



Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2010

MARKING SCHEME

**MATHEMATICS
(PROJECT MATHS)**

HIGHER LEVEL

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Introduction

The Higher Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared a common Paper 1 with the examination for all other candidates. The marking scheme used for Paper 1 was identical for the two groups.

This document contains the complete marking scheme for both papers for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.

Marking scheme for Paper 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
 - Blunders - mathematical errors/omissions (-3)
 - Slips - numerical errors (-1)
 - Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
 - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
 - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
 - a mark between zero and the attempt mark is never awarded.
3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.
4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.
5. The phrase “and stops” means that no more work is shown by the candidate.
6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.
7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.
8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.
9. The *same* error in the *same* section of a question is penalised *once* only.
10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.
11. A serious blunder, omission or misreading results in the attempt mark at most.
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	20 (5, 10, 5) marks	Att (2, 3, 2)
Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (a) (5, 5) marks Att (2, 2)

1. (a) $x^2 - 6x + t = (x + k)^2$, where t and k are constants.
Find the value of k and the value of t .

(a) **Equating coefficients** 5 marks Att 2
Values 5 marks Att 2

1 (a)

$$x^2 - 6x + t = (x + k)^2 \Rightarrow x^2 - 6x + t = x^2 + 2kx + k^2.$$

$$\therefore 2k = -6 \text{ and } t = k^2 \Rightarrow k = -3 \text{ and } t = 9.$$

Or

(a) **Perfect square** 5 marks Att 2
Values 5 marks Att 2

1(a)

$$\begin{aligned}x^2 - 6x + t &= (x + k)^2 \\(x^2 - 6x + t) &\text{ is a perfect square} \\(x - 3)^2 &= x^2 - 6x + 9 \\ \Rightarrow k &= -3 \text{ and } t = 9\end{aligned}$$

Blunders (-3)

- B1 Expansion $(x + a)^2$ once only
- B2 Not like-to-like in equating coefficients
- B3 Indices

Part (b)

20 (5, 10, 5) marks

Att (2, 3, 2)

(b) Given that p is a real number, prove that the equation $x^2 - 4px - x + 2p = 0$ has real roots.

(b) Equation arranged 5 marks Att 2

Correct substitution in $b^2 - 4ac$ 10 marks Att 3

Finish 5 marks Att 2

1 (b) $x^2 - 4px - x + 2p = 0 \Rightarrow x^2 + x(-4p - 1) + 2p = 0.$

$$b^2 - 4ac = (-4p - 1)^2 - 4(2p) = 16p^2 + 8p - 8p + 1 = 16p^2 + 1 \geq 0 \text{ for all } p.$$

\therefore Roots are real.

Blunders (-3)

B1 Expansion of $(a + b)^2$ once only

B2 Incorrect value a

B3 Incorrect value b

B4 Incorrect value c

B5 Inequality sign

B6 Indices

B7 Incorrect deduction or no deduction

Part (c)

20 (5, 5, 5, 5) marks

Att (2, 2, 2, 2)

(c) $(x-2)$ and $(x+1)$ are factors of $x^3 + bx^2 + cx + d$.

(i) Express c in terms of b .

(ii) Express d in terms of b .

(iii) Given that b, c and d are three consecutive terms in an arithmetic sequence, find their values.

$f(2)$ and $f(-1)$
 c in terms of b
 d in terms of b
Values

5 marks
5 marks
5 marks
5 marks

Att 2
Att 2
Att 2
Att 2

1 (c) (i)

$$(x-2) \text{ is a factor} \Rightarrow f(2) = 0. \quad \therefore 8 + 4b + 2c + d = 0 \Rightarrow 4b + 2c + d = -8.$$

$$(x+1) \text{ is a factor} \Rightarrow f(-1) = 0. \quad \therefore -1 + b - c + d = 0 \Rightarrow b - c + d = 1.$$

$$\therefore 3b + 3c = -9 \Rightarrow b + c = -3 \Rightarrow c = -b - 3.$$

1 (c) (ii) By part (i)

$$4b + 2c + d = -8$$

$$\frac{2b - 2c + 2d = 2}{6b + 3d = -6} \Rightarrow 2b + d = -2 \Rightarrow d = -2b - 2.$$

1 (c) (iii) An arithmetic sequence $b, c, d \Rightarrow c - b = d - c \Rightarrow 2c = b + d$.

$$\therefore -2b - 6 = b - 2b - 2 \Rightarrow b = -4.$$

$$\therefore c = 1 \text{ and } d = 6.$$

Blunders (-3)

B1 Indices

B2 Deduction root from factor

B3 Statement of AP

Slips (-1)

S1 Numerical

Worthless

W1 Geometric Sequence

Or

Division & remainder = 0	5 marks	Att 2
c in terms of b	5 marks	Att 2
d in terms of b	5 marks	Att 2
Values	5 marks	Att 2

1 (c) (i)

$$(x-2)(x+1) = (x^2 - x - 2) \quad \text{factor}$$

1 (c) (ii)

$$\begin{array}{r}
 x + (b+1) \\
 x^2 - x - 2 \overline{) x^3 + bx^2 + cx + d} \\
 \underline{x^3 - x^2 - 2x} \\
 (b+1)x^2 + (c+2)x + d \\
 \underline{(b+1)x^2 - (b+1)x - 2(b+1)} \\
 (c+2)x + (b+1)x + d + 2(b+1) = 0
 \end{array}$$

since $(x^2 - x - 2)$ is a factor

$$[(c+2) + (b+1)]x + [d + 2(b+1)] = (0)x + (0)$$

Equating Coefficients

$$(i) \quad b + c + 3 = 0 \Rightarrow c = -3 - b$$

$$(ii) \quad d + 2b + 2 = 0 \Rightarrow d = -2b - 2$$

1 (c) (iii) As in previous solution

Blunders (-3)

B1 $(x-2)(x+1)$ once only

B2 Indices

B3 Not like-to-like when equating coefficients

Slips (-1)

S1 Not changing sign when subtracting

Attempts

A1 Any effort at division

Worthless

W1 Geometric sequence

Other linear factor & multiplication	5 marks	Att 2
c in terms of b	5 marks	Att 2
d in terms of b	5 marks	Att 2
Values	5 marks	Att 2

1 (c) (i) (ii)

$$(x-2)(x+1) = (x^2 - x - 2) \text{ factor}$$

$$(x^2 - x - 2)\left(x - \frac{d}{2}\right) = x^3 + bx^2 + cx + d$$

$$x^3 - x^2 - 2x - \frac{dx^2}{2} + \frac{dx}{2} + d = x^3 + bx^2 + cx + d$$

$$x^3 + \left(-\frac{d}{2} - 1\right)x^2 + \left(-2 + \frac{d}{2}\right)x + d = x^3 + (b)x^2 + (c)x + (d)$$

Equating Coefficients

$$\begin{aligned} \text{(i)} \quad &: -2 + \frac{d}{2} = c \\ &-4 + d = 2c \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad &: -\frac{d}{2} - 1 = b \\ &-d - 2 = 2b \\ &-2b - 2 = d \end{aligned}$$

Put this value of d into (i)

$$\begin{aligned} \text{(i)} \quad &-4 + (-2b - 2) = 2c \\ &-4 - 2b - 2 = 2c \\ &-6 - 2b = 2c \\ &c = -3 - b \end{aligned}$$

1 (c) (iii) As in previous solution

Blunders (-3)

B1 Indices

B2 $(x-2)(x+1)$ once only

B3 Not like to like when equating coefficients

Attempts

A1 Other factors not linear in (1) only

Worthless

W1 Geometric sequence

QUESTION 2

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	20 (10, 10) marks	Att (3, 3)
Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (a) **10 (5, 5) marks** **Att(2, 2)**

(a) Solve the simultaneous equations

$$2x + 3y = 0$$

$$x + y + z = 0$$

$$3x + 2y - 4z = 9.$$

(a) One unknown **5 marks** **Att 2**
Other values **5 marks** **Att 2**

2 (a)

$$4x + 4y + 4z = 0$$

$$3x + 2y - 4z = 9$$

$$\hline 7x + 6y = 9$$

$$4x + 6y = 0$$

$$\hline 3x = 9 \Rightarrow x = 3. \therefore y = -2 \text{ and } z = -1.$$

Blunders (-3)

B1 Multiplying one side of equation only

B2 Not finding 2nd value, having found 1st value

B3 Not finding 3rd value, having found other two

Slips (-1)

S1 Numerical

S1 Not changing sign when subtracting

Worthless

W1 Trial and error only

Part (b)**20(10, 10) marks****Att (3, 3)**

- (b) The equation $x^2 - 12x + 16 = 0$ has roots α^2 and β^2 , where $\alpha > 0$ and $\beta > 0$.
- (i) Find the value of $\alpha\beta$.
- (ii) Hence, find the value of $\alpha + \beta$.

(b) (i) Value of $\alpha\beta$ **10 marks****Att 3****(b) (ii) Value of $(\alpha + \beta)$** **10 marks****Att 3****2 (b) (i)**

$$\alpha^2 \beta^2 = 16 \Rightarrow \alpha\beta = 4.$$

2 (b) (ii)

$$\alpha^2 + \beta^2 = 12 \text{ and } \alpha\beta = 4.$$

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta = 12 + 8 = 20.$$

$$\therefore \alpha + \beta = \sqrt{20} = 2\sqrt{5}.$$

Blunders (-3)

B1 Indices

B2 Incorrect sum

B3 Incorrect product

B4 Incorrect statements

B5 Excess value each time

Slips (-1)

S1 Numerical

Part (c)**20 (5, 5, 5, 5) marks****Att (2, 2, 2, 2)**

- (c) (i) Prove that for all real numbers a and b ,
- $$a^2 - ab + b^2 \geq ab.$$
- (ii) Let a and b be non-zero real numbers such that $a + b \geq 0$.
- Show that $\frac{a}{b^2} + \frac{b}{a^2} \geq \frac{1}{a} + \frac{1}{b}$.

(c) (i)	5 marks	Att 2
(ii) Factors	5 marks	Att 2
Use of part (i)	5 marks	Att 2
Finish	5 marks	Att 2

2 (c) (i)

$$(a-b)^2 \geq 0 \Rightarrow a^2 - 2ab + b^2 \geq 0.$$

$$\therefore a^2 - ab + b^2 \geq ab.$$

2 (c) (ii)

$$\frac{a}{b^2} + \frac{b}{a^2} = \frac{a^3 + b^3}{a^2 b^2} = \frac{(a+b)(a^2 - ab + b^2)}{a^2 b^2}.$$

But $\frac{(a+b)(a^2 - ab + b^2)}{a^2 b^2} \geq \frac{ab(a+b)}{a^2 b^2}$, by part (i)

$$\frac{ab(a+b)}{a^2 b^2} = \frac{a+b}{ab} = \frac{a}{ab} + \frac{b}{ab} = \frac{1}{b} + \frac{1}{a}.$$

$$\therefore \frac{a}{b^2} + \frac{b}{a^2} \geq \frac{1}{a} + \frac{1}{b}.$$

OR**2 (c) (ii)**

$$\frac{a}{b^2} + \frac{b}{a^2} \geq \frac{1}{a} + \frac{1}{b}$$

Multiply across by $a^2 b^2$, which is positive:

$$\Leftrightarrow a^3 + b^3 \geq ab^2 + ba^2$$

$$\Leftrightarrow (a+b)(a^2 - ab + b^2) \geq ab(a+b)$$

$$\Leftrightarrow a^2 - ab + b^2 \geq ab, \quad \text{since } a+b \geq 0$$

true, by part (i).

