



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2011

Marking Scheme

**Mathematics
(Project Maths)**

Higher Level

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Introduction

The Higher Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared some content with the examination for all other candidates. The marking scheme used for the shared content was identical for the two groups.

This document contains the complete marking scheme for both papers for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics
(Project Maths – Phase 2)

Paper 1

Higher Level

Friday 10 June Afternoon 2:30 – 4:30

300 marks

Model Solutions – Paper 1

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	3 questions

Answer questions as follows:

In Section A, answer **all four** questions

In Section B, answer **both** Question 5 **and** Question 6

In Section C, answer **any two** of the three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

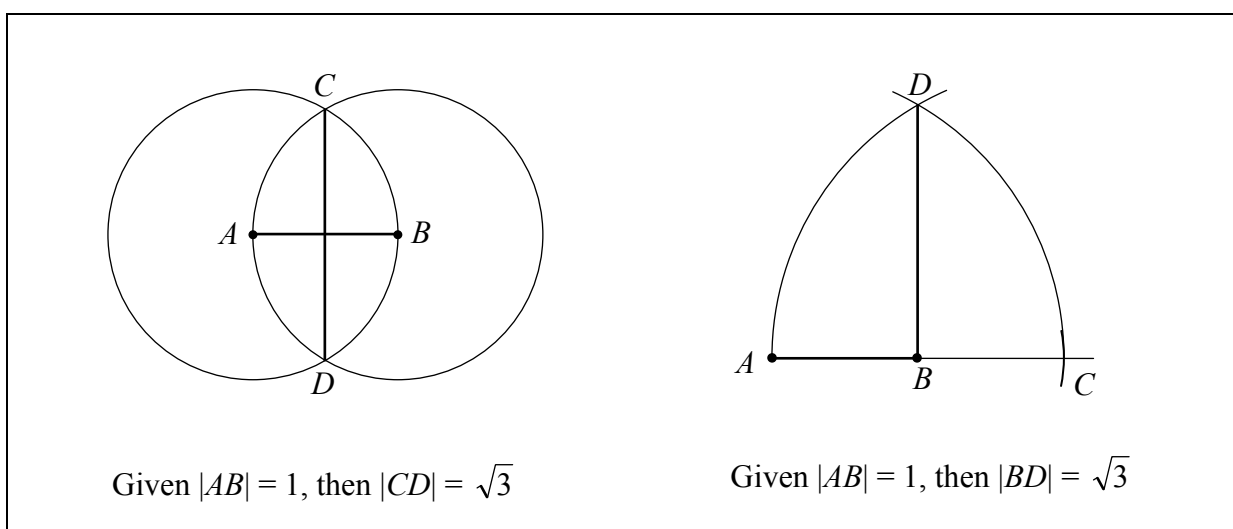
Answer **all four** questions from this section.

Question 1**(25 marks)**

(a) Explain what it means to say that $\sqrt{3}$ is not a rational number.

- It cannot be written as a fraction of integers.
- It cannot be written in the form $\frac{a}{b}$, $a, b \in \mathbf{Z}, b \neq 0$.
- There do not exist integers a and b such that $\left(\frac{a}{b}\right)^2 = 3$.
- The decimal expansion of $\sqrt{3}$ is infinite with no repeating pattern.

(b) Given a line segment of length one unit, show clearly how to construct a line segment of length $\sqrt{3}$ units, using only a compass and straight edge.



(c) Solve the equation $x^2 - 2\sqrt{3}x - 9 = 0$, giving your answers in the form $a\sqrt{3}$, where $a \in \mathbb{Q}$.

$$\begin{aligned}
 x &= \frac{2\sqrt{3} \pm \sqrt{(-2\sqrt{3})^2 - 4(1)(-9)}}{2(1)} = \frac{2\sqrt{3} \pm \sqrt{48}}{2} \\
 &= \frac{2\sqrt{3} \pm 4\sqrt{3}}{2} \\
 &= \sqrt{3} \pm 2\sqrt{3} \\
 x &= -\sqrt{3} \text{ or } x = 3\sqrt{3}
 \end{aligned}$$

Or

$$x^2 - 2\sqrt{3}x - 9 = 0$$

$$x^2 - 2\sqrt{3}x + 3 - 12 = 0$$

$$(x - \sqrt{3})^2 = 12$$

$$x - \sqrt{3} = \pm\sqrt{12}$$

$$x = \sqrt{3} \pm 2\sqrt{3}$$

$$x = -\sqrt{3} \text{ or } x = 3\sqrt{3}$$

Or

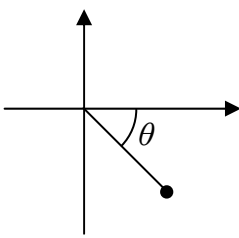
$$(x + \sqrt{3})(x - 3\sqrt{3}) = 0$$

$$x = -\sqrt{3} \text{ or } x = 3\sqrt{3}$$

Question 2

(25 marks)

- (a) (i)** Write the complex number $1-i$ in polar form.

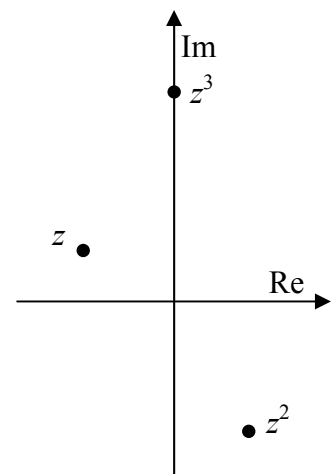
$1-i = \sqrt{2} \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right)$ <p style="text-align: center;">Or</p> $1-i = \sqrt{2} \left(\cos(-45^\circ) + i \sin(-45^\circ) \right)$ <p style="text-align: center;">Or</p> $1-i = \sqrt{2} \left(\cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right) \right)$ <p style="text-align: center;">Or</p> $1-i = \sqrt{2} \left(\cos(315^\circ) + i \sin(315^\circ) \right)$	 <p style="margin-left: 20px;"> $\theta = -\frac{\pi}{4}$ $r = \sqrt{2}$ </p>
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- (ii)** Use De Moivre's theorem to evaluate $(1-i)^9$, giving your answer in rectangular form.

$\begin{aligned} (1-i)^9 &= \sqrt{2}^9 \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right)^9 \\ &= (\sqrt{2})^9 \left(\cos\left(-\frac{9\pi}{4}\right) + i \sin\left(-\frac{9\pi}{4}\right) \right) \\ &= 16\sqrt{2} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \\ &= 16 - 16i \end{aligned}$
--

- (b)** A complex number z has modulus greater than 1. The three numbers z , z^2 , and z^3 are shown on the Argand diagram. One of them lies on the imaginary axis, as shown.

- (i)** Label the points on the diagram to show which point corresponds to which number.
- (ii)** Find θ , the argument of z .



$\begin{aligned} z^3 \text{ has argument } 3\theta &= (90+360n)^\circ \\ \text{so } z \text{ has argument } \theta &= (30+120n)^\circ. \\ z \text{ in 2}^{\text{nd}} \text{ quadrant} &\Rightarrow \theta = 150^\circ \text{ or } \frac{5\pi}{6}. \end{aligned}$
--

Question 3**(25 marks)**

- (a) The cubic function $f : x \mapsto x^3 + 7x^2 + 17x + 15$ has one integer root and two complex roots. Find all three roots.

Integer root must be one of: $\pm 1, \pm 3, \pm 5$. Trial and error yields:

$$f(-3) = (-3)^3 + 7(-3)^2 + 17(-3) + 15 = 0$$

$\therefore -3$ is a root of f , and therefore $x + 3$ is a factor of $f(x)$

$$\begin{array}{r} x^2 + 4x + 5 \\ x + 3 \overline{) x^3 + 7x^2 + 17x + 15} \\ \underline{x^3 + 3x^2} \\ 4x^2 + 17x + 15 \\ \underline{4x^2 + 12x} \\ 5x + 15 \\ \underline{5x + 15} \\ 0 \end{array}$$

Solve $x^2 + 4x + 5 = 0$ to find the two complex roots.

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{-4}}{2} \\ &= \frac{-4 \pm 2i}{2} \\ &= -2 \pm i \end{aligned}$$

Or

$$\begin{aligned} x^2 + 4x + 5 &= 0 \\ (x + 2)^2 &= -1 \\ x + 2 &= \pm i \\ x &= -2 \pm i \end{aligned}$$

Roots of f are: $-3, -2 + i, -2 - i$

- (b) Using part (a), or otherwise, solve the equation $(x - 2)^3 + 7(x - 2)^2 + 17(x - 2) + 15 = 0$.

$$\begin{array}{lll} x - 2 = -3, & x - 2 = -2 + i, & x - 2 = -2 - i \\ x = -1, & x = i, & x = -i \end{array}$$

Question 4**(25 marks)**

In a science experiment, a quantity $Q(t)$ was observed at various points in time t . Time is measured in seconds from the instant of the first observation. The table below gives the results.

t	0	1	2	3	4
$Q(t)$	2.920	2.642	2.391	2.163	1.957

Q follows a rule of the form $Q(t) = Ae^{-bt}$, where A and b are constants.

- (a) Use any two of the observations from the table to find the value of A and the value of b , correct to three decimal places.

$$Q(0) = Ae^0 = 2.92$$

$$\therefore A = 2.920$$

$$Q(t) = 2.92e^{-bt}$$

$$Q(1) = 2.92e^{-b} = 2.642$$

$$e^{-b} = \frac{2.642}{2.92}$$

$$-b = \log_e \frac{2.642}{2.92}$$

$$b = 0.100$$

- (b) Use a different observation from the table to verify your values for A and b .

$$Q(t) = 2.92e^{-0.1t}$$

$$Q(2) = 2.92e^{-0.2} = 2.391$$

From the table, $Q(2) = 2.391$, thus verifying the values for A and b .

- (c) Show that $Q(t)$ is a constant multiple of $Q(t-1)$, for $t \geq 1$.

$$\frac{Q(t)}{Q(t-1)} = \frac{Ae^{-bt}}{Ae^{-b(t-1)}} \\ = e^{-b} \quad (\text{a constant})$$

Or

$$\frac{Q(t-1)}{Q(t)} = e^b$$

Or

$$Q(t) = 2.92e^{-0.1t} \\ Q(t-1) = 2.92e^{-0.1(t-1)} \\ \frac{Q(t)}{Q(t-1)} = \frac{2.92e^{-0.1t}}{2.92e^{-0.1(t-1)}} = \frac{1}{e^{0.1}}$$

- (d) Find the value of the constant k for which $Q(t+k) = \frac{1}{2}Q(t)$, for all $t \geq 0$.

Give your answer correct to two decimal places.

$$Q(t+k) = \frac{1}{2}Q(t) \\ Ae^{-b(t+k)} = \frac{1}{2}Ae^{-bt} \\ 2e^{-b(t+k)} = e^{-bt} \\ 2e^{-bk} = 1 \\ e^{bk} = 2 \\ bk = \log_e 2 \\ k = \frac{1}{b} \log_2 e \\ k = 10 \log_e 2 \\ k \approx 6.93$$

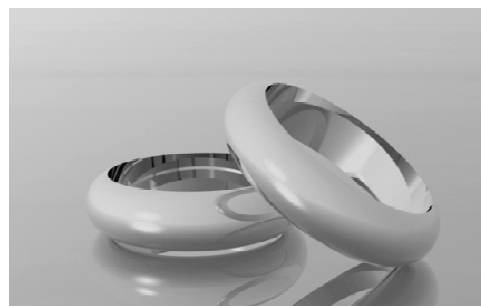
Answer **both** Question 5 **and** Question 6.

