which the masses will collide.

The masses coalesce on colliding.

(ii) Find the greatest height which the combined mass will reach.

2. **(a)** Two straight roads intersect at an angle of 30°.
Car A is moving along one road towards the intersection with a uniform speed of 6 m s^{-1}.
Car B is moving along the other road towards the intersection with a uniform speed of 8 m s^{-1}.

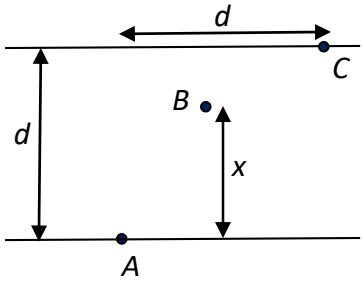


(i) Find the velocity of B relative to A.

A reaches the intersection 5 seconds before B.

(ii) Find the distance of each car from the intersection when they are nearest to each other.

(b) A woman can swim at $\frac{5}{8}$ m s^{-1} in still water.
She swims across a river of width d.
She sets out at right angles to the bank.
The current flows parallel to the straight banks.
It flows at $\frac{1}{2}$ m s^{-1} for the first part of her journey and at 1 m s^{-1} for the remainder of her journey.
She starts from point A, the current changes when she reaches point B and she lands at point C.



She finds that when she has reached the other side, she has drifted downstream a distance equal to the width of the river.
B is a distance x from the bank, as shown in the diagram.

Find x in terms of d.

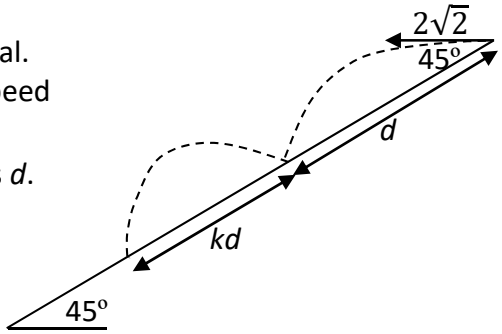
3. **(a)** A particle is projected from a point P with speed u m s^{-1} at an angle α to the horizontal.

(i) Show that the range of the particle is $\frac{2u^2 \sin\alpha \cos\alpha}{g}$.

The particle is 24·5 m above the horizontal ground after 5 seconds and it strikes the ground 235·2 m from P.

(ii) Find the value of u.

(b) A plane is inclined at an angle of 45° to the horizontal.
A particle is projected down the plane with initial speed $2\sqrt{2}$ m s^{-1} at an angle of 45° to the inclined plane.
The range along the inclined plane of the first hop is d.



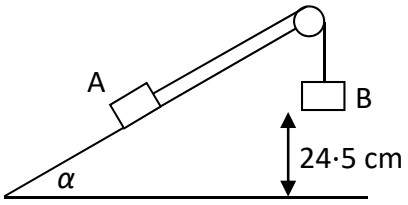
The plane of projection is vertical and contains the line of greatest slope.

(i) Find the value of d.

The coefficient of restitution between the particle and the inclined plane is 0·4.
The range of the second hop is kd.

(ii) Find the value of k.

4. **(a)** A block A of mass 10m on a smooth plane inclined at an angle α with the horizontal, where $\tan\alpha = \frac{3}{4}$, is connected by a light inextensible string which passes over a smooth pulley to a second block B of mass 10m. B is 24·5 cm above an inelastic horizontal floor, as shown in the diagram.



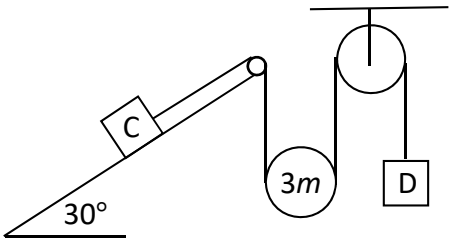
The system is released from rest.

Find

(i) the acceleration of B

(ii) the time that B remains in contact with the floor.

(b) A particle C of mass 2m rests on a rough plane which is inclined at 30° to the horizontal.
The coefficient of friction between C and the plane is $\frac{\sqrt{3}}{21}$. A light inextensible string which passes under a smooth movable pulley of mass 3m connects C to a particle D of mass m, as shown in the diagram.

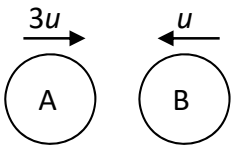


The system is released from rest. C moves up the plane.

(i) Show, on separate diagrams, the forces acting on the moveable pulley and on each of the masses.

(ii) Find in terms of m the tension in the string.

5. **(a)** A smooth sphere A of mass m, moving with speed $3u$ on a smooth horizontal table collides directly with a smooth sphere B of mass $2m$, moving in the opposite direction with speed u.
The directions of motion of A and B are reversed by the collision.



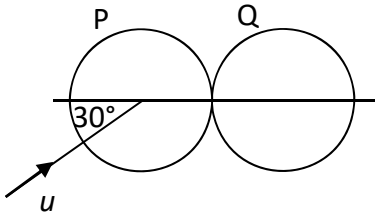
The coefficient of restitution between A and B is e.

(i) Find the speed, in terms of u and e, of each sphere after the collision.

Subsequently B hits a wall at right angles to the line of motion of A and B.
The coefficient of restitution between B and the wall is $\frac{1}{2}$.
After B rebounds from the wall there is a further collision between A and B.

(ii) Show that $\frac{1}{8} < e < \frac{1}{4}$.

(b) A smooth sphere P has mass m_1 and speed u.
It collides obliquely with a smooth sphere Q, of mass m_2, which is at rest.
Before the collision the direction of P makes an angle of 30° to the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is e.

