# Coimisiún na Scrúduithe Stáit
# State Examinations Commission

## LEAVING CERTIFICATE EXAMINATION, 2007

## MATHEMATICS – HIGHER LEVEL

**PAPER 2 ( 300 marks )**

**MONDAY, 11 JUNE – MORNING, 9:30 to 12:00**

Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

**WARNING: Marks will be lost if all necessary work is not clearly shown.**

**Answers should include the appropriate units of measurement, where relevant.**

## SECTION A

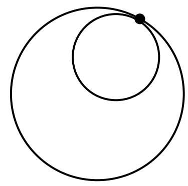
**Answer FIVE questions from this section.**

**1.** **(a)** The following parametric equations define a circle:

$x = 5 + 7\cos\theta, \quad y = 7\sin\theta,$ where $\theta \in \mathbf{R}$.

What is the Cartesian equation of the circle?

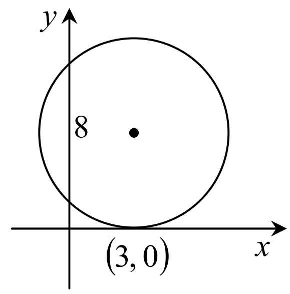
**(b)** $x^2 + y^2 - 4x - 6y + 5 = 0$ and $x^2 + y^2 - 6x - 8y + 23 = 0$ are two circles.



**(i)** Prove that the circles touch internally.

**(ii)** Find the coordinates of the point of contact of the two circles.

**(c)** A circle has its centre in the first quadrant.
The $x$-axis is a tangent to the circle at the point (3, 0).
The circle cuts the $y$-axis at points that are 8 units apart.
Find the equation of the circle.




**2.** **(a)** $\vec{x} = -2\vec{i} + 5\vec{j}$ and $\vec{x}\vec{y} = -6\vec{i} - 8\vec{j}$. Express $\vec{y}$ in terms of $\vec{i}$ and $\vec{j}$.

**(b)** $\vec{a} = 5\vec{i}$ and $\vec{b} = \sqrt{3}\,\vec{i} + 3\vec{j}$.

**(i)** Show that $\overrightarrow{ab}$ is not perpendicular to $\vec{b}$.

**(ii)** Find the value of the real number $k$, given that $\vec{c} = k\vec{b}$ and $\overrightarrow{ac} \perp \vec{b}$.

**(c)** $\vec{p} = 3\vec{i} + 4\vec{j}$ and $\vec{q} = 5\vec{i} + 12\vec{j}$.

$\vec{r} = \dfrac{65t}{16}\left(\dfrac{\vec{p}}{|\vec{p}|} + \dfrac{\vec{q}}{|\vec{q}|}\right)$, where $t > 0$.

**(i)** Express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

**(ii)** Find $\vec{p}.\vec{r}$ and $\vec{q}.\vec{r}$.

**(iii)** Hence, show that $r$ is on the bisector of $\angle poq$, where $o$ is the origin.

**3.** **(a)** Find the area of the triangle with vertices $(1,1)$, $(8,-5)$ and $(5,-2)$.

**(b)** $f$ is the transformation $(x, y) \to (x', y')$, where $x' = 4x + 2y$ and $y' = -3x - y$.
$K$ is the line $x + y = 0$.

**(i)** Show that $K$ is its own image under $f$.

**(ii)** $p\ (1, -1)$ and $q\ (3, -3)$ are two points.
Find the ratio $|pq| : |f(p)f(q)|$, giving your answer in its simplest form.

**(c)** Consider the equation $k(3x - 5y + 6) + l(5x - 7y + 4) = 0$, where $k, l \in \mathbf{R}$.

**(i)** Show that for all $k$ and $l$, the given equation represents a line passing through the point of intersection of $3x - 5y + 6 = 0$ and $5x - 7y + 4 = 0$.

