Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2014

Marking Scheme

Mathematics
(Project Maths – Phase 3)

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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2014. M329

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 1

Higher Level

Friday 6 June   Afternoon 2:00 – 4:30

300 marks

Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

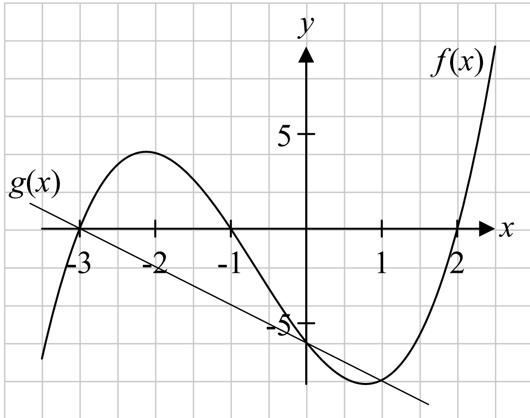
Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) The graph of a cubic function $f(x)$ cuts the x-axis at $x=-3$, $x=-1$ and $x=2$, and the y-axis at $(0, -6)$, as shown.

Verify that $f(x)$ can be written as
$$f(x) = x^3 + 2x^2 - 5x - 6.$$



$x=-3, \quad x=-1, \quad x=2$

$f(x) = (x+3)(x+1)(x-2) = x^3 + 2x^2 - 5x - 6$

OR

$f(x) = x^3 + 2x^2 - 5x - 6$

$f(-3) = -27 + 18 + 15 - 6 = 0 \Rightarrow (x+3)$ is a factor

$f(-1) = -1 + 2 + 5 - 6 = 0 \Rightarrow (x+1)$ is a factor

$f(2) = 8 + 8 - 10 - 6 = 0 \Rightarrow (x-2)$ is a factor

$f(x) = (x+3)(x+1)(x-2) = x^3 + 2x^2 - 5x - 6$

(b) **(i)** The graph of the function $g(x) = -2x - 6$ intersects the graph of the function $f(x)$ above. Let $f(x) = g(x)$ and solve the resulting equation to find the co-ordinates of the points where the graphs of $f(x)$ and $g(x)$ intersect.

$$f(x) = g(x)$$
$$x^3 + 2x^2 - 5x - 6 = -2x - 6$$
$$\Rightarrow x^3 + 2x^2 - 3x = 0$$
$$\Rightarrow x(x^2 + 2x - 3) = 0$$
$$\Rightarrow x(x-1)(x+3) = 0$$
$$\Rightarrow x = 0, \quad x = 1, \quad x = -3$$
$$\Rightarrow y = -6, \;\; y = -8, \;\; y = 0$$

Points: $(-3, 0), (0, -6), (1, -8)$

(ii) Draw the graph of the function $g(x) = -2x - 6$ on the diagram above.

$$g(x) = -2x - 6$$
$$g(-3) = -2(-3) - 6 = 6 - 6 = 0 \Rightarrow (-3, 0)$$
$$g(0) = -2(0) - 6 = -6 \Rightarrow (0, -6)$$

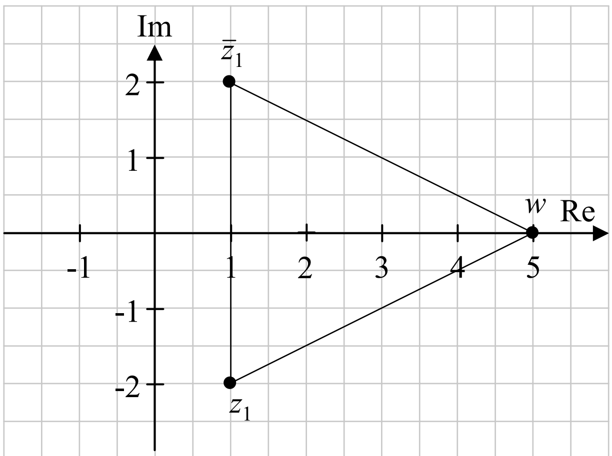
Question 2 **(25 marks)**

Let $z_1 = 1 - 2i$, where $i^2 = -1$.

(a) The complex number z_1 is a root of the equation $2z^3 - 7z^2 + 16z - 15 = 0$.
Find the other two roots of the equation.

> $z_1 = 1 - 2i$ a root $\Rightarrow \bar{z}_1 = 1 + 2i$ a root.
> $(z - 1 + 2i)(z - 1 - 2i) = z^2 - 2z + 5$, a factor
> Hence, $(z^2 - 2z + 5)(az + b) = 2z^3 - 7z^2 + 16z - 15$
> Equate coefficients: $a = 2$ and $b - 2a = -7 \quad \Rightarrow \quad b = -3$
> Third factor: $2z - 3 \quad \Rightarrow \quad z = \frac{3}{2}$
>
> Or
>
> $(2z^3 - 7z^2 + 16z - 15) \div (z^2 - 2z + 5) = 2z - 3$
> Third factor: $2z - 3 \quad \Rightarrow \quad z = \frac{3}{2}$
> Other roots: $z_2 = 1 + 2i$, $z_3 = \frac{3}{2}$

(b) **(i)** Let $w = z_1.\bar{z}_1$, where $\bar{z}_1$ is the conjugate of z_1. Plot z_1, $\bar{z}_1$ and w on the Argand diagram and label each point.

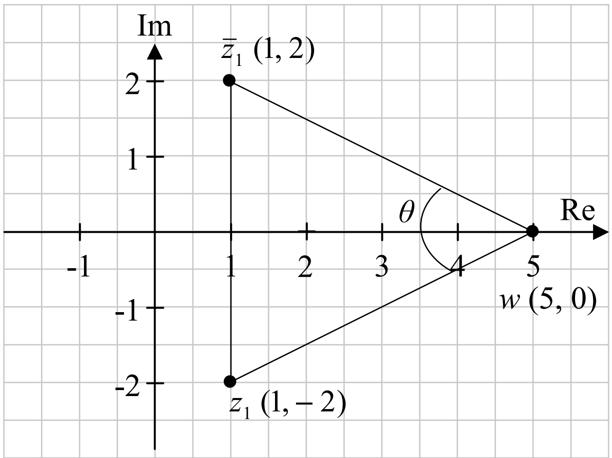


> $w = (1 - 2i)(1 + 2i)$
> $= 5$

(ii) Find the measure of the acute angle, $\bar{z}_1 w z_1$, formed by joining $\bar{z}_1$ to w to z_1 on the diagram above. Give your answer correct to the nearest degree.

$$\tan \tfrac{1}{2} \angle \bar{z}_1 w z_1 = \tfrac{2}{4} \quad \Rightarrow \quad \tfrac{1}{2} |\angle \bar{z}_1 w z_1| = 26 \cdot 57 \quad \Rightarrow \quad |\angle \bar{z}_1 w z_1| = 53 \cdot 14 \approx 53°$$

OR



$$|z_1 w| = \sqrt{(0+2)^2 + (5-1)^2} = \sqrt{16+4} = \sqrt{20}$$

$|z_1 w| = \sqrt{20}$ $\quad |\bar{z}_1 w| = \sqrt{20}$ $\quad |\bar{z}_1 z_1| = 4$

Cosine rule:

$$4^2 = (\sqrt{20})^2 + (\sqrt{20})^2 - 2(\sqrt{20})(\sqrt{20})\cos\theta$$

$$40\cos\theta = 24$$

$$\cos\theta = \frac{24}{40} = 0{\cdot}6$$

$$|\theta| = 53{\cdot}13 \approx 53°$$

Question 3 **(25 marks)**

(a) Prove, by induction, that the sum of the first n natural numbers, $1+2+3+\cdots+n$, is $\frac{n(n+1)}{2}$.

To Prove: $P(n)=1+2+3+\cdots+n=\frac{n(n+1)}{2}$

$P(1)$: $1 = \frac{1(1+1)}{2}=1$, True

Assume P(n) is true for n = k, and prove P(n) is true for n = k + 1.

n = k: $1+2+3+\cdots+k=\frac{k(k+1)}{2}$

To prove P(k+1) $=\frac{(k+1)}{2}(k+2)$

L.H.S. $=1+2+3+\cdots+k+(k+1)=\frac{k(k+1)}{2}+(k+1)=\frac{k(k+1)}{2}+\frac{2(k+1)}{2}$

$=\frac{(k+1)}{2}(k+2)=$ R.H.S

But P(1) is true, so P(2) is true etc.
Hence, P(n) is true for all n.

(b) Hence, or otherwise, prove that the sum of the first n even natural numbers, $2+4+6+\cdots+2n$, is n^2+n.

a = 2 and d = 2.
$S_n=\frac{n}{2}\left(2a+(n-1)d\right)=\frac{n}{2}\left(4+(n-1)2\right)=\frac{n}{2}\left(2n+2\right)=n^2+n$

OR

$S_n = 2+4+6+\ldots\ldots\ldots\ldots.+2n$
$= 2(1+2+3+\ldots\ldots\ldots..+n)$
$= 2\left[\frac{n(n+1)}{2}\right]$
$= n(n+1)$
$= n^2+n$

(c) Using the results from **(a)** and **(b)** above, find an expression for the sum of the first n odd natural numbers in its simplest form.

$$1 + 2 + 3 + \cdots + 2n = \frac{2n(2n+1)}{2} = 2n^2 + n$$
$$\Rightarrow (1 + 3 + 5 + \cdots n \text{ terms}) + (2 + 4 + 6 + \cdots n \text{ terms}) = 2n^2 + n$$
$$\Rightarrow (1 + 3 + 5 + \cdots n \text{ terms}) + (n^2 + n) = 2n^2 + n$$
$$\Rightarrow 1 + 3 + 5 + \cdots n \text{ terms} = 2n^2 + n - (n^2 + n) = n^2$$

OR

$$S_A = 1 + 2 + 3 + \ldots\ldots\ldots\ldots\ldots + (2n-1) + (2n) \quad = 2n^2 + n$$
$$S_B = 2 + 4 + 6 + 8 + \ldots\ldots\ldots + 2n \quad = n^2 + n$$
$$S_A - S_B = 1 + 3 + 5 + \ldots\ldots\ldots + (2n-1) \quad = n^2$$

Question 4 **(25 marks)**

(a) Differentiate the function $2x^2-3x-6$ with respect to x from first principles.

$$f(x)=2x^2-3x-6$$
$$f(x+h)=2(x+h)^2-3(x+h)-6=2x^2+4xh+2h^2-3x-3h-6$$
$$f(x+h)-f(x)=4xh+2h^2-3h$$
$$\underset{h\to 0}{Limit}\left(\frac{f(x+h)-f(x)}{h}\right)=\underset{h\to 0}{Limit}\left(\frac{4xh+2h^2-3h}{h}\right)=4x-3$$

(b) Let $f(x)=\dfrac{2x}{x+2}$, $x\neq -2$, $x\in\mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y=f(x)$ is $\dfrac{1}{4}$.

$$f(x)=\frac{2x}{x+2}$$
Let $u(x)=2x\Rightarrow u'(x)=2$ and $v(x)=x+2\Rightarrow v'(x)=1$
$$f'(x)=\frac{(x+2)(2)-2x(1)}{(x+2)^2}=\frac{4}{(x+2)^2}$$
$$f'(x)=\frac{1}{4}\Rightarrow\frac{4}{(x+2)^2}=\frac{1}{4}$$
$$\Rightarrow 16=(x+2)^2$$

$\Rightarrow x+2=4$ or $x+2=-4$
$\Rightarrow x=2$ or $x=-6$

or

$x^2+4x-12=0$
$(x-2)(x+6)=0$
$\Rightarrow x-2=0$ or $x+6=-0$
$\Rightarrow x=2$ or $x=-6$

$$f(-6)=\frac{-12}{-6+2}=3 \text{ and } f(2)=\frac{4}{2+2}=1$$
Points $(-6,3)$ and $(2,1)$

Question 5 **(25 marks)**

(a) Find $\int 5\cos 3x\,dx$.

$$\int 5\cos 3x\,dx = \tfrac{5}{3}\sin 3x + c$$

(b) The slope of the tangent to a curve $y = f(x)$ at each point (x, y) is $2x - 2$.
The curve cuts the x-axis at $(-2, 0)$.

(i) Find the equation of $f(x)$.

$$\int dy = \int (2x-2)dx$$
$$\Rightarrow y = x^2 - 2x + c$$
At $x = -2, y = 0 \quad \Rightarrow \quad 0 = 4 + 4 + c \quad \Rightarrow \quad c = -8$
Hence, $y = x^2 - 2x - 8$

(ii) Find the average value of f over the interval $0 \le x \le 3, x \in \mathbb{R}$.