Blunders (-3)

- B1 Expansion $(a-b)^2$ once only
- B2 Factors $a^3 + b^3$
- B3 Indices
- B4 Inequality sign
- B5 Incorrect deduction or no deduction

Slips (-1)

S1 Numerical

Attempts

A1 $a^3 + b^3 = (a+b)(a^2 + b^2)$

Worthless

W1 Particular values

(c) (i)	5 marks	Att 2
(ii) Common denominator	5 marks	Att 2
Factorised	5 marks	Att 2
Finish	5 marks	Att 2

2 (c) (i)

$$\begin{aligned}(a^2 - ab + b^2) \geq ab, & \Leftrightarrow (a^2 - ab + b^2) - ab \geq 0. \\ (a^2 - ab + b^2) - ab &= a^2 - 2ab + b^2 \\ &= (a - b)^2 \\ &\geq 0\end{aligned}$$

2 (c) (ii)

$$\begin{aligned}\frac{a}{b^2} + \frac{b}{a^2} \geq \frac{1}{a} + \frac{1}{b}, & \Leftrightarrow \left(\frac{a}{b^2} + \frac{b}{a^2}\right) - \left(\frac{1}{a} + \frac{1}{b}\right) \geq 0. \\ \left(\frac{a}{b^2} + \frac{b}{a^2}\right) - \left(\frac{1}{a} + \frac{1}{b}\right) &= \frac{a^3 + b^3 - ab^2 - a^2b}{a^2b^2} \\ &= \frac{(a^3 - a^2b) - (ab^2 - b^3)}{a^2b^2} \\ &= \frac{a^2(a - b) - b^2(a - b)}{a^2b^2} \\ &= \frac{(a - b)[a^2 - b^2]}{a^2b^2} \\ &= \frac{(a - b)[(a - b)(a + b)]}{(ab)^2} \\ &= \frac{(a - b)^2(a + b)}{(ab)^2} \geq 0, \text{ since } a + b \geq 0\end{aligned}$$

Blunders (-3)

B1 Indices

B2 Inequality Sign

B3 Factors $(a^2 - b^2)$ once only

B4 Incorrect deduction or no deduction

Worthless

W1 Particular values

QUESTION 3

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	20 (5, 5, 10) marks	Att (2, 2, 3)
Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (a) **10 (5, 5) marks** **Att(2, 2)**

(a) Find x and y such that

$$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 32 \end{pmatrix}.$$

Inverse of A evaluated **5 marks** **Att 2**
Finish **5 marks** **Att 2**

3 (a)

$$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 32 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1} \begin{pmatrix} 20 \\ 32 \end{pmatrix}.$$

$$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{18-20} \begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 20 \\ 32 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -8 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$$

Or

One unknown **5 marks** **Att 2**
Other unknown **5 marks** **Att 2**

3 (a)

(i) $3x + 4y = 20.6 \Rightarrow 18x + 24y = 120$
(ii) $5x + 6y = 32.4 \Rightarrow 20x + 24y = 128$

$$\begin{array}{r} \underline{-2x = -8} \\ x = 4 \end{array}$$

(i) $3x + 4y = 20$
 $12 + 4y = 20$
 $4y = 8 \Rightarrow y = 2$

Blunders (-3)

- B1 Formula for inverse
- B2 Matrix multiplication

Slips (-1)

- S1 Each incorrect element in matrix multiplication
- S2 Numerical
- S3 Not changing sign when subtracting

(b) Let $z_1 = s + 8i$ and $z_2 = t + 8i$, where $s \in \mathbb{R}, t \in \mathbb{R}$ and $i^2 = -1$.

(i) Given that $|z_1| = 10$, find the values of s .

(ii) Given that $\arg(z_2) = \frac{3\pi}{4}$, find the value of t .

(b) (i) Values for modulus

5 marks

Att 2

Values of s

5 marks

Att 2

(ii) Value of t

10 marks

Att 3

3 (b) (i) $|s + 8i| = 10 \Rightarrow \sqrt{s^2 + 64} = 10 \Rightarrow s^2 = 36. \therefore s = \pm 6.$

3 (b) (ii) $\tan \frac{3\pi}{4} = \frac{8}{t} \Rightarrow -t = 8 \Rightarrow t = -8.$

Or

3 (b) (i) $z_1 = s + 8i \Rightarrow |z_1| = 10$

$$\sqrt{s^2 + 64} = 10$$

$$s^2 + 64 = 100$$

$$s^2 = 36$$

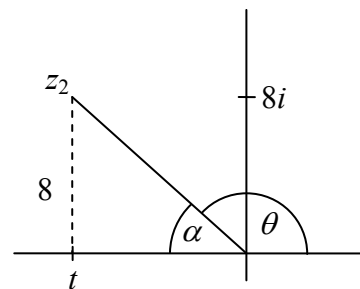
$$s = \pm 6$$

3 (b) (ii)

$$\tan \alpha = \tan \frac{\pi}{4} = 1$$

$$\Rightarrow \frac{8}{|t|} = 1$$

$$|t| = 8 \Rightarrow t = -8$$



$$\theta = \frac{3\pi}{4} \Rightarrow \alpha = \frac{\pi}{4}$$

Blunders (-3)

B1 Formula for modulus

B2 Indices

B3 Only one value for s

B4 Diagram for z_2 once only

B5 Incorrect argument

B6 Trig Definition

B7 Mod Values

B8 $\tan \frac{3\pi}{4} = 1$

Slips (-1)

S1 Trig value

S2 Numerical

(c) (i) Use De Moivre's theorem to find, in polar form, the five roots of the equation

$$z^5 = 1.$$

(ii) Choose one of the roots w , where $w \neq 1$. Prove that $w^2 + w^3$ is real.

(c) (i) $z = cis \frac{2n\pi}{5}$

5 marks

Att 2

Five roots

5 marks

Att 2

(c) (ii) $w^2 + w^3$ as sum of cos and sin

5 marks

Att 2

Show real

5 marks

Att 2

3 (c) (i)

$$z = (\cos 0 + i \sin 0)^{\frac{1}{5}} = \cos\left(\frac{0 + 2n\pi}{5}\right) + i \sin\left(\frac{0 + 2n\pi}{5}\right), \text{ for } n = 0, 1, 2, 3, 4.$$

$$n = 0 \Rightarrow z_0 = 1.$$

$$n = 1 \Rightarrow z_1 = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}.$$

$$n = 2 \Rightarrow z_2 = \cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}.$$

$$n = 3 \Rightarrow z_3 = \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}.$$

$$n = 4 \Rightarrow z_4 = \cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}.$$

3 (c) (ii)

$$\text{Let } w = z_1 = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}.$$

$$\therefore w^2 + w^3 = \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}\right)^2 + \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}\right)^3$$

$$= \cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5} + \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$$

$$= \left(\cos \frac{6\pi}{5} + \cos \frac{4\pi}{5}\right) + i \left(\sin \frac{6\pi}{5} + \sin \frac{4\pi}{5}\right)$$

$$= \left(2 \cos \pi \cos \frac{\pi}{5}\right) + i \left(2 \sin \pi \cos \frac{\pi}{5}\right)$$

$$= -2 \cos \frac{\pi}{5} + i(0),$$

$$= -2 \cos \frac{\pi}{5}, \text{ which is real}$$

Blunders (-3)

B1 Formula De Moivre once only

B2 Application De Moivre

B3 Indices

B4 Trig Formula
B5 Polar formula once only
B6 i

Slips (-1)

S1 Trig value
S2 Root omitted

Note: Must show $(0)i$

Attempt

A1 Use of decimals in c(ii)

Worthless

W1 $w=1$ used in c(ii)

QUESTION 4

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	15 (5, 5, 5) marks	Att (2, 2, 2)
Part (c)	25 (5, 5, 5, 5, 5) marks	Att (2, 2, 2, 2, 2)

Part (a) **10 (5, 5)marks** **Att (2, 2)**

(a) Write the recurring decimal $0.474747\dots$ as an infinite geometric series and hence as a fraction.

(a) Series **5 marks** **Att 2**
Fraction **5 marks** **Att 2**

4 (a)

$$0.474747\dots = \frac{47}{100} + \frac{47}{100^2} + \frac{47}{100^3} + \dots$$
$$= \frac{a}{1-r} = \frac{\frac{47}{100}}{1-\frac{1}{100}} = \frac{47}{99}$$

Blunders (-3)

- B1 Infinity formula once only
- B2 Incorrect a
- B3 Incorrect r

Slips (-1)

- S1 Numerical

Part (b)**15 (5, 5, 5) marks****Att (2, 2, 2)****(b)** In an arithmetic sequence, the fifth term is -18 and the tenth term is 12 .**(i)** Find the first term and the common difference.**(ii)** Find the sum of the first fifteen terms of the sequence.**(b) (i) Terms in a and d** **5 marks****Att 2****Values of a and d** **5 marks****Att 2****(b) (ii) Sum****5 marks****Att 2****4 (b) (i)**

$$T_5 = -18 \Rightarrow a + 4d = -18$$

$$T_{10} = 12 \Rightarrow \underline{a + 9d = 12}$$

$$-5d = -30 \Rightarrow d = 6 \text{ and } a = -42$$

4 (b) (ii)

$$S_n = \frac{n}{2} \{2a + (n-1)d\}. \therefore S_{15} = \frac{15}{2} \{-84 + 14(6)\} = \frac{15}{2}(0) = 0.$$

Blunders (-3)

B1 Term of A.P.

B2 Formula A.P. once only (term)

B3 Incorrect a B4 Incorrect d

B5 Formula for sum arithmetic series once only

Slips (-1)

S1 Numerical

Worthless

W1 Treats as G.P.

Part (c)

25 (5, 5, 5, 5, 5) marks

Att (2, 2, 2, 2, 2)

(c) (i) Show that $(r+1)^3 - (r-1)^3 = 6r^2 + 2$.

(ii) Hence, or otherwise, prove that $\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$.

(iii) Find $\sum_{r=1}^{30} (3r^2 + 1)$.

(c) (i)

5 marks

Att 2

4 (c) (i) $(r+1)^3 - (r-1)^3 = r^3 + 3r^2 + 3r + 1 - (r^3 - 3r^2 + 3r - 1) = 6r^2 + 2$.

OR

4 (c) (i)

$$\begin{aligned}(r+1)^3 - (r-1)^3 &= [(r+1) - (r-1)][(r+1)^2 + (r+1)(r-1) + (r-1)^2] \\ &= [r+1 - r+1][r^2 + 2r + 1 + r^2 - 1 + r^2 - 2r + 1] \\ &= (2)(3r^2 + 1) \\ &= 6r^2 + 2\end{aligned}$$

Blunders (-3)

- B1 Expansion of $(r+1)^3$ once only
- B2 Expansion of $(r-1)^3$ once only
- B3 Formula $a^3 - b^3$
- B4 Indices
- B5 Expansion of $(r+1)^2$ once only
- B6 Expansion of $(r-1)^2$ once only
- B7 Binomial expansion once only

(c) (ii) Cancellation & addition shown
Prove

5 marks
5 marks

Att 2
Att 2

4 (c) (ii)

$$\begin{aligned} \cancel{2^3} - 0^3 &= 6(1^2) + 2 \\ \cancel{3^3} - 1^3 &= 6(2^2) + 2 \\ \cancel{4^3} - \cancel{2^3} &= 6(3^2) + 2 \\ &\vdots \\ &\vdots \end{aligned}$$

$$\begin{aligned} \cancel{(n-1)^3} - \cancel{(n-3)^3} &= 6(n-2)^2 + 2 \\ n^3 - \cancel{(n-2)^3} &= 6(n-1)^2 + 2 \\ (n+1)^3 - \cancel{(n-1)^3} &= 6n^2 + 2 \end{aligned}$$

$$(n+1)^3 + n^3 - 1 = 6 \sum_{r=1}^n r^2 + 2n$$

$$\begin{aligned} \sum_{r=1}^n r^2 &= \frac{1}{6}(n^3 + 3n^2 + 3n + 1 + n^3 - 1 - 2n) = \frac{1}{6}(2n^3 + 3n^2 + n) \\ &= \frac{n(2n^2 + 3n + 1)}{6} = \frac{n(n+1)(2n+1)}{6} \end{aligned}$$

OR

4 (c) (ii) Prove by induction that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n}{6}(n+1)(2n+1)$

P(1): Test $n = 1$: $\frac{1}{6}(2)(3) = 1 \Rightarrow$ True for $n = 1$.