Question 5**(50 marks)**

Gold jewellery is made from a gold alloy – that is, a mixture of pure gold and other metals. The purity of the material is measured by its “carat rating”, given by the formula

$$c = \frac{24m_g}{m_t}$$

where c = carat rating
 m_g = mass of gold in the material
 m_t = total mass of the material.



A jeweller is recycling old gold jewellery. He has the following old jewellery in stock:
 147 grams of 9-carat gold
 85 grams of 18-carat gold.

He can melt down this old jewellery and mix it in various proportions to make new jewellery of different carat values. The value of the old jewellery is equal to the value of its gold content only. Gold is valued at €36 per gram.

(a) What is the total value of the jeweller’s stock of old jewellery?

The formula tells us that 9-carat gold is $\frac{9}{24}$ gold, 18-carat gold is $\frac{18}{24}$ gold, etc.

\therefore Gold content of jewellery in stock is $\frac{9}{24}(147) + \frac{18}{24}(85) = 118.875$ grams.

\therefore Value of jewellery = $118.875 \times €36 = €4279.50$.

(b) The jeweller wants to make a 15-carat gold pendant weighing 21 grams. He melts down some 9-carat gold and some 18-carat gold to do this. How many grams of each should he use in order to get the 21 grams of 15-carat gold?

Use x grams of 9-carat and y grams of 18 carat.

Total mass $\Rightarrow x + y = 21 \dots$ (I)

Gold content $\Rightarrow \frac{9}{24}x + \frac{18}{24}y = \frac{15}{24}(21) \Rightarrow x + 2y = 35 \dots$ (II)

(II) – (I) $\Rightarrow y = 14 \Rightarrow x = 7$.

Ans: 7 grams of 9-carat gold and 14 grams of 18-carat gold.

Or

Carat value of mixture is weighted average of carat value of constituent parts.
 15 divides [9, 18] in the ratio 2:1, so we need $\frac{2}{3}$ of 18-carat and $\frac{1}{3}$ of 9-carat.
 \therefore 7 grams of 9-carat gold and 14 grams of 18-carat gold.

- (c) The other metals in the gold alloy are copper and silver. The colour of the alloy depends on the ratio of copper to silver. In all of the old jewellery, the amount of silver is equal to the amount of copper. The jeweller has a stock of pure silver that he can add to any mixture. He wants to make an item that:
- weighs 48 grams
 - is of 15-carat gold purity
 - has twice as much silver as copper.

- (i) How many grams of copper will this item contain?

15-carat gold is $\frac{15}{24}$ gold $\Rightarrow \frac{15}{24} \times 48 = 30$ grams gold \Rightarrow 18 grams copper & silver.

Twice as much silver as copper \Rightarrow 12 grams silver and 6 grams copper.

Answer: 6 grams of copper.

- (ii) How many grams of each type of stock (9-carat gold, 18-carat gold, and pure silver) should the jeweller use in order to make this item?

Use x grams of 9-carat, y grams of 18-carat, and z grams of pure silver.

Total mass $\Rightarrow x + y + z = 48 \dots$ (I)

Gold content $\Rightarrow \frac{9}{24}x + \frac{18}{24}y + 0z = \frac{15}{24}(48) \Rightarrow x + 2y = 80 \dots$ (II)

Twice as much silver as copper $\Rightarrow z = 6$. (Old jewellery contributes equal quantities of copper and silver, so excess of silver over copper comes from pure silver only.)

\therefore (I) becomes $x + y = 42$. Subtract from (II) to get $y = 38$, giving $x = 4$.

Answer: 4 grams of 9-carat gold, 38 grams of 18-carat gold, and 6 grams of pure silver.

OR

Use x grams of 9-carat, y grams of 18-carat, and z grams of pure silver.

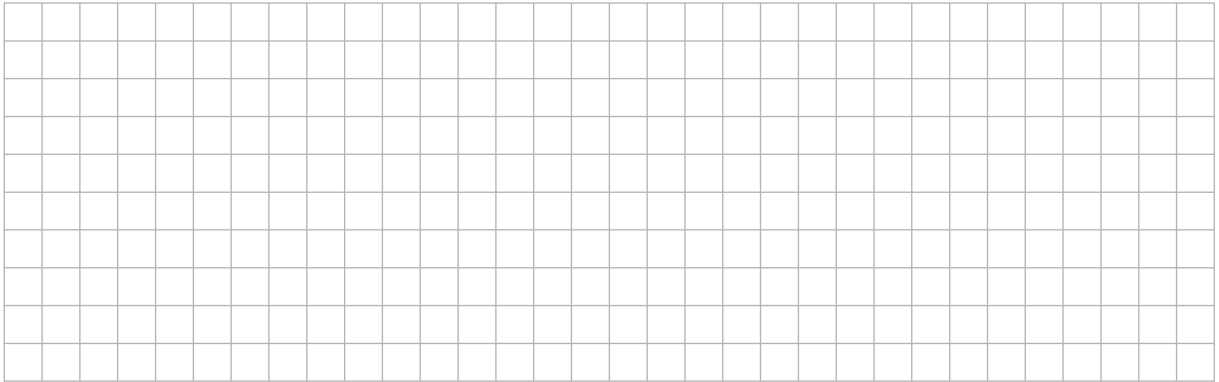
Total mass $\Rightarrow x + y + z = 48 \dots$ (I)

Gold content $\Rightarrow \frac{9}{24}x + \frac{18}{24}y + 0z = 30 \Rightarrow x + 2y = 80 \dots$ (II)

Copper content $\Rightarrow \frac{15}{48}x + \frac{3}{24}y + 0z = 6 \Rightarrow 5x + 2y = 96$ (III)

Subtract (II) from (III) to get $4x = 16 \Rightarrow x = 4, \Rightarrow y = 38 \Rightarrow z = 6$.

Answer: 4 grams of 9-carat gold, 38 grams of 18-carat gold, and 6 grams of pure silver.



- (d) A large jewellery business makes and sells 14-carat gold wedding rings, weighing an average of 5 grams each. The cost of producing each ring is €135 plus the value of the gold. The manager has noted that the more they charge for the rings, the fewer they sell. In particular:
- if they charge €200, they sell an average of twenty per month
 - for each additional €20 charged, the number sold drops by one per month.
- (i) Taking the price charged as €(200 + 20x), find an expression in x for the monthly profit from these rings.

Cost of gold per ring = $\frac{14}{24} \times 5 \times 36 = €105 \Rightarrow$ production cost is $105 + 135 = €240$

profit per ring = selling price – production costs = €(20x – 40)
 number sold per month = 20 – x.

\therefore monthly profit in euro is $(20 - x)(20x - 40)$.

- (ii) Find the range of selling prices for which the monthly profit is at least €1600.

$(20 - x)(20x - 40) \geq 1600$

$\therefore x^2 - 22x + 120 \leq 0$

$(x - 10)(x - 12) \leq 0$

$10 \leq x \leq 12$

\therefore Range of selling prices is:

$[200 + 20(10), 200 + 20(12)]$

That is, from €400 to €440.

Question 6**(50 marks)**

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual-payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash-value* option pays a smaller amount than this.

- (a) If the amount of the first annual payment is A , write down, in terms of A , the amount of the second, third, fourth and 26th payments.

1st payment (now):	<u> A </u>
2nd payment:	<u> $A(1.04)$ </u>
3rd payment:	<u> $A(1.04)^2$ </u>
4th payment:	<u> $A(1.04)^3$ </u>
.	.
26th payment:	<u> $A(1.04)^{25}$ </u>

- (b) The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of A .

$$\begin{aligned}
 \text{Prize} &= A[1 + 1.04 + (1.04)^2 + \dots + (1.04)^{25}] \\
 &= A \left[\frac{1(1.04^{26} - 1)}{(1.04 - 1)} \right] \\
 &= 44.31174462A
 \end{aligned}$$

- (c) Find, correct to the nearest dollar, the value of A that corresponds to an advertised jackpot prize of \$21.5 million.

$$\begin{aligned}
 44.31174462A &= 21\,500\,000 \\
 A &= \$485,199 \text{ (to the nearest dollar).}
 \end{aligned}$$

(d) A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4.78%. We want to find the cash value of the prize referred to in part (c).

(i) Complete the table below to show the actual amount and the present value of each of the first three annual payments.

payment number	time to payment (years)	actual amount	present value
1	0	485,199	485,199
2	1	504,606.96	481,587.10
3	2	524,791.24	478,002.02

(ii) Write down, in terms of n , an expression for the present value of the n th annual payment.

$$\frac{485199(1.04)^{n-1}}{(1.0478)^{n-1}} \quad \text{or} \quad 485199 \left(\frac{1.04}{1.0478} \right)^{n-1}$$

(iii) Find the amount of prize money payable under the *cash-value* option. That is, find the total of the present values of the 26 annual payments.

Give your answer in millions, correct to one decimal place.

$$\begin{aligned}
 S_{26} &= 485,199 + 485,199 \left(\frac{1.04}{1.0478} \right) + \dots + 485,199 \left(\frac{1.04}{1.0478} \right)^{26-1} \\
 &= 485199 \left[\frac{1 - \left(\frac{1.04}{1.0478} \right)^{26}}{1 - \left(\frac{1.04}{1.0478} \right)} \right] \\
 &= \$11.5 \text{ million}
 \end{aligned}$$

(e) The jackpot described in parts (c) and (d) above was won by an Irish woman earlier this year. She chose the *cash-value* option. After tax, she received \$7.9 million. What percentage of tax was charged on her winnings?

$$\text{Tax: } \$11.5 \text{ million} - \$7.9 \text{ million}$$

$$= \$3.6 \text{ million}$$

$$\text{Percentage tax: } \frac{3.6}{11.5} \times 100 = 31.3\%$$

Section C**Functions and Calculus (old syllabus)****100 marks**

Answer **any two** of the three questions from this section.

Model solutions for questions 7, 8, and 9 are incorporated into the marking scheme for section C.
See page 32.

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	5
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5		
10 mark scale		0, 6, 10	0, 6, 8, 10	0, 4, 5, 8, 10	
15 mark scale			0, 9, 13, 15	0, 8, 10, 13, 15	
20 mark scale		0, 12, 20	0, 10, 18, 20	0, 10, 16, 18, 20	0, 5, 10, 14, 17, 20
25 mark scale				0, 15, 20, 23, 25	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

- (a) 5B
- (b) 10C
- (c) 10C

Question 2

- (a) (i) 15C
- (a) (ii) 5C
- (b) (i)&(ii) 5C

Question 3

- (a) 20E
- (b) 5B

Question 4

- (a) 15C
- (b) 5B
- (c)&(d) 5C

Section B

Question 5

- (a) 20D
- (b) 5B
- (c) (i)&(ii) 20D
- (d) (i)&(ii) 5C

Question 6

- (a) 20B
- (b) 10C
- (c) 10C
- (d)(i)&(ii) 5C
- (d)(iii)&(e) 5C

Detailed marking notes

Section A

Question 1

(a) Scale 5B (0, 3, 5).

Partial credit:

- Its decimal expansion is infinite.
- Its decimal expansion continues on forever.
- $1.732\dots$ and continues on forever.