Average value: $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$

$$\frac{1}{3-0}\int_0^3 \left(x^2 - 2x - 8\right)dx = \frac{1}{3}\left[\frac{x^3}{3} - x^2 - 8x\right]_0^3$$
$$= \frac{1}{3}\left[\frac{27}{3} - 9 - 24\right] = -8$$

Question 6 **(25 marks)**

The n^{th} term of a sequence is $T_n = \ln a^n$, where $a > 0$ and a is a constant.

(a) **(i)** Show that T_1, T_2, and T_3 are in arithmetic sequence.

$T_1 = \ln a, \quad T_2 = \ln a^2 = 2\ln a, \quad T_3 = \ln a^3 = 3\ln a.$
$T_2 - T_1 = 2\ln a - \ln a = \ln a$
$T_3 - T_2 = 3\ln a - 2\ln a = \ln a$
$T_3 - T_2 = T_2 - T_1.$ Hence, terms are in arithmetic sequence.

(ii) Prove that the sequence is arithmetic and find the common difference.

$T_n = \ln a^n = n\ln a,$
$T_{n-1} = \ln a^{n-1} = (n-1)\ln a.$
$T_n - T_{n-1} = n\ln a - (n-1)\ln a = \ln a,$ (a constant).
Hence, the sequence is arithmetic.

Common difference: $T_n - T_{n-1} = \ln a$

(b) Find the value of a for which $T_1 + T_2 + T_3 + \cdots + T_{98} + T_{99} + T_{100} = 10\,100$.

$T_1 + T_2 + T_3 + \cdots + T_{98} + T_{99} + T_{100} = 10\,100$
$\Rightarrow \ln a + 2\ln a + 3\ln a + \cdots + 100\ln a = 10\,100$
$\Rightarrow \frac{100}{2}[2\ln a + (100-1)\ln a] = 10\,100$
$\Rightarrow 50[101\ln a] = 10\,100$
$\Rightarrow 5050\ln a = 10\,100$
$\Rightarrow \ln a = 2$
$\Rightarrow a = e^2 = 7 \cdot 389$

(c) Verify that, for all values of a,

$$\left(T_1+T_2+T_3+\cdots+T_{10}\right)+100d=\left(T_{11}+T_{12}+T_{13}+\cdots+T_{20}\right),$$

where d is the common difference of the sequence.

$$\begin{aligned}\left(T_1+T_2+T_3+\cdots+T_{10}\right)+100d &= (T_1+10d)+(T_2+10d)+(T_3+10d)+\cdots+(T_{10}+10d)\\ &= T_{11}+T_{12}+T_{13}+\cdots+T_{20}.\end{aligned}$$

OR

$$\begin{aligned}\left(T_1+T_2+T_3+\cdots+T_{10}\right)+100d &= (\ln a+2\ln a+3\ln a+\cdots+10\ln a)+100\ln a\\ &= \tfrac{10}{2}\left(2\ln a+(10-1)\ln a\right)+100\ln a\\ &= 5\left(11\ln a\right)+100\ln a\\ &= 155\ln a\end{aligned}$$

$$\begin{aligned}\left(T_{11}+T_{12}+T_{13}+\cdots+T_{20}\right) &= 11\ln a+12\ln a+13\ln a+\cdots+20\ln a\\ &= \tfrac{10}{2}\left(22\ln a+(10-1)\ln a\right)\\ &= 5\left(31\ln a\right)\\ &= 155\ln a\end{aligned}$$

Hence, L.H.S = R.H.S

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(40 marks)**

(a) Three natural numbers a, b and c, such that $a^2 + b^2 = c^2$, are called a Pythagorean triple.

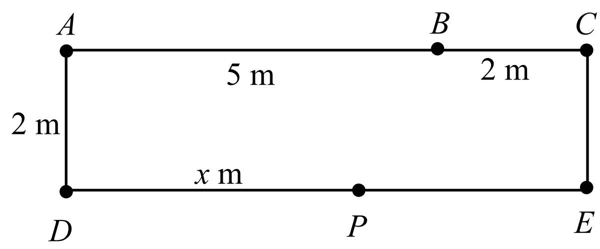
(i) Let $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$.
Pick one natural number n and verify that the corresponding values of a, b and c form a Pythagorean triple.

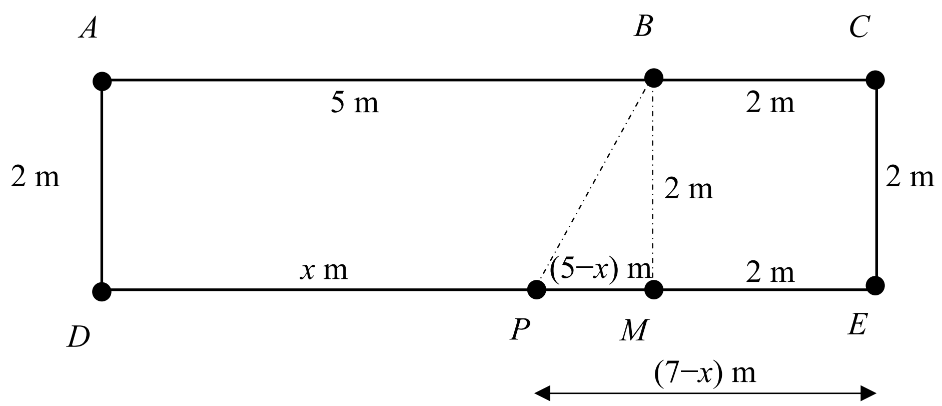
> Let $n = 1$:
> $a = 2n+1 \Rightarrow a = 2(1)+1 = 3$
> $b = 2n^2 + 2n \Rightarrow b = 2(1)^2 + 2(1) = 4$
> $c = 2n^2 + 2n + 1 \Rightarrow c = 2(1)^2 + 2(1) + 1 = 5$
>
> $3^2 + 4^2 = 5^2 \Rightarrow a^2 + b^2 = c^2$

(ii) Prove that $a = 2n+1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$, where $n \in \mathbb{N}$, will always form a Pythagorean triple.

> $a^2 = (2n+1)^2 = 4n^2 + 4n + 1$
> $b^2 = (2n^2 + 2n)^2 = 4n^4 + 8n^3 + 4n^2$
> $a^2 + b^2 = 4n^4 + 8n^3 + 8n^2 + 4n + 1$
>
> $c^2 = (2n^2 + 2n + 1)^2$
> $= 4n^4 + 8n^3 + 8n^2 + 4n + 1$
> $= a^2 + b^2$

(b) *ADEC* is a rectangle with $|AC| = 7$ m and $|AD| = 2$ m, as shown.
B is a point on $[AC]$ such that $|AB| = 5$ m.
P is a point on $[DE]$ such that $|DP| = x$ m.





(i) Let $f(x) = |PA|^2 + |PB|^2 + |PC|^2$.
Show that $f(x) = 3x^2 - 24x + 86$, for $0 \le x \le 7$, $x \in \mathbb{R}$.

$$|PM| = |PE| - |ME|$$
$$= (7 - x) - 2$$
$$= (5 - x)$$

$$f(x) = |PA|^2 + |PB|^2 + |PC|^2$$
$$= \left[|PD|^2 + |DA|^2\right] + \left[|PM|^2 + |MB|^2\right] + \left[|PE|^2 + |EC|^2\right]$$
$$= x^2 + 2^2 + \left((5 - x)^2 + 2^2\right) + \left((7 - x)^2 + 2^2\right)$$
$$= x^2 + 4 + 25 - 10x + x^2 + 4 + 49 - 14x + x^2 + 4$$
$$= 3x^2 - 24x + 86$$

(ii) The function $f(x)$ has a minimum value at $x = k$.
Find the value of k and the minimum value of $f(x)$.

$$f(x) = 3x^2 - 24x + 86$$
$$f'(x) = 6x - 24$$
$$f''(x) = 6 > 0 \quad \Rightarrow \quad \text{minimum}$$
$$f'(x) = 0 \Rightarrow 6x - 24 = 0 \Rightarrow x = 4 = k$$
$$f(4) = 3(4)^2 - 24(4) + 86 = 38$$

OR

$$\begin{aligned} f(x) &= 3x^2 - 24x + 86 \\ &= 3\left(x^2 - 8x + \frac{86}{3}\right) \\ &= 3\left[\left(x^2 - 8x + 16\right) + \frac{38}{3}\right] \\ &= 3\left[\left(x-4\right)^2 + \frac{38}{3}\right] \end{aligned}$$

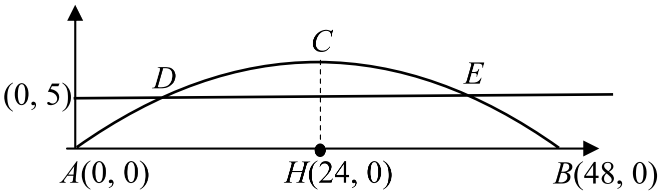
At $x = 4 \Rightarrow$ minimum value for $f(x)$

$$\begin{aligned} f(4) &= 3x^2 - 24x + 86 \\ &= 3(4)^2 - 24(4) + 86 \\ &= 48 - 96 + 86 \\ &= 38 \end{aligned}$$

Question 8 **(50 marks)**

In 2011, a new footbridge was opened at Mizen Head, the most south-westerly point of Ireland.

The arch of the bridge is in the shape of a parabola, as shown. The length of the span of the arch, $[AB]$, is 48 metres.



(a) Using the co-ordinate plane, with $A(0, 0)$ and $B(48, 0)$, the equation of the parabola is $y = -0 \cdot 013x^2 + 0 \cdot 624x$. Find the co-ordinates of C, the highest point of the arch.

$$y = -0 \cdot 013x^2 + 0 \cdot 624x$$

$$\Rightarrow \frac{dy}{dx} = -0 \cdot 026x + 0 \cdot 624 = 0 \Rightarrow x = 24$$

$$y = -0 \cdot 013x^2 + 0 \cdot 624x = -0 \cdot 013(24)^2 + 0 \cdot 624(24) = 7 \cdot 488.$$

$C(24,\ 7 \cdot 488)$

OR

Max height at C when x = 24

$$y = -0 \cdot 013x^2 + 0 \cdot 624x$$
$$= -0 \cdot 013(24)^2 + (0 \cdot 624)(24)$$
$$= 7 \cdot 488$$

$C(24,\ 7 \cdot 488)$

(b) The perpendicular distance between the walking deck, $[DE]$, and $[AB]$ is 5 metres.
Find the co-ordinates of D and of E. Give your answers correct to the nearest whole number.

Equation DE: $y = 5$

Equation of the parabola: $y = -0 \cdot 013x^2 + 0 \cdot 624x$.

$$5 = -0 \cdot 013x^2 + 0 \cdot 624x$$

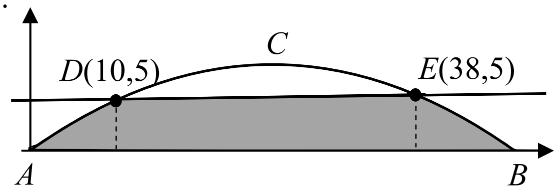
$$\Rightarrow 0 \cdot 013x^2 - 0 \cdot 624x + 5 = 0$$

$$x = \frac{0 \cdot 624 \pm \sqrt{0 \cdot 624^2 - 4(0 \cdot 013)5}}{2(0 \cdot 013)} = \frac{0 \cdot 624 \pm 0 \cdot 360}{0 \cdot 026}$$

$$x = 37 \cdot 8 \ \text{ or } \ x = 10 \cdot 15$$

$D(10, 5)$, $E(38, 5)$

(c) Using integration, find the area of the shaded region, *ABED,* shown in the diagram below. Give your answer correct to the nearest whole number.