P(k): Assume true for $n = k$: $\Rightarrow S_k = \frac{k}{6}(k+1)(2k+1)$

To prove: $S_{k+1} = \frac{k+1}{6}(k+2)(2k+3)$

Proof: $S_{k+1} = 1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{k}{6}(k+1)(2k+1) + (k+1)^2$, using P(k)

$$\begin{aligned} &= \frac{(k+1)}{6}[k(2k+1) + 6(k+1)] \\ &= \frac{(k+1)}{6}[2k^2 + k + 6k + 6] \\ &= \frac{k+1}{6}[2k^2 + 7k + 6] \\ &= \frac{k+1}{6}[(k+2)(k+3)] \end{aligned}$$

\Rightarrow Formula true for $n = (k+1)$ if true for $n = k$

It is true for $n = 1 \Rightarrow$ true for all n

* Must show three terms at start and two at finish or vice versa in first method.

Blunders (-3)

- B1 Indices
- B2 Cancellation must be shown or implied
- B3 Term omitted
- B4 Expansion $(n+1)^3$ once only

(c) (iii) Substitution of $r = 30$ and $r = 10$

5 marks

Att 2

Sum

5 marks

Att 2

4 (c) (iii)

$$\begin{aligned}\sum_{r=11}^{30} (3r^2 + 1) &= 3 \sum_1^{30} r^2 - 3 \sum_1^{10} r^2 + 30 - 10 \\ &= \frac{3(30)(31)(61)}{6} - \frac{3(10)(11)(21)}{6} + 20 = 28365 - 1155 + 20 = 27230.\end{aligned}$$

Blunders (-3)

- B1 Formula
- B2 Not $(\Sigma 30 - \Sigma 10)$
- B3 Value n

Slips (-1)

- S1 Numerical

QUESTION 5

Part (a)	10 (5, 5)marks	Att(2, 2)
Part (b)	20 (5, 5, 10) marks	Att (2, 2, 3)
Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (a) **10 (5, 5)marks** **Att (2, 2)**

(a) Solve $\log_2(x+6) - \log_2(x+2) = 1$.

(a) Log law applied **5 marks** **Att 2**
Value **5 marks** **Att 2**

$$\begin{aligned}\log_2(x+6) - \log_2(x+2) &= 1. \\ \therefore \log_2\left(\frac{x+6}{x+2}\right) &= 1 \Rightarrow \frac{x+6}{x+2} = 2 \\ \therefore 2x+4 &= x+6 \Rightarrow x = 2.\end{aligned}$$

Blunders (-3)

B1 Log laws

B2 Indices

Part (b)

20 (5, 5, 10) marks

Att (2, 2, 3)

(b) Use induction to prove that

$$2 + (2 \times 3) + (2 \times 3^2) + (2 \times 3^3) + \dots + (2 \times 3^{n-1}) = 3^n - 1,$$

where n is a positive integer.

Part (b) $P(1)$

5 marks

Att 2

$P(k)$

5 marks

Att 2

$P(k+1)$

10 marks

Att 3

5 (b)

Test for $n = 1$, $P(1) = 3^1 - 1 = 2$.

\therefore True for $n = 1$.

Assume $P(k)$. (That is, assume true for $n = k$.)

i.e., assume $S_k = 3^k - 1$, where S_k is the sum of the first k terms.

Deduce $P(k+1)$. (That is, deduce truth for $n = k+1$.)

i.e. deduce that $S_{k+1} = 3^{k+1} - 1$.

Proof: $S_{k+1} = S_k + T_{k+1} = 3^k - 1 + 2 \times 3^k = 3(3^k) - 1 = 3^{k+1} - 1$.

\therefore True for $n = k+1$.

So, $P(k+1)$ is true whenever $P(k)$ is true. Since $P(1)$ is true, then, by induction, $P(n)$ is true, for all positive integers n .

Blunders (-3)

B1 Indices

B2 Not T_{k+1} added to each side

B3 Not $n = 1$

Worthless

W1 $P(0)$

(c) (i) Expand $\left(x + \frac{1}{x}\right)^2$ and $\left(x + \frac{1}{x}\right)^4$.

(ii) Hence, or otherwise, find the value of $x^4 + \frac{1}{x^4}$, given that $x + \frac{1}{x} = 3$.

(c) (i) $\left(x + \frac{1}{x}\right)^2$ 5 marks Att 2

$\left(x + \frac{1}{x}\right)^4$ 5 marks Att 2

(c) (ii) Terms collected 5 marks Att 2
Value 5 marks Att 2

5 (c) (i)

$$\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}.$$

$$\begin{aligned} \left(x + \frac{1}{x}\right)^4 &= x^4 + {}^4C_1 x^3 \left(\frac{1}{x}\right) + {}^4C_2 x^2 \left(\frac{1}{x}\right)^2 + {}^4C_3 x \left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^4 \\ &= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}. \end{aligned}$$

OR

$$\begin{aligned} \left(x + \frac{1}{x}\right)^4 &= \left[\left(x + \frac{1}{x}\right)^2\right]^2 \\ &= \left[\left(x^2 + \frac{1}{x^2}\right) + 2\right]^2 \\ &= \left(x^2 + \frac{1}{x^2}\right)^2 + 2(2)\left(x^2 + \frac{1}{x^2}\right) + 4 \\ &= x^4 + 2 + \frac{1}{x^4} + 4x^2 + \frac{4}{x^2} + 4 \\ &= x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4} \end{aligned}$$

5 (c) (ii)

$$\left(x + \frac{1}{x}\right)^4 = 81 = x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4} = \left(x^4 + \frac{1}{x^4}\right) + 4\left(x^2 + \frac{1}{x^2}\right) + 6$$

$$\therefore x^4 + \frac{1}{x^4} = 75 - 4\left(x^2 + \frac{1}{x^2}\right).$$

$$\text{But } x^2 + 2 + \frac{1}{x^2} = 9 \Rightarrow x^2 + \frac{1}{x^2} = 7.$$

$$\therefore x^4 + \frac{1}{x^4} = 75 - 28 = 47.$$

Blunders (-3)

B1 Binomial Expansion once only

B2 Indices

B3 Value $\binom{n}{r}$ or no $\binom{n}{r}$

B4 $x^0 \neq 1$

B5 Expansion $\left(x + \frac{1}{x}\right)^2$ once only

B6 Expansion $\left(x + \frac{1}{x}\right)^4$ once only

B7 Value $\left(x^2 + \frac{1}{x^2}\right)$ or no value $\left(x^2 + \frac{1}{x^2}\right)$

OR

**(c) (ii) Roots
Value**

**5 marks
5 marks**

**Att 2
Att 2**

5 (c) (ii)

$$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$x^4 - 7x^2 + 1 = 0$$

$$x^2 = \frac{7 \pm 3\sqrt{5}}{2}$$

$$x^4 + \frac{1}{x^4} = \left(\frac{7 + 3\sqrt{5}}{2}\right)^2 + \left(\frac{2}{7 + 3\sqrt{5}}\right)^2$$

$$= \frac{94 + 42\sqrt{5}}{4} + \frac{4}{94 + 42\sqrt{5}}$$

$$= \frac{2209 + 987\sqrt{5}}{47 + 21\sqrt{5}} \cdot \frac{47 - 21\sqrt{5}}{47 - 21\sqrt{5}}$$

$$= \frac{103823 + 46389\sqrt{5} - 46389\sqrt{5} - 103635}{2209 - 2205}$$

$$= 47$$

Similarly, when $x^2 = \frac{7 - 3\sqrt{5}}{2}$, $x^4 + \frac{1}{x^4} = 47$.

Note: must test two roots.

Blunders (-3)

B1 Roots formula once only

B2 Indices

B3 Expansion $\left(x + \frac{1}{x}\right)^2$ once only

Attempts

A1 Decimals used

QUESTION 6

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	20 (5, 5, 10) marks	Att (2, 2, 3)
Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (a) **10 (5, 5) marks** **Att (2, 2)**

- (a) The equation $x^3 + x^2 - 4 = 0$ has only one real root.
Taking $x_1 = \frac{3}{2}$ as the first approximation to the root, use the Newton-Raphson method to find x_2 , the second approximation.

(a) Differentiation **5 marks** **Att 2**
Value **5 marks** **Att 2**

6 (a)

$$x_2 = f\left(\frac{3}{2}\right) - \frac{f\left(\frac{3}{2}\right)}{f'\left(\frac{3}{2}\right)}.$$

$$f(x) = x^3 + x^2 - 4 \Rightarrow f\left(\frac{3}{2}\right) = \frac{27}{8} + \frac{9}{4} - 4 = \frac{13}{8}.$$

$$f'(x) = 3x^2 + 2x \Rightarrow f'\left(\frac{3}{2}\right) = \frac{27}{4} + 3 = \frac{39}{4}.$$

$$\therefore x_2 = \frac{3}{2} - \frac{\frac{13}{8}}{\frac{39}{4}} = \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \frac{4}{3}.$$

Blunders (-3)

B1 Newton-Raphson formula once only

B2 Differentiation

B3 Indices

B4 $x_1 \neq \frac{3}{2}$

Part (b)

20 (5, 5, 10) marks

Att (2, 2, 3)

(b) Parametric equations of a curve are:

$$x = \frac{2t-1}{t+2}$$

$$y = \frac{t}{t+2}, \text{ where } t \in \mathbb{R} \setminus \{-2\}.$$

(i) Find $\frac{dy}{dx}$.

(ii) What does your answer to part **(i)** tell you about the shape of the graph?

(b)(i) $\frac{dx}{dt}$ or $\frac{dy}{dt}$

5 marks

Att 2

$\frac{dy}{dx}$

5 marks

Att 2

6 (b) (i)

$$x = \frac{2t-1}{t+2} \Rightarrow \frac{dx}{dt} = \frac{(t+2)2 - (2t-1)1}{(t+2)^2} = \frac{5}{(t+2)^2}.$$

$$y = \frac{t}{t+2} \Rightarrow \frac{dy}{dt} = \frac{1(t+2) - t(1)}{(t+2)^2} = \frac{2}{(t+2)^2}.$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{2}{(t+2)^2} \cdot \frac{(t+2)^2}{5} = \frac{2}{5}.$$

OR

(b) (i) Elimination of t

5 marks

Att 2

$\frac{dy}{dx}$

5 marks

Att 2

6 (b) (i)

$$x = \frac{2t-1}{t+2}$$

$$\Rightarrow t = \frac{(-2x-1)}{(x-2)}$$

$$y = \frac{t}{t+2}$$

$$t = \frac{-2y}{y-1}$$

$$t = \frac{(-2x-1)}{(x-2)} = \frac{(-2y)}{(y-1)}$$

$$\Rightarrow 2x+1 = 5y$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{5}$$

Blunders (-3)

B1 Indices

B2 Differentiation

B3 Incorrect $\frac{dy}{dx}$

Attempts

A1 Error in differentiation formula

(b) (ii)

10 marks

Att 3

6 (b) (ii)

Since the slope is constant, it is a (subset of a) straight line.

If “line” is not mentioned in the answer, can only get Att 3 at most.

