Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2014

Marking Scheme

Applied Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

1. **(a)** Two cars, P and Q, travel with the same constant velocity 15 m s^{-1} along a straight level road. The front of car P is 24 m behind the rear of car Q. At a given instant both cars decelerate, P at 4 m s^{-2} and Q at 5 m s^{-2}.

(i) Find, in terms of *t*, the distance between the cars *t* seconds later.

(ii) Find the distance between the cars when they are at rest.

(i)	$s = ut + \frac{1}{2}at^2$		
	$s_Q = 15t + \frac{1}{2}(-5)t^2$	5	
	$s_P = 15t + \frac{1}{2}(-4)t^2$	5	
	$d = 24 + s_Q - s_P$		
	$= 24 - \frac{1}{2}t^2$	5	
(ii)	$v^2 = u^2 + 2as$		
	$0 = 15^2 + 2(-5)s_Q$		
	$\Rightarrow s_Q = 22{\cdot}5$	5	
	$0 = 15^2 + 2(-4)s_P$		
	$\Rightarrow s_P = 28{\cdot}125$	5	
	$d = 24 + s_Q - s_P$		
	$= 24 + 22{\cdot}5 - 28{\cdot}125$		
	$= 18{\cdot}375$ m	5	30

1. **(b)** At a particular instant a car of mass 1200 kg is towing a trailer of mass 450 kg on a level road at a speed of 25 m s^{-1} when the engine exerts a constant power of 50 kW. Friction and air resistance amount to 930 N on the car and 200 N on the trailer.

(i) Find the acceleration of the car at this instant.

(ii) Calculate the maximum speed at which the car (without the trailer) could travel up an incline of $\sin^{-1}\frac{1}{10}$ against the same resistance with the engine working at the same rate.

(i) $P = Tv$

$$\Rightarrow T = \frac{50000}{25} = 2000$$ 5

$$2000 - 930 - 200 = 1650a$$ 5

$$\Rightarrow a = \frac{29}{55} = 0{\cdot}527 \text{ m s}^{-2}$$ 5

(ii) $T - 930 - 1200g\sin\alpha = 1200 \times 0$

$$T - 930 - 120g = 0$$

$$T = 2106$$

$$P = Tv$$

$$v = \frac{50000}{2106}$$

$$= 23{\cdot}74 \text{ m s}^{-1}$$ 5 20

2. **(a)** Three ships X, Y and Z are observed from a coastguard station at half-hour intervals. With distances measured in kilometres and speeds in kilometres per hour, they have the following displacement and uniform velocity vectors:

Ship	X	Y	Z
Time	14:00	14:30	15:00
Displacement	$2\vec{i} + 7\vec{j}$	$6\vec{i} + 9\vec{j}$	$12\vec{i} + 9\vec{j}$
Velocity	$4\vec{i} + 5\vec{j}$	$3\vec{i} + 4\vec{j}$	$2\vec{i} + 6\vec{j}$

(i) Prove that if X and Z continue with their uniform velocities they will collide. Find the time of the collision.

At the instant of the collision ship Y changes course and then proceeds directly to the scene of the collision at its original speed.

(ii) Find the time, to the nearest minute, at which Y will arrive at the scene of the collision.

(i) $2\vec{i} + 7\vec{j} + \left(4\vec{i} + 5\vec{j}\right) t = 12\vec{i} + 9\vec{j} + \left(2\vec{i} + 6\vec{j}\right)(t-1)$ 5

$2 + 4t = 12 + 2(t-1)$ 5

$t = 4$

$\Rightarrow \text{time} = 18:00$ 5

(ii) $t = 18:00$

$\bar{r}_X = 2\vec{i} + 7\vec{j} + \left(4\vec{i} + 5\vec{j}\right) 4$

$= 18\vec{i} + 27\vec{j}$

$\bar{r}_Y = 6\vec{i} + 9\vec{j} + \left(3\vec{i} + 4\vec{j}\right)(3{\cdot}5)$

$= 16{\cdot}5\vec{i} + 23\vec{j}$

$|XY| = \sqrt{(18 - 16{\cdot}5)^2 + (27 - 23)^2}$

$= 4{\cdot}272 \text{ km}$

$t_1 = \dfrac{4{\cdot}272}{5} \times 60 = 51$

$\Rightarrow \text{time} = 18:51$ 5 **20**

2. **(b)** At noon ship B is 300 km south of ship A.
Ship A is travelling southwest at $24\sqrt{2}$ km h^{-1}. Ship B is travelling due west at 31 km h^{-1}.