$$\text{Area } ABED = \int_0^{10} y\,dx + \text{Area of rectangle} + \int_{38}^{48} y\,dx$$

$$= 2\int_0^{10} y\,dx + (38-10)\times 5$$

$$= 2\int_0^{10} (-0{\cdot}013x^2 + 0{\cdot}624x)dx + 140$$

$$= 2\left[\frac{-0{\cdot}013x^3}{3} + \frac{0{\cdot}624x^2}{2}\right]_0^{10} + 140$$

$$= 2\left[-\frac{0{\cdot}013(10)^3}{3} + \frac{0{\cdot}624(10)^2}{2} - 0\right] + 140$$

$$= 2\left[-\frac{13}{3} + 31{\cdot}2\right] + 140$$

$$= 193{\cdot}7$$

$$\approx 194 \text{ m}^2$$

OR

Contd…

Area under curve between A and B:

$$= \int_0^{48} (-0 \cdot 013x^2 + 0 \cdot 624x)\,dx$$

$$= \left[\frac{-0 \cdot 013x^3}{3} + \frac{0 \cdot 624x^2}{2} \right]_0^{48}$$

$$= \left[-\frac{0 \cdot 013(48)^3}{3} + \frac{0 \cdot 624(48)^2}{2} - 0 \right]$$

$$= 239 \cdot 616$$

Translate curve vertically downwards and find area under the curve between *D* and *E*:

$$= \int_{10}^{38} (-0 \cdot 013x^2 + 0 \cdot 624x - 5)\,dx$$

$$= \left[\frac{-0 \cdot 013x^3}{3} + \frac{0 \cdot 624x^2}{2} - 5x \right]_{10}^{38}$$

$$= \left[-\frac{0 \cdot 013(38)^3}{3} + \frac{0 \cdot 624(38)^2}{2} - 5(38) \right] - \left[\frac{-0 \cdot 013(10)^3}{3} + \frac{0 \cdot 624(10)^2}{2} - 5(10) \right]$$

$$= 22 \cdot 7493 + 23 \cdot 133$$

$$= 45 \cdot 8826$$

$$\begin{aligned} \text{Shaded area} &= 239 \cdot 616 - 45 \cdot 8826 \\ &= 193 \cdot 73 \\ &\approx 194 \text{ m}^2 \end{aligned}$$

(d) Write the equation of the parabola in part **(a)** in the form $y-k=p(x-h)^2$, where k, p, and h are constants.

$$\begin{aligned} y &= -0\cdot013x^2 + 0\cdot624x \\ &= -0\cdot013\left(x^2 - 48x\right) \\ &= -0\cdot013\left(x^2 - 48x + (-24)^2 - (-24)^2\right) \\ &= -0\cdot013(x-24)^2 + 7\cdot488 \\ \Rightarrow y - 7\cdot488 &= -0\cdot013(x-24)^2 \end{aligned}$$

(e) Using what you learned in part **(d)** above, or otherwise, write down the equation of a parabola for which the coefficient of x^2 is -2 and the co-ordinates of the maximum point are $(3,-4)$.

Given function: coefficient of x^2, $-0\cdot013$; maximum point $(24, 7\cdot488)$
New function: coefficient of x^2, -2; maximum point $(3,-4)$
Function: $y+4=-2(x-3)^2$

OR

Given function: $y-7\cdot488=-0\cdot013(x-24)^2$

$$y-(\text{max height}) = (\text{coefficient of } x^2)(x-x_{\text{max}})^2$$

New parabola: max height: -4
coefficient of x^2: -2
x_{max} 3

$$y-(-4)=-2(x-3)^2$$
$$y+4=-2(x-3)^2$$

Question 9 **(60 marks)**

Ciarán is preparing food for his baby and must use cooled boiled water. The equation $y = Ae^{kt}$ describes how the boiled water cools. In this equation:

- t is the time, in minutes, from when the water boiled,
- y is the *difference* between the water temperature and room temperature at time t, measured in degrees Celsius,
- A and k are constants.

The temperature of the water when it boils is $100°C$ and the room temperature is a constant $23°C$.

(a) Write down the value of the temperature difference, y, when the water boils, and find the value of A.

$y = 100 - 23 = 77$ at $t = 0$

$y = Ae^{kt} \Rightarrow 77 = Ae^{0} \Rightarrow A = 77$

(b) After five minutes, the temperature of the water is $88°C$.
Find the value of k, correct to three significant figures.

At $t = 5$, $y = 88 - 23 = 65$

$y = 77e^{kt} \Rightarrow 65 = 77e^{5k} \Rightarrow 5k = \ln \frac{65}{77} = -0 \cdot 169418$

$\Rightarrow k = -0 \cdot 03388 \approx -0 \cdot 0339$

(c) Ciarán prepares the food for his baby when the water has cooled to $50°C$. How long does it take, correct to the nearest minute, for the water to cool to this temperature?

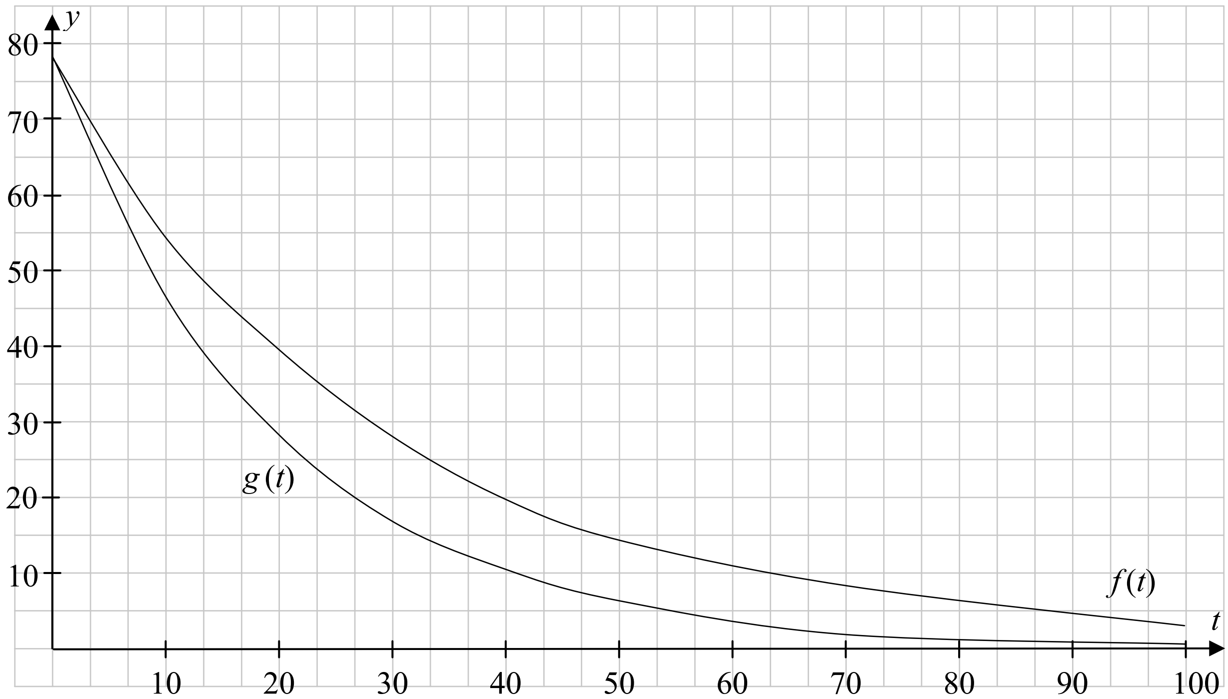
$y = 50 - 23 = 27$

$27 = 77e^{-0 \cdot 0339t} \Rightarrow 0 \cdot 0339t = \ln \frac{77}{27} = 1 \cdot 047969$

$\Rightarrow t = 30 \cdot 9 \approx 31$ minutes

(d) Using your values for A and k, sketch the curve $f(t) = Ae^{kt}$ for $0 \le t \le 100,\ t \in \mathbb{R}$.

$t = 0 \Rightarrow y = 77e^{-0\cdot0339(0)} = 77\,;$ (0, 77)
$t = 10 \Rightarrow y = 77e^{-0\cdot0339(10)} = 54\cdot9\,;$ (10, 55)
$t = 20 \Rightarrow y = 77e^{-0\cdot0339(20)} = 39\cdot1\,;$ (20, 39)
$t = 30 \Rightarrow y = 77e^{-0\cdot0339(30)} = 27\cdot9\,;$ (30, 28)
$t = 40 \Rightarrow y = 77e^{-0\cdot0339(40)} = 19\cdot8\,;$ (40, 20)
$t = 50 \Rightarrow y = 77e^{-0\cdot0339(50)} = 14\cdot1\,;$ (50, 14)
$t = 60 \Rightarrow y = 77e^{-0\cdot0339(60)} = 10\cdot1\,;$ (60, 10)
$t = 70 \Rightarrow y = 77e^{-0\cdot0339(70)} = 7\cdot2\,;$ (70, 7)
$t = 80 \Rightarrow y = 77e^{-0\cdot0339(80)} = 5\cdot1\,;$ (80, 5)
$t = 90 \Rightarrow y = 77e^{-0\cdot0339(90)} = 3\cdot6\,;$ (90, 4)
$t = 100 \Rightarrow y = 77e^{-0\cdot0339(100)} = 2\cdot6\,;$ (100, 3)



(e) **(i)** On the same diagram, sketch a curve $g(t) = Ae^{mt}$, showing the water cooling at a *faster* rate, where A is the value from part **(a)**, and m is a constant. Label each graph clearly.

(ii) Suggest one possible value for m for the sketch you have drawn and give a reason for your choice.

Test $m = -0 \cdot 02$, $m = k = -0.0339$ and $m = -0 \cdot 05$

$m = -0 \cdot 02$, $t = 10 \Rightarrow y = 77e^{-0 \cdot 02(10)} = 63 \cdot 0$
$m = k = -0 \cdot 0339 \Rightarrow y = 54 \cdot 9$ (from table)
$m = -0 \cdot 05$, $t = 10 \Rightarrow y = 77e^{-0 \cdot 05(10)} = 46 \cdot 7$

Any value of $m < k$ for faster decay.

(f) **(i)** Find the rates of change of the function $f(t)$ after 1 minute and after 10 minutes. Give your answers correct to two decimal places.

$$y = 77e^{-0 \cdot 0339t} \Rightarrow \frac{dy}{dt} = -2 \cdot 6103e^{-0 \cdot 0339t}$$

$$t = 1, \ \frac{dy}{dt} = -2 \cdot 6103e^{-0 \cdot 0339} = -2 \cdot 52$$

$$t = 10, \ \frac{dy}{dt} = -2 \cdot 6103e^{-0 \cdot 339} = -1 \cdot 86$$

(ii) Show that the rate of change of $f(t)$ will always increase over time.

$$\frac{d^2y}{dt^2} = 0 \cdot 088e^{-0 \cdot 0339t} > 0 \Rightarrow \frac{dy}{dt} \text{ is increasing}$$

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 3, 5	0, 3, 4, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 5, 7, 10	0, 3, 7, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 7, 10, 15	0, 5, 9, 12, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	15C
(b)(i)	5C
(b)(ii)	5C

Question 2

(a)	5D
(b)(i)	10C
(b)(ii)	10C

Question 3

(a)	10D
(b)	10D
(c)	5B

Question 4

(a)	15D
(b)	10D

Question 5

(a)	5B
(b)(i)	10C
(b)(ii)	10C

Question 6

(a)(i)	10C
(a)(ii)	5C
(b)	5C
(c)	5C

Section B

Question 7

(a)(i)	10B
(a)(ii)	10D
(b)(i)	5D
(b)(ii)	15C

Question 8

(a)	15C
(b)	10C
(c)	10D
(d)	10C
(e)	5B

Question 9

(a)	10C
(b)	10C
(c)	10C
(d)	15C
(e)	5C
(f)(i)	5C
(f)(ii)	5C

Detailed marking notes

Section A

Question 1

(a) Scale 15C (0, 7, 10, 15)
Low Partial Credit:
- Only one value verified
- Recognising one factor

High Partial Credit:
- Writing $(x+3)(x+1)(x-2)$
- Two relevant roots tested

(b)(i) Scale 5C (0, 3, 4, 5)
Low Partial Credit:
- Equations correct when $f(x)=g(x)$
- Cubic equation not factorised

High Partial Credit:
- Roots identified

(b)(ii) Scale 5C (0, 3, 4, 5)
Low Partial Credit:
- One point found in $g(x)$
- Only one point indicated on graph

High Partial Credit:
- Two points identified
- Two points plotted but no graph drawn

Question 2

(a) Scale 5D (0, 2, 3, 4, 5)

Low Partial Credit:
- Identifies another root
- Forms an equation

Mid Partial Credit:
- Works with correct quadratic factor
- Indicates division of quadratic into cubic

High Partial Credit:
- Finds third factor

(b)(i) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Plots one point correctly
- Finds $\bar{z}_1$

High Partial Credit:
- Points plotted but not labelled or labelled incorrectly
- Two points plotted and labelled
- Calculates w

(b)(ii) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Length of any one side of triangle calculated correctly
- Correct definition of trig ratio
- Correct $\cos$ rule
- Recognises the half-angle

High Partial Credit:
- $\cos$ value calculated but angle not found
- $\tan$ value of half-angle calculated

Question 3

(a) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:
- One correct step in induction
- Statement $P(1)$ true