Part (c)**20(5, 5, 5, 5) marks****Att (2, 2, 2, 2)****(c)** A curve is defined by the equation $x^2y^3 + 4x + 2y = 12$.**(i)** Find $\frac{dy}{dx}$ in terms of x and y .**(ii)** Show that the tangent to the curve at the point $(0, 6)$ is also the tangent to it at the point $(3, 0)$.**(c) (i) Differentiation** **5 marks** **Att 2****Isolate $\frac{dy}{dx}$** **5 marks** **Att 2****(c) (ii) Equation 1st Tangent** **5 marks** **Att 2****Equation 2nd Tangent** **5 marks** **Att 2****6 (c) (i)**

$$x^2y^3 + 4x + 2y = 12 \Rightarrow x^2 \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 2x + 4 + 2 \frac{dy}{dx} = 0.$$

$$\therefore \frac{dy}{dx}(3x^2y^2 + 2) = -2xy^3 - 4 \Rightarrow \frac{dy}{dx} = \frac{-2xy^3 - 4}{3x^2y^2 + 2}.$$

6 (c) (ii)

$$\frac{dy}{dx} = \frac{-2xy^3 - 4}{3x^2y^2 + 2}$$

Slope of tangent at $(0, 6)$ is $\frac{-4}{2} = -2$.Equation of tangent at $(0, 6)$ is $y - 6 = -2x \Rightarrow 2x + y = 6$.Slope of tangent at $(3, 0)$ is $\frac{-4}{2} = -2$.Equation of tangent at $(3, 0)$ is $y = -2(x - 3) \Rightarrow 2x + y = 6$. \therefore same tangent.*Blunders (-3)*

B1 Differentiation

B2 Indices

B3 Incorrect value of x or no value of x in slopeB4 Incorrect value of y or no value of y in slope

B5 Equation of tangent

B6 Incorrect conclusion or no conclusion

Slips (-1)

S1 Numerical

Attempts

A1 Error in differentiation formula

A2 $\frac{dy}{dx} = 3x^2y^2 \frac{dy}{dx} + 4 + 2 \frac{dy}{dx} \rightarrow$ and uses the three $\left(\frac{dy}{dx}\right)$ term**OR**

(c) (ii)

10 marks

Att 3

6 (c) (ii)

$$\frac{dy}{dx} = \frac{-2xy^3 - 4}{3x^2y^2 + 2}$$

Slope of tangent at $A(0, 6)$ is $\frac{-4}{2} = -2 = m_1$

Slope of tangent at $B(3, 0)$ is $\frac{-4}{2} = -2 = m_2$

Slope of the line $[AB]$ is $m_3 = \frac{-6}{3} = -2$

So, $m_1 = m_2 = m_3 = -2$

\Rightarrow the line through A and B is the tangent at both points.

Blunders (-3)

B1 Slope omitted

B2 Incorrect deduction or no deduction

QUESTION 7

Part (a)	10 (5, 5) marks	Att (2, 2)
Part (b)	20 (10, 10) marks	Att (3, 3)
Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (a) **10 (5, 5) marks** **Att (2, 2)**

(a) Differentiate x^2 with respect to x from first principles.

$f(x+h) - f(x)$ simplified **5 marks** **Att 2**

Finish **5 marks** **Att 2**

7 (a)

$$f(x) = x^2 \Rightarrow f(x+h) = (x+h)^2.$$

$$\begin{aligned} \frac{dy}{dx} &= \text{Limit}_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \text{limit}_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \text{limit}_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \text{limit}_{h \rightarrow 0} (2x + h) = 2x. \end{aligned}$$

Blunders (-3)

B1 $f(x+h)$

B2 Indices

B3 Expansion of $(x+h)^2$ once only

B4 $h \rightarrow \infty$

B5 No limits shown or implied or no indication of $h \rightarrow 0$

(b) Let $y = \frac{\cos x + \sin x}{\cos x - \sin x}$.

(i) Find $\frac{dy}{dx}$.

(ii) Show that $\frac{dy}{dx} = 1 + y^2$.

(b) (i) Differentiation

10 marks

Att 3

(ii) Show

10 marks

Att 3

7 (b) (i)

$$y = \frac{\cos x + \sin x}{\cos x - \sin x} \Rightarrow \frac{dy}{dx} = \frac{(\cos x - \sin x)(-\sin x + \cos x) - (\cos x + \sin x)(-\sin x - \cos x)}{(\cos x - \sin x)^2}$$

$$\frac{dy}{dx} = \frac{(\cos x - \sin x)^2 + (\cos x + \sin x)^2}{(\cos x - \sin x)^2} = \frac{2}{(\cos x - \sin x)^2}$$

7 (b) (ii)

$$\frac{dy}{dx} = \frac{(\cos x - \sin x)^2 + (\cos x + \sin x)^2}{(\cos x - \sin x)^2} = 1 + \frac{(\cos x + \sin x)^2}{(\cos x - \sin x)^2} = 1 + y^2$$

OR

7 (b) (i) & 7 (b) (ii)

$$y = \frac{\cos x + \sin x}{\cos x - \sin x} = (\cos x + \sin x) \cdot (\cos x - \sin x)^{-1}$$

$$\frac{dy}{dx} = (\cos x + \sin x) \left[-1 \cdot (\cos x - \sin x)^{-2} (-\sin x - \cos x) \right] + (\cos x - \sin x)^{-1} (-\sin x + \cos x)$$

$$= \frac{(\cos x + \sin x)^2}{(\cos x - \sin x)^2} + \frac{\cos x - \sin x}{\cos x - \sin x}$$

$$= \left(\frac{\cos x + \sin x}{\cos x - \sin x} \right)^2 + 1$$

$$= y^2 + 1$$

Blunders (-3)

B1 Differentiation

B2 Indices

B3 Trig formula

Attempts

A1 Error in differentiation Formula

Worthless

W1 Integration

(c) The function $f(x) = (1+x)\log_e(1+x)$ is defined for $x > -1$.

(i) Show that the curve $y = f(x)$ has a turning point at $\left(\frac{1-e}{e}, -\frac{1}{e}\right)$.

(ii) Determine whether the turning point is a local maximum or a local minimum.

(c) (i) $f'(x)$	5 marks	Att 2
Value of x	5 marks	Att 2
Value of y	5 marks	Att 2
(c) (ii) Turning points	5 marks	Att 2

7 (c) (i)

$$f(x) = (1+x)\log_e(1+x) \Rightarrow f'(x) = (1+x) \cdot \left(\frac{1}{1+x}\right) + \log_e(1+x) = 1 + \log_e(1+x).$$

$$f'(x) = 0 \Rightarrow \log_e(1+x) = -1 \Rightarrow 1+x = e^{-1}. \therefore x = \frac{1}{e} - 1 = \frac{1-e}{e}.$$

$$y = \left(\frac{1}{e}\right) \log_e\left(\frac{1}{e}\right) \Rightarrow y = \frac{1}{e}(-\log_e e) = -\frac{1}{e}. \text{ So turning point is } \left(\frac{1-e}{e}, -\frac{1}{e}\right).$$

OR

7 (c) (i) $f'(x) = [\log_e(1+x)] + 1$

$$\begin{aligned} \text{At } x = \frac{1-e}{e}, f'(x) &= \log_e\left(1 + \frac{1-e}{e}\right) + 1 = \log_e\left(\frac{e+1-e}{e}\right) + 1 = \log_e\left(\frac{1}{e}\right) + 1 \\ &= [\log_e(1) - \log_e(e)] + 1 \\ &= 0 - 1 + 1 = 0. \end{aligned}$$

$$\text{So } f'(x) = 0 \text{ at } x = \frac{1-e}{e}.$$

$$\text{Also, at } x = \frac{1-e}{e}, y = \left(\frac{1}{e}\right) \log_e\left(\frac{1}{e}\right) \Rightarrow y = \frac{1}{e}(-\log_e e) = -\frac{1}{e}.$$

$$\text{So turning point is } \left(\frac{1-e}{e}, -\frac{1}{e}\right).$$

7 (c) (ii)

$$f''(x) = \frac{1}{1+x} \Rightarrow f''\left(\frac{1-e}{e}\right) = \frac{1}{1 + \frac{1-e}{e}} = \frac{e}{1} = e > 0. \therefore \left(\frac{1-e}{e}, -\frac{1}{e}\right) \text{ is a local minimum.}$$

Blunders (-3)

B1 Differentiation

B2 $f'(x) \neq 0$

B3 Indices

B4 Incorrect deduction or no deduction

Slips (-1)

S1 $\log_e e \neq 1$

Attempts

A1 Error in differentiation formula

Worthless

W1 Integration

QUESTION 8

Part (a)	10 marks	Att3
Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)
Part (c)	20 (5, 5, 10) marks	Att (2, 2, 3)

Part (a) **10 marks** **Att 3**

(a) Find $\int (\sin 2x + e^{4x}) dx$.

(a) **10 marks** **Att 3**

8 (a)

$$\int (\sin 2x + e^{4x}) dx = -\frac{1}{2} \cos 2x + \frac{1}{4} e^{4x} + c$$

Blunders (-3)

B1 Integration

B2 No 'c'

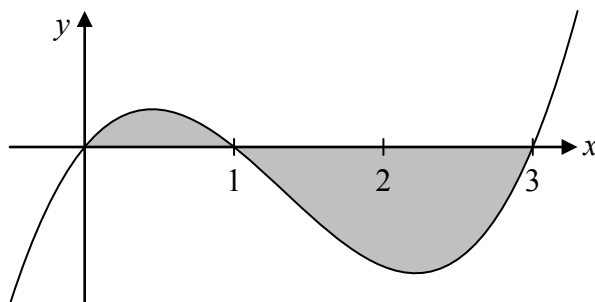
Attempts

A1 Only 'c' correct \Rightarrow Att 3

Worthless

W1 Differentiation instead of integration

(b) The curve $y = 12x^3 - 48x^2 + 36x$ crosses the x -axis at $x = 0$, $x = 1$ and $x = 3$, as shown.



Calculate the total area of the shaded regions enclosed by the curve and the x -axis.

(b) First area	5 marks	Att 2
Second area	5 marks	Att 2
Total Area	5 marks	Att 2

8 (b)

$$\text{Required area} = \left| \int_0^1 (12x^3 - 48x^2 + 36x) dx \right| + \left| \int_1^3 (12x^3 - 48x^2 + 36x) dx \right|$$

$$\left| \int_0^1 (12x^3 - 48x^2 + 36x) dx \right| = \left| 3x^4 - 16x^3 + 18x^2 \right|_0^1 = |3 - 16 + 18| = 5.$$

$$\begin{aligned} \left| \int_1^3 (12x^3 - 48x^2 + 36x) dx \right| &= \left| 3x^4 - 16x^3 + 18x^2 \right|_1^3 \\ &= |(243 - 432 + 162) - (3 - 16 + 18)| = |-27 - 5| = 32 \end{aligned}$$

\therefore the required area is $5 + 32 = 37$.

Blunders (-3)

- B1 Integration
- B2 Indices
- B3 Error in area formula
- B4 Incorrect order in applying limits
- B5 Not calculating substituted limits
- B6 Uses $\pi \int y dx$ for area formula

Attempts

- A1 Uses volume formula
- A2 Uses y^2 in formula

Worthless

- W1 Wrong area formula and no work

(c) (i) Find, in terms of a and b

$$I = \int_a^b \frac{\cos x}{1 + \sin x} dx.$$

(ii) Find in terms of a and b

$$J = \int_a^b \frac{\sin x}{1 + \cos x} dx.$$

(iii) Show that if $a + b = \frac{\pi}{2}$, then $I = J$.