(b) Scale 10C (0, 6, 8, 10).

Low partial credit:

- Perpendicular bisector of line segment constructed.

High partial credit:

- Right angled triangle constructed, with sides of lengths 1, $\sqrt{3}$ and 2 indicated.
- $\cos 60^\circ = \frac{\sqrt{3}}{2}$ and uses this information to draw, using protractor and ruler a triangle with sides $\sqrt{3}$, 1, and 2.
- Graphs the parabola $y = x^2$ and using the graph indicates $\sqrt{3}$ on the x -axis.

Full credit:

- Accept answer with length $\sqrt{3}$ not indicated.

(c) Scale 10C (0, 6, 8, 10)

Low partial credit:

- Attempt at factorising e.g. $(x - \quad)(x + \quad)$
- Quadratic formula written down.
- $a = 1$, $b = -2\sqrt{3}$, $c = -9$

High partial credit:

- Correct substitution into formula.
- Substitutes $x = a\sqrt{3}$ into the equation and fails to finish correctly.

Question 2

(a) (i) Scale 15C (0, 9, 13, 15)

Low partial credit:

- r calculated correctly.
- θ calculated correctly.
- $1 - i$, plotted on the Argand diagram.

High partial credit

- Reference angle is correct, but complex number plotted in the wrong quadrant and continues.
- Both r and θ calculated correctly.

(ii) Scale 5C (0, 3, 4, 5)

Accept answers from (i)

Low partial credit:

- Statement of De Moivre's theorem.

High partial credit

- $(\sqrt{2})^9 \left(\left[\cos\left(-\frac{9\pi}{4}\right) + i \sin\left(-\frac{9\pi}{4}\right) \right] \right)$ or $(\sqrt{2})^9 \left(\left[\cos\left(\frac{63\pi}{4}\right) + i \sin\left(\frac{63\pi}{4}\right) \right] \right)$
- Solves $z = (1 - i)^{\frac{1}{9}}$ to find one solution in rectangular form.

(b) (i)&(ii) Scale 5C (0, 3, 4, 5)

Full Credit: (i) and (ii) correct.

High Partial: One of (i) and (ii) correct.

Low Partial: Any correct step in either (i) or (ii)

(i)

Partial credit:

- One correct.
- $\sqrt{a^2 + b^2} > 1$
- $z^2 = r^2(\cos 2\theta + i \sin 2\theta)$
- Label modulus lengths on the diagram.
- Takes a complex number with modulus greater than 1 and squares, e.g. $(1 - i)^2$

(ii)

Partial credit:

- Protractor used to measure, θ .
- Statement such as z^3 has no real part.
- Expands $(a + ib)^2$.
- Takes a complex number with modulus greater than 1 and squares, e.g. $(1 - i)^2$
- Oversimplified if z is on the i -axis giving an argument of $\frac{\pi}{2}$. Merits partial credit.
- $z^2 = r^2(\cos 2\theta + i \sin 2\theta)$

Question 3

(a) Scale 20E (0, 5, 10, 14, 17, 20)

Low partial credit:

- Attempt at finding integer root. e.g. finds $f(-1)$ and stops.
- Factors 15 into 3 and 5 and stops.

Low middle partial credit:

- Shows $f(-3) = 0$ with or without work.
- Correct long division, but uses incorrect linear factor.

Upper middle partial credit

- Correct long division.

High partial credit:

- $-3, \frac{-4 \pm \sqrt{-4}}{2}$

(b) Scale 5B (0, 3, 5)

Accept answers from part (a).

Partial credit

- Any attempt at expanding $(x - 2)^3$ or $(x - 2)^2$.
- Multiplying $17(x - 2)$.
- $x = -1$ and stops.
- $x = \pm i$ and stops.
- Trial and error $f(1), f(2)$, etc.

Question 4

(a) Scale 15C (0, 9, 13, 15).

Low partial credit

- Some substitution for t and/or $Q(t)$.
- Writes for example, $Q(0) = 2 \cdot 92$ and stops.

High partial credit

- Finds A and stops.
- Correctly substitutes two values from the table.
- Incorrect A and finds b .
- Finds e^b or e^{-b} and stops.

(b) Scale 5B (0, 3, 5).

Accept answers from (a).

Partial credit:

- Substitutes value for A and/or b into $Q(t)$.
- Uses the table to write down some value of $Q(t)$. e.g. $Q(2) = 2 \cdot 391$ (provided its not a value used in part (a)).
- Substitutes but draws no conclusion.

(c)&(d) Scale 5C (0, 3, 4, 5)

Full Credit: (c) and (d) correct.

High Partial: One of (c) and (d) correct.

Low Partial: Any correct step in either (c) or (d) (as detailed below)

Relevant steps for (c) are, e.g.:

- Finds $Q(t-1)$ either in terms of A and b or in terms of t .
- Writes $kQ(t) = Q(t-1)$.
- Writes an expression for $\frac{Q(t)}{Q(t-1)}$ but fails to show that this simplifies to a constant.
- Shows for example, $\frac{Q(2)}{Q(1)} = \frac{Q(3)}{Q(2)}$

Relevant steps for (d) are, e.g.:

- Finds $Q(t+k)$ either in terms of A , b and k , or in terms of k and t .
- Substitutes for A and/or b .
- Equates $Q(t+k)$ and $\frac{1}{2}Q(t)$ plus some work towards eliminating t .

Section B

Question 5

(a) Scale 20 D (0, 10, 16, 18, 20)

Low partial credit

- Substitutes 147 and/or 85 into formula and stops.

Mid partial credit:

- Finds correctly either the amount of gold in the 9 carat jewellery or 18 carat jewellery.

High partial credit:

- Finds correctly the amount of gold in the 9 carat and 18 carat jewellery.
- Finds the total amount of gold.

(b) Scale 5B (0, 3, 5)

Partial credit:

- Finds mass of gold, 13.125 *grams* in 15 carat pendant.
- Writes $9 = \frac{24m_g}{m_t}$ or $18 = \frac{24m_g}{m_t}$ or $15 = \frac{24m_g}{m_t}$

(c) (i)&(ii) Scale 20D (0, 10, 16, 18, 20)

Full Credit: (i) and (ii) correct.

High Partial: One of (i) and (ii) correct.

Mid Partial: Two relevant steps in either (i) or (ii).

Low Partial: Any correct step in either (i) or (ii) or both.

Relevant steps for (i) are, e.g.:

- Divides 15 or 48 in the ratio 2:1
- Writes $15 = \frac{24m_g}{48}$ and stops.
- Calculate the mass of gold, m_g in the ring (= two relevant steps)

Relevant steps for (ii) are, e.g.:

- Some work with formula, $c = \frac{24m_g}{m_t}$
- Writes down a correct equation (= two relevant steps)

(d) (i)&(ii) Scale 5C (0, 3, 4, 5)

Full Credit: (i) and (ii) correct.

High Partial: One of (i) and (ii) correct.

Low Partial: Any correct step in either (i) or (ii)

Relevant steps for (i) are, e.g.:

- Finds the mass of gold in the ring.
- Finds the total cost of producing the ring.
- Expression with $20 - x$.

Relevant steps for (ii) are, e.g.:

- Sets up inequality using answer from (i).
- $10 \leq x \leq 12$ or equivalent if profit function from (i) is quadratic.
- $x = 10$ or $x = 12$

Question 6

(a) Scale 20B (0, 12, 20)

Partial credit:

- $\frac{A}{1.04}, \frac{A}{1.04^2}$ etc.
- $A \times \frac{4}{100}$
- Incorrect percentage e.g. 0.4%.

(b) Scale 10 C(0, 6, 8, 10)

Low partial credit:

- Prize = Answer from (a) and stops.
- Correct formula, $S_n = \frac{a(1-r^n)}{1-r}$.

High partial credit:

- $A \left[\frac{1(1.04^{25} - 1)}{1.04 - 1} \right]$

Note: Accept Prize = $A \left[\frac{1(1.04^{26} - 1)}{1.04 - 1} \right]$

(c) Scale 10 C (0, 6, 8, 10).

Low partial credit:

- $A = \frac{21,500,000}{1.04^{25}}$.

High partial credit:

- $A = \frac{21.5}{44.312}$

(d) (i)&(ii) Scale 5C (0, 3, 4, 5)

Full Credit: (i) and (ii) correct.

High Partial: One of (i) and (ii) correct.

Low Partial: Any correct step in either (i) or (ii)

Relevant steps for (i) are, e.g.:

- Finds actual amount of each payment and stops.
- $P = \frac{F}{(1+i)^t}$
- Any correct entry in the table.
- Finds present value of A each time, rather than present value of $A, A(1.04)$ and $A(1.04)^2$.

Relevant steps for (ii) are, e.g.:

- Formula for T_n of a geometric sequence.
- $485,199(1.04)^{n-1}$
- $\frac{485,199(1.04)^n}{(1.0478)^n}$
- $\frac{485,199}{1.0478^{n-1}}$, based on answer from (i).
- $A \frac{1.04^{n-1}}{1.0478^{n-1}}$

Note: Accept answers from (i) e.g. $\frac{485,199}{(1.0478)^n}$

(d)(iii) & (e) Scale 5C (0, 3, 4, 5)

Full Credit: (d)(iii) and (e) correct.

High Partial: One of (d)(iii) and (e) correct.

Low Partial: Any correct step in either (d)(iii) or (e)

Relevant steps for (d)(iii) are, e.g.:

- Formula for S_n of a geometric series.

- Finds $485,199 \left[\frac{1 - \left(\frac{1.04}{1.0478} \right)^{25}}{1 - \frac{1.04}{1.0478}} \right]$

Note: Accept answers from (ii). e.g. $S_{26} = 485,199 + \frac{485,199}{1.0478} + \frac{485,199}{1.0478^2} + \dots + \frac{485,199}{1.0478^{25}}$

Relevant steps for (e) are, e.g.:

- Any attempt at finding a relevant percentage.
- Finds tax.

Marking scheme – Paper 1, Section C

General Guidelines for Examiners – Paper 1, Section C

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2, ...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work of merit is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 7

Part (a)	15 marks	Att 5
Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)
Part (c)	15 (5, 5, 5) marks	Att (2, 2, 2)

Part (a) **15 marks** **Att 5**

(a) Differentiate $\cos^2 x$ with respect to x .

7 (a)

$$f(x) = \cos^2 x \Rightarrow f'(x) = -2\cos x \sin x.$$

Blunders (-3)

B1 Differentiation

Attempts

A1 Error in differentiation formula (chain rule)

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

7 (b) The equation of a curve is $y = e^{-x^2}$.

(i) Find $\frac{dy}{dx}$.

(ii) Find the co-ordinates of the turning point of the curve.

(iii) Determine whether this turning point is a local maximum or a local minimum.

Part (b) (i)	5 marks	Att 2
(ii) $f'(x) = 0$	5 marks	Att 2
Turning point	5 marks	Att 2
(iii)	5 marks	Att 2

7 (b) (i)

$$\frac{dy}{dx} = e^{-x^2} (-2x).$$

7 (b) (ii)

$$\frac{dy}{dx} = 0 \Rightarrow e^{-x^2} (-2x) = 0 \Rightarrow x = 0 \text{ and } y = 1. \text{ Turning point is } (0, 1).$$

7 (b) (iii)

$$\frac{d^2y}{dx^2} = e^{-x^2} (-2x)(-2x) - 2e^{-x^2} = e^{-x^2} (4x^2 - 2).$$

$$\text{For } x = 0, \frac{d^2y}{dx^2} = -2e^0 = -2 < 0 \Rightarrow (0, 1) \text{ is a local maximum.}$$

Blunders (-3)

B1 Indices

B2 Differentiation

B3 $e^{-x^2} = 0$

B4 No 2nd differential

Attempts

A1 Error in differentiation formula (chain rule)

Note: Over simplified work in (i) can lead to attempt at most in (ii) and (iii).