Mid Partial Credit:
- Uses $(k+1)$ term on LHS
- Some work with $(k+1)$ term

High Partial Credit:
- Correct RHS
- No conclusion

(b) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:
- Recognising 2 as common factor
- $\frac{n}{2}(n+1)$

Mid Partial Credit:
- Correct equation from (a)
- Taking 2 out of series

High Partial Credit:
- Work not completed

OR

(b) Scale 10D (0, 3, 7, 8, 10) [when series tested as an AP]

Low Partial Credit:
- Recognition of $a = 2$
- Recognition of $d = 2$
- Correct AP formula only

Mid Partial Credit:
- Some substitution into correct formula

High Partial Credit:
- Work not fully simplified
- Answer not in required form
- S_n missing

(c) Scale 5B (0, 3, 5)

Partial Credit:
- $S_B - S_A$ indicated
- $S_B + S_A$
- Use of correct series from (b)

Note: Must use result from (a) and (b) here

Question 4

(a) Scale 15D (0, 5, 9, 12, 15)

Low Partial Credit:

- Introduces $f(x+h)$

Mid Partial Credit:

- $f(x+h)-f(x)$ expressed (need not be simplified)
- RHS only

High Partial Credit:

- $\underset{h\to 0}{Limit}\dfrac{f(x+h)-f(x)}{h}$ (need not be simplified)

(b) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:

- Either $\dfrac{du}{dx}$ or $\dfrac{dv}{dx}$ correct
- No differentiation but writes $f'(x)=\dfrac{1}{4}$

Mid Partial Credit:

- $f'(x)$ correct but not simplified

High Partial Credit:

- Correct values of x from students work

Question 5

(a) Scale 5B (0, 3, 5)

Partial Credit:

- Some correct integration
- Integrand does not contain c
- c only

(b)(i) Scale 10C (0, 5, 7, 10)

Low Partial Credit:

- Some correct integration
- Integrand does not contain c
- c only
- $\frac{dy}{dx} = 2x - 2$ or $\frac{dy}{dx} =$ slope of tangent

High Partial Credit:

- Substitutes (−2, 0) but c not simplified

Note: <u>must</u> have 'c' in equation to get high partial marks

(b)(ii) Scale 10C (0, 5, 7, 10)

Low Partial Credit:

- Correct formula only
- Some correct integration
- Indication of integration with correct limits
- If only values used e.g. $f(0)$, $f(1)$, $f(2)$ etc. when $0 \leq x \leq 3$, give Low Partial Credit for two or more values

High Partial Credit:

- Limits inserted into function but not calculated
- $\frac{1}{(b-a)}$ missing from formula

Note: NO CREDIT – differentiation
NO CREDIT – no integration

Question 6

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

(a)(i) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Only one term correct

High Partial Credit:
- Either $(T_2 - T_1)$ or $(T_3 - T_2)$ correct

(a)(ii) Scale 5C (0, 3, 4, 5)

Low Partial Credit:
- Uses two consecutive general terms
- Recognition of common difference and no more

High Partial Credit:
- Shows series arithmetic but does not state common difference

(b) Scale 5C (0, 3, 4, 5)

Low Partial Credit:
- Writes three or more terms in form of n and $\ln a$
- Correct AP formula stated
- Correct T_n formula

High Partial Credit:
- Correct substitution into formula
- $\ln a = 2$ and does not finish

Note: accept $a = e^2$ for full marks

(c) Scale 5C (0, 3, 4, 5)

Low Partial Credit:
- Recognising $T_{11} = T_1 + 10d$ or similar work

High Partial Credit:
- LHS correct in terms of $\ln a$
- RHS correct in terms of $\ln a$

Note: log is not needed in first solution box

Question 7

(a)(i) Scale 10B (0, 5, 10)
Partial Credit:
- Correct substitution of chosen value
- Not squaring values

Note: Allow 10 marks for $n = 0$ and correct work in (a)(i)

(a)(ii) Scale 10D (0, 3, 7, 8, 10)
Low Partial Credit:
- a^2 or b^2 or c^2 expressed in terms of n

Mid Partial Credit:
- Any two terms

High Partial Credit:
- Three terms fully squared
- $(a^2 + b^2)$ fully worked out in terms of n

Notes for (a)(i) and (a)(ii):
- Mark particular case with scheme for (a)(i) wherever it occurs
- Mark general case with scheme for (a)(ii) wherever it occurs

(b)(i) Scale 5D (0, 2, 3, 4, 5)
Low Partial Credit:
- Expression for either $|PA|^2$ or $|PB|^2$ or $|PC|^2$ in terms of x
- Any appropriate construction line, e.g. the line PM

Mid Partial Credit:
- Correct expression of two sides in terms of x

High Partial Credit:
- Correct expression of three sides in terms of x
- Correct expression of function in x not simplified

(b)(ii) Scale 15C (0, 7, 10, 15)
Low Partial Credit:
- Stating $f'(x) = 0$ with no work
- Any correct differentiation

High Partial Credit:
- Finding value of x

OR

(b)(ii) Scale 15C (0, 7, 10, 15)
Low Partial Credit:
- 3 as factor

High Partial Credit:
- Finding value of x

Question 8

(a) Scale 15C (0, 7, 10, 15)

Low Partial Credit:
- Identifies $x = 24$
- Any correct differentiation

High Partial Credit:
- Substitutes $x = 24$ in $f(x)$

(b) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Recognition of line $y = 5$

High Partial Credit:
- Values in quadratic formula
- Gets x values only

Notes: - Accept $x = 10$ and $x = 38$ by trial and error from correct quadratic for full marks
- NO CREDIT - uses $x = 5$

(c) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:
- Any area formula
- Area under curve from A to B
- Area under curve from D to E
- Correct limits

Mid Partial Credit:
- Any correct integration
- One correct area only
- Area under curve from A to B minus area under curve from D to E

High Partial Credit:
- Limits substituted but not evaluated (must state that area of rectangle is 140)
- Both areas

OR

(c) Scale 10D (0, 3, 7, 8, 10)

Low Partial Credit:
- Any one area

Mid Partial Credit:
- Translation

High Partial Credit:
- 3^{rd} area

(d) Scale 10C (0, 5, 7, 10)

Low Partial Credit:

- Common factor of $-0{\cdot}013$ identified
- Attempt at equating like to like
- Attempt at completing square

High Partial Credit:

- Values of two of the constants found
- p correct and completion of square correct

(e) Scale 5B (0, 3, 5)

Partial Credit:

- One correct value for the equivalent of k, p or h in equation

Question 9

(a) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Value of y only
- Some use of 100 and/or 23

High Partial Credit:
- Correct substitution into equation
- A calculated from incorrect y

(b) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Value of y only
- Some use of 88 and/or 23

High Partial Credit:
- Correct expression for k
- k calculated from incorrect y

(c) Scale 10C (0, 5, 7, 10)

Low Partial Credit:
- Value of y only

High Partial Credit:
- Correct expression for t
- t calculated from incorrect y

(d) Scale 15C (0, 7, 10, 15)

Low Partial Credit:
- Any one point identified
- Graph of correct shape, even if no point correct or no point calculated
- Accept candidates value of k

Note: all graphs may not be the same, due to different values of A and k

High Partial Credit:
- Three points correctly plotted, but graph incomplete or no graph

Note: do not accept straight line graph

(e)(i) and (e)(ii)

Scale 5C (0, 3, 4, 5)

Low Partial Credit:
- Any attempt at similar graph
- No graph but correct deduction

High Partial Credit:
- Correct graph plotted but graph incomplete, or no graph

(f)(i) Scale 5C (0, 3, 4, 5)

Low Partial Credit:
- Indication of differentiation i.e. $\frac{dy}{dt}$, $\frac{dx}{dt}$ or $f'(t)$ (i.e. differentiation with respect to *t*)
- Treats *e* as *x* in differentiation

High Partial Credit:
- One value of $\frac{dy}{dt}$ indicated

(f)(ii) Scale 5C (0, 3, 4, 5)

Low Partial Credit:
- Attempt at 2^{nd} derivative
- Attempt at deduction from numerical values

High Partial Credit:
- Shows 2^{nd} derivative positive

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 2

Higher Level

Monday 9 June   Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7 to 9.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

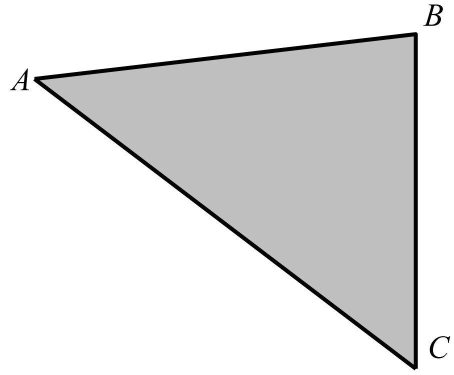
Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

The lengths of the sides of a flat triangular field ACB are,
$|AB| = 120$ m, $|BC| = 134$ m and $|AC| = 150$ m.

(a) **(i)** Find $|\angle CBA|$. Give your answer, in degrees, correct to two decimal places.



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{120^2 + 134^2 - 150^2}{2(120)(134)}$$
$$= \frac{9856}{32160}$$
$$= 0 \cdot 306468$$
$$\Rightarrow B = 72 \cdot 15°$$

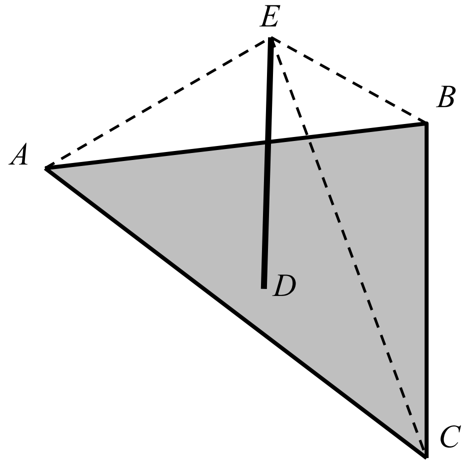
(ii) Find the area of the triangle ACB correct to the nearest whole number.

$$\text{Area } \Delta ABC = \tfrac{1}{2} ac \sin B = \tfrac{1}{2}(120)(134)\sin 72 \cdot 15$$
$$= 7652 \cdot 97$$
$$\approx 7653 \text{ m}^2$$

Or

$$\text{Area } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$$
$$= \sqrt{202(202-134)(202-150)(202-120)}$$
$$= \sqrt{58570304} = 7653 \cdot 12$$
$$\approx 7653 \text{ m}^2$$

(b) A vertical mast, $[DE]$, is fixed at the circumcentre, D, of the triangle. The mast is held in place by three taut cables $[EA]$, $[EB]$ and $[EC]$. Explain why the three cables are equal in length.



Circumcentre at $D \Rightarrow |AD| = |BD| = |CD|$

Each of the triangles *EAD*, *EBD*, *ECD* is right-angled at *D* and has the two sides, the base and the perpendicular, equal.
Hence, by theorem of Pythagoras, the third side of each, the hypotenuse (the cables), must be equal.

Question 2 **(25 marks)**

(a) Prove that $\cos 2A = \cos^2 A - \sin^2 A$.

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos 2A = \cos(A+A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A$$

(b) The diagram shows part of the circular end of a running track with three running lanes shown. The centre of each of the circular boundaries of the lanes is at O.

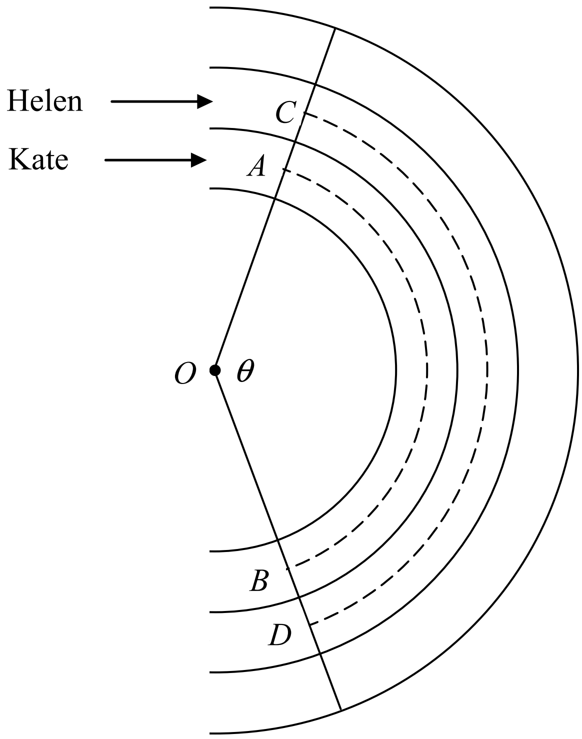
Kate runs in the middle of lane 1, from A to B as shown.