(c) (i)

5 marks

Att 2

(ii)

5 marks

Att 2

(iii)

10 marks

Att 2

8 (c) (i)

$$I = \int_a^b \frac{\cos x}{1 + \sin x} dx. \quad \text{Let } u = 1 + \sin x \quad \therefore du = \cos x dx.$$

$$I = \int_{1+\sin a}^{1+\sin b} \frac{du}{u} = [\log_e u]_{1+\sin a}^{1+\sin b} = \log_e(1 + \sin b) - \log_e(1 + \sin a).$$

$$I = \log_e \left(\frac{1 + \sin b}{1 + \sin a} \right).$$

8 (c) (ii)

$$J = \int_a^b \frac{\sin x}{1 + \cos x} dx. \quad \text{Let } u = 1 + \cos x \quad \therefore du = -\sin x dx.$$

$$J = \int_{1+\cos a}^{1+\cos b} \frac{-du}{u} = -[\log_e u]_{1+\cos a}^{1+\cos b} = -\log_e(1 + \cos b) + \log_e(1 + \cos a).$$

$$J = \log_e \left(\frac{1 + \cos a}{1 + \cos b} \right).$$

8 (c) (iii)

When $a + b = \frac{\pi}{2}$, then

$$I = \log_e \left(\frac{1 + \sin b}{1 + \sin a} \right) = \log_e \left(\frac{1 + \sin \left(\frac{\pi}{2} - a \right)}{1 + \sin \left(\frac{\pi}{2} - b \right)} \right) = \log_e \left(\frac{1 + \cos a}{1 + \cos b} \right) = J.$$

Blunders (-3)

- B1 Integration
- B2 Differentiation
- B3 Trig Formula
- B4 Logs
- B5 Limits
- B6 Incorrect order in applying limits
- B7 Not calculating substituted limits
- B8 Not changing limits
- B9 Incorrect deduction or no deduction

Slips (-1)

- S1 Numerical
- S2 Trig value



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**MATHEMATICS (PROJECT MATHS)
PAPER 2**

HIGHER LEVEL



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State Examinations Commission

Leaving Certificate Examination 2010

Mathematics (Project Maths)

Paper 2

Higher Level

Monday 14 June Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

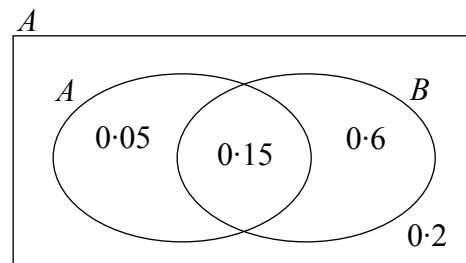
Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

Answer **all six** questions from this section.

Question 1**(25 marks)**

Two events A and B are such that $P(A) = 0.2$, $P(A \cap B) = 0.15$ and $P(A' \cap B) = 0.6$.

(a) Complete this Venn diagram.



(b) Find the probability that neither A nor B happens.

$$0.2.$$

or

$$P(A \cup B)' = 1 - P(A \cup B) = 1 - (0.05 + 0.15 + 0.6) = 0.2$$

(c) Find the conditional probability $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{0.15}{0.75} = 0.2.$$

(d) State whether A and B are independent events and justify your answer.

$$A \text{ and } B \text{ are independent events as, } P(A|B) = P(A) = 0.2.$$

or

$$A \text{ and } B \text{ are independent events as, } P(A)P(B) = (0.2)(0.75) = 0.15 = P(A \cap B).$$

Question 2

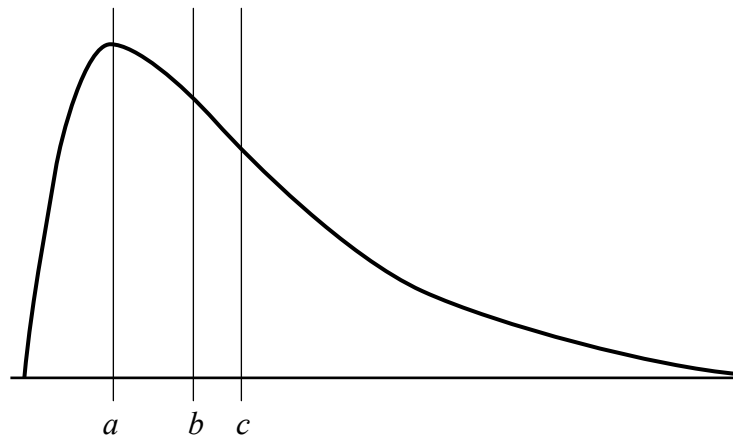
(25 marks)

- (a) The back-to-back stem-and-leaf diagram below shows data from two samples. The corresponding populations are assumed to be identical in shape and spread. Use the *Tukey quick test* to test, at the 5% significance level, the hypothesis that the populations have the same average.

8	6	6	<u>3</u>	<u>0</u>	3	4							
	5	3	2	0	4	4	1	5	5	7	7	8	9
		2	0	0	6	1	<u>4</u>	<u>9</u>					
					7	<u>2</u>	<u>4</u>						
					8	<u>1</u>							
key: 6 3 means 36						key: 8 1 means 81							

Upper tail = 5.
 Lower tail = 3.
 Tail count = 5 + 3 = 8.
 $8 \geq 7$. Therefore significant at the 5% level.
 Conclusion: we reject the null hypothesis and conclude that the populations do not have the same average.

- (b) The diagram below shows a skewed frequency distribution. Vertical lines have been drawn through the mean, mode and median. Identify which is which by inserting the relevant letters in the spaces below.

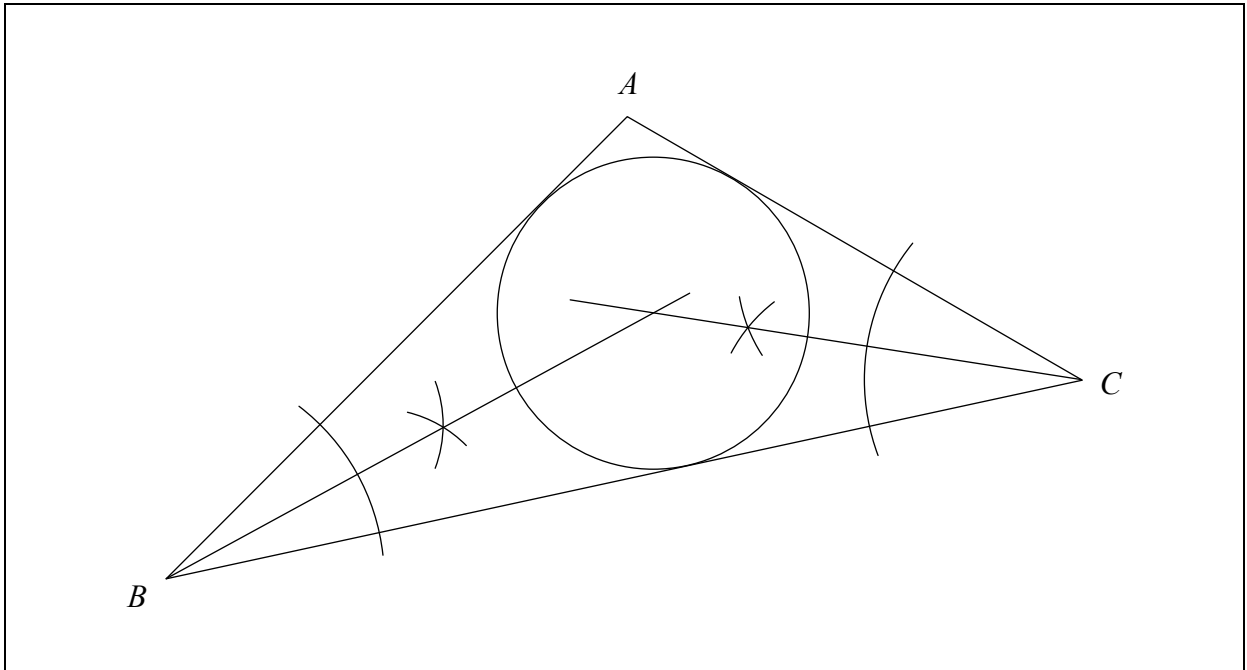


mean = c mode = a median = b

Question 3

(25 marks)

- (a) Construct the incircle of the triangle ABC below using only a compass and straight edge. Show all construction lines clearly.

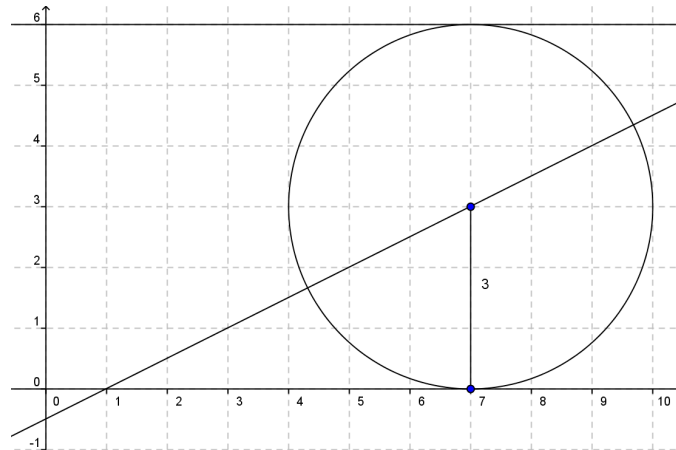


- (b) An equilateral triangle has sides of length 2 units. Find the area of its incircle.

$$\tan 30^\circ = \frac{r}{1}$$
$$r = \frac{1}{\sqrt{3}}$$
$$A = \pi(r)^2$$
$$A = \pi\left(\frac{1}{\sqrt{3}}\right)^2$$
$$A = \frac{\pi}{3} \text{ square units.}$$

Question 4**(25 marks)**

- (a) The centre of a circle lies on the line $x - 2y - 1 = 0$. The x -axis and the line $y = 6$ are tangents to the circle. Find the equation of this circle.



$$r = 3$$

$$\text{centre: } (h, 3)$$

$$h - 2(3) - 1 = 0$$

$$h = 7$$

Equation of circle:

$$(x - 7)^2 + (y - 3)^2 = 3^2$$

$$(x - 7)^2 + (y - 3)^2 = 9$$

or

$$x^2 + y^2 - 14x - 6y + 49 = 0$$

- (b) A different circle has equation $x^2 + y^2 - 6x - 12y + 41 = 0$. Show that this circle and the circle in part (a) touch externally.

$$x^2 + y^2 - 6x - 12y + 41 = 0.$$

$$\text{centre: } (3, 6); \quad \text{radius} = \sqrt{9 + 36 - 41} = \sqrt{4} = 2.$$

$$\text{Distance between centres: } \sqrt{(7 - 3)^2 + (3 - 6)^2} = \sqrt{25} = 5$$

Sum of radii: $3 + 2 = 5 = \text{distance between centres.}$
 \therefore circles touch externally.

Question 5

(25 marks)

(a) Solve the equation $\cos 3\theta = \frac{1}{2}$, for $\theta \in \mathbb{R}$, (where θ is in radians).