Part (c)

15 (5, 5, 5) marks

Att (2, 2, 2)

7 (c) The function f is defined as $x \rightarrow \frac{2x}{x+1}$, where $x \in \mathbf{R} \setminus \{-1\}$.

- (i) Find the equations of the asymptotes of the curve $y = f(x)$.
- (ii) P and Q are distinct points on the curve $y = f(x)$. The tangent at Q is parallel to the tangent at P . The co-ordinates of P are $(1, 1)$.
Find the co-ordinates of Q .
- (iii) Verify that the point of intersection of the asymptotes is the midpoint of $[PQ]$.

Part (c) (i)

5 marks

Att 2

(ii)

5 marks

Att 2

(iii)

5 marks

Att 2

7 (c) (i)

$x = -1$ is the vertical asymptote.

$$\lim_{x \rightarrow \infty} \frac{2x}{x+1} = \lim_{x \rightarrow \infty} \frac{2}{1 + \frac{1}{x}} = 2 \Rightarrow y = 2 \text{ is a horizontal asymptote.}$$

7 (c) (ii)

$$f'(x) = \frac{2(x+1) - 2x(1)}{(x+1)^2} = \frac{2}{(x+1)^2}. \text{ Slope at } P(1, 1) = \frac{2}{4} = \frac{1}{2}.$$

$$\text{Slope at } Q = \frac{1}{2} \Rightarrow \frac{2}{(x+1)^2} = \frac{1}{2} \Rightarrow (x+1)^2 = 4.$$

$$\therefore x+1 = \pm 2 \Rightarrow x = 1 \text{ or } x = -3. \therefore Q \text{ is } (-3, 3).$$

OR

$$\begin{aligned}
 (x+1)^2 &= 4 \\
 x^2 + 2x + 1 - 4 &= 0 \\
 x^2 + 2x - 3 &= 0 \\
 (x+3)(x-1) &= 0 \\
 \Rightarrow x+3 &= 0 \quad \text{or} \quad x-1 = 0 \\
 x &= -3 \quad \text{or} \quad x = 1 \\
 \downarrow & \qquad \qquad \downarrow \\
 Q(-3,3) & \qquad \qquad P(1,1)
 \end{aligned}$$

7 (c) (iii) Asymptotes intersect at $(-1, 2)$,
 $P(1, 1)$ and $Q(-3, 3)$.
 Mid-point of $[PQ]$ is $(-1, 2)$.

Blunders (-3)

- B1 Asymptotes
- B2 Limits
- B3 Differentiation
- B4 Indices
- B5 Formula for mid-point line

Slips (-1)

- S1 Numerical

Attempts

- A1 Error in differentiation formula

Note: Cannot get 2nd 5 marks in (c) (ii) if slope at Q not equal to slope at P .

QUESTION 8

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	25 (10, 10, 5) marks	Att (3, 3, 2)
Part (c)	15 (10, 5) marks	Att (3, 2)

Part (a) **10 (5, 5) marks** **Att (2, 2)**

8 (a) Find the slope of the tangent to the curve $x^2 + y^3 = x - 2$ at the point $(3, -2)$.

Differentiation **5 marks** **Att 2**
Slope **5 marks** **Att 2**

8 (a)

$$2x + 3y^2 \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1 - 2x}{3y^2}. \therefore \text{Slope of tangent at } (3, -2) = \frac{-5}{12}.$$

Blunders (-3)

- B1 Differentiation
- B2 Indices
- B3 Incorrect value of x or no value of x in slope
- B4 Incorrect value of y or no value of y in slope

Slips (-1)

- S1 Numerical

Attempts

- A1 Error in differentiation formula

- A2 $\frac{dy}{dx} = 2x + 3y^2 \frac{dy}{dx} = 1$ and uses the two $\left(\frac{dy}{dx}\right)$ terms

Part (b)

25 (10, 10, 5) marks

Att (3, 3, 2)

8 (b) A curve is defined by the parametric equations

$$x = \frac{t-1}{t+1} \quad \text{and} \quad y = \frac{-4t}{(t+1)^2}, \quad \text{where } t \neq -1.$$

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

(ii) Hence find $\frac{dy}{dx}$, and express your answer in terms of x .

Part (b) $\frac{dx}{dt}$

10 marks

Att 3

$\frac{dy}{dt}$

10 marks

Att 3

$\frac{dy}{dx}$

5 marks

Att 2

8 (b) (i)

$$\frac{dx}{dt} = \frac{1(t+1) - 1(t-1)}{(t+1)^2} = \frac{2}{(t+1)^2}.$$

$$\frac{dy}{dt} = \frac{-4(t+1)^2 + 4t(2)(t+1)}{(t+1)^4} = \frac{-4(t+1) + 8t}{(t+1)^3} = \frac{4(t-1)}{(t+1)^3}.$$

8 (b) (ii)

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{4(t-1)}{(t+1)^3} \times \frac{(t+1)^2}{2} = \frac{2(t-1)}{t+1} = 2x.$$

Blunders (-3)

B1 Differentiation

B2 Indices

B3 Error in getting $\frac{dy}{dx}$

Attempts

A1 Error in differentiation formula

Part (c)

15 (10, 5) marks

Att (3, 2)

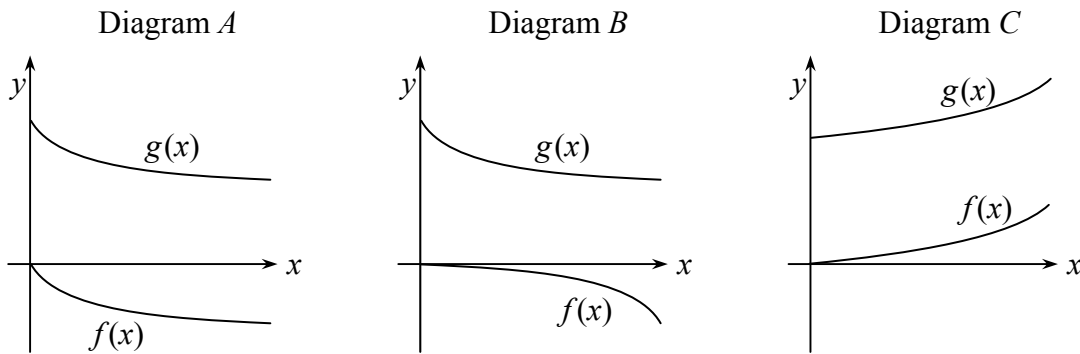
(c) The functions f and g are defined on the domain $\mathbb{R} \setminus \{-1, 0\}$ as follows:

$$f : x \rightarrow \tan^{-1}\left(\frac{-x}{x+1}\right) \quad \text{and} \quad g : x \rightarrow \tan^{-1}\left(\frac{x+1}{x}\right).$$

(i) Show that $f'(x) = \frac{-1}{2x^2 + 2x + 1}$.

(ii) It can be shown that $f'(x) = g'(x)$.

One of the three diagrams A, B, or C below represents parts of the graphs of f and g . Based only on the derivatives, state which diagram is the correct one, and state also why each of the other two diagrams is incorrect.



c(i)

10 marks

Att 3

8 (c) (i)

$$f(x) : x \rightarrow \tan^{-1}\left(\frac{-x}{x+1}\right)$$

$$f'(x) = \frac{1}{1 + \left(\frac{-x}{x+1}\right)^2} \times \frac{-1(x+1) + x(1)}{(x+1)^2} = \frac{(x+1)^2}{x^2 + 2x + 1 + x^2} \times \frac{-1}{(x+1)^2} = \frac{-1}{2x^2 + 2x + 1}$$

OR

$$y = \tan^{-1}\left(\frac{-x}{x+1}\right)$$

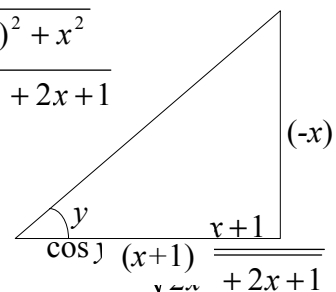
$$\tan y = \frac{-x}{x+1}$$

$$\sec^2 y \cdot \frac{dy}{dx} = \frac{(x+1)(-1) - (-x)(1)}{(x+1)^2}$$

$$\frac{1}{\cos^2 y} \cdot \frac{dy}{dx} = \frac{-x-1+x}{(x+1)^2}$$

$$\frac{1}{\cos^2 y} \cdot \frac{dy}{dx} = \frac{-1}{(x+1)^2}$$

$$\begin{aligned} &\sqrt{(x+1)^2 + x^2} \\ &= \sqrt{2x^2 + 2x + 1} \end{aligned}$$



$$\cos^2 y = \frac{(x+1)^2}{2x^2 + 2x + 1}$$

(...)

$$\begin{aligned}\frac{dy}{dx} &= \frac{-\cos^2 y}{(x+1)^2} \\ &= \frac{-1}{(x+1)^2} \cdot \frac{(x+1)^2}{2x^2 + 2x + 1} \\ &= \frac{-1}{2x^2 + 2x + 1}\end{aligned}$$

Blunders (-3)

- B1 Differentiation
- B2 Indices
- B3 Error in value of $\tan y$
- B4 Error in value of $\cos y$
- B5 Sides of triangle once only

Attempts

- A1 Error in differentiation formula and hence Att 2 at most in simplification

Part (c) (ii)

5 marks

Att 2

8 (c) (ii)

Diagram *A* is correct.

It cannot be Diagram *B*, as these curves are not “parallel” (i.e. identical up to a vertical shift, which is necessary because their derivatives are equal for all x).

It cannot be Diagram *C* as these graphs are increasing, whereas they should be decreasing, because their derivatives are negative for $x > 0$.

OR

$$\text{Given } f'(x) = g'(x)$$

$$\Rightarrow m_1 = m_2 \quad (\text{same slopes})$$

\Rightarrow parallel curves

$$f'(x) = \frac{-1}{2x^2 + 2x + 1} < 0 \quad \text{when } x > 0$$

\Rightarrow Both $f(x)$ and $g(x)$ are decreasing functions.

Diagram *A*: correct

Diagram *B*: not parallel curves

Diagram *C*: increasing curves

Blunders (-3)

- B1 Incorrect statement

QUESTION 9

Part (a)	15 marks	Att 5
Part (b)	25 (5, 5, 5, 5, 5) marks	Att (2, 2, 2, 2, 2)
Part (c)	10 (5, 5) marks	Att (2, 2)

Part (a) **15 marks** **Att 5**

9 (a) Find $\int (x^3 + \sqrt{x}) dx$.

9 (a)

$$\int (x^3 + \sqrt{x}) dx = \frac{1}{4}x^4 + \frac{2}{3}x^{\frac{3}{2}} + c.$$

Blunders (-3)

B1 Integration

B2 Indices

B3 No 'c'

Part (b) **25 (5, 5, 5, 5, 5) marks** **Att (2, 2, 2, 2, 2)**

9 (b) (i) Evaluate $\int_0^2 \frac{x+1}{x^2+2x+2} dx$.