**(ii)** Find the relationship between $k$ and $l$ for which the given equation represents a line of slope 2.

**(iii)** If $k = 1$, what line through the point of intersection cannot be represented by the given equation? Justify your answer.

**4.** **(a)** Show that $(\cos A + \sin A)^2 = 1 + \sin 2A$.

**(b)** Find all the solutions of the equation

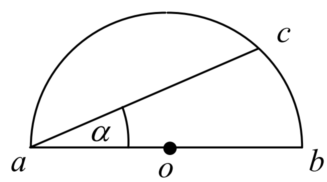
$$6\cos^2 x + \sin x - 5 = 0, \text{ where } 0^\circ \le x \le 360^\circ.$$

Give the solutions correct to the nearest degree.

**(c)** $[ab]$ is the diameter of a semicircle of centre $o$ and radius-length $r$.
$[ac]$ is a chord such that $|\angle cab| = \alpha$, where $\alpha$ is in radian measure.

**(i)** Find $|ac|$ in terms of $r$ and $\alpha$.

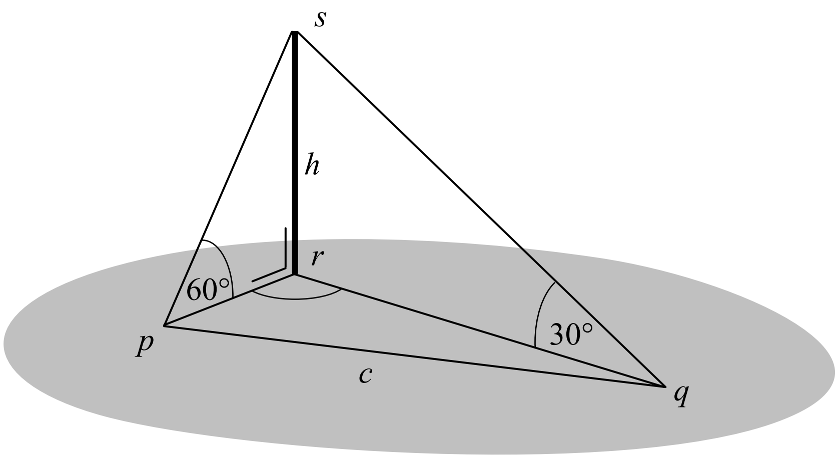
**(ii)** $[ac]$ bisects the area of the semicircular region.
Show that $2\alpha + \sin 2\alpha = \frac{\pi}{2}$.



**5.** **(a)** Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 2x}{\sin 3x}$.

**(b)** Using the formula $\cos(A+B) = \cos A\cos B - \sin A\sin B$, derive a formula for $\cos(A-B)$ and hence prove that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

**(c)** *p, q* and *r* are three points on horizontal ground.
[*sr*] is a vertical pole of height *h* metres.
The angle of elevation of *s* from *p* is 60° and the angle of elevation of *s* from *q* is 30°.
$|pq| = c$ metres.
Given that $3c^2 = 13h^2$, find $|\angle prq|$.



**6.** **(a)** Six people, including Mary and John, sit in a row.

**(i)** How many different arrangements of the six people are possible?

**(ii)** In how many of these arrangements are Mary and John next to each other?

**(b)** $\alpha$ and $\beta$ are the roots of the quadratic equation $px^2 + qx + r = 0$.
$u_n = l\alpha^n + m\beta^n$, for all $n \in \mathbf{N}$.
Show that $pu_{n+2} + qu_{n+1} + ru_n = 0$, for all $n \in \mathbf{N}$.

**(c)** $w$ white discs and $r$ red discs are placed in a box. Two of the discs are drawn at random from the box. The probability that both discs are red is $p$.

**(i)** Find $p$ in terms of $w$ and $r$.

**(ii)** When $w = 1$, find the value of $r$ for which $p = \frac{1}{2}$.

**(iii)** There are other values of $w$ and $r$ that also give $p = \frac{1}{2}$.

The next smallest such value of $w$ is even.

