Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2003

MATHEMATICS — HIGHER LEVEL

PAPER 2
(300 marks)

MONDAY, 9 JUNE — MORNING, 9:30 to 12:00

Attempt **FIVE** questions from Section **A** and **ONE** question from Section **B**.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

SECTION A

Answer FIVE questions from this section.

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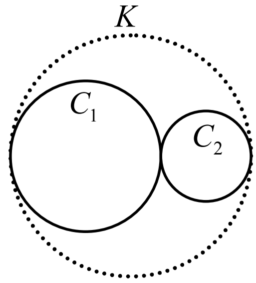
1. **(a)** For all values of $t \in \mathbf{R}$, the point $\left(\dfrac{3-3t^2}{1+t^2}, \dfrac{6t}{1+t^2}\right)$ lies on the circle $x^2 + y^2 = r^2$.

Find r, the radius of the circle.

(b) $C_1: x^2 + y^2 + 2x - 2y - 23 = 0$ and
$C_2: x^2 + y^2 - 14x - 2y + 41 = 0$ are two circles.



(i) Prove that C_1 and C_2 touch externally.

(ii) K is a third circle.
Both C_1 and C_2 touch K internally.
Find the equation of K.

(c) The line $ax + by = 0$ is a tangent to the circle $x^2 + y^2 - 12x + 6y + 9 = 0$ where $a, b \in \mathbf{R}$ and $b \neq 0$.

(i) Show that $\dfrac{a}{b} = -\dfrac{3}{4}$.

(ii) Hence, or otherwise, find the co-ordinates of the point of contact.

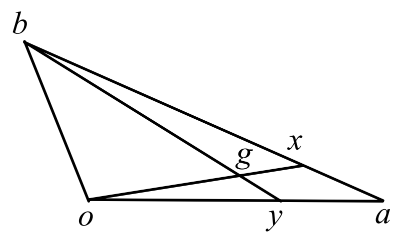
2. **(a)** $oabc$ is a parallelogram where o is the origin, $\vec{a} = 3\vec{i} - \vec{j}$ and $\vec{b} = 4\vec{i} + 3\vec{j}$.
Express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) $\vec{p} = 2\vec{i} + \vec{j}$, $\vec{q} = 3\vec{i} + k\vec{j}$, $\vec{r} = 3\vec{i} + t\vec{j}$ where $k, t \in \mathbf{R}$ and o is the origin.

(i) Given that $\vec{p} \perp \vec{q}$, calculate the value of k.

(ii) Given that $|\angle por| = 45°$, calculate the two possible values of t.

(c) oab is a triangle where o is the origin.



(i) x is a point on $[ab]$ such that $|ax| : |xb| = 1 : 3$.
Express $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$.

(ii) y is a point on $[oa]$ such that $|oy| : |ya| = 2 : 1$.
Express $\overrightarrow{by}$ in terms of $\vec{a}$ and $\vec{b}$.

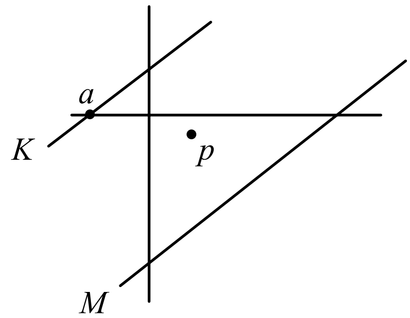
(iii) $[ox]$ and $[by]$ intersect at g.
Given that $\vec{g} = m\vec{x}$ and $\overrightarrow{bg} = n\overrightarrow{by}$ where $m, n \in \mathbf{R}$, find the value of m and the value of n.

3. **(a)** f is the transformation $(x, y) \to (x', y')$ where $x' = x + y$ and $y' = x - y$.
L is the line $4x - 2y - 1 = 0$.

Find the equation of $f(L)$, the image of L under f.

(b) K is the line $3x - 4y + 9 = 0$.
The point $a(-3, 0)$ is on K.

The line M is parallel to K.
The point $p(2, -1)$ is midway between K and M.



(i) Find the equation of M.

(ii) Calculate the distance between K and M.

(iii) Calculate the measure of the acute angle between ap and K.
Give your answer correct to the nearest degree.