(i) Find the magnitude and direction of the velocity of B relative to A.

A and B can exchange signals when they are within *d* km of each other.

(ii) If the ships can exchange signals for 2.8 hours, find the value of *d*.

(i)

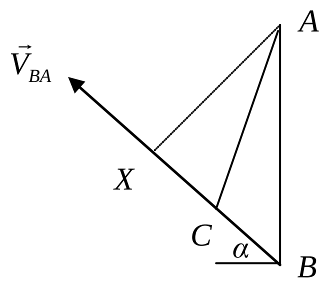
$$\vec{V}_A = -24\vec{i} - 24\vec{j}$$

$$\vec{V}_B = -31\vec{i}$$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = -7\vec{i} + 24\vec{j}$$ 5

$$\left|\vec{V}_{BA}\right| = \sqrt{(-7)^2 + (24)^2} = 25$$ 5

$$\alpha = \tan^{-1}\left(\frac{24}{7}\right) = 73{\cdot}74°$$ 5



(ii)

$$|AX| = 300\cos\alpha$$

$$= 84$$ 5

$$|CX| = \vec{V}_{BA}(1{\cdot}4)$$

$$= 35$$ 5

$$d = |AC| = \sqrt{84^2 + 35^2}$$

$$= 91 \text{ km.}$$ 5

30

3. **(a)** A particle is projected from a point on the horizontal ground with speed u m s^{-1} at an angle $30°$ to the horizontal. The particle is at a height of 7.35 m above the horizontal ground at times t_1 and t_2 seconds.

If $t_2 - t_1 = 1.5$ seconds, find the value of u.

$$\vec{r} = (u\cos 30.t\ )\vec{i} + \left(u\sin 30.t - \tfrac{1}{2}gt^2\right)\vec{j}$$ 5

$$r_{\vec{j}} = 7{\cdot}35$$

$$u\sin 30.t - \tfrac{1}{2}gt^2 = 7{\cdot}35$$ 5

$$\tfrac{1}{2}ut - \tfrac{1}{2}gt^2 = 7{\cdot}35$$

$$gt^2 - ut + 14{\cdot}7 = 0$$

$$t = \frac{u \pm \sqrt{u^2 - 58{\cdot}8g}}{2g}$$ 5

$$t_1 = \frac{u - \sqrt{u^2 - 58{\cdot}8g}}{2g}, \quad t_2 = \frac{u + \sqrt{u^2 - 58{\cdot}8g}}{2g}$$

$$t_2 - t_1 = \frac{\sqrt{u^2 - 58{\cdot}8g}}{g} = 1{\cdot}5$$

$$u^2 - 58{\cdot}8g = 2{\cdot}25g^2$$

$$u^2 = 792{\cdot}33$$

$$u = 28{\cdot}15 \text{ m s}^{-1}$$ 5

20

3. **(b)** A particle is projected up an inclined plane with initial speed *u*.
The line of projection makes an angle *α* with the inclined plane and the plane is inclined at an angle *β* to the horizontal.
The plane of projection is vertical and contains the line of greatest slope.
The particle is moving horizontally when it strikes the inclined plane.

(i) Show that $\tan\alpha = \dfrac{\tan\beta}{1+2\tan^2\beta}$.

(ii) Hence or otherwise, show that $\tan\alpha < \tan\beta$.