By investigating the even numbers in turn, find this value of $w$ and the corresponding value of $r$.

**7.** **(a)** **(i)** How many different selections of four letters can be made from the letters of the word FLORIDA ?

**(ii)** How many of these selections contain at least one vowel?

**(b)** Two dice are thrown.

**(i)** What is the probability of getting two identical numbers or a total of five?

**(ii)** What is the probability that the product of the two numbers thrown is at least twice their sum?

**(c)** **(i)** Find, in terms of $a$ and $d$, the mean of the first seven terms of an arithmetic sequence with first term $a$ and common difference $d$.

**(ii)** Show that the standard deviation of these seven numbers is $2d$.

## SECTION B

**Answer ONE question from this section.**

**8.** **(a)** $p$ and $q$ are real numbers such that $p + q = 1$.
Find the value of $p$ that maximizes the product $pq$.

**(b)** **(i)** Derive the Maclaurin series for $f(x) = (1+x)^m$ up to and including the term containing $x^3$.

**(ii)** Given that the general term of the series $f(x)$ is

$$\frac{m(m-1)(m-2).........(m-r+1)}{r!}x^r,$$

show that the series converges for $-1 < x < 1$.

**(c)** Evaluate $\displaystyle\int_0^1 \tan^{-1}x\,dx$.

**9.** **(a)** Two events $E_1$ and $E_2$ are independent. $P(E_1) = \frac{1}{5}$ and $P(E_2) = \frac{1}{7}$. Find

**(i)** $P(E_1 \cap E_2)$

**(ii)** $P(E_1 \cup E_2)$.

**(b)** Five unbiased coins are tossed.

**(i)** Find the probability of getting three heads and two tails.

**(ii)** The five coins are tossed eight times. Find the probability of getting three heads and two tails exactly four times.
Give your answer correct to three decimal places.

**(c)** The amounts due on monthly mobile phone bills are normally distributed with mean €53 and standard deviation €15.

**(i)** If a bill is chosen at random, find the probability that the amount due is between €47 and €74.

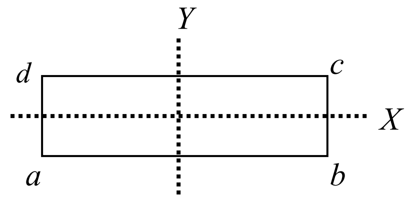
**(ii)** A random sample of 900 bills is taken. Find the probability that the mean amount due on the bills in the sample is greater than €53·30.

**10.** **(a)** For each of the following, give a reason why it is not a group.

**(i)** The set of natural numbers under subtraction.

**(ii)** The set of real numbers under multiplication.

**(b)** $G = \{I_\pi, R_{180^\circ}, S_X, S_Y\}$ is the set of symmetries of the rectangle *abcd*.




**(i)** Show that $G$ is a group under composition.
You may assume that composition of symmetries is associative.

**(ii)** Find $Z(G)$, the centre of the group.

**(c)** Use Lagrange's theorem to prove that

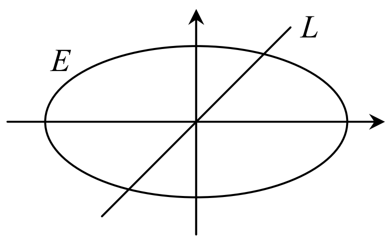
**(i)** any group of prime order is cyclic.

**(ii)** the order of any element of a finite group $G$ divides the order of $G$.

**11.** **(a)** Find the eccentricity of the ellipse with equation $\frac{x^2}{64} + \frac{y^2}{48} = 1$.

**(b)** Prove that a similarity transformation maps the orthocentre of a triangle onto the orthocentre of the image of the triangle.

**(c)** $E$ is the ellipse $\frac{x^2}{4} + y^2 = 1$ and $L$ is the line $y = x$.
Using a transformation that maps $E$ to the unit circle, or otherwise, find the equation of the diameter that is conjugate to $L$ in $E$.



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