(iv) $b(x, y)$ is a point on K such that $|ab| = 15$ and $x > 0$.
Find the value of x and the value of y.

4. **(a)** The circumference of a circle is 30π cm.
The area of a sector of the circle is 75 cm^2.

Find, in radians, the angle in this sector.

(b) Find all the solutions of the equation

$$\sin 2x + \sin x = 0$$

in the domain $0° \leq x \leq 360°$.

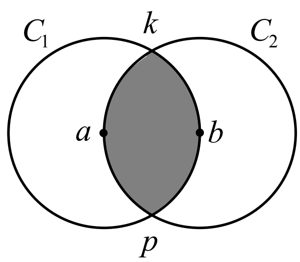
(c) C_1 is a circle with centre a and radius r.

C_2 is a circle with centre b and radius r.

C_1 and C_2 intersect at k and p.

$a \in C_2$.

$b \in C_1$.

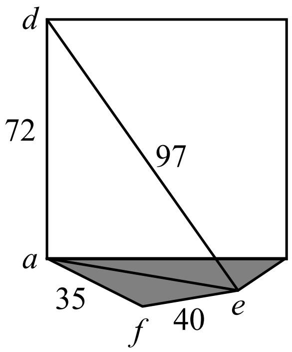


(i) Find, in radians, the measure of angle kap.

(ii) Calculate the area of the shaded region.
Give your answer in terms of r and π.

5. **(a)** Find the value of $\sin 15°$ in surd form.

(b) a, f and e are points on horizontal ground.
d is a point on a vertical wall directly above a.



$|ad| = 72$ m, $|de| = 97$ m,
$|af| = 35$ m and $|fe| = 40$ m.

(i) Calculate $|ae|$.

(ii) Hence, calculate $|\angle afe|$.

(c) **(i)** Using the identity $\cos(A - B) = \cos A \cos B + \sin A \sin B$, or otherwise, prove:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

(ii) Prove:

$$\sin(A + B)\sin(A - B) = (\sin A + \sin B)(\sin A - \sin B).$$

6. **(a)** Eight people, including Kieran and Anne, are available to form a committee.
Five people must be chosen for the committee.

(i) In how many ways can the committee be formed if both Kieran and Anne must be chosen?

(ii) In how many ways can the committee be formed if neither Kieran nor Anne can be chosen?

(b) **(i)** Solve the difference equation $u_{n+2} - 4u_{n+1} + 3u_n = 0$, where $n \geq 0$, given that $u_0 = -2$ and $u_1 = 4$.

(ii) Verify that the solution you have obtained in **(i)** satisfies the difference equation.

(c) Ten discs, each marked with a different whole number from 1 to 10, are placed in a box.
Three of the discs are drawn at random (without replacement) from the box.

(i) What is the probability that the disc with the number 7 is drawn?

(ii) What is the probability that the three numbers on the discs drawn are odd?

(iii) What is the probability that the product of the three numbers on the discs drawn is even?

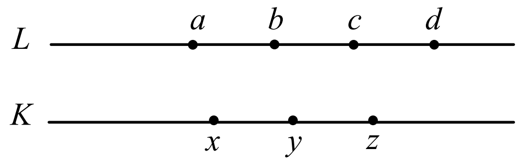
(iv) What is the probability that the smallest number on the discs drawn is 4?

7. **(a)** Five cars enter a car park. There are exactly five vacant spaces in the car park.

(i) In how many different ways can the five cars park in the vacant spaces?

(ii) Two of the cars leave the car park without parking.
In how many different ways can the remaining three cars park in the five vacant spaces?

(b)



L and K are distinct parallel lines.

a, b, c and d are points on L such that $|ab| = |bc| = |cd| = 1$ cm.

x, y and z are points on K such that $|xy| = |yz| = 1$ cm.

(i) How many different triangles can be constructed using three of the named points as vertices?

(ii) How many different quadrilaterals can be constructed using four of the named points as vertices?

(iii) How many different parallelograms can be constructed using four of the named points as vertices?

(iv) If one quadrilateral is constructed at random, what is the probability that it is *not* a parallelogram?

(c) The mean of the real numbers a and b is $\overline{x}$.
The standard deviation is σ.

(i) Express σ in terms of a, b and $\overline{x}$.