Helen runs in the middle of lane 2, from C to D as shown.

Helen runs 3 m further than Kate.

$|\angle AOB| = |\angle COD| = \theta$ radians.

If each lane is 1·2 m wide, find θ.



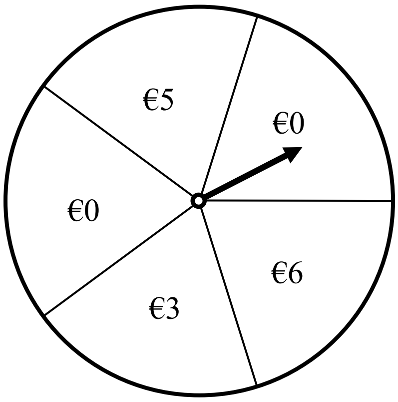
Kate: $|AB| = s_1 = |OA|\theta = r\theta$

Helen: $|CD| = s_2 = (|OA| + 1\cdot 2)\theta = (r + 1\cdot 2)\theta$

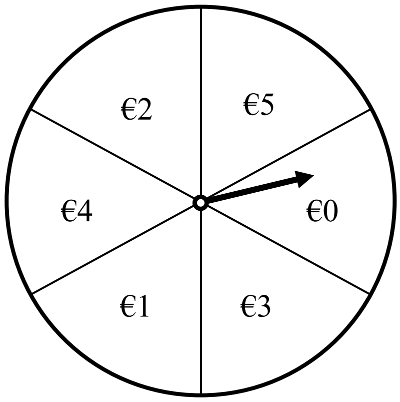
$s_1 + 3 = s_2$
$\Rightarrow r\theta + 3 = r\theta + 1\cdot 2\theta$
$\Rightarrow 1\cdot 2\theta = 3$
$\Rightarrow \theta = 2\cdot 5$ radians

Question 3 **(25 marks)**

Two different games of chance, shown below, can be played at a charity fundraiser. In each game, the player spins an arrow on a wheel and wins the amount shown on the sector that the arrow stops in. Each game is fair in that the arrow is just as likely to stop in one sector as in any other sector on that wheel.



Game A



Game B

(a) John played Game A four times and tells us that he has won a total of €8. In how many different ways could he have done this?

5, 3, 0, 0;	3, 5, 0, 0;	0, 5, 3, 0;	0, 3, 5, 0;	
5, 0, 3, 0;	3, 0, 5, 0;	0, 5, 0, 3;	0, 3, 0, 5;	12 ways
5, 0, 0, 3;	3, 0, 0, 5;	0, 0, 5, 3;	0, 0, 3, 5.	

(b) To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for the charity? Give a reason for your answer.

Expected outcome $E(X) = \sum x.P(x)$

Game A: $E(X) = 0\left(\frac{2}{5}\right) + 3\left(\frac{1}{5}\right) + 5\left(\frac{1}{5}\right) + 6\left(\frac{1}{5}\right) = 2\frac{4}{5}$

Game B: $E(X) = 0\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) = 2\frac{3}{6} = 2\frac{1}{2}$

Game B - it pays out less money.

Or

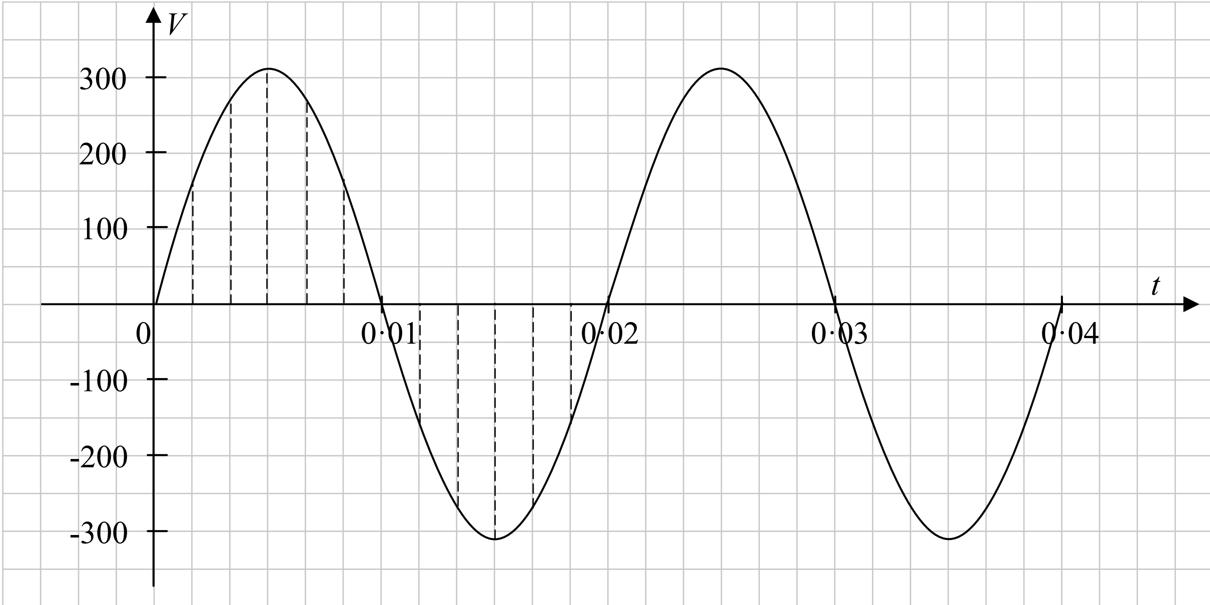
On average, over the long term:
Game A pays out €14 for every €15 taken in.
Game B pays out €15 for every €18 taken in.
Game B – it pays out a smaller proportion on the money taken in.

(c) Mary plays Game B six times. Find the probability that the arrow stops in the €4 sector exactly twice.

P(stops in €4 sector exactly twice) $= \binom{6}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4 = 0 \cdot 2$

Question 4 **(25 marks)**

The graph below shows the voltage, V, in an electric circuit as a function of time, t. The voltage is given by the formula $V = 311\sin(100\pi t)$, where V is in volts and t is in seconds.



(a) **(i)** Write down the range of the function.

Range: $[-311, 311]$

(ii) How many complete periods are there in one second?

$$\frac{100\pi}{2\pi} = 50 \text{ periods per second}$$

Or

Time for 1 period = 0·02 seconds

Number of periods in 1 second $= \dfrac{1}{0 \cdot 02} = 50$

(b) **(i)** The table below gives the voltage, correct to the nearest whole number, at equally spaced intervals from t_0 to t_{12} over one complete period (as shown by the dashed lines on the diagram). Use the entries given in the table and the properties of the function to complete the table.

T	t_1	t_2	t_3	t_4	t_5	$t_6 = 0\cdot01$	t_7	t_8	t_9	t_{10}	t_{11}	$t_{12} = 0\cdot02$
V	156	269	311	269	156	0	−156	−269	−311	−269	−156	0

(ii) Using a calculator, or otherwise, calculate the standard deviation, σ, of the twelve values of V in the table, correct to the nearest whole number.

$$\sigma = 219\cdot 89 = 220$$

(c) **(i)** The standard deviation, σ, of closely spaced values of any function of the form $V = a\sin(bt)$, over 1 full period, is given by $k\sigma = V_{\max}$, where k is a constant that does not depend on a or b, and $V_{\max}$ is the maximum value of the function. Use the function $V = 311\sin(100\pi t)$ to find an approximate value for k correct to three decimal places.

$$k = \frac{V_{\max}}{\sigma} = \frac{311}{220} \approx 1\cdot 414$$

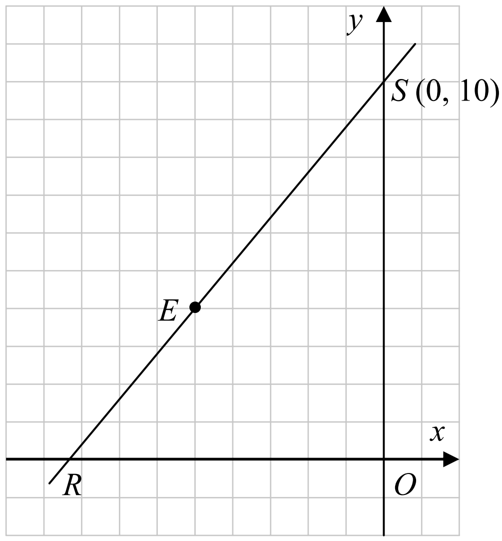
(ii) Using your answer in part **(c) (i)**, or otherwise, find the value of b required so that the function $V = a\sin(bt)$ has 60 complete periods in one second and the approximate value of a so that it has a standard deviation of 110 volts.

$$\frac{b}{2\pi} = 60 \Rightarrow b = 120\pi = 377$$

$$k\sigma = V_{\max} \quad \Rightarrow \quad V_{\max} = 1\cdot 414 \times 110 = 155\cdot 54 \Rightarrow a = 156$$

Question 5 **(25 marks)**

The line RS cuts the x-axis at the point R and the y-axis at the point $S(0, 10)$, as shown. The area of the triangle ROS, where O is the origin, is $\frac{125}{3}$.



(a) Find the co-ordinates of R.

$\text{Area } ROS = \frac{1}{2}|RO|.|OS| = \frac{125}{3}$
$\Rightarrow \frac{1}{2}|RO|(10) = \frac{125}{3}$
$\Rightarrow |RO| = \frac{25}{3}$
$R\left(-\frac{25}{3}, 0\right)$

(b) Show that the point $E(-5, 4)$ is on the line RS.

$\text{Slope RS} = \dfrac{10-0}{0+\frac{25}{3}} = \dfrac{6}{5}$ $\quad\text{Slope ES} = \dfrac{10-4}{0+5} = \dfrac{6}{5}$ $\quad\text{Slope ER} = \dfrac{4-0}{-5+\frac{25}{3}} = \dfrac{6}{5}$

Any two slopes correct $\Rightarrow (-5, 4) \in RS$

Or

RS: $y - 10 = \frac{6}{5}(x - 0) \Rightarrow 6x - 5y + 50 = 0$

$6(-5) - 5(4) + 50 = -30 - 20 + 50 = 0 \Rightarrow (-5, 4) \in RS$

(c) A second line $y = mx + c$, where m and c are positive constants, passes through the point E and again makes a triangle of area $\frac{125}{3}$ with the axes. Find the value of m and the value of c.

$y = mx + c$ cuts x-axis at $P\left(-\frac{c}{m}, 0\right)$ and cuts y-axis at Q(0, c)

$\text{Area } \Delta POQ = \frac{1}{2}\left|0 - \left(-\frac{c}{m}\right)c\right| = \frac{1}{2}\left|\frac{c^2}{m}\right| = \frac{125}{3} \Rightarrow m = \frac{3c^2}{250}$

$(-5, 4) \in y = mx + c \Rightarrow 4 = -5m + c \Rightarrow 4 = -5\left(\frac{3c^2}{250}\right) + c \Rightarrow 3c^2 - 50c + 200 = 0$

$\Rightarrow (3c - 20)(c - 10) = 0 \Rightarrow c = \frac{20}{3} \text{ or } c = 10$

$c = \frac{20}{3}$

Hence, $m = \dfrac{3c^2}{250} = \dfrac{3\left(\frac{20}{3}\right)^2}{250} = \dfrac{400}{750} = \dfrac{8}{15}$

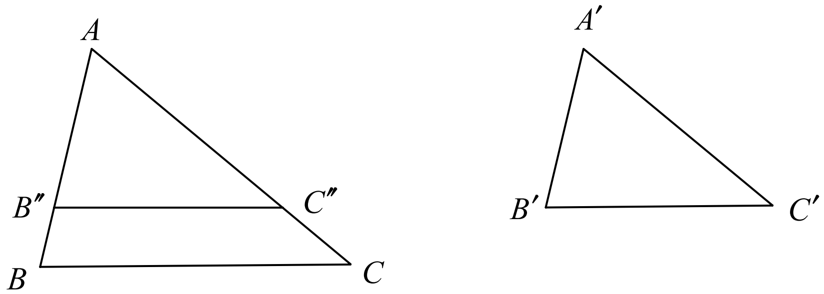
Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) Prove that, if two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$



Given: The similar triangles ΔABC and $\Delta A'B'C'$.

To Prove: $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$.