(i)

$$r_j = 0 \qquad 5$$

$$u\sin\alpha \times t - \tfrac{1}{2} g\cos\beta \times t^2 = 0$$

$$t = \frac{2u\sin\alpha}{g\cos\beta} \qquad 5$$

$$v_{\vec{i}} = u\cos\alpha - g\sin\beta \times t$$

$$= u\cos\alpha - 2u\sin\alpha\tan\beta \qquad 5$$

$$v_{\vec{j}} = u\sin\alpha - g\cos\beta \times t$$

$$= -u\sin\alpha \qquad 5$$

$$\tan\ell = \frac{-v_{\vec{j}}}{v_{\vec{i}}}$$

$$\tan\beta = \frac{u\sin\alpha}{u\cos\alpha - 2u\sin\alpha\tan\beta}$$

$$\tan\beta = \frac{\tan\alpha}{1-2\tan\alpha\tan\beta}$$

$$\tan\beta - 2\tan\alpha\tan^2\beta = \tan\alpha$$

$$\tan\beta = \tan\alpha + 2\tan\alpha\tan^2\beta$$

$$= \tan\alpha\left(1+2\tan^2\beta\right)$$

$$\tan\alpha = \frac{\tan\beta}{1+2\tan^2\beta} \qquad 5$$

(ii)

$$1+2\tan^2\beta > 1$$

$$\tan\alpha = \frac{\tan\beta}{1+2\tan^2\beta} < \tan\beta \qquad 5$$

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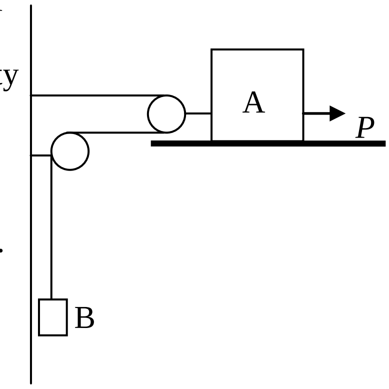
4. **(a)** A block A of mass 4 kg, can slide on a rough horizontal table. It is connected inelastically to a pulley system from which B, a mass of 8 kg, hangs freely under gravity by a light inelastic string, as shown in the diagram.
A horizontal force P of 320 N is applied to the mass A, which then moves in the direction of P.

The coefficient of friction between A and the table is $\frac{4}{7}$.

Find **(i)** the acceleration of A

(ii) the tension in the string connected to B.



(i)

$$320 - 2T - \frac{4}{7}(4g) = 4f \qquad 5$$

$$T - 8g = 8(2f) \qquad 5$$

$$320 - 2T - \frac{16g}{7} = 4f$$

$$2T - 16g = 32f$$

$$320 - \frac{16g}{7} - 16g = 36f$$

$$\Rightarrow f = 3{\cdot}91 \text{ m s}^{-2} \qquad 5$$

(ii)

$$T - 8g = 16f$$

$$T = 140{\cdot}96 \text{ N} \qquad 5$$

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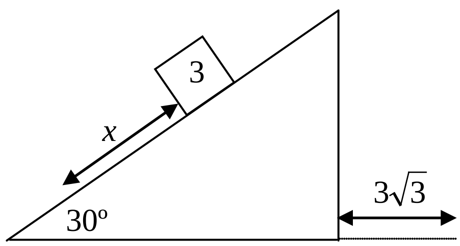
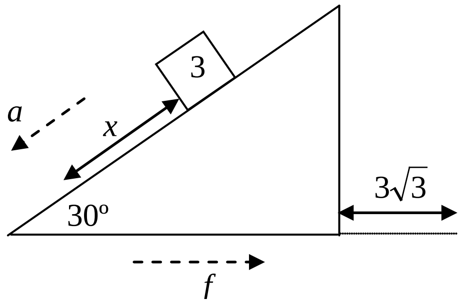
4. **(b)** A smooth wedge, of mass 8 kg and slope 30°, rests on a smooth horizontal surface.
A particle of mass 3 kg is placed on the smooth inclined face of the wedge.
The system is released from rest.

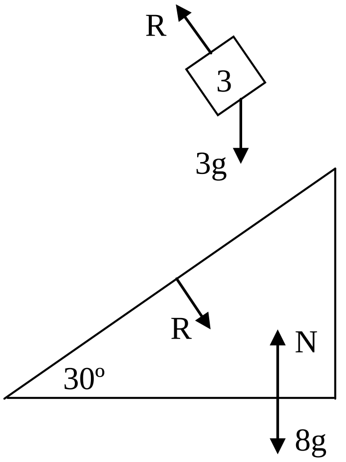
(i) Show, on separate diagrams, the forces acting on the wedge and on the particle.