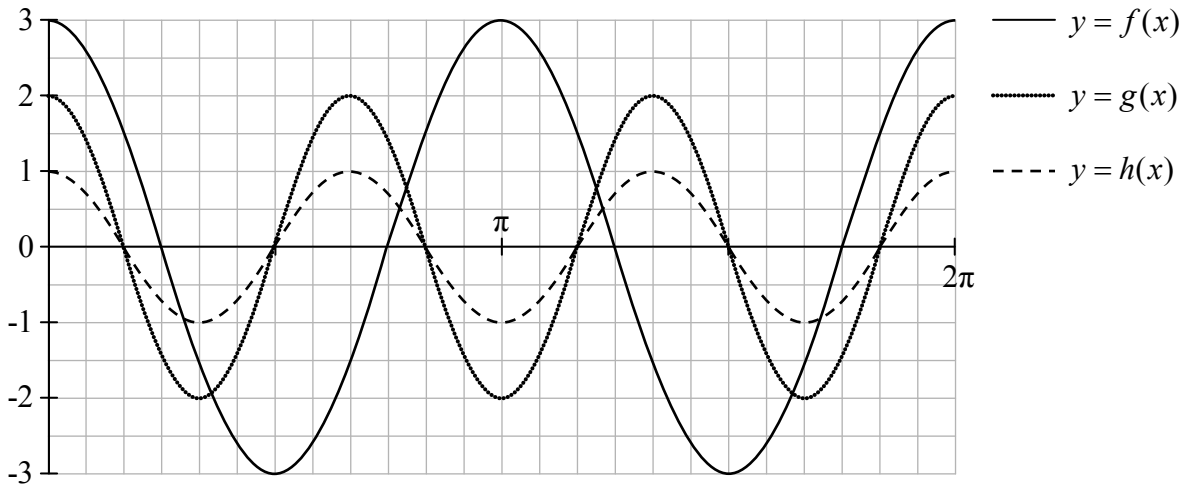
$$3\theta = \frac{\pi}{3} + 2n\pi, \text{ or } 3\theta = \frac{5\pi}{3} + 2n\pi, \text{ where } n \in \mathbb{Z}$$

$$\therefore \theta = \frac{\pi}{9} + \frac{2n\pi}{3}, \text{ or } \theta = \frac{5\pi}{9} + \frac{2n\pi}{3}, \text{ where } n \in \mathbb{Z}.$$

(b) The graphs of three functions are shown on the diagram below. The scales on the axes are not labelled. The three functions are:

- $x \rightarrow \cos 3x$
- $x \rightarrow 2 \cos 3x$
- $x \rightarrow 3 \cos 2x$

Identify which function is which, and write your answers in the spaces below the diagram.



$f : x \rightarrow 3 \cos 2x$

$g : x \rightarrow 2 \cos 3x$

$h : x \rightarrow \cos 3x$

(c) Label the scales on the axes in the diagram in part (b).

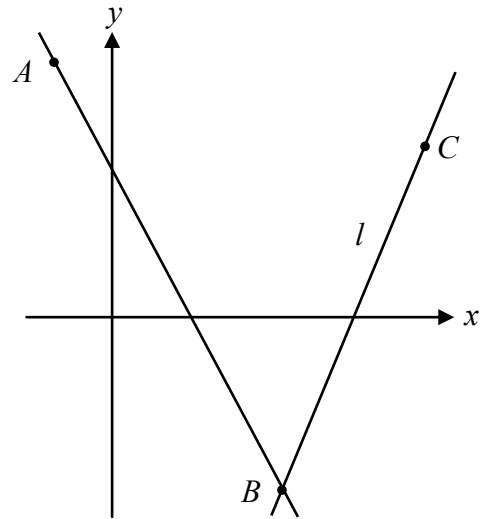
Question 6

(25 marks)

Three points A , B and C have co-ordinates:

$$A(-2,9), B(6,-6) \text{ and } C(11,6).$$

The line l passes through B and has equation $12x - 5y - 102 = 0$.



(a) Verify that C lies on l .

$$\begin{aligned} 12(11) - 5(6) - 102 &= 0. \\ 132 - 30 - 102 &= 0 \\ 0 &= 0 \\ \therefore C \text{ lies on } l. \end{aligned}$$

(b) Find the slope of AB , and hence find $\tan(\angle ABC)$, as a fraction.

$$\begin{aligned} \text{Slope of } AB &= m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 9}{6 + 2} = -\frac{15}{8} \\ \text{Slope of } l \text{ (i.e., slope of } BC) &= m_2 = \frac{12}{5} \\ \tan(\angle ABC) &\pm \frac{m_1 - m_2}{1 + m_1 m_2} = \pm \frac{-\frac{15}{8} - \frac{12}{5}}{1 + \left(-\frac{15}{8}\right)\left(\frac{12}{5}\right)} = \pm \frac{171}{140}. \\ \text{But } |\angle ABC| &\leq 90^\circ. \quad \therefore \tan(\angle ABC) = \frac{171}{140}. \end{aligned}$$

(c) Find the vectors \overrightarrow{BC} and \overrightarrow{BA} in terms of \vec{i} and \vec{j} .

$$\begin{aligned} \vec{a} &= -2\vec{i} + 9\vec{j} \\ \vec{b} &= 6\vec{i} - 6\vec{j} \\ \vec{c} &= 11\vec{i} + 6\vec{j} \\ \overrightarrow{BC} &= \vec{c} - \vec{b} & \overrightarrow{BA} &= \vec{a} - \vec{b} \\ \overrightarrow{BC} &= (11\vec{i} + 6\vec{j}) - (6\vec{i} - 6\vec{j}) & \overrightarrow{BA} &= (-2\vec{i} + 9\vec{j}) - (6\vec{i} - 6\vec{j}) \\ \overrightarrow{BC} &= 5\vec{i} + 12\vec{j} & \overrightarrow{BA} &= -8\vec{i} + 15\vec{j} \end{aligned}$$

- (d) Use the dot product to find $\cos(\angle ABC)$ and show that the answer is consistent with the answer to part (b).

$$\begin{aligned}\cos \theta &= \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| |\vec{v}_2|} \\ \cos(\angle ABC) &= \frac{(5\vec{i} + 12\vec{j}) \cdot (-8\vec{i} + 15\vec{j})}{|5\vec{i} + 12\vec{j}| | -8\vec{i} + 15\vec{j} |} \\ &= \frac{-40 + 180}{\sqrt{5^2 + 12^2} \sqrt{(-8)^2 + (15)^2}} \\ &= \frac{140}{(13)(17)} \\ &= \frac{140}{221}.\end{aligned}$$

Consistent because, for θ acute,

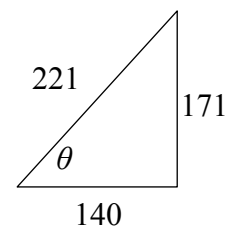
$$\cos \theta = \frac{140}{221} \Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta} = \frac{171}{221} \Rightarrow \tan \theta = \frac{\left(\frac{171}{221}\right)}{\left(\frac{140}{221}\right)} = \frac{171}{140}$$

OR

$$\text{Consistent because } \cos \theta = \frac{140}{221} \Rightarrow \sec^2 \theta = \frac{48841}{19600}, \text{ and } \tan \theta = \frac{171}{140} \Rightarrow 1 + \tan^2 \theta = \frac{48841}{19600}$$

OR

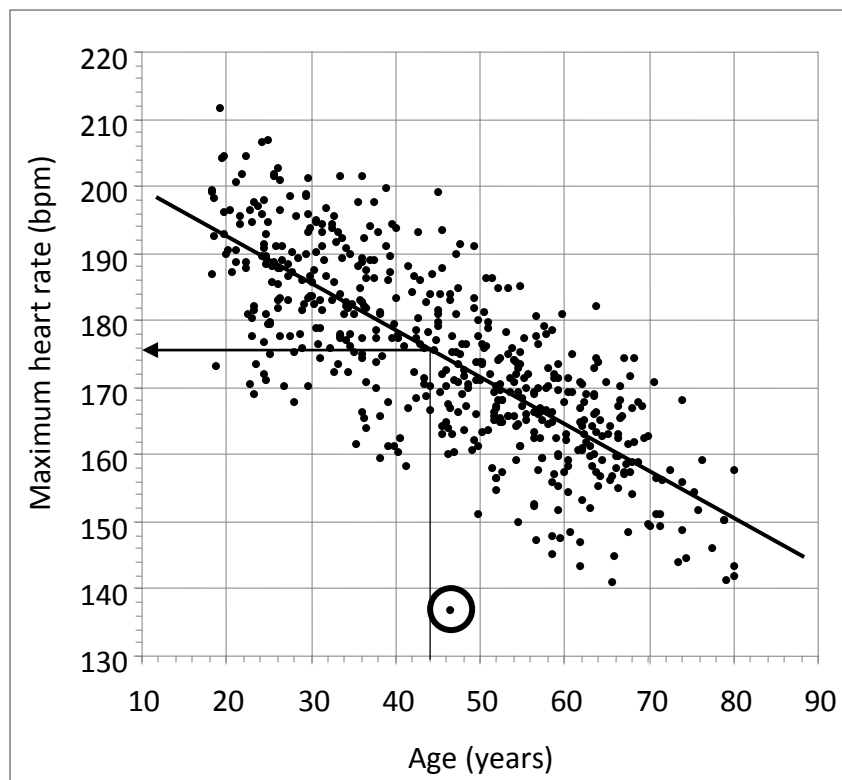
Consistent because this right-angled triangle is consistent with Pythagoras' theorem, as $221^2 = 140^2 + 171^2$.



Answer Question 7, Question 8, and **either** Question 9A **or** Question 9B.

Question 7**Probability and Statistics****(50 marks)**

A person's *maximum heart rate* is the highest rate at which their heart beats during certain extreme kinds of exercise. It is measured in beats per minute (bpm). It can be measured under controlled conditions. As part of a study in 2001, researchers measured the maximum heart rate of 514 adults and compared it to each person's age. The results were like those shown in the scatter plot below.



Source: Simulated data based on: Tanaka H, Monaghan KD, and Seals DR. *Age-predicted maximal heart rate revisited*, J. Am. Coll. Cardiol. 2001;37:153-156.

- (a) From the diagram, estimate the correlation coefficient.

Answer:

-0.75

- (b) Circle the *outlier* on the diagram and write down the person's age and maximum heart rate.

Age = 47 years

Max. heart rate = 137 bpm

- (c) The line of best fit is shown on the diagram. Use the line of best fit to estimate the maximum heart rate of a 44-year-old person.

Answer:

176 bpm

- (d) By taking suitable readings from the diagram, calculate the slope of the line of best fit.

Possible Readings

(10, 200) and (90, 144).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{144 - 200}{90 - 10} = -\frac{56}{80} = -\frac{7}{10} \text{ or } m = -0.7.$$

- (e) Find the equation of the line of best fit and write it in the form: $MHR = a - b \times (\text{age})$, where MHR is the maximum heart rate.

$$y - y_1 = m(x - x_1)$$

$$y - 200 = -0.7(x - 10)$$

$$y = -0.7x + 207$$

$$MHR = 207 - 0.7 \times (\text{age})$$

- (f) The researchers compared their new rule for estimating maximum heart rate to an older rule. The older rule is: $MHR = 220 - \text{age}$. The two rules can give different estimates of a person's maximum heart rate. Describe how the level of agreement between the two rules varies according to the age of the person. Illustrate your answer with two examples.

For young adults the old rule gives a greater MHR than the new rule.

Adult aged 20

$$MHR = 220 - 20 = 200 \text{ bpm (Old rule)}$$

$$MHR = 207 - 0.7(20) = 193 \text{ bpm (New Rule)}$$

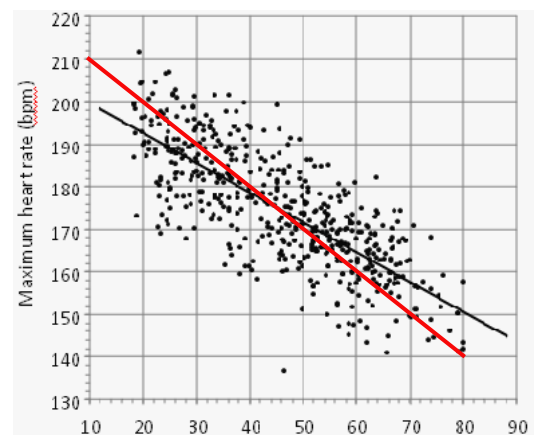
Towards middle age there is a greater agreement between the rules.