Construction: Mark B'' on $|AB|$ such that $|AB''| = |A'B'|$.
Mark C'' on $|AC|$ such that $|AC''| = |A'C'|$.
Join $B''\ C''$

Proof: $\Delta AB''C''$ is congruent to $\Delta A'B'C'$. . SAS

$\Rightarrow |\angle AB''C''| = |\angle ABC|$

$\Rightarrow B''C'' \parallel BC$ ... corresponding angles

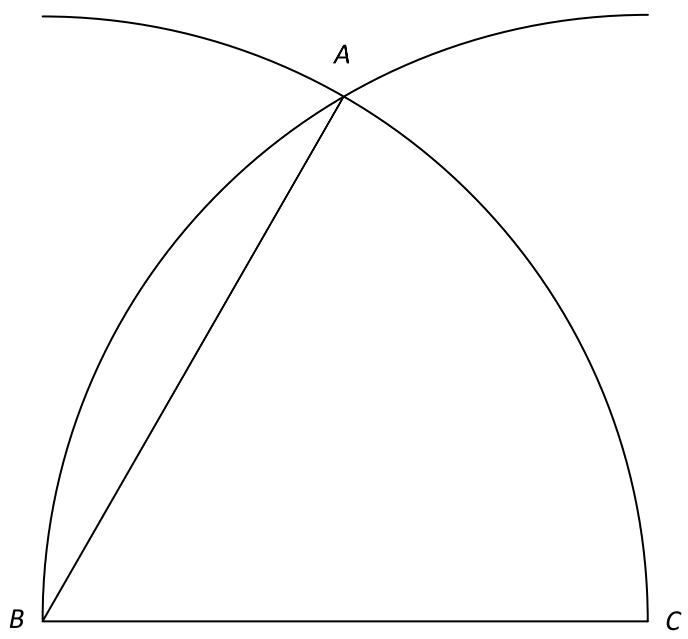
Hence, $\frac{|AB|}{|AB''|} = \frac{|AC|}{|AC''|}$...Theorem

$\Rightarrow \frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|}$.

Similarly, $\frac{|BC|}{|B'C'|} = \frac{|AB|}{|A'B'|}$

Hence, $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$.

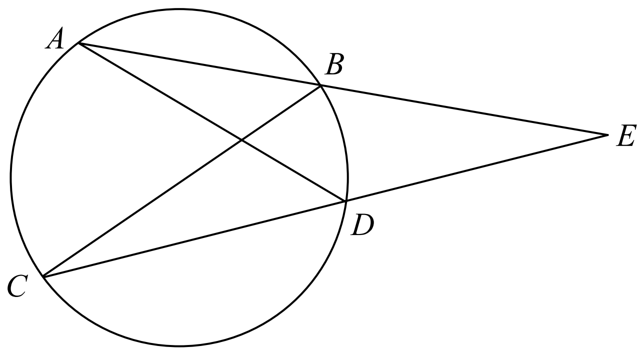
(b) Given the line segment $[BC]$, construct, without using a protractor or set square, a point A such that $|\angle ABC| = 60°$. Show your construction lines.



OR

Question 6B

$[AB]$ and $[CD]$ are chords of a circle that intersect externally at E, as shown.



(a) Name two similar triangles in the diagram above and give reasons for your answer.

ΔADE and ΔBCE are similar

$|\angle EAD| = |\angle BCE|$, on arc BD

$|\angle DEA| = |\angle CEB|$, same angle

$|\angle ADE| = |\angle EBC|$, third angle

Also (i) ΔAXB and ΔDXC are similar, where $AD \cap CB = \{X\}$
and (ii) ΔAXC and ΔBXD are similar, where $AD \cap CB = \{X\}$

(b) Prove that $|EA|.|EB| = |EC|.|ED|$.

ΔADE and ΔBCE are similar.

Hence, $\dfrac{|EA|}{|EC|} = \dfrac{|ED|}{|EB|}$

$\Rightarrow |EA|.|EB| = |EC|.|ED|$

(c) Given that $|EB| = 6\cdot25$, $|ED| = 5\cdot94$ and $|CB| = 10$, find $|AD|$.

$$\frac{|ED|}{|EB|} = \frac{|AD|}{|CB|} \Rightarrow \frac{5\cdot94}{6\cdot25} = \frac{|AD|}{10}$$

$$\Rightarrow |AD| = \frac{5\cdot94\times10}{6\cdot25} = 9\cdot504$$

Section B Contexts and Applications 150 marks

Answer **all three** questions from this section.

Question 7 **(45 marks)**

Table 1 below gives details of the number of males (M) and females (F) aged 15 years and over at work, unemployed, or not in the labour force for each year in the period 2004 to 2013.

Table 1

Labour Force Statistics 2004 to 2013 - Persons aged 15 years and over (000's)

Year	At work			Unemployed			Not in labour force			Total
	M	F	Total	M	F	Total	M	F	Total	
2004	1045.9	738.9	1784.8	79.6	31.6	111.2	457.1	854.2	1311.3	3207.3
2005	1087.3	779.7	1867.0	81.3	33.5	114.8	459.5	846.6	1306.1	3287.9
2006	1139.8	815.1	1954.9	80.6	38.1	118.7	457.6	844.9	1302.5	3376.1
2007	1184.0	865.6	2049.6	84.3	39.2	123.5	472.4	852.7	1325.1	3498.2
2008	1170.9	889.5	2060.4	106.3	41.0	147.3	494.8	872.5	1367.3	3575.0
2009	1039.8	863.5	1903.3	234.0	82.4	316.4	505.6	874.9	1380.5	3600.2
2010	985.1	843.5	1828.6	257.6	98.2	355.8	529.2	884.6	1413.8	3598.2
2011	970.2	843.2	1813.4	260.7	103.4	364.1	540.1	881.5	1421.6	3599.1
2012	949.6	823.8	1773.4	265.2	108.0	373.2	546.5	896.9	1443.4	3590.0
2013	974.4	829.0	1803.4	227.7	102.3	330.0	557.8	895.0	1452.8	3586.2

(*Source: Central Statistics Office http://www.cso.ie*)

(a) Suggest two categories of people, aged 15 years and over, who might not be in the labour force.

Students, Retired, Stay at home persons, Disabled

(b) Find the median and the interquartile range of the total persons at work over the period.

Median: $\frac{1}{2}(1828\cdot 6 + 1867\cdot 0) = 1847\cdot 8$

IQR: $1954\cdot 9 - 1803\cdot 4 = 151\cdot 5$

(c) The following data was obtained from Table 1. The percentages of persons aged 15 years and over at work, unemployed, or not in the labour force for the year 2006 are given below.

		At work	Unemployed	Not in the labour force
Persons aged 15 years and over	2006	57·9%	3·5%	38·6%
	2011			

(i) Complete the table for the year 2011. Give your answers correct to one decimal place.

		At work	Unemployed	Not in the labour force
Persons aged 15 years and over	2006	57·9%	3·5%	38·6%
	2011	50·4%	10·1%	39·5%

(ii) A census in 2006 showed that there were 864 449 persons in the population aged under 15 years of age. The corresponding number in the 2011 census was 979 590. Assuming that none of these persons are in the labour force, complete the table below to give the percentages of the *total population* at work, unemployed, or not in the labour force for the year 2011.

		At work	Unemployed	Not in the labour force
Total population	2006	46·1%	2·8%	51·1%
	2011	39·6%	8·0%	52·4%

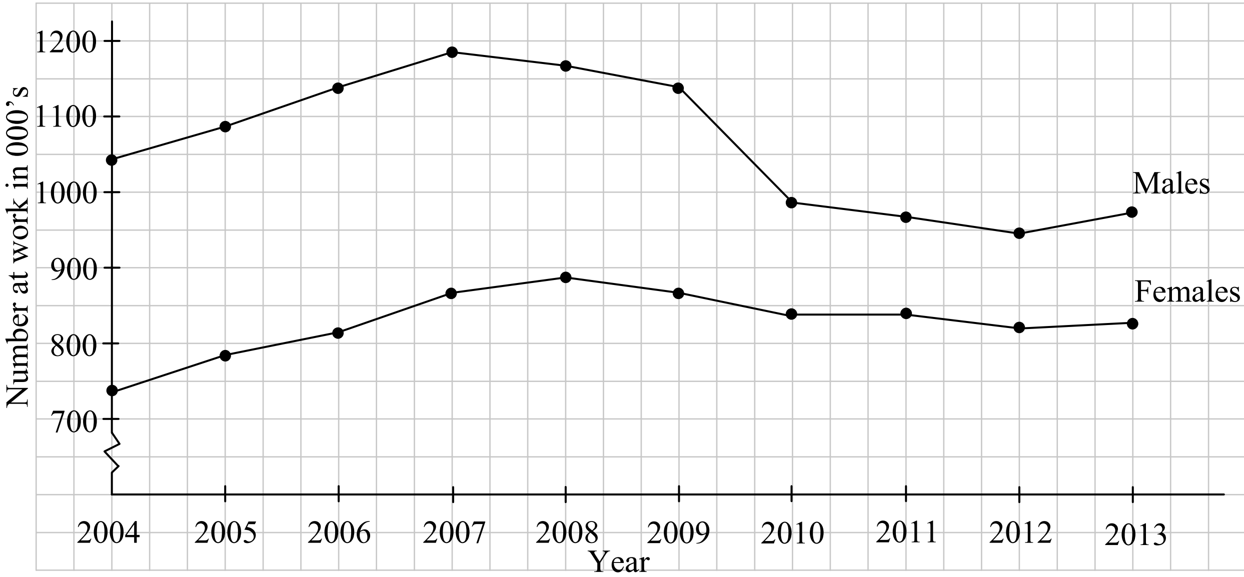
(iii) A commentator states that "The changes reflected in the data from 2006 to 2011 make it more difficult to balance the Government's income and expenditure."
Do you agree with this statement? Give two reasons for your answer based on your calculations above.

Yes
Percentage at work down, so reduced taxes collected, so income reduced.
Percentage not in workforce up, so increased expenditure on supports, pensions etc.

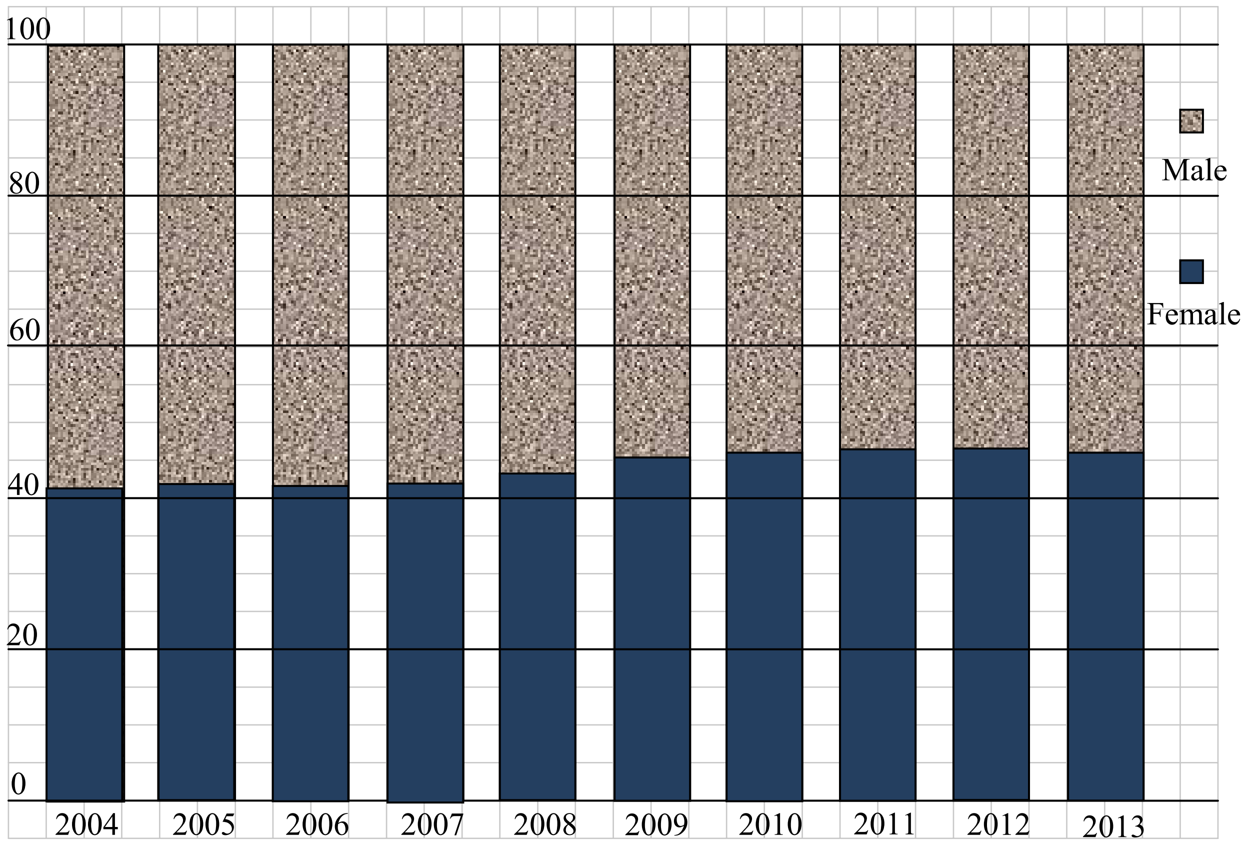
Note: Answer here depends on candidate's answers in previous sections

(d) Liam and Niamh are analysing the number of males and the number of females at work over the period 2004 to 2013.