The wedge moves $3\sqrt{3}$ cm in t seconds and in this time the particle moves a distance x cm relative to the wedge.

(ii) Find the value of t and the value of x.





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$$3g\sin 30 = 3(a - f\cos 30)$$
$$3g\cos 30 - R = 3f\sin 30$$
$$R\sin 30 = 8f$$

$$R\sin 30 = 8f \qquad \Rightarrow R = 16f$$ 5

$$3g\cos 30 - R = 3f\sin 30$$
$$3g\left(\frac{\sqrt{3}}{2}\right) - 16f = \frac{3f}{2} \qquad \Rightarrow f = \frac{3\sqrt{3}g}{35}$$ 5

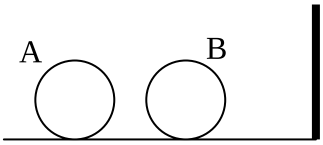
$$\frac{3g}{2} = 3\left(a - \frac{3\sqrt{3}g}{35} \times \frac{\sqrt{3}}{2}\right)$$
$$a = \frac{22g}{35}$$ 5

$$\frac{3\sqrt{3}}{100} = 0 + \frac{1}{2}\left(\frac{3\sqrt{3}g}{35}\right)t^2 \qquad \Rightarrow t = \sqrt{\frac{7}{10g}}$$

$$x = 0 + \frac{1}{2}\left(\frac{22g}{35}\right)\left(\frac{7}{10g}\right) = 0{\cdot}22 \text{ m}$$ 5

30

5. **(a)** A smooth sphere A, of mass $2m$, moving with speed u collides directly with a smooth sphere B, of mass $7m$, which is at rest. B then collides with a vertical wall which is perpendicular to the direction of motion of the spheres.



The coefficient of restitution is $\frac{1}{2}$ for all collisions.

(i) Show that the spheres will not collide for a second time.

(ii) What is the total loss of kinetic energy due to the impacts?

(i) PCM $2m(u) + 7m(0) = 2mv_1 + 7mv_2$ 5

NEL $v_1 - v_2 = -\frac{1}{2}(u-0)$ 5

$v_1 = \frac{-u}{6}, \quad v_2 = \frac{u}{3}$ 5

$v_3 = -ev_2$

$= \frac{-u}{6}$

$\Rightarrow v_1 = v_3$ 5

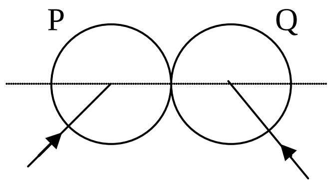
(ii) $KE_B = \frac{1}{2}(2m)u^2 = mu^2$

$KE_A = \frac{1}{2}(2m)\frac{u^2}{36} + \frac{1}{2}(7m)\frac{u^2}{36}$

$= \frac{1}{8}mu^2$

$KE_B - KE_A = \frac{7}{8}mu^2$ 5 25

5. **(b)** A smooth sphere P, of mass $2m$, collides with a smooth sphere Q, of mass m. The velocity of P is $3u\,\vec{i} + 4u\,\vec{j}$ and the velocity of Q is $-4u\,\vec{i} + 3u\,\vec{j}$. When they collide their line of centres is parallel to the unit vector $\vec{i}$.



The impact causes a loss of kinetic energy equal to $\dfrac{25mu^2}{2}$.

(i) Find the coefficient of restitution between the spheres.

(ii) If the magnitude of the impulse imparted to each sphere due to the collision is kmu, find the value of k.