(ii) Evaluate $\int_0^2 \frac{x^2+2x+2}{x+1} dx$.

Part (b) (i) Correct substitution	5 marks	Att 2
Integration	5 marks	Att 2
Finish	5 marks	Att 2

9 (b) (i)

$$\int_0^2 \frac{x+1}{x^2+2x+2} dx. \quad \text{Let } u = x^2 + 2x + 2 \Rightarrow du = (2x+2)dx.$$

$$= \frac{1}{2} \int_2^{10} \frac{du}{u} = \frac{1}{2} [\log_e u]_2^{10} = \frac{1}{2} [\log_e 10 - \log_e 2] = \frac{1}{2} \log_e 5 = \log_e \sqrt{5}.$$

Blunders (-3)

B1 Integration

B2 Differentiation

B3 Logs

B4 Limits

B5 Incorrect order in applying limits

B6 Not calculating substituted limits

B7 Not changing limits

Slips (-1)

S1 Numerical

Part (b) (ii) Integration

5 marks

Att 2

Finish

5 marks

Att 2

9 (b) (ii)

$$\begin{aligned}\therefore \int_0^2 \frac{x^2 + 2x + 2}{x+1} dx &= \int_0^2 \frac{(x+1)^2 + 1}{x+1} dx = \int_0^2 \left((x+1) + \frac{1}{x+1} \right) dx \\ &= \left[\frac{1}{2}x^2 + x + \log_e(x+1) \right]_0^2 = 2 + 2 + \log_e 3 = 4 + \log_e 3.\end{aligned}$$

OR

$$\begin{aligned}\int \frac{x^2 + 2x + 2}{x+1} dx &= \int \left[(x+1) + \frac{1}{x+1} \right] dx \\ &= \int (x+1) dx + \int \frac{1}{x+1} dx \\ &= \frac{1}{2}x^2 + x + \log_e(x+1) + C\end{aligned}$$
$$\begin{array}{r}x+1 \overline{)x^2 + 2x + 2} \\ \underline{x^2 + x} \\ x + 2 \\ \underline{x + 1} \\ 1\end{array}$$

Finish as above

Blunders (-3)

B1 Integration

B2 Differentiation

B3 Logs

B4 Limits

B5 Incorrect order in applying limits

B6 Not calculating substituted limits

B7 Not changing limits

Slips (-1)

S1 Numerical

S2 Not changing sign when subtracting in division

Part (c)

10 (5, 5) marks

Att (2, 2)

9 (c) Use integration methods to establish the formula $A = \pi r^2$ for the area of a disc of radius r .

Set up
Finish

5 marks
5 marks

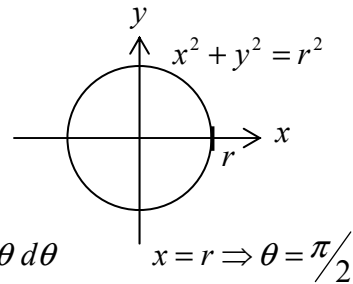
Att 2
Att 2

9 (c)

$x^2 + y^2 = r^2$ is a circle, centre $(0, 0)$, radius = r .

$$\text{Area of disc} = A = 4 \int_0^r \sqrt{r^2 - x^2} dx$$

Let $x = r \sin \theta \Rightarrow dx = r \cos \theta d\theta$.



$$\therefore A = 4 \int_0^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 \theta} \cdot r \cos \theta d\theta = 4 \int_0^{\frac{\pi}{2}} \sqrt{r^2 (1 - \sin^2 \theta)} \cdot r \cos \theta d\theta$$

$$= 4 \int_0^{\frac{\pi}{2}} r^2 \cos^2 \theta d\theta = (4r^2) \frac{1}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{2}}$$

$x = 0 \Rightarrow \theta = 0$

$$= 2r^2 \left[\left(\frac{\pi}{2} + \sin \pi \right) - (0 + \sin 0) \right]$$

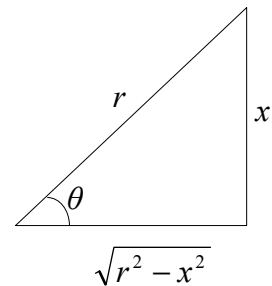
$$\therefore A = 2r^2 \left(\frac{\pi}{2} \right) = \pi r^2.$$

OR

$$\frac{x}{r} = \sin \theta \Rightarrow x = r \sin \theta$$

$$\frac{dx}{d\theta} = r \cos \theta \Rightarrow dx = r \cos \theta d\theta$$

From diagram: $\cos \theta = \frac{\sqrt{r^2 - x^2}}{r} \Rightarrow r \cos \theta = \sqrt{r^2 - x^2}$



$$A = 4 \int_0^r \sqrt{r^2 - x^2} dx$$

$$= 4 \int (r \cos \theta) \cdot (r \cos \theta) \cdot d\theta$$

$$= 4 \int r^2 \cos^2 \theta \cdot d\theta \text{ etc.}$$

Blunders (-3)

- B1 Integration
- B2 Differentiation
- B3 Trig formula
- B4 Indices

- B5 Limits
- B6 Incorrect order in applying limits
- B7 Not calculating substituted limits
- B8 Not changing limits
- B9 Definition of $\sin \theta$
- B10 Definition of $\cos \theta$

Slips (-1)

- S1 Numerical
- S2 Trig value or no trig value

Attempts

- A1 Error in differentiation formula or rules of integration

Worthless

- W1 $x = r \sin \theta$ or $x = r \cos \theta$ not used in integration: 0 marks for 2nd 5



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

Monday 13 June Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

- (a) A random variable X follows a normal distribution with mean 20 and standard deviation 5. Find $P(14 \leq X \leq 26)$.

$$z = \frac{x - \mu}{\sigma}$$

$$z_1 = \frac{14 - 20}{5} = -1.2$$

$$z_2 = \frac{26 - 20}{5} = 1.2$$

$$P(14 \leq X \leq 26) = P(-1.2 \leq z \leq 1.2)$$

$$\begin{aligned} P(-1.2 \leq z \leq 1.2) &= 1 - 2P(z > 1.2) \\ &= 1 - 2[1 - P(z \leq 1.2)] \\ &= 2P(z \leq 1.2) - 1 \\ &= 0.7698 \end{aligned}$$

- (b) There are 16 girls and 8 boys in a class. Half of these 24 students study French. The probability that a randomly selected girl studies French is 1.5 times the probability that a randomly selected boy studies French. How many of the boys in the class study French?

Let x = number of boys, who study French.

$\therefore 12 - x$ = number of girls who study French.

$$\frac{12 - x}{16} = 1.5 \left(\frac{x}{8} \right)$$

$$96 - 8x = 24x$$

$$32x = 96$$

$$x = 3$$

Three boys study French.

Question 2

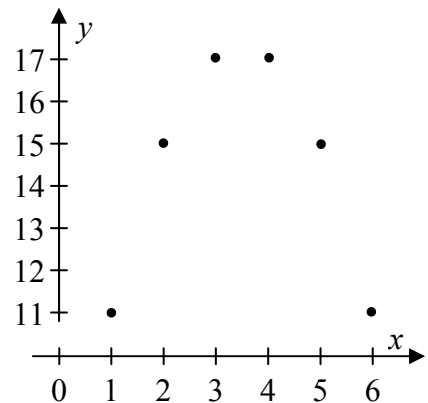
(25 marks)

- (a) Explain, with the aid of an example, what is meant by the statement:
“Correlation does not imply causality.”

A positive correlation between two variables does not mean that one is necessarily causing the other. For example, in a primary school there might be a correlation between reading ability and shoe size, but big feet don't make you read better and reading doesn't make your feet grow! In this case, both variables are connected to age – a ‘confounding factor’

- (b) The data given in the table below and represented in the scatter diagram are pairs of observations of the variables x and y .

x	1	2	3	4	5	6
y	11	15	17	17	15	11



- (i) Calculate the correlation coefficient.

Answer: _____ 0 _____

- (ii) What kind of relationship, if any, do the observed data suggest exists between x and y ?

There is no linear relationship, but the pattern suggests a quadratic relationship.

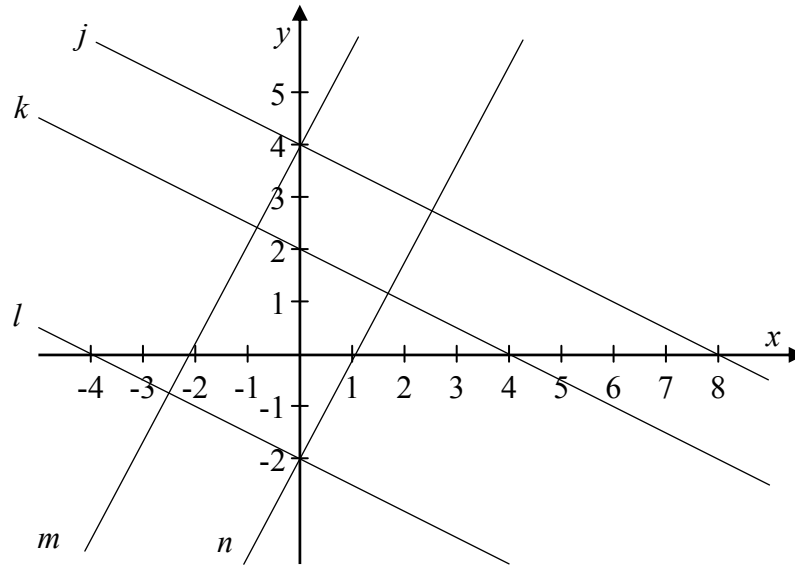
Or

There appears to be a non-linear relationship between x and y .

Question 3

(25 marks)

In the co-ordinate diagram shown, the lines j , k , and l are parallel, and so are the lines m and n . The equations of four of the five lines are given in the table below.



Equation	Line
$x + 2y = -4$	l
$2x - y = -4$	m
$x + 2y = 8$	j
$2x - y = 2$	n

- (a) Complete the table, by matching four of the lines to their equations.

$$\begin{aligned}
 x + 2y = -4 &\Rightarrow y = -\frac{1}{2}x - 2 && \rightarrow l \\
 2x - y = -4 &\Rightarrow y = 2x + 4 && \rightarrow m \\
 x + 2y = 8 &\Rightarrow y = -\frac{1}{2}x + 4 && \rightarrow j \\
 2x - y = 2 &\Rightarrow y = 2x - 2 && \rightarrow n
 \end{aligned}$$

- (b) Hence, insert scales on the x -axis and y -axis.

Shown above

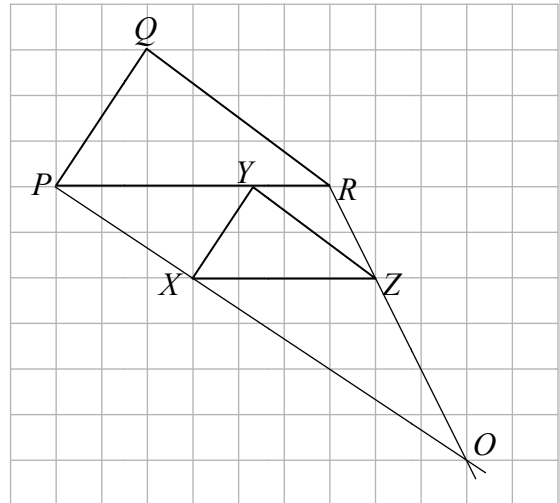
- (c) Hence, find the equation of the remaining line, given that its x -intercept and y -intercept are both integers.