Liam draws the following chart using data obtained from Table 1.



Niamh also uses data from Table 1 and calculates the number of females at work as a percentage of the total number of persons at work and then draws the following chart.



(i) Having examined both charts, a commentator states "females were affected just as much as males by the downturn in employment." Do you agree or disagree with this statement? Give a reason for your conclusion.

> Disagree.
> Male unemployment declined from 2007, female from 2008.
> Greater decline in number of males employed.

(ii) Which, if any, of the two charts did you find most useful in reaching your conclusion above? Give a reason for your answer.

> Liam's graph shows trend over time as well as the numbers.
> Niamh's graph only shows percentage in workforce and gives no information about actual numbers.

(iii) Use the data in Table 1, for the years 2012 and 2013 only, to predict the percentage of persons, aged 15 years and over, who will be at work in 2014.

2012	49·4% at work	
2013	50·3% at work	(+0·9%)
2014	51·2% at work	

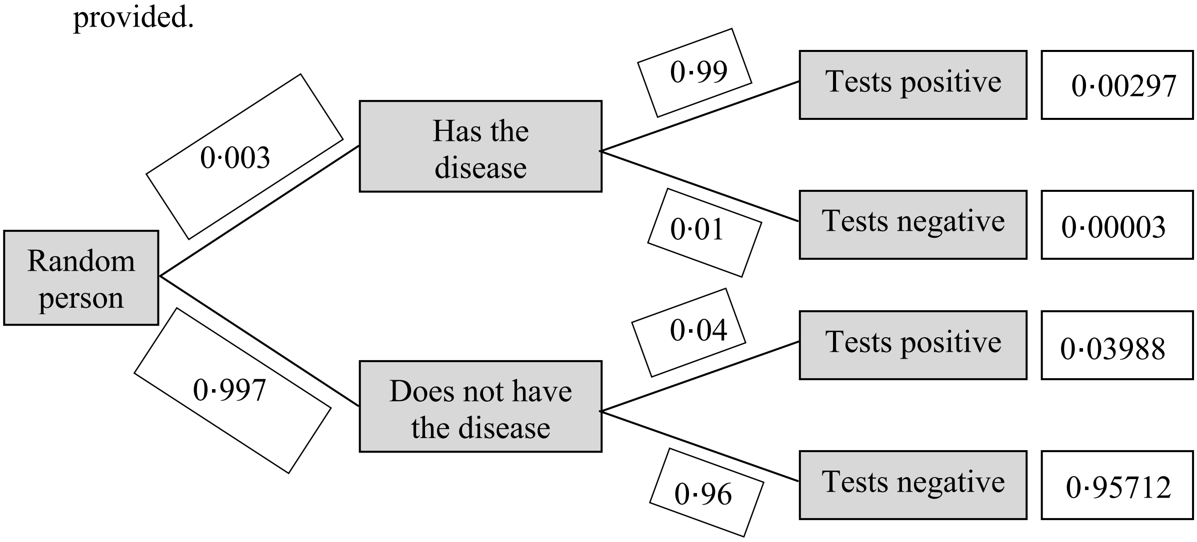
Note: Candidates not required to round to any particular number of places of decimals

Question 8 **(45 marks)**

Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative).

It is estimated that 0·3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

(a) **(i)** Write the probability associated with each branch of the tree diagram in the blank boxes provided.



(ii) Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

$$\text{P(Positive test)} = 0\cdot 00297 + 0\cdot 03988 = 0\cdot 04285$$

(iii) A person tests positive for the disease. What is the probability that the person actually has the disease. Give your answer correct to three significant figures.

$$\text{P(Has disease|positive test)} = \frac{0\cdot 00297}{0\cdot 04285} = 0\cdot 0693$$

(iv) The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

Test is not very useful.
A person who tests positive has the disease only 7% of the time.

(b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two new drugs, *A* and *B*, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug *A* was successful in the case of 296 patients. The company claims that Drug *A* is more successful in treating the condition than the generic drug.

(i) Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

H_0: The new drug is not more successful than the generic drug.
$p = 0\cdot 51$

95% margin of error $= \dfrac{1}{\sqrt{500}} = 0\cdot 045$

The success rate for the new drug is $\dfrac{296}{500} = 0\cdot 592$.

This is outside the interval $[0\cdot 51 - 0\cdot 045,\ 0\cdot 51 + 0\cdot 045] = [0\cdot 465,\ 0\cdot 555]$

Result is significant, reject the null hypothesis.

There is evidence to conclude that the new drug is more successful than the generic.

Or

H_0: The new drug is not more successful than the generic drug.
H_1: The new drug is more successful than the generic drug.
$p = 0\cdot 51$

95% margin of error $= \dfrac{1}{\sqrt{500}} = 0\cdot 045$

The success rate for the new drug is $\dfrac{296}{500} = 0\cdot 592$.

The 95% confidence interval for the population is
$0\cdot 592 - 0\cdot 045 < p < 0\cdot 592 + 0\cdot 045 = 0\cdot 547 < p < 0\cdot 637$

$p = 0\cdot 51$ is outside this interval.

Result is significant, reject the null hypothesis.

There is evidence to conclude that the new drug is more successful than the generic

(ii) The null hypothesis was accepted for Drug *B*. Estimate the greatest number of patients in that trial who could have been successfully treated with Drug *B*.

The result must lie in the interval $[0\cdot 465, 0\cdot 555]$

Thus, $\frac{n}{500} < 0\cdot 555 \Rightarrow n < 277\cdot 5$

Hence, 277 patients.

Or

$k - 0\cdot 045 < 0\cdot 51 < k + 0\cdot 045$

$\Rightarrow k - 0\cdot 045 < 0\cdot 51$

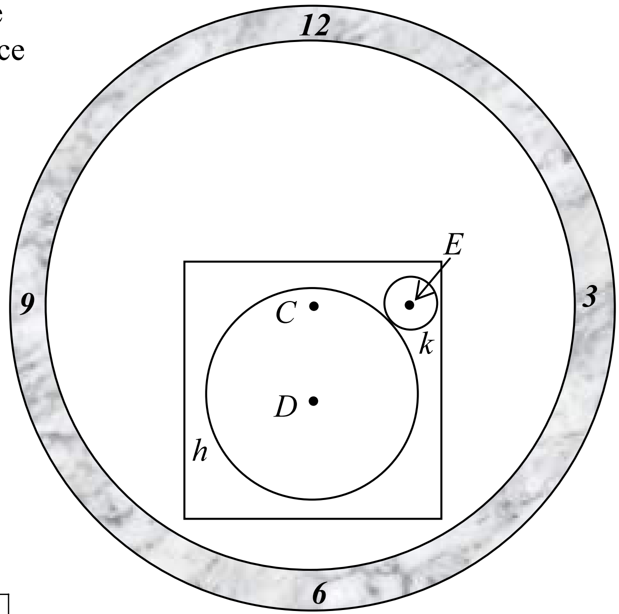
$\Rightarrow k < 0\cdot 555$

Number of patients $< 0\cdot 555 \times 500 = 277\cdot 5$

Hence, 277 patients.

Question 9 **(60 marks)**

(a) The diagram shows a circular clock face, with the hands not shown. The square part of the clock face is glass so that the mechanism is visible. Two circular cogs, h and k, which touch externally are shown.



The point C is the centre of the clock face. The point D is the centre of the larger cog, h, and the point E is the centre of the smaller cog, k.

(i) In suitable co-ordinates the equation of the circle h is

$$x^2 + y^2 + 4x + 6y - 19 = 0.$$

Find the radius of h and the co-ordinates of its centre, D.

$r_1 = \sqrt{4+9+19} = \sqrt{32} = 4\sqrt{2}$

$D(-2, -3)$

(ii) The point E has co-ordinates $(3, 2)$. Find the radius of the circle k.

$|DE| = \sqrt{(3+2)^2 + (2+3)^2} = \sqrt{50} = 5\sqrt{2}$

$r_1 + r_2 = |DE| \Rightarrow 4\sqrt{2} + r_2 = 5\sqrt{2} \Rightarrow r_2 = \sqrt{2}$

(iii) Show that the distance from $C(-2, 2)$ to the line DE is half the length of $[DE]$.

Slope $DE = \dfrac{2+3}{3+2} = 1$

Equation $DE : y + 3 = 1(x+2) \Rightarrow x - y - 1 = 0$

Distance from C to DE: $p = \left|\dfrac{-2-2-1}{\sqrt{1+1}}\right| = \left|\dfrac{5}{\sqrt{2}}\right| = \dfrac{5\sqrt{2}}{2} = \dfrac{1}{2}|DE|$

(iv) The translation which maps the midpoint of $[DE]$ to the point C maps the circle k to the circle j. Find the equation of the circle j.

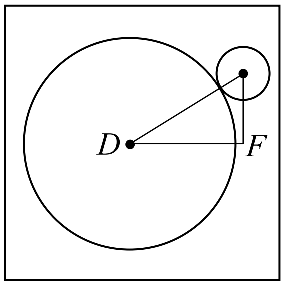
Midpoint $[DE] = \left(\frac{-2+3}{2}, \frac{-3+2}{2}\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$

$\left(\frac{1}{2}, -\frac{1}{2}\right) \to (-2, 2)$ maps $(3,\ 2) \to \left(\frac{1}{2}, \frac{9}{2}\right)$

$j: \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \left(\sqrt{2}\right)^2 = 2$

$4x^2 + 4y^2 - 4x - 36y + 74 = 0$

(v) The glass square is of side length l. Find the smallest whole number l such that the two cogs, h and k, are fully visible through the glass.



$D(-2, -3), \quad F(3, -3)$

$|DF| = 5$

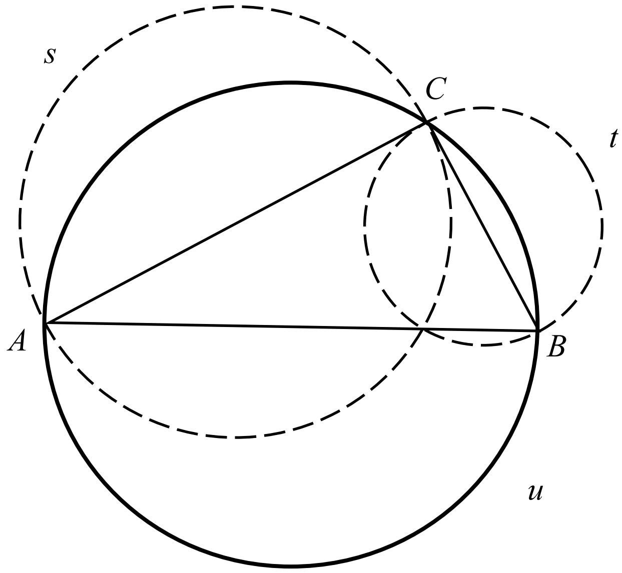
Length: $r_1 + |DF| + r_2 = 4\sqrt{2} + 5 + \sqrt{2} = 5\sqrt{2} + 5 = 12 \cdot 07$

$l = 13$

(b) The triangle ABC is right-angled at C.

The circle s has diameter $[AC]$ and the circle t has diameter $[CB]$.

(i) Draw the circle u which has diameter $[AB]$.



(ii) Prove that in any right-angles triangle ABC, the area of the circle u equals the sum of the areas of the circles s and t.