(i) PCM $\quad 2m(3u) + m(\text{-}4\text{u}) = 2mv_1 + mv_2$ $\quad$ 5

NEL $\quad v_1 - v_2 = -e\,(3u + 4u)$ $\quad$ 5

$$v_1 = \frac{u(2-7e)}{3}$$

$$v_2 = \frac{2u(1+7e)}{3}$$

$$\tfrac{1}{2}(2m)(9u^2) + \tfrac{1}{2}(m)(16u^2) - \tfrac{1}{2}(2m)(v_1)^2 - \tfrac{1}{2}(m)(v_2)^2 = \tfrac{25}{2}mu^2$$ 5

$$34 - \frac{2}{9}\left(4 - 28e + 49e^2\right) - \frac{4}{9}\left(1 + 14e + 49e^2\right) = 25$$

$$294e^2 = 69$$

$$\Rightarrow e = \sqrt{\frac{23}{98}} = 0{\cdot}484$$ 5

(ii)

$$\text{I} = mv_2 - m(-4u)$$

$$\text{I} = \frac{2mu(1+7e)}{3} + 4mu$$

$$\text{I} = 6.93mu$$

$$\Rightarrow k = 6{\cdot}93$$ 5 $\quad$ 25

6. **(a)** A particle of mass 5 kg is suspended from a fixed point by a light elastic string. The mass is set vibrating vertically with simple harmonic motion, making 4 oscillations per second.

(i) Find the elastic constant of the string.

The mass is now removed from the string.

(ii) Find the force required to stretch the string through 1.5 cm from its natural length.

(*i*)

$$T_0 = 5g$$

$$ke = 5g$$

$$F = 5g - T_1$$
$$= 5g - k(e + x)$$
$$= -kx \qquad 5$$

$$a = \frac{F}{m} = -\frac{k}{5}x$$

$$\Rightarrow \omega = \sqrt{\frac{k}{5}} \qquad 5$$

$$\frac{\omega}{2\pi} = 4 \qquad 5$$

$$\sqrt{\frac{k}{5}} = 8\pi$$

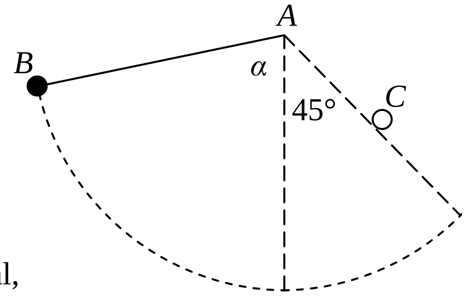
$$\Rightarrow k = 3158{\cdot}27 \qquad 5$$

(*ii*)

$$F = ke_1$$
$$= 3158{\cdot}27 \times 0{\cdot}015$$
$$= 47{\cdot}4 \text{ N} \qquad 5$$

25

6. **(b)** A particle of mass *m*, is suspended by a light inextensible string *AB* of length 2*d*. The end *A* is fixed and the particle is released from rest when *AB* makes an angle α with the downward vertical through *A*.



When *B* has risen again to a height so that *AB* makes an angle of 45º with the downward vertical, the midpoint of the string comes into contact with a small horizontal peg *C*.

(i) If $\cos\alpha = \frac{1}{4}$ find, in terms of *d*, the speed of the particle at the moment that the string touches the peg.

(ii) Find, in terms of *m*, the tension in the string when the particle reaches the same height as the peg.

(i)
$$\tfrac{1}{2}mv^2 = mg\{2d\cos 45 - 2d\cos\alpha\}$$
$$v^2 = 2g\{\sqrt{2}\,d - \tfrac{1}{2}d\}$$
$$v^2 = gd\{2\sqrt{2} - 1\}$$
5

$$v = \sqrt{gd\,(2\sqrt{2}-1)} = 4.233\sqrt{d}$$
5

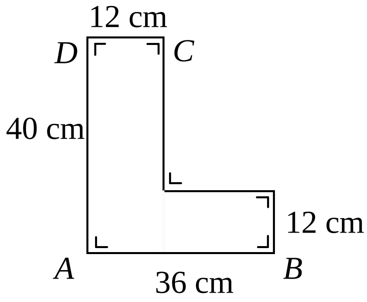
(ii)
$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m(v_1)^2 + mg\{d\cos 45\}$$
5

$$(v_1)^2 = v^2 - 2gd\cos 45$$
$$= gd\{2\sqrt{2}-1\} - \sqrt{2}\,gd$$
$$= gd\{\sqrt{2}-1\} = 4.06d$$
5

$$T = \frac{m(v_1)^2}{d}$$
$$= \frac{mgd\{\sqrt{2}-1\}}{d}$$
$$\Rightarrow T = mg\{\sqrt{2}-1\} = 4.06m$$
5

25

7. **(a)** A letter L is made from a sheet of uniform thin plastic, with dimensions as shown in the diagram.