Equation of k : $y = -\frac{1}{2}x + 2$
 or
 $x + 2y = 4$

Question 4

(25 marks)

Two triangles are drawn on a square grid as shown. The points $P, Q, R, X,$ and Z are on vertices of the grid, and the point Y lies on $[PR]$. The triangle PQR is an enlargement of the triangle XYZ .



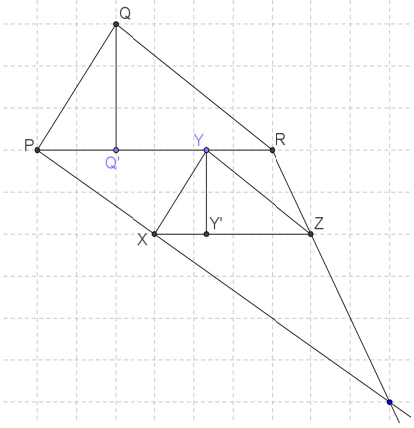
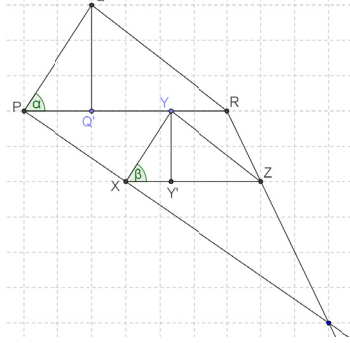
- (a) Calculate the scale factor of the enlargement, showing your work.

$$\frac{|PR|}{|XZ|} = \frac{6}{4} = \frac{3}{2}$$

- (b) By construction or otherwise, locate the centre of enlargement on the diagram above.

Shown as O above.

- (c) Calculate $|YR|$ in grid units.

 $ Y'Z' = \frac{2}{3} Q'R = \frac{2}{3}(4) = \frac{8}{3}$ $ YR = \frac{8}{3} - 1 = \frac{5}{3}$	 $\tan \alpha = \frac{3}{2}$ $\alpha = \beta$ $\frac{2}{ XY' } = \frac{3}{2}$ $ XY' = \frac{4}{3}$ $ YR = 3 - \frac{4}{3} = \frac{5}{3}$
---	--

Question 5**(25 marks)**

The line $x + 3y = 20$ intersects the circle $x^2 + y^2 - 6x - 8y = 0$ at the points P and Q .
Find the equation of the circle that has $[PQ]$ as diameter.

$$\text{Line} \Rightarrow x = 20 - 3y$$

$$\therefore (20 - 3y)^2 + y^2 - 6(20 - 3y) - 8y = 0$$

$$9y^2 - 120y + 400 + y^2 - 120 + 18y - 8y = 0$$

$$10y^2 - 110y + 280 = 0$$

$$y^2 - 11y + 28 = 0$$

$$(y - 7)(y - 4) = 0$$

$$y = 7 \quad \text{or} \quad y = 4$$

$$x = -1 \quad \text{or} \quad x = 8$$

$$P(-1, 7) \quad \text{and} \quad Q(8, 4)$$

$$\text{Centre is midpoint of } [PQ]: \quad C\left(\frac{7}{2}, \frac{11}{2}\right)$$

$$r = \sqrt{\left(\frac{7}{2} + 1\right)^2 + \left(\frac{11}{2} - 7\right)^2}$$

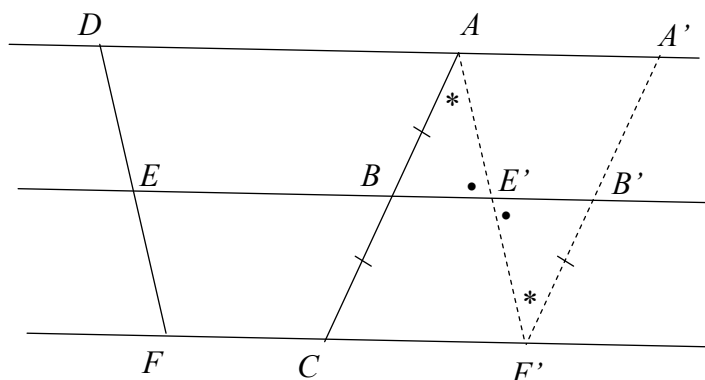
$$= \sqrt{20 \cdot 25 + 2 \cdot 25}$$

$$= \sqrt{22 \cdot 5} \quad \text{or} \quad \sqrt{\frac{45}{2}}$$

$$\text{Equation: } \left(x - \frac{7}{2}\right)^2 + \left(y - \frac{11}{2}\right)^2 = \frac{45}{2}$$

Question 6**(25 marks)**Answer **either** 6A **or** 6B.**Question 6A**

Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:

Given: $AD \parallel BE \parallel CF$, as in the diagram, with $|AB| = |BC|$

To prove:

$$|DE| = |EF|$$

Construction:

Draw $AE' \parallel DE$, cutting EB at E' and CF at F'
 Draw $F'B' \parallel AB$, cutting EB at B' , as in the diagram.

Proof:

$$|B'F'| = |BC| \quad \text{(opposite sides in a parallelogram)}$$

$$= |AB| \quad \text{(by assumption)}$$

$$|\angle BAE'| = |\angle E'F'B'| \quad \text{(alternate angles)}$$

$$|\angle AE'B| = |\angle F'E'B'| \quad \text{(vertically opposite angles)}$$

$$\therefore \triangle ABE' \text{ is congruent to } \triangle F'B'E' \quad \text{(ASA)}$$

Therefore $|AE'| = |F'E'|$.

$$\text{But } |AE'| = |DE| \text{ and } |F'E'| = |FE| \quad \text{(opposite sides in a parallelogram)}$$

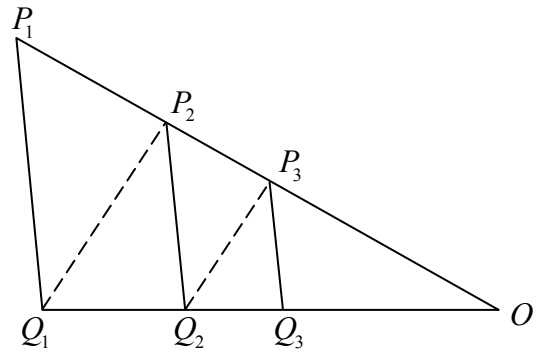
$$\therefore |DE| = |EF|.$$

OR

Question 6B

In the diagram, P_1Q_1 , P_2Q_2 , and P_3Q_3 are parallel and so also are Q_1P_2 and Q_2P_3 .

Prove that $|P_1Q_1| \times |P_3Q_3| = |P_2Q_2|^2$.



$$\frac{|OP_3|}{|OP_2|} = \frac{|OQ_2|}{|OQ_1|} \quad (P_3Q_2 \parallel P_2Q_1) *$$

$$\frac{|OP_2|}{|OP_1|} = \frac{|OQ_2|}{|OQ_1|} \quad (P_2Q_2 \parallel P_1Q_1)$$

$$\therefore \frac{|OP_3|}{|OP_2|} = \frac{|OP_2|}{|OP_1|}$$

$$\frac{|OP_2|}{|OP_1|} = \frac{|P_2Q_2|}{|P_1Q_1|} \quad (\text{Similar triangles})$$

$$\frac{|OP_3|}{|OP_2|} = \frac{|P_3Q_3|}{|P_2Q_2|} \quad (\text{Similar triangles})$$

$$\therefore \frac{|P_2Q_2|}{|P_1Q_1|} = \frac{|P_3Q_3|}{|P_2Q_2|}$$

$$\therefore |P_1Q_1| \times |P_3Q_3| = |P_2Q_2|^2$$

* In the printed copy of the scheme, “ Q_2 ” incorrectly read “ Q_3 ”. This web version has been amended to accurately reflect the scheme as used by examiners.

Answer Question 7 and Question 8.

Question 7

(75 marks)

- (a) Some students are using a database of earthquakes to investigate the times between the occurrences of serious earthquakes around the world. They extract information about all of the earthquakes in the 20th century that caused at least 1000 deaths. There are 115 of these.

The students wonder whether there are patterns in the timing of these earthquakes, so they look at the number of days between each successive pair of these earthquakes.

They make the following table, showing the number of earthquakes for which the time interval from the previous earthquake is as shown.

Time in days from previous earthquake	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800	800 – 1000	1000 – 1300
Number of earthquakes	31	24	12	14	8	7	5	6	5	3

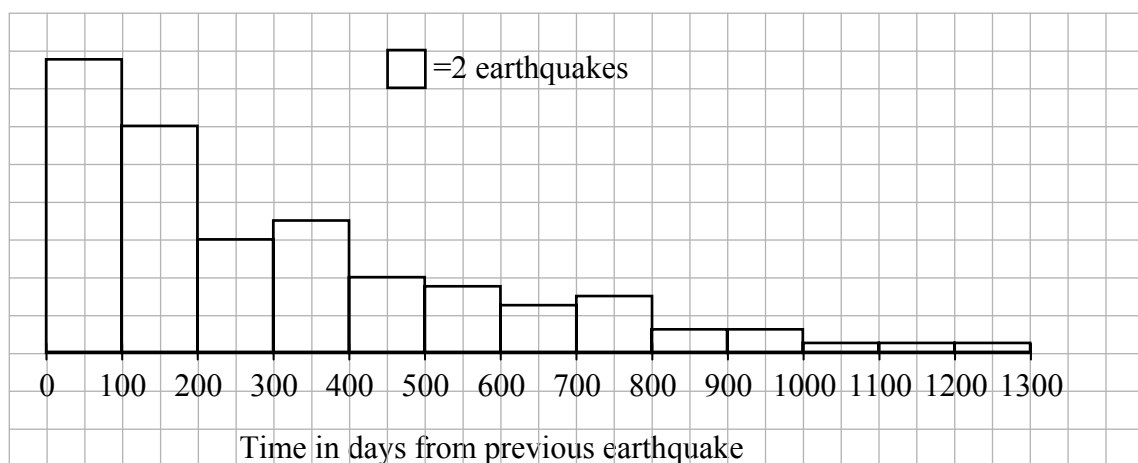
[Source: National geophysical data center, significant earthquake database: www.ngdc.noaa.gov]

- (i) Create a suitable graphical representation of the distribution.

Histogram.

First, divide up unequal intervals, and estimate an allocation of the data (optional step).

800 – 900	900 – 1000	1000 – 1100	1100 – 1200	1200 – 1300
2.5	2.5	1	1	1



- (ii) Describe the distribution. Your description should refer to the shape of the distribution and should include an estimate of the median.

The distribution is skewed to the right. (Or, e.g., there is a lot of data to the left, and it tails off to the right, etc.)

The median is approximately 220 days.

- (iii) The mean time between these earthquakes is 309 days and the standard deviation is 277 days. Suppose that such an earthquake has just occurred and that we want to find the probability that the time to the next one will be between 100 and 200 days. Explain why it would **not** be correct to use standard normal distribution tables (*z*-tables) to do this.

Because the distribution is not normal.

- (iv) Based on the information presented in this question so far, what is the best estimate for the probability described in part (iii) above? Explain your reasoning.