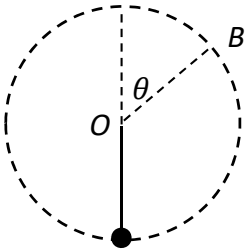
Prove that P will turn through a right-angle if $4m_1 = (3e - 1)m_2$.

6. **(a)** A particle D of mass m is suspended from a fixed point by a light elastic string of natural length ℓ and elastic constant $\frac{6mg}{\ell}$.
Initially D rests in equilibrium with the string vertical.
The particle is now pulled down a vertical distance $\frac{1}{3}\ell$ below its equilibrium position and released from rest.

(i) Show that D moves with simple harmonic motion.

(ii) In terms of ℓ, find the greatest speed of D while the string remains taut.

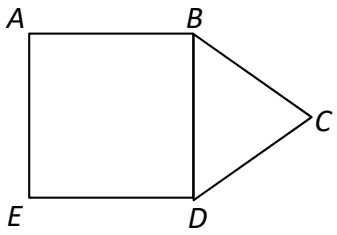
(b) A particle P is attached to one end of a light inextensible string of length d.
The other end of the string is attached to a fixed point O.
The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed $\sqrt{3gd}$.
The particle moves in a vertical circle.
The string becomes slack when P is at the point B.
OB makes an angle θ with the upward vertical.



(i) Show that $\cos\theta = \frac{1}{3}$.

(ii) In terms of d, find the greatest height of P above B in the subsequent motion.

7. **(a)** A uniform lamina *ABCDE* is formed by joining square *ABDE* with triangle *BCD*, as shown in the diagram.
BCD is an isosceles triangle.



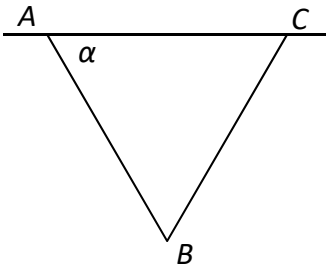
$|AB| = 24$ cm and $|BC| = 20$ cm.

(i) Find the distance of the centre of gravity of the lamina from *AE*.

The lamina is freely suspended from *B* and hangs in equilibrium.

(ii) Find the angle which *BD* makes with the vertical.

(b) Two uniform rods *AB* and *BC*, of equal length, and of weights $2W$ and W respectively are smoothly hinged at *B*.



A and *C* are hinged to a horizontal beam.

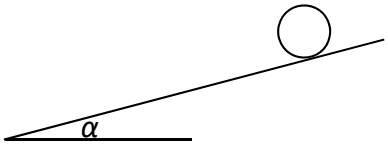
The rods are in a vertical plane with *B* below *AC*.

$|\angle BAC| = \alpha$.

Find the horizontal and vertical components of the reaction of the hinge at *B* on the rod *AB*.

8. **(a)** Prove that the moment of inertia of a uniform disc, of mass *m* and radius *r* about an axis through its centre, perpendicular to its plane, is $\frac{1}{2}mr^2$.

(b) A wheel consists of a uniform circular disc of radius *r*.
The wheel rolls down a rough inclined plane without slipping.
The plane is inclined at an angle α to the horizontal.



(i) Show that the acceleration of the wheel is $\frac{2}{3}g\sin\alpha$.

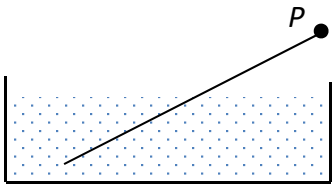
The coefficient of friction between the wheel and the inclined plane is 0·2.

(ii) Find the maximum value of α.

9. **(a)** A U-tube whose limbs are vertical and of equal length contains mercury of relative density 13·6.
The surface of the mercury is 15 cm from the top of each limb.
The cross-sectional area of the U-tube is 10 cm^2.
100 cm^3 of oil, of relative density 0·68, is poured into one limb. The surface of the oil is x cm from the top of the limb.

Find the value of x.

(b) A uniform rod, of length ℓ and weight W, is freely hinged at the point P.
The rod is free to move about a horizontal axis through P.
The other end of the rod is immersed in water.
The relative density of the rod is 0·64.
The rod is in equilibrium and is inclined as shown in the diagram.



Find

(i) the length of the immersed part of the rod in terms of ℓ

(ii) the reaction at the hinge in terms of W.

[Density of water = 1000 kg m^{-3}]

10. **(a)** One method of dyeing a piece of cloth is to immerse it in a container which has P grams of dye dissolved in a fixed volume of water.

The cloth absorbs the dye at a rate proportional to the mass of dye remaining.

$$\frac{dx}{dt} = k(P - x)$$

where t is time in seconds, x is the mass of dye absorbed by the cloth and $k = \frac{1}{50}$.

(i) Find the time taken to dye a piece of cloth if a mass of $\frac{5}{8}P$ needs to be absorbed to reach the desired colour.
(Note: $\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a + bx| + c$)

An alternative method is to keep the mass of dye present in the water constant at P grams by continuously adding dye throughout the process.

(ii) Find the time taken to dye the piece of cloth to the desired colour using this method.

(b) A particle P travelling in a straight line has a deceleration of $4v^{n+1}$ m s^{-2}, where n (> 0) is a constant and v is its speed at time t (> 0).

P has an initial speed of u.

(i) Find an expression for v in terms of u, n and t.

(ii) When n = 3 obtain an expression for the speed of P when it has travelled a distance of 3 m from its initial position.

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Leaving Certificate Examination – Higher Level

Applied Mathematics

2 hours 30 minutes