(i) Find the distance of its centre of mass from each of the lines *AB* and *AD*.

This letter L is to be used for a shop sign.
It is held in position by nails at *D* and *C*.

The nail at *C* breaks so that the letter is freely suspended from *D*.

(ii) What angle will the line *AD* make with the vertical when the letter is hanging in equilibrium?

(i)

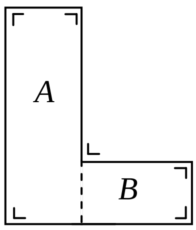
A :	480m	(6, 20)	
B :	288m	(24, 6)	5
L :	768m	$(\bar{x}, \bar{y})$	

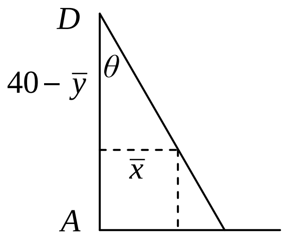
$$768\text{m}\,\bar{x} = 480\text{m}(6) + 288\text{m}(24)$$

$$\Rightarrow \bar{x} = 12{\cdot}75 \qquad 5$$

$$768\text{m}\,\bar{y} = 480\text{m}(20) + 288\text{m}(6)$$

$$\Rightarrow \bar{y} = 14{\cdot}75 \qquad 5$$





(ii)

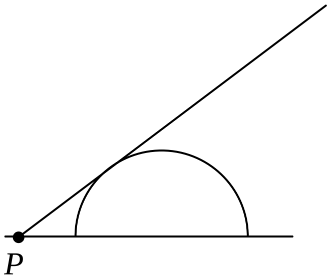
$$\tan\theta = \frac{\bar{x}}{40-\bar{y}}$$

$$= \frac{12.75}{40-14.75} = 0.50495$$

$$\theta = 26.79° \qquad 5$$

20

7. **(b)** A uniform rod of length 3.4 m and weight 85 N is smoothly hinged at *P* to a rough horizontal floor. The rod rests on the smooth curved surface of a hemisphere whose plane face is on the floor.



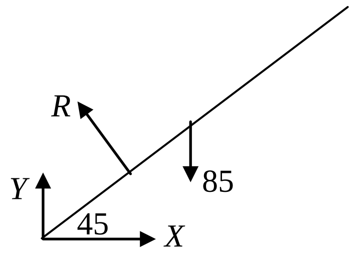
The rod is in equilibrium inclined at 45° to the horizontal. The hemisphere, of weight 44 N and radius 1 m, is in limiting equilibrium.

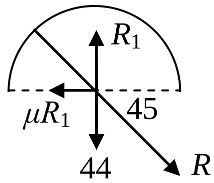
(i) Show on separate diagrams the forces acting on the rod and on the hemisphere.

(ii) Find the coefficient of friction between the hemisphere and the floor.

(iii) Find the magnitude and direction of the reaction at *P*.

(i)





5
5

(ii)

$$R(1) = 85\cos 45 \times 1.7$$

$$R = 102{\cdot}18$$ 5

$$\mu R_1 = R\cos 45$$

$$\mu(R\sin 45 + 44) = R\cos 45$$

$$\mu = 0{\cdot}62$$ 5

(iii)

$$X = R\sin 45 = 72{\cdot}25$$

$$Y = 85 - R\cos 45 = 12{\cdot}75$$

$$F = \sqrt{72{\cdot}25^2 + 12{\cdot}75^2}$$

$$= 73{\cdot}4 \text{ N}$$ 5

$$\tan\alpha = \frac{12.75}{72.25} = 0{\cdot}1765$$

$$\alpha = 10{\cdot}0°$$ 5

30

8. **(a)** Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre, perpendicular to its plane, is $\frac{1}{2}mr^2$.

$$\text{Let } M = \text{mass per unit area}$$

$$\text{mass of element} = M\{2\pi x\,dx\} \qquad 5$$

$$\text{moment of inertia of the element} = M\{2\pi x\,dx\}x^2 \qquad 5$$

$$\text{moment of inertia of the disc} = 2\pi M \int_0^r x^3\,dx \qquad 5$$

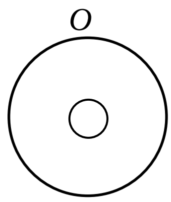
$$= 2\pi M\left[\frac{x^4}{4}\right]_0^r \qquad 5$$

$$= M\pi\frac{r^4}{2}$$

$$= \tfrac{1}{2}mr^2 \qquad 5 \qquad 25$$

8. **(b)** A uniform disc, of mass M and radius r, oscillates as a compound pendulum about a horizontal axis perpendicular to its plane through a point O on its circumference.