Triangle ABC is right-angled:

$$|AB|^2 = |AC|^2 + |CB|^2$$

$$\Rightarrow \frac{\pi}{4}\left(|AB|^2\right) = \frac{\pi}{4}\left(|AC|^2 + |CB|^2\right)$$

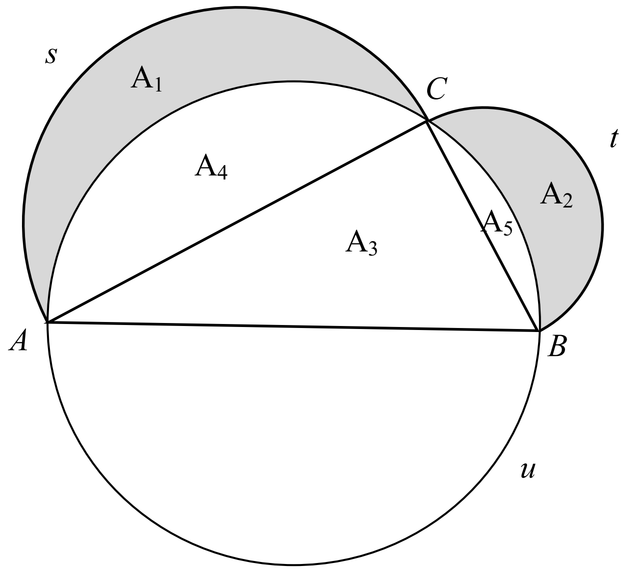
$$\Rightarrow \pi\left(\frac{|AB|}{2}\right)^2 = \pi\left(\frac{|AC|}{2}\right)^2 + \pi\left(\frac{|CB|}{2}\right)^2$$

Thus, area of u = area of s + area of t.

(iii) The diagram shows the right-angled triangle ABC and arcs of the circles s, t and u.

Each of the shaded areas in the diagram is called a lune, a crescent-shaped area bounded by arcs of the circles.

Prove that the sum of the areas of the two shaded lunes is equal to the area of the triangle ABC.



$$\frac{1}{2} \text{ area of } u = \frac{1}{2}(\text{ area of } s + \text{area of } t)$$

$$\Rightarrow A_3 + A_4 + A_5 = (A_1 + A_4) + (A_2 + A_5)$$

$$\Rightarrow A_3 = A_1 + A_2$$

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales		0, 2, 5	0, 2, 3, 5		
10 mark scales		0, 5, 10	0, 3, 7, 10	0, 2, 5, 8, 10	
15 mark scales			0, 5, 10, 15	0, 4, 7, 11, 15	

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)(i)	15D
(a)(ii)	5C
(b)	5C

Question 2

(a)	15C
` (b)	10D

Question 3

(a)	10 C
(b)	5C
(c)	10C

Question 4

(a)	10C
(b)	10C
(c)	5C

Question 5

(a)	10C
(b)	10C
(c)	5C

Question 6A

(a)	5B
	5B
	10C
(b)	5B

Question 6B

(a)	10C
(b)	10C
(c)	5C

Section B

Question 7

(a)	10B
(b)	5C
(c) (i)	5B
(c) (ii)	5B
(c) (iii)	5B
(d) (i)	5B
(d) (ii)	5B
(d) (ii)	5C

Question 8

(a) (i)	10C
(a) (ii)	5C
(a) (iii)	5C
(a) (iv)	5B
(b) (i)	15D
(b) (ii)	5B

Question 9

(a) (i)	10C
(a) (ii)	10C
(a) (iii)	10C
(a) (iv)	10C
(a) (v)	5C
(b) (i)	5B
(b) (ii)	5C
(b) (iii)	5C

Detailed Marking Notes

Section A

Question 1 **(25 marks)**

(a)(i) Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:
- Identifies Cosine Rule formula

Mid Partial Credit:
- All values correctly inserted

High Partial Credit:
- $Cos\angle CBA$ evaluated but angle not found
- Substantially correct work with one non arithmetic error

(ii) Scale 5C(0, 2, 3, 5)

Low Partial Credit:
- Relevant area formula
- Effort at finding a perpendicular height

High Partial Credit
- Substantially correct work with one non arithmetic error
- Values correctly inserted

(b) Scale 5C(0, 2, 3, 5)

Low Partial Credit:
- Recognises $|AD| = |DB| = |DC|$ (any two)
- Recognises one relevant right angle
- Indicates some understanding of circumcentre of a triangle

High Partial Credit
- Recognises $|AD| = |DB| = |DC|$ and relevant right angles but fails to conclude fully
- Clearly identifies two congruent triangles but does not make reference to the remaining triangle

Question 2 **(25 marks)**

(a) Scale 15C (0, 5, 10, 15)

Low Partial Credit:

- Relevant compound angle formula
- Tested with one or more values for A

High Partial Credit

- Expansion correct but not tidied

(b) Scale 10D (0, 2 ,5, 8, 10)

Low Partial Credit:

- Correct formula for finding either arc

Mid Partial Credit

- One or both arcs expressed correctly

High Partial Credit

- θ not fully evaluated
- $|CD| - |AB|$ = 3 or equivalent statement
- Substantially correct with one non arithmetic error

Question 3 **(25 marks)**

(a) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Some reference to €3 and €5

High Partial Credit

- Listing with not more than five omitted

(b) Scale 5C (0, 2, 3, 5)

Low Partial Credit

- One partially accurate statement
- Expected outcome formula

High Partial Credit

- Correct answer but inaccurate / only partially correct supporting evidence
- Expected outcome of both Game A and Game B calculated but incorrect or no conclusion.

(c) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Establishes probability of stopping on €4 sector once
- Establishes probability of not stopping on €4 sector once
- Effort to express a relevant binomial expansion

High Partial Credit

- Omits $\binom{6}{2}$
- Indices incorrectly assigned

Question 4 **(25 marks)**

(a)(i) (ii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Some reference to 311 or -311
- Some indication that term' range' understood
- Ranges (other than correct one) between $\pm 300, \pm 320$ (consistent)
- Some reference to how long it takes to complete a cycle
- Time for one period found

High Partial Credit:

- Correct range
- Correct number of periods

(b)(i) (ii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- At least three further entries in table correct
- Formula for standard deviation

High Partial Credit:

- Table correct
- Errors in table (with at least three additional entries) but standard deviation correct from candidates work

(c)(i) (ii) Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- k isolated in formula
- Value(s) entered in formula
- $\dfrac{b}{2\pi}$ written

High Partial Credit:

- k or b or a found

Question 5 **(25 marks)**

(a) Scale 10C (0, 3, 7, 10)
Low Partial Credit:
Relevant area of triangle formula

High Partial Credit:
- $|OR|$ found but x ordinate of R not stated
- Substantially correct work with one error

(b) Scale 10C (0, 3, 7, 10)
Low Partial Credit:
- Effort at finding one slope
- Effort at finding equation of *RS*

High Partial Credit:
- Relevant conclusion not stated or implied
- *E* inserted into equation of *RS* but relevant conclusion not stated or implied

(c) Scale 5C (0, 2 ,3, 5)
Low Partial Credit:
- Effort at finding intercept on one or both axes
- Effort at inserting $(-5,\ 4)$ into $y = mx + c$

High Partial Credit:
- Either *c* or *m* found

Question 6A **(25 marks)**

(a) **Diagram / Given** : Scale 5B (0, 2, 5)

Partial Credit:

- Effort at *Diagram* or *Given*

Construction: Scale 5B (0, 2, 5)

Partial Credit:

- Construction attempted
- Construction not explained or explanation incomplete

Proof: Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- More than one critical step omitted but still some substantial work of merit

High Partial Credit:

- Proof completed with one critical step omitted

(b) Scale 5B (0, 2, 5)

Partial Credit:

- Arc AC and/or arc AB
- Effort at drawing arc from B

Question 6B **(25 marks)**

(a) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Triangles named

High Partial Credit:

- Two pairs of angles in relevant triangles identified but justification incomplete
- Two pairs of angles identified with justification but triangles not named

(b) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- Relevant triangles identified
- Partly correct ratio

High Partial Credit:

- Correct ratio established but fails to complete

(c) Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- Effort at establishing ratio

High Partial Credit:

- Ratio established and values entered

Question 7 **(45 marks)**

(a) Scale 10B (0, 5, 10)
Partial Credit:
- One correct category only

(b) Scale 5C (0, 2, 3, 5)
Low Partial Credit:
- Effort at listing numbers in order of magnitude
- Identifying either 1828·6 or 1867
- Some indication of understanding of the term 'median'
- Identifying 1803·4 and/ or 1954·9 or equivalent as relating to quartiles
- Some indication of understanding of interquartile range

High Partial Credit:
- Median correct or interquartile range correct only

(c)(i) Scale 5B (0, 2, 5)
Partial Credit:
- One or two correct (no work shown)
- All incorrect but sum to 100%
- Correct numerator or denominator chosen for one category- work shown

(ii) Scale 5B (0, 2, 5)
Partial Credit:
- One or two correct (no work shown)
- All incorrect but sum to 100%
- Correct total population - shown
- Correct numerator or denominator chosen for one category- work shown

(iii) **Note:** Answer here depends on candidate's answers in previous sections
Scale 5B (0, 2, 5)
Partial Credit:
- One reason only
- Reasons contradictory

(d)(i) Scale 5B (0, 2, 5)
Partial Credit:
- Incomplete or contradictory statement

(ii) Scale 5B (0, 2 ,5)
Partial Credit:
- Stating none, one or both without explanation

(iii) Scale 5C (0, 2, 3 , 5)
Low Partial Credit:
- Effort at finding increase in number from 2012 to 2103
- Effort at finding % increase between 2012 and 2013

High Partial Credit:
- Total number predicted at work in 2014
- Percentage increase calculated

Note: Candidates not required to round to any particular number of places of decimals

Question 8 **(45 marks)**

(a)(i) Scale 10C (0, 3, 7, 10)

Low Partial Credit:

- One element entered correctly
- One column correct
- Some indication that values lie between 0 and 1

High Partial Credit:

- Two columns correct

(ii) Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- One correct value chosen
- Addition of values indicated
- Configured correctly but no values entered
- Answer outside range

High Partial Credit:

- Correct values chosen but operator incorrect

(iii) Scale 5C (0, 2, 3, 5)

Low Partial Credit:

- One or both correct value(s) chosen only
- Configured correctly but values not entered
- Answer outside range

High Partial Credit:

- Correct values chosen but incorrect operator leading to an answer within range

(iv) Scale 5B (0, 2, 5)

Partial Credit:

- Reason incorrect or incomplete

(b)(i) Scale 15D(0, 4, 7, 11, 15)

Low Partial Credit:

- One relevant step e.g. null hypothesis stated only
- Some work towards margin of error

Mid Partial Credit:

- Margin of error or observed proportion
- Margin of error and observed proportion found but fails to continue

High Partial Credit:

- Failure to state null hypothesis correctly
- Failure to contextualise answer (e.g. Stops at reject Null Hypothesis)

(b)(ii) Scale 5B (0, 2, 5)

Partial Credit:

- $\dfrac{n}{500}$ and stops
- Recognises interval where result must lie
- Some relevant work

Question 9 **(60 marks)**

(a)(i) Scale 15C (0, 5, 10, 15)

Low Partial Credit:
- Effort at relating one or more coefficients of given equation to general equation of circle
- Effort at completing square(s)

High Partial Credit:
- Either radius or centre correct
- Substantive work with one critical error

(ii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- Effort at finding $|DE|$
- Length of line segment formula
- Indicates some understanding of $r_1 + r_2 = |DE|$

High Partial Credit:
- $r_1 + r_2 = |DE|$ or equivalent with known values substituted

(iii) Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- Slope DE
- Equation DE and stops
- Formula for slope and /or equation of DE
- Perpendicular distance formula

High Partial Credit:
- Values inserted into perpendicular distance formula
- No conclusion stated or implied

(iv) Scale 5C (0, 2, 3, 5)

Low Partial Credit:
- Effort to find midpoint of DE
- Centre found from scaled drawing

High Partial Credit:
- Centre of j found and inserted into equation of circle i.e radius omitted

(v) Scale 5C (0, 2, 3, 5)
Low Partial Credit:
- Effort to find F
- Indication length $r_1 + r_2 + |DF|$ (or equivalent)

High Partial Credit:
- F found

(b)(i) Scale 5B (0, 2, 5)
Partial Credit:
- Circle containing A and B but lacking in accuracy

(ii) Scale 5C (0, 2, 3, 5)
Low Partial Credit:
- Pythagoras stated or implied
- Effort at finding area of s or t or u

High Partial Credit
- Correct expression for area of any circle

  e.g area $u = \frac{\pi}{4}\left(|AB|^2\right)$ or $\pi\left(\frac{|AB|}{2}\right)^2$

(iii) Scale 5C (0, 2, 3, 5)
Low Partial Credit:
- Statement using result from (b)(ii)
- Recognising half the area of s or half the area t can be expressed in terms of two component areas
- Recognising half area of u can be expressed in terms of three components

High Partial Credit
- Correct expression for two of the relevant areas

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc $\times$ 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] $\times$ 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

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