For older people the new rule gives a greater MHR than the old rule.

Adult aged 70

$$MHR = 220 - 70 = 150 \text{ bpm}$$

$$MHR = 207 - 0.7(70) = 158 \text{ bpm}$$



- (g) A particular exercise programme is based on the idea that a person will get most benefit by exercising at 75% of their estimated MHR . A 65-year-old man has been following this programme, using the old rule for estimating MHR . If he learns about the researchers' new rule for estimating MHR , how should he change what he is doing?

He should exercise a bit more intensely.

Using the old rule he exercises to 75% of $(220 - 65) = 116$ bpm.

Using the new rule he can exercise to 75% of $(207 - 0.7 \times 65) = 121$ bpm.

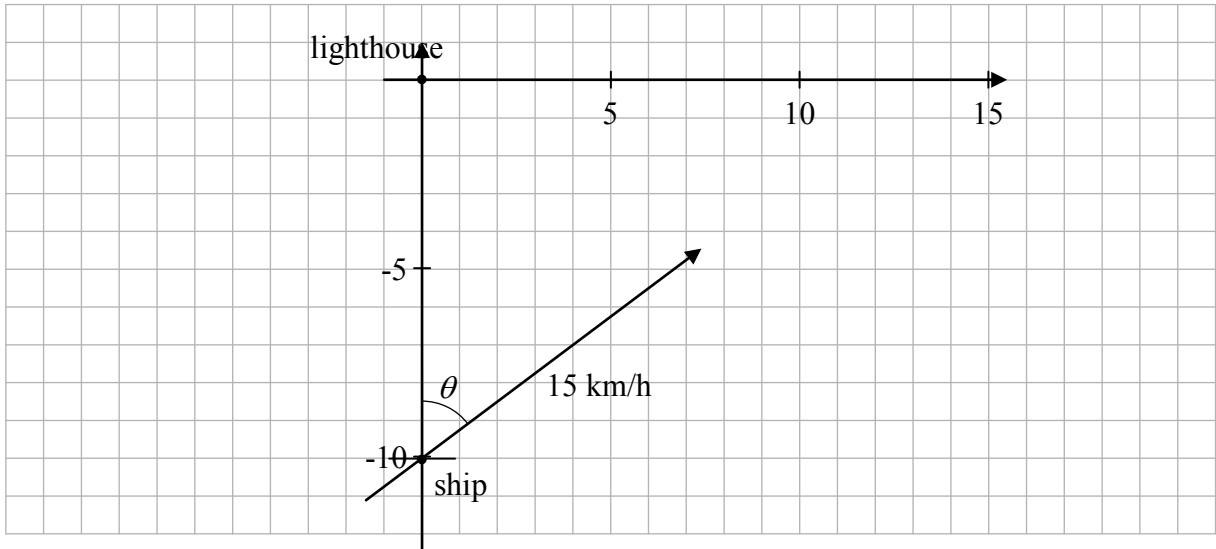
Question 8

Geometry and Trigonometry

(50 marks)

A ship is 10 km due South of a lighthouse at noon.

The ship is travelling at 15 km/h on a bearing of θ , as shown below, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.



- (a) On the diagram above, draw a set of co-ordinate axes that takes the lighthouse as the origin, the line East-West through the lighthouse as the x -axis, and kilometres as units.
- (b) Find the equation of the line along which the ship is moving.

$\tan \theta = \frac{4}{3}$ $\therefore m = \frac{3}{4}$ $y + 10 = \frac{3}{4}(x - 0)$ $4y + 40 = 3x$ $3x - 4y - 40 = 0$	Or	$y = mx + c$ $y = \frac{3}{4}x - 10$
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- (c) Find the shortest distance between the ship and the lighthouse during the journey.

$\sin \theta = \frac{d}{10}$ $\frac{4}{5} = \frac{d}{10}$ $5d = 40$ $x = 8 \text{ km}$	Or	$d = \frac{ ax_1 + by_1 + c }{\sqrt{a^2 + b^2}}$ $d = \frac{ 3(0) - 4(0) - 40 }{\sqrt{3^2 + (-4)^2}}$ $d = 8 \text{ km.}$	
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- (d) At what time is the ship closest to the lighthouse?

$$\tan \theta = \frac{8}{x}$$

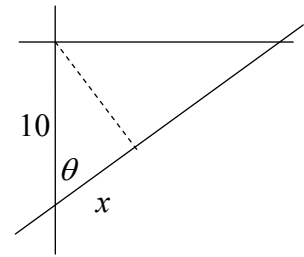
$$\frac{4}{3} = \frac{8}{x}$$

$$4x = 24$$

$$x = 6 \text{ km.}$$

$$\text{Time} = \frac{6}{15} = 0.4 \text{ hours} = 24 \text{ minutes.}$$

\therefore closest to the lighthouse at 12:24 pm



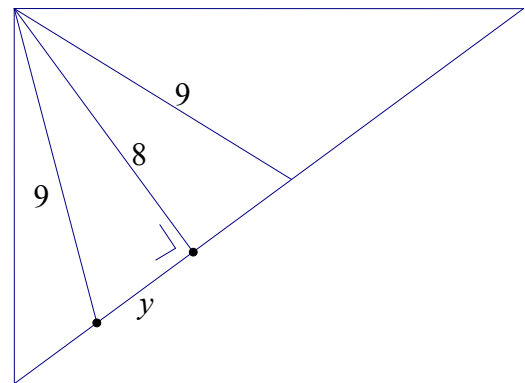
- (e) Visibility is limited to 9 km. For how many minutes in total is the ship visible from the lighthouse?

$$8^2 + y^2 = 9^2$$

$$y^2 = 81 - 64$$

$$y^2 = 17$$

$$y = \sqrt{17}$$



Distance travelled by the ship while visible from the lighthouse is $2y = 2\sqrt{17}$ km.

$$\text{Time} = \frac{2\sqrt{17}}{15} \text{ hours.}$$

$$= 8\sqrt{17} \text{ minutes or } 32.98 \text{ minutes} \approx 33 \text{ minutes.}$$

Question 9A**Probability and Statistics****(50 marks)**

A factory manufactures aluminium rods. One of its machines can be set to produce rods of a specified length. The lengths of these rods are normally distributed with mean equal to the specified length and standard deviation equal to 0.2 mm.

The machine has been set to produce rods of length 40 mm.

- (a) What is the probability that a randomly selected rod will be less than 39.7 mm in length?

$$\begin{aligned}P(X < 39.7) &= P\left(Z < \frac{39.7 - 40}{0.2}\right) = P(Z < -1.5) \\ &= P(z > 1.5) \\ &= 1 - P(Z \leq 1.5) \\ &= 1 - 0.9332 \\ &= 0.0668\end{aligned}$$

- (b) Five rods are selected at random. What is the probability that at least two of them are less than 39.7 mm in length?

Binomial distribution with $n = 5$, $p = 0.0668$, $q = 0.9332$.

$$\begin{aligned}P(X \geq 2) &= 1 - P(X < 2) = 1 - [P(X = 1) + P(X = 0)] \\ &= 1 - \left[\binom{5}{1} (0.0668)(0.9332)^4 + \binom{5}{0} (0.9332)^5 \right] \\ &= 0.03895.\end{aligned}$$

Or

$$\begin{aligned}P(X \geq 2) &= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \\ &= \binom{5}{2} (0.0668)^2 (0.9332)^3 + \binom{5}{3} (0.0668)^3 (0.9332)^2 + \binom{5}{4} (0.0668)^4 (0.9332) + \binom{5}{5} (0.0668)^5 \\ &= 0.03895\end{aligned}$$

- (c) The operators want to check whether the setting on the machine is still accurate. They take a random sample of ten rods and measure their lengths. The lengths in millimetres are:

39.5	40.0	39.7	40.2	39.8
39.7	40.2	39.9	40.1	39.6

Conduct a hypothesis test at the 5% level of significance to decide whether the machine's setting has become inaccurate. You should start by clearly stating the null hypothesis and the alternative hypothesis, and finish by clearly stating what you conclude about the machine.

$$H_0 : \mu = 40 \text{ mm (null hypothesis)}$$

$$H_1 : \mu \neq 40 \text{ mm (alternative hypothesis)}$$

$$\sigma_{\bar{x}} = \frac{0.2}{\sqrt{10}} = 0.0632456$$

Observed value of $\bar{x} = 39.87$

$$\therefore \text{Observed } z = \frac{39.87 - 40}{0.0632456} = -2.055$$

The critical values for the test are ± 1.96

As $-2.055 < -1.96$, we reject the null hypothesis at the 5% level of significance and we conclude that the machine setting has become inaccurate.

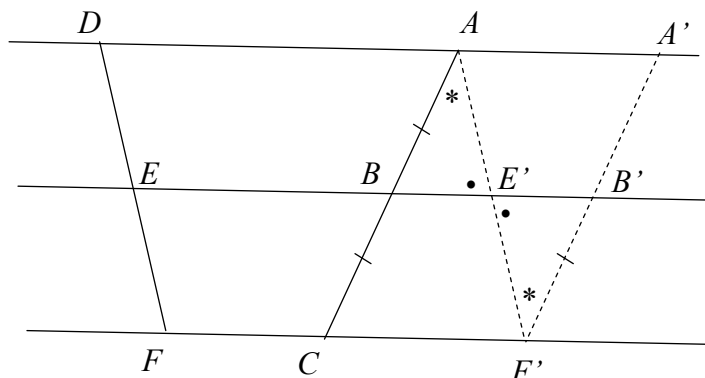
Question 9B

Geometry and Trigonometry

(50 marks)

- (a) Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:



Given: $AD \parallel BE \parallel CF$, as in the diagram, with $|AB| = |BC|$

To prove: $|DE| = |EF|$

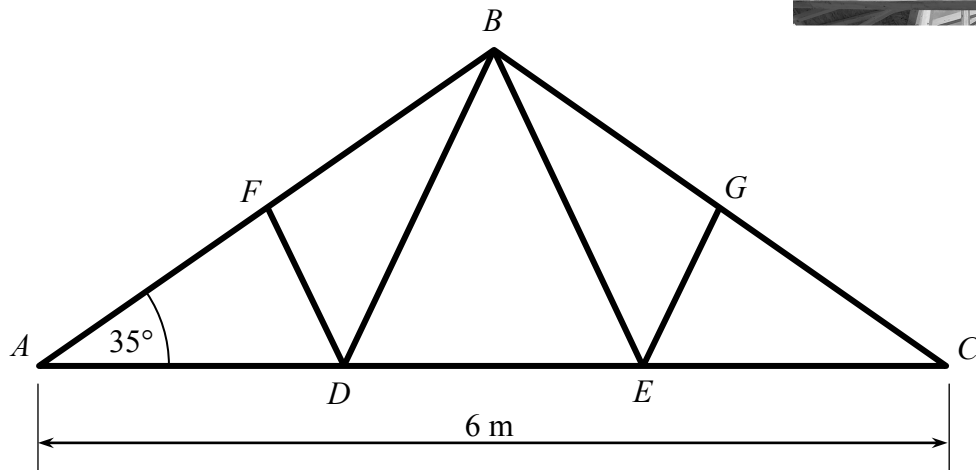
Construction:

Draw $AE' \parallel DE$, cutting EB at E' and CF at F'
 Draw $F'B' \parallel AB$, cutting EB at B' , as in the diagram.