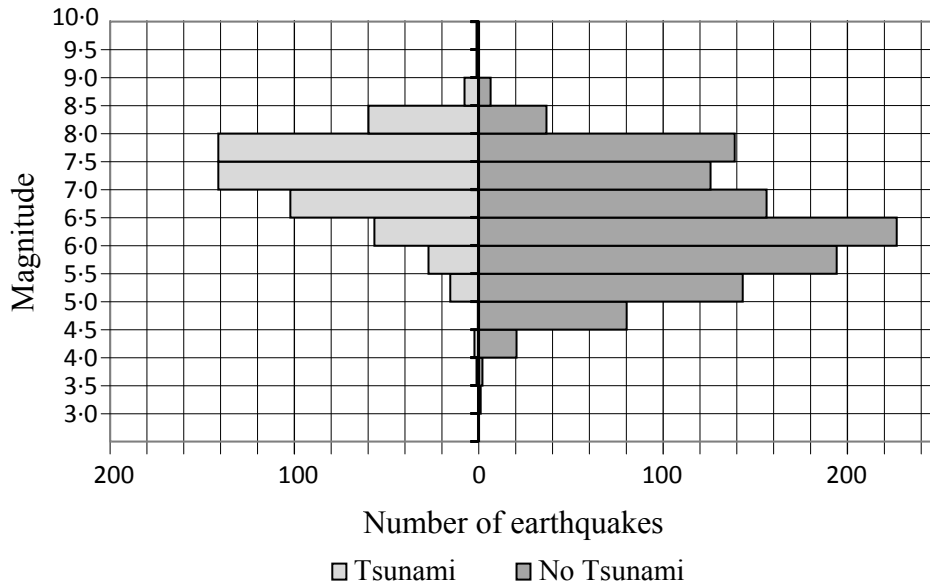
The best estimate is to assume the probability is as reflected in the proportion of such intervals in the historical data.

$$\frac{24}{115} \approx 0.2$$

- (v) As stated at the beginning, the students chose to analyse earthquake timings by looking at the time intervals between the occurrences of a particular type of earthquake. Suggest a different way that they could have looked at the data in the database in order to try to find patterns in the timing of earthquakes.

- They could have looked at the number of earthquakes each year, or some other interval of time (e.g. distribution of earthquakes per decade, per year, etc.)
- They could have redefined serious earthquakes as earthquakes greater than a certain magnitude; earthquakes in less populated areas are not included.
- The data set could have been broadened to include less serious earthquakes. This could result in a different pattern.

- (b) The students heard a reporter saying that “strong earthquakes will cause large destructive ocean waves called tsunamis, while weaker ones will not”. They decide to check this. They draw two histograms back to back, one showing the magnitudes of the earthquakes that caused tsunamis, and the other showing the magnitudes of those that did not. They use all of the suitable data from the 20th century that were recorded in this particular database.



- (i) Comment on the reporter’s statement, using information from the diagram to support your answer, and suggest a more accurate statement.

The statement is too deterministic – strong earthquakes don’t always cause tsunamis and weak ones sometimes do. A better statement would be “Strong earthquakes are more likely to cause tsunamis than weaker ones.”

- (ii) By taking suitable readings from the diagram, estimate the probability that an earthquake of magnitude between 6.5 and 7.0 will cause a tsunami.

About 103 of these did and about 156 didn’t. So probability is $\frac{103}{259} \approx 0.4$

- (iii) Consider the next six earthquakes of magnitude at least 7.5. Find an estimate for the probability that at least four of them will cause a tsunami, assuming that these six events are independent of each other.

$$\text{Tsunami : } 142 + 60 + 8 = 210$$

$$\text{No tsunami: } 139 + 36 + 7 = 182$$

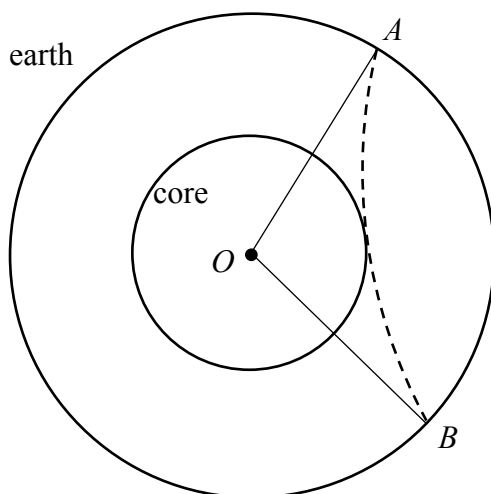
$$p \approx \frac{210}{392} \approx 0.54 *$$

$$\binom{6}{4}(0.54)^4(0.46)^2 + \binom{6}{5}(0.54)^5(0.46) + (0.54)^6 = 0.421$$

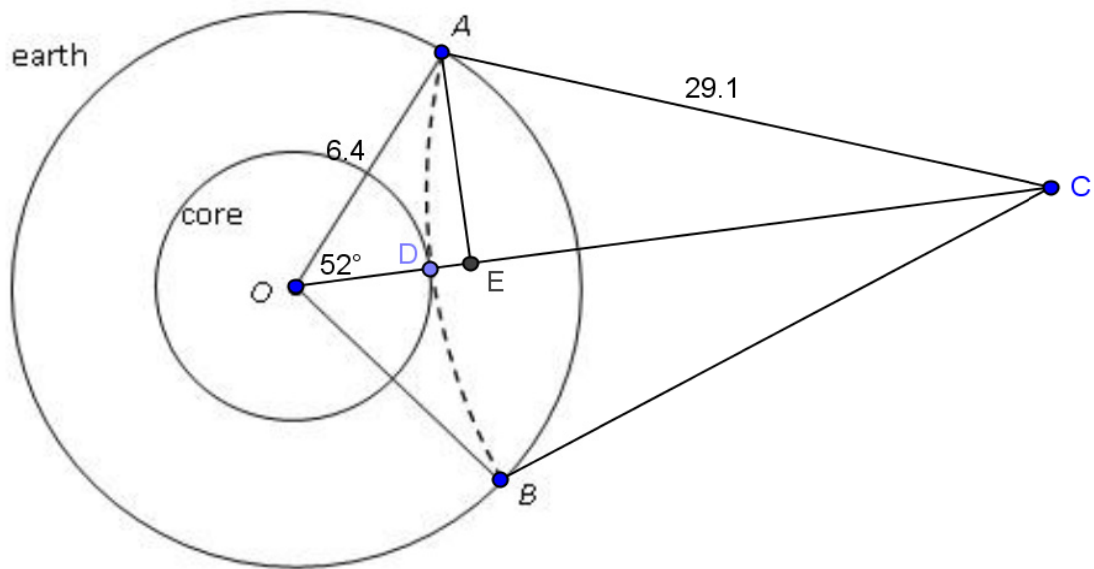
- (c) Scientists use information about seismic waves from earthquakes to find out about the internal structure of the earth.

The diagram below represents a circular cross-section of the earth. The dashed curve represents the path of a seismic wave travelling through the earth from an earthquake near the surface at A to a monitoring station at B . The radius of the earth is 6.4 units and the path of this wave is a circular arc of radius 29.1 units, where 1 unit = 1000 km. Based on information from other stations, it is known that this particular path just touches the earth's core. The angle AOB measures 104° , where O is the centre of the earth.

Find the radius of the earth's core.
(There is space for work on the next page.)



* In the printed copy of the scheme, "210" incorrectly read "310". This web version has been amended to accurately reflect the scheme as used by examiners.



$$|AE| = 6.4 \sin 52 = 5.04$$

$$|OE| = 6.4 \cos 52 = 3.94$$

$$|CE| = \sqrt{29.1^2 - 5.04^2} = 28.66$$

$$|DE| = 29.1 - 28.66 = 0.44$$

$$|OD| = 3.94 - 0.44 = 3.5 \text{ units (or 3500 km).}$$

Or

Let $|OC| = x$. Then, in triangle OAC we have: $(29.1)^2 = (6.4)^2 + x^2 - 2(6.4)x \cos 52^\circ$

$$x^2 - 7.88x - 805.85 = 0$$

$$x = \cancel{24.7} \quad x = 32.6$$

$$|OE| = 32.6 - 29.1 = 3.5 \text{ units (or 3500 km).}$$

Question 8

(75 marks)

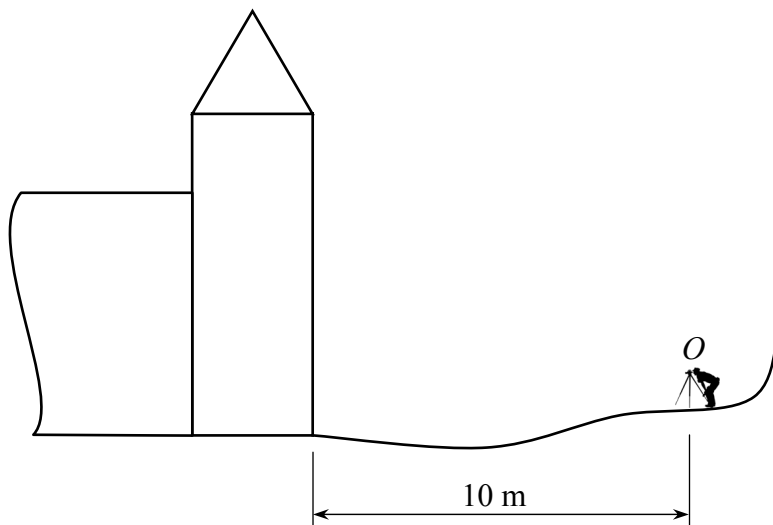
- (a) A tower that is part of a hotel has a square base of side 4 metres and a roof in the form of a pyramid. The owners plan to cover the roof with copper. To find the amount of copper needed, they need to know the total area of the roof.

A surveyor stands 10 metres from the tower, measured horizontally, and makes observations of angles of elevation from the point O as follows:

The angle of elevation of the top of the roof is 46° .

The angle of elevation of the closest point at the bottom of the roof is 42° .

The angle of depression of the closest point at the bottom of the tower is 9° .



- (i) Find the vertical height of the roof.

In the solid triangle:

$$\tan 42^\circ = \frac{y}{10}$$

$$y = 10 \tan 42$$

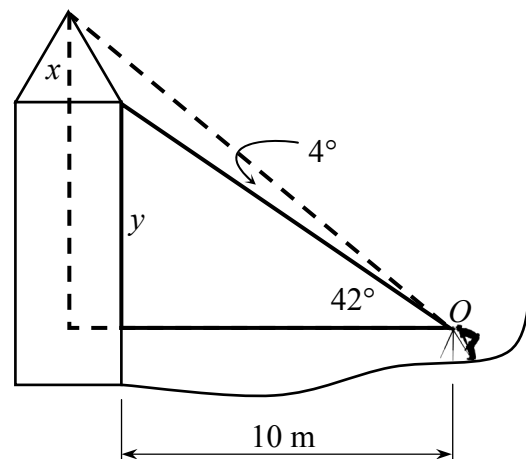
$$y = 9.004$$

In the dashed triangle:

$$\tan 46^\circ = \frac{x+y}{12}$$

$$x+y = 12 \tan 46$$

$$x = 12.426 - 9.004 = 3.42 \text{ m}$$

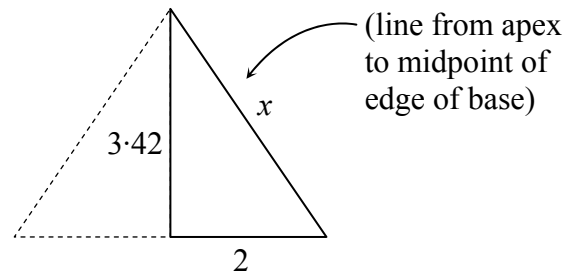


- (ii) Find the total area of the roof.

$$x = \sqrt{3.42^2 + 2^2}$$

$$x = 3.964$$

$$\text{Area} = 4 \left(\frac{1}{2} (4)(3.964) \right) = 31.71 \text{ m}^2$$



- (iii) If all of the angles observed are subject to a possible error of $\pm 1^\circ$, find the range of possible areas for the roof.