(ii) Hence, express σ in terms of a and b only.

(iii) Show that $\overline{x}^2 - \sigma^2 = ab$.

SECTION B

Answer ONE question from this section.

8. **(a)** Use integration by parts to find $\int xe^{-5x}\,dx$.

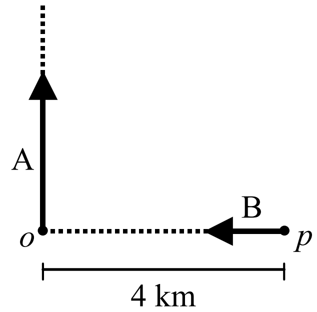
(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series.

(i) Derive the Maclaurin series for $f(x) = \log_e(1+x)$ up to and including the term containing x^4.

(ii) Write down the general term and use the Ratio Test to show that the series converges for $-1 < x < 1$.

(c) The point p is 4 km due east of the point o.

At noon, A leaves o and travels north at a steady speed of 12 km/h. At the same time, B leaves p and travels towards o at a steady speed of 6 km/h.



(i) Write down expressions in x for the distances that A and B will each have travelled at x minutes after noon.

(ii) Find an expression in x for the distance that B will be from A at x minutes after noon.

(iii) At how many minutes after noon will B be closest to A?

9. **(a)** z is a random variable with standard normal distribution.
Calculate $P(-2.13 < z \leq 1.46)$.

(b) Whenever Anne's mobile phone rings, the probability that she answers the call is $\frac{3}{4}$.

A friend phones Anne six times.

(i) What is the probability that she misses all the calls?

(ii) What is the probability that she misses the first two calls and answers the others?

(iii) What is the probability that she answers exactly one of the calls?

(iv) What is the probability that she answers at least two of the calls?

(c) In a newspaper advertisement, a driving school claimed that 80% of its clients passed their driving test on their first attempt.

1000 people who had attended the school and who had taken the test for the first time were randomly selected.

Find, at the 5% level of significance, the interval in which the number who passed should lie in order that the claim made in the advertisement be accepted.

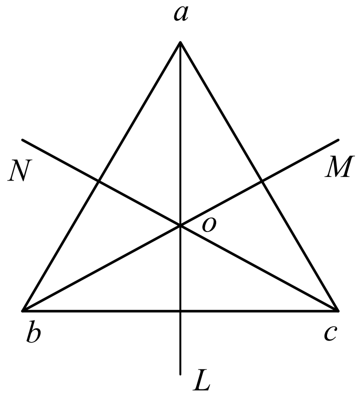
10. **(a)** $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$M = \{A, B, C, D\}$ is a cyclic group under matrix multiplication.

Verify that A is a generator of M.

(b) abc is an equilateral triangle.
L, M and N, the perpendicular bisectors of the sides, intersect at o.

$D_3 = \{I_\pi, R_{120°}, R_{240°}, S_L, S_M, S_N\}$, under composition, is the symmetry group of abc.



(i) Investigate whether $S_L \circ S_M = S_M \circ S_L$.

(ii) Write down the centralizer of $R_{120°}$.

(c) $P = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \right\}$

P is a group under composition of permutations.
D_3 is the symmetry group of the equilateral triangle abc, as described in **(b)**.

$f : D_3 \to P$ is an isomorphism where $f(R_{120°}) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$.

Find $f(R_{120°}{}^{-1})$, justifying your answer.

11. **(a)** The polar of a point p with respect to the circle $x^2 + y^2 = 9$ is $2x - 5y = 27$.

Given that the polar of the point (x_1, y_1) with respect to the circle $x^2 + y^2 = r^2$ is $xx_1 + yy_1 = r^2$, find the coordinates of p.

(b) f is a similarity transformation.
f maps the angle θ onto the angle ϕ.

Prove that θ and ϕ are equal in measure.

(c) **(i)** [cd] is a diameter of the ellipse $\dfrac{x^2}{100} + \dfrac{y^2}{25} = 1$ where c is the point $\left(5\sqrt{2}, \dfrac{5}{2}\sqrt{2}\right)$.

Find the equation of the tangent to the ellipse at c.

(ii) The diameter [st] is conjugate to the diameter [cd].
Find the equation of st and the coordinates of the points s and t.

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