Proof: $|B'F'| = |BC|$ (opposite sides in a parallelogram)
 $= |AB|$ (by assumption)
 $|\angle BAE'| = |\angle E'F'B'|$ (alternate angles)
 $|\angle AE'B| = |\angle F'E'B'|$ (vertically opposite angles)
 $\therefore \triangle ABE'$ is congruent to $\triangle F'B'E'$ (ASA)
 Therefore $|AE'| = |F'E'|$.
 But $|AE'| = |DE|$ and $|F'E'| = |FE|$ (opposite sides in a parallelogram)
 $\therefore |DE| = |EF|$.

- (b) Roofs of buildings are often supported by frameworks of timber called *roof trusses*.

A quantity surveyor needs to find the total length of timber needed in order to make the triangular truss shown below.



The length of $[AC]$ is 6 metres, and the pitch of the roof is 35° , as shown.
 $|AD| = |DE| = |EC|$ and $|AF| = |FB| = |BG| = |GC|$.

- (i) Calculate the length of $[AB]$, in metres, correct to two decimal places.

$$|AH| = 3 \text{ m}$$

$$\cos 35^\circ = \frac{3}{|AB|}$$

$$|AB| = \frac{3}{\cos 35^\circ} \approx 3.66232$$

$$|AB| = 3.66 \text{ m (to 2 decimal places)}$$

- (ii) Calculate the total length of timber required to make the truss.

$$|FD|^2 = 1.83^2 + 2^2 - 2(1.83)(2) \cos 35^\circ$$

$$= 1.352707.$$

$$|FD| = 1.163 \text{ m}$$

$$|BD|^2 = 2^2 + 3.66^2 - 2(2)(3.66) \cos 35^\circ$$

$$= 5.403214.$$

$$|BD| = 2.325 \text{ m.}$$

OR Similar triangles $\Rightarrow |BE| = 2|FD|$

$$\text{Total length required} = 6 + 2(3.662) + 2(1.163) + 2(2.325) = 20.296 = 20.3 \text{ m.}$$

Marking scheme for Paper 2

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5	
10 mark scale		0, 4, 10	0, 3, 8, 10	0, 3, 5, 8, 10
15 mark scale			0, 5, 10, 15	0, 6, 9, 13, 15
20 mark scale			0, 7, 13, 20	0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

- (a) 10B
- (b) 5B
- (c) 5C
- (d) 5B

Question 2

- (a) 15C
- (b) 10B

Question 3

- (a) 20C
- (b) 5C

Question 4

- (a) 15C
- (b) 10C

Question 5

- (a) 5C
- (b) 10B
- (c) 10B

Question 6

- (a) 5B
- (b) slope AB : 5B
tan $\angle ABC$: 5C
- (c) 5B
- (d) 5C

Section B

Question 7

- (a) 5B
- (b) outlier: 5A
age & MHR: 5B
- (c) 5B
- (d) 5B
- (e) 10C
- (f) 10C
- (g) 5B

Question 8

- (a) 5B*
- (b) 10C
- (c) 10C*
- (d) 10C
- (e) 15C

Question 9A

- (a) 10C*
- (b) 15D*
- (c) test: 20D
conclusion: 5B

Question 9B

- (a) diag. & given: 5B
construction: 5B
proof: 10D
- (b) (i) 10C*
(ii) $|FD|$ 5B
 $|BD|$ 10B
finish 5B*

Detailed marking notes

Section A

Question 1

(a) Scale 10B

Partial credit:

- $P(A$ but not $B)$ and no other correct work.

Note: Do not penalise omission of $P((A \cup B)')$ from diagram.

(b) Scale 5B

Partial credit:

- $1 -$ (some relevant probability), (provided answer lies in $[0, 1]$)
- Correct expression, but not executed accurately (provided answer lies in $[0, 1]$).

(c) Scale 5C

Low partial credit:

- $P(B)$ found correctly from candidates own work in (a).
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$

High partial credit:

- Correct expression, but not executed accurately (provided answer lies in $[0, 1]$).
- $P(B|A)$ found correctly.

(d) Scale 5B

Partial credit:

- States events are independent with incomplete or unsatisfactory justification or no justification.
- Defines or explains independence.

Note 1: Errors in previous work may lead to $P(A).P(B) \neq P(A \cap B)$ and candidate stating that events are not independent. This will merit full credit for part (d).

Note 2: No credit for statements or calculations related to mutually exclusive events. For example, $P(A \cap B) \neq 0 \Rightarrow$ events are not independent.

Question 2

(a) Scale 15C

Low partial credit:

- Listing the numbers from both sets.
- Indicates a number or some numbers in the tail.
- States the critical value is 7 for 5% significance.
- States a correct null and alternative hypothesis.

High partial credit

- Correct tail count.

(b) Scale 10B

Partial credit:

- One correct.

Question 3

(a) Scale 20C

Low partial credit:

- One properly constructed bisector.
- Circle by trial and error, but within tolerance.

High partial credit:

- Incentre properly constructed.
- Outside tolerance using template supplied.

(b) Scale 5C

Low partial credit

- Finds 30° .
- Constructs incircle.
- Counts squares (provided $0.9 \leq \text{Area} \leq 1.2$).

High partial credit:

- Finds radius from accurate diagram and finishes. ($0.9 \leq \text{Area} \leq 1.2$).
- Writes $\tan 30^\circ$ as a decimal and proceeds correctly i.e. answer as a decimal without first writing it as $\frac{\pi}{3}$.

Question 4

(a) Scale 15C

Low partial credit

- Draws a diagram.
- Some relevant work.

High partial credit

- Finds the centre and radius length.

(b) Scale 10C

Low partial credit:

- Finds one of: centre, radius and distance.

High partial credit

- Finds two of: centre, radius and distance.

Full Credit:

- Finds all three and concludes.

Question 5

(a) Scale 5C

Low partial credit

- Reference angle found.
- $\theta = \frac{\pi}{9}$.
- Diagram drawn (angles indicated on a unit circle)

High partial credit:

- Two or more correct solutions.

Full Credit:

$$\theta = \frac{\pi}{9} + \frac{2n\pi}{3} \text{ and } \theta = \frac{5\pi}{9} + \frac{2n\pi}{3}$$

(i.e., take it as implied in the context that $n \in \mathbb{Z}$.)

(b) Scale 10B

Partial credit:

- One correct.

(c) Scale 10B

Partial credit:

- One axis scaled correctly.

Question 6

(a) Scale 5B

Partial credit:

- Some correct substitution.
- B substituted. (No credit for substituting A)

(b) **Slope of AB :** Scale 5B

Partial credit:

- Correct substitution.

$\tan \angle ABC$: Scale 5C

Low partial credit

- Slope of BC.
- Correct substitution.
- Finds the equation of AB.
- Finds $\angle ABC$ without using the slope of AB , but must continue to find $\tan \angle ABC$.

High partial credit:

- Leaves the answer as $\pm \frac{171}{140}$
- $\tan \angle ABC = -\frac{171}{140}$.
- $\tan \angle ABC = \pm 1.22$.

(c) Scale 5B.

Partial credit:

- \vec{a}, \vec{b} or \vec{c} written in terms of \vec{i} and \vec{j} .

(d) Scale 5C

Low partial credit

- Correct substitution.

High partial credit:

- Finds $\cos \angle ABC$ correctly.
- Uses calculator to verify answer; i.e., finds $|\angle ABC|$ and hence verifies.
- Accept $221^2 = 171^2 + 140^2$ to show answer is consistent with (b).

Section B

Question 7

(a) Scale 5B

Partial credit:

- Direction correct, but outside tolerance. Tolerance: $-0.9 \leq r \leq -0.6$
- Direction is incorrect, but within tolerance.

(b) **Identify outlier:** Scale 5A.

Age and MHR: Scale 5B

Partial credit:

- Age only correct. Tolerance: $45 \leq A \leq 49$
- MHR only correct. Tolerance: $135 \leq MHR \leq 139$

(c) Scale 5B

Partial Credit:

- Line drawn on the diagram, but no answer given.
Tolerance: $172 \leq MHR \leq 178$.

(d) Scale 5B

Note: tolerance on the slope, m is $-0.8 \leq m \leq -0.6$.

Partial credit

- Slope formula with some substitution.
- Negative number.
- Error in reading points. e.g., (0, 200).
- Answer that is within tolerance but without work.

(e) Scale 10C

Low partial credit

- Correctly substitutes values into formula.

High partial credit:

- Equation written in the form $y = mx + c$.
- Incorrect intercept and finishes.

(f) Scale 10C

Low partial credit

- One example.
- Incomplete explanation. (“The rules sometimes agree.”)

High partial credit:

- Two examples and no correct explanation.
- Correct explanation and one example.

Examples of acceptable explanations.

“Old rule overestimates young people and underestimates old people.”

“Agrees well in the middle but not at the ends.”

(g) Scale 5B.

Partial credit:

- Finds 75% of some MHR.

Question 8

(a) Scale 5B*

Partial credit:

- Uses ship as origin.
- Error in scales.

(b) Scale 10C

High partial

- Solution with one error. Example: $m = \frac{4}{3}$.

Low partial

- Any correct relevant work.

(c) Scale 10C*

Low partial credit:

- Draws perpendicular line from lighthouse to line.

High partial credit:

- Sets up a correct equation.
- Reads co-ordinates from the diagram.

(d) Scale 10C.

Low partial credit:

- Estimated answer between 12:20 pm and 12:40 pm.
- Finds distance.

High partial credit

- Finds the time taken but fails to finish.

(e) Scale 15C

Low partial credit:

- Sets up a correct equation.
- Circle drawn with lighthouse as centre.

High partial credit:

- Finds $\sqrt{17}$.
- Answer given in hours.

Question 9A

(a) Scale 10C*

Low partial credit

- Finds $z = 1.5$ or $z = -1.5$.

High partial credit:

- Finds $P(Z < 1.5)$.

Note: Accept rounding to two decimal places.

(b) Scale 15D*

Low partial credit

- Calculates $1 - p$.

Mid partial credit:

- Any correct term.

High partial credit:

- Two correct terms but fails to subtract from 1.
- All terms, but not evaluated.

(c) **Test:** Scale 20D

Low partial credit

- Correctly states null and alternative hypotheses.
- Calculates correctly the mean of the sample.
- States critical values for the test.

Mid partial credit:

- Calculates standard error of mean correctly.

High partial credit:

- Correct value for z .
- Does a one-tailed test. ($-2.055 < -1.645$)

Contextualised conclusion: Scale 5B

Partial credit:

- Non-contextualised conclusion

Question 9B

(a) **Diagram and Given:** Scale 5B

Partial credit:

- One correct.

Construction: Scale 5B

- Construction without explanation.

Proof: Scale 10D

Low partial credit

- Any correct line.

Mid partial credit:

- Proof with two errors (see note 1 below).

High partial credit:

- Proof with one error (see note 1 below).

Note 1: errors in proofs:

- Failing to justify step(s): one error.
- Omitting a critical line: one error.

Note 2: alternative proofs:

The model solutions show the anticipated proof, as given in the geometry course document. Other proofs are acceptable, provided that they are sound and are consistent with that course document. That is, provided that:

- the proof uses only the terms and axioms that are in the course document
- the proof relies **only** on results that are listed **earlier** in the document than the one being proved
- all steps, including constructions, are properly justified.

(b) **Part (i)** Scale 10C*.

Low partial credit

- Divide $[AC]$ into 2, 2, 2.
- Any relevant work.

High partial credit:

- Sets up a correct equation.

Part (ii)

|FD|: Scale 5B

Partial credit:

- Any relevant work.

|BD|: Scale 10B

Partial credit:

- Any relevant work.

Total: Scale 5B*

Partial credit:

- Omits one or more numbers from the sum.

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. $198 \text{ marc} \times 5\% = 9.9 \Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle $[300 - \text{bunmharc}] \times 15\%$, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