Maximum possible area of roof given by:

Angle of elevation of bottom = 41°

Angle of elevation of top = 47°

$$\therefore \text{Height} = 12 \tan 47 - 10 \tan 41 = 4.18 \text{ m}$$

$$x = \sqrt{4.18^2 + 2^2} = 4.634 \text{ m}$$

$$\text{Area} = 37.07 \text{ m}^2$$

Minimum possible area of roof given by:

Angle of elevation of bottom = 43°

Angle of elevation of top = 45°

$$\therefore \text{Height} = 12 \tan 45 - 10 \tan 43 = 2.675 \text{ m}$$

$$x = \sqrt{2.675^2 + 2^2} = 3.34 \text{ m}$$

$$\text{Area} = 26.72 \text{ m}^2$$

$$26.72 \text{ m}^2 \leq \text{area of roof} \leq 37.07 \text{ m}^2.$$

- (b) Twenty five students each measure and record a particular angle of elevation, in degrees, each using his or her own home-made clinometer. The results are as follows:

24	20	22	15	70
15	16	15	16	15
18	16	21	21	73
16	20	12	18	20
18	18	14	22	18

- (i) Find what you consider to be the best estimate of the true value of the angle, explaining your reasoning.

The data include two extreme outliers: 70 and 73. These probably arise from students reading the complementary angle ($90^\circ - \theta$) on their clinometers. The most reasonable course of action is either to correct them or discard them.

- Change 70 to 20 and 73 to 17. Then the mean = 17.88° is the best estimate for the true value of the angle.
- Exclude 70 and 73 as outliers. Then the mean = 17.83° .
- Use the median, as it less influenced by outliers. Median = 18, (irrespective of treatment of outliers).

- (ii) Based on previous experience, a teacher has claimed that, in these circumstances, half of all students will measure the angle correctly to within two degrees. Taking these students to be a simple random sample, and assuming the true value of the angle is the one you calculated in part (i), is there sufficient evidence to reject the teacher's claim at the 5% level of significance?

H_0 : Half of all students will measure the angle correctly to within 2° .

95% margin of error for sample of size 25: $\frac{1}{\sqrt{25}} = \frac{1}{5} = 0.2$.

Reject H_0 if sample proportion lies outside 0.5 ± 0.2 .

Taking 17.88° (or 17.83°) as the best estimate, then 8 students measured correctly to within 2°

Sample proportion: $\frac{8}{25} = 0.32$. \therefore Do not reject H_0 . Answer: No.

Or

Taking 18° as the best estimate, then 12 students measured correctly to within 2°

Sample proportion: $\frac{12}{25} = 0.48$. \therefore Do not reject H_0 . Answer: No.

Or

Confidence interval at 5% level of significance = 0.32 ± 0.2 or 0.48 ± 0.2 . In either case, this interval contains the hypothesised proportion of 0.5, so do not reject H_0 .

Or

Exact (binomial) test with $p = 0.5$, $n = 25$ yields acceptance for r from 8 to 17 inclusive.

Marking Scheme – Paper 2

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	5
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5		
10 mark scale		0, 6, 10	0, 6, 8, 10	0, 4, 5, 8, 10	
15 mark scale			0, 9, 13, 15	0, 8, 10, 13, 15	
20 mark scale		0, 12, 20	0, 10, 18, 20	0, 10, 16, 18, 20	0, 5, 10, 14, 17, 20
25 mark scale				0, 15, 20, 23, 25	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

- (a) 20D
- (b) 5B

Question 2

- (a) 10C
- (b) (i) 10B
- (ii) 5B

Question 3

- (a) 15C
- (b) 5C
- (c) 5B

Question 4

- (a) 15B
- (b) 5B
- (c) 5B

Question 5

- Intersection: 15D
- Radius & Centre: 5B
- Equation: 5B

Question 6A

- diag. & given: 5B
- construction: 10B
- proof: 10C

Question 6B

25C

Section B

Question 7

- (a) (i) 20C
- (ii) 10B
- (iii) 5B
- (iv) 5C
- (v) 10B
- (b) (i) 5B
- (ii) 5B
- (iii) 5C
- (c) 10D

Question 8

- (a) (i) 25D
- (ii) 10C
- (iii) 5C
- (b) (i) 25C
- (ii) 10C

Detailed marking notes

Section A

Question 1

(a) Scale 20D (0, 10, 16, 18, 20).

Low partial credit:

- Writes formula, $z = \frac{x - \mu}{\sigma}$ and stops.
- Identifies $\mu = 20$ and $\sigma = 5$

Mid partial credit:

- Substitutes correctly into formula.

High partial credit

- Finds $P(z \leq 1.2)$.

(b) Scale 5B (0, 3, 5).

Partial credit:

- Any correct probability.
- Any correct probability in terms of x .
- $12 - x$

Question 2

(a) Scale 10C (0, 6, 8, 10)

Low partial credit:

- Explains correlation.

High partial credit

- Correct explanation, without example.

(b) (i) Scale 10B (0, 6, 10)

Partial credit:

- Estimate $-0.3 \leq r \leq 0.3$

(ii) Scale 5B (0, 3, 5)

Partial credit:

- No relationship.
- 'Weak relationship'
- Symmetric.

Question 3

(a) Scale 15 C (0, 9, 13, 15)

Low partial credit:

- One correct.
- Writes any of the equations in the form $y = mx + c$.

High partial credit

- Two correct.

(b) Scale 5C (0, 3, 4, 5)

Low partial credit

- Scaled but incorrect scaling on both axes.

High partial credit.

- One axis scaled correctly.

(c) Scale 5B (0, 3, 5)

Partial credit:

- Correct slope. $m = -\frac{1}{2}$.
- Coordinates, (0, 2) and (4, 0) indicated on a diagram.
- $y = \frac{1}{2}x + 2$
- $x + 2y = c$

Question 4

(a) Scale 15B (0, 9, 15).

Partial credit

- Scale factor = $\frac{2}{3}$ or 0.67.
- Uses ruler to measure lengths and finds ratio.

(b) Scale 5B (0, 3, 5).

Partial credit:

- Joins together two corresponding vertices.
- Joins together two corresponding circumcentres or incentres.

(c) Scale 5B (0, 3, 5)

Accept ratio from part (i)

Partial credit

- Estimate from diagram.
- Similar or equiangular triangles mentioned.
- Perpendiculars from Q to $[PR]$ and/or Y to $[XZ]$ shown on the diagram.

Question 5

Intersection of line and circle: Scale 15D (0, 8, 10, 13, 15)

Low partial credit

- Writes x in terms of y and stops.

Mid Partial:

- Substitutes to form quadratic.

High partial credit:

- Solves for y .

Radius and centre: Scale 5B (0, 3, 5).

Partial credit:

- Finds one of radius, centre or diameter.

Equation. Scale 5B (0, 3, 5)

Partial credit:

- Sign error.
- Does not square r .

Question 6A

Diagram & Given: Scale 5B (0, 3, 5)

Partial Credit:

- Diagram or given

Construction: Scale 10B (0, 6, 10)

Partial Credit:

- Construction not explicit

Main body of proof: Scale 10C (0, 6, 8, 10)

High partial credit:

- Correct proof with one critical step omitted.

Low partial credit:

- More than one critical step omitted, but there is still some substantial work of merit.

Question 6B

Scale 25C (0, 15, 22, 25)

Low partial credit:

- Any relevant proportion, i.e. statement of equality of two ratios.

High Partial credit:

- Two proportions at least one having a ratio that does not include O .

Section B

Question 7

(a) (i) Scale 20C (0, 10, 18, 20)

Accept any reasonable way of handling the last two intervals.

Accept any other reasonable representation e.g. frequency polygon, box plot.

Low partial credit:

- Scaled Axis.

High Partial credit:

- Representation with some minor errors.

(a) (ii) Scale 10B (0, 6, 10)

Accept $200 < \text{median} < 300$.

Partial credit:

- Describes shape without giving median, or gives median without describing shape.
- No median found but statement such as “mean greater than median”

(a) (iii) Scale 5B (0, 3, 5)

Partial credit:

- The distribution is skewed and no mention of normal distributions.
- Not symmetrical.

(a) (iv) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Some probability provided $0 < P(x) < 1$.

High partial credit:

- Probability without explanation.

(a) (v) Scale 10B (0, 6, 10)

Partial credit:

- Suggestion that is not related to the timing of earthquakes.
- Look at another database containing information on earthquakes.

(b) (i) Scale 5B (0, 3, 5)

Partial credit:

- Comment on reporters statement with no alternative statement.
- Some true statement e.g. ‘Strong earthquakes under the ocean cause tsunamis.’

(b) (ii) Scale 5B (0, 3, 5)

Accept 100 to 110 for *tsunami* readings and 150 to 160 for *no tsunami*

Partial credit:

- Some relevant readings from the diagram, but no probability calculated.
- Mishandles information.
- Gives reading from row above or row below.

(b) (iii) Scale 5C (0, 3, 4, 5)

Accept any reasonable p with work.

Low partial credit:

- Any term of binomial distribution calculated.

High partial credit:

- Two terms of the binomial distribution calculated.

(c) Scale 10 D(0, 4, 5, 8, 10)

Low partial credit

- Diagram with 104° indicated.

Mid Partial:

- Some work on angles, e.g. Finds 38° or 52° .

High partial credit:

- Finds the correct measure of one side in a relevant diagram.

Question 8

(a) (i) Scale 25D (0, 15, 20, 23, 25)

Accept height from floor to apex.

Low partial credit:

- Diagram with some relevant information.

Mid partial credit:

- Calculates the height to the base of the roof.
- Uses some relevant trigonometry.

High Partial credit:

- Calculates the height to the base of the roof and calculates height to apex.

(a) (ii) Scale 10C (0, 6, 8, 10)

Accept answers from (i)

Low partial credit:

- Cross section of the roof showing height.

High partial credit:

- Calculates slant height.
- Shows cross section with height and base length of roof.

(a) (iii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Area formula
- Finds new heights.
- Any correct work towards finding maximum and minimum areas.

High partial credit:

- Finds maximum and minimum areas but mishandles angles.

(b) (i) Scale 25C (0, 15, 22, 25)

Low partial credit:

- Calculates mean and includes 70 and 73.

High partial credit:

- Error in calculating median.
- Mode given.

(b) (ii) Scale 10C (0, 6, 8, 10)

Low partial credit:

- Sample proportion, or margin of error calculated.

High partial credit:

- Both of the above calculated or confidence interval.

Marcanna Breise as ucht Freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ghnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ghnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. $198 \text{ marc} \times 5\% = 9.9 \Rightarrow \text{bónas} = 9 \text{ marc}$.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle $[300 - \text{bunmharc}] \times 15\%$, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