(i) Find the period of small oscillations.



A mass $0.2M$ is removed by drilling a circular hole through the centre of the disc.

(ii) The effect of removing a mass $0.2M$ is to increase the period of small oscillations by a factor of $\dfrac{4}{\sqrt{k}}$.

Find k.

(i)

$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{\frac{1}{2}Mr^2 + Mr^2}{Mgr}}$$ 5,5

$$T = 2\pi\sqrt{\frac{3r}{2g}}$$ 5

(ii)

$$\frac{M}{\pi r^2} = \frac{0{\cdot}2M}{\pi x^2}$$

$$x^2 = 0{\cdot}2r^2$$

$$I = \left\{\tfrac{1}{2}Mr^2 - \tfrac{1}{2}(0.2M)x^2\right\} + (M - 0{\cdot}2M)r^2$$

$$= 1{\cdot}28Mr^2$$ 5

$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{1.28Mr^2}{(M - 0{\cdot}2M)gr}}$$

$$T = 2\pi\sqrt{\frac{8r}{5g}}$$

$$2\pi\sqrt{\frac{8r}{5g}} = \frac{4}{\sqrt{k}} \times 2\pi\sqrt{\frac{3r}{2g}}$$

$$k = 15$$ 5

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9. **(a)** When equal volumes of two substances are mixed the density of the mixture is 4000 kg m^{-3}. When equal masses of the same two substances are mixed the density of the mixture is 3840 kg m^{-3}.

Find the density of each of the substances.

$$m_1 + m_2 = m_M$$

$$\rho_1 \times V + \rho_2 \times V = 4000 \times (2V)$$

$$\rho_1 + \rho_2 = 8000$$ 5

$$V_1 + V_2 = V_M$$

$$\frac{m}{\rho_1} + \frac{m}{\rho_2} = \frac{2m}{3840}$$

$$\frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{2}{3840}$$ 5

$$\frac{1}{\rho_1} + \frac{1}{8000 - \rho_1} = \frac{2}{3840}$$ 5

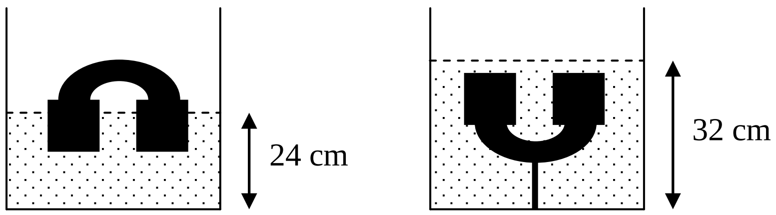
$$\frac{8000}{\rho_1(8000 - \rho_1)} = \frac{1}{1920}$$

$$\rho_1^{\,2} - 8000\rho_1 + 15360000 = 0$$

$$\rho_1 = 3200 \quad \text{or} \quad 4800$$

$$\rho_1 = 3200 \qquad \rho_2 = 4800$$ 5 20

9. **(b)** A cube of side x cm contains water to a height of 18 cm. A solid body of weight 120 N is placed in the cube. It floats and the water level rises to 24 cm.



The body is then completely submerged and tied by a string to the bottom of the cube. The water level rises to 32 cm.

Find **(i)** the value of x

(ii) the tension in the string

(iii) the density of the body.

[Density of water = 1000 kg m^{-3}]

(i) $B = W$

$1000\left(x^2 \times 10^{-4} \times 0.06\right)g = 120$ 5

$x^2 = 2040.8$

$x = 45.2$ 5

(ii) $T + \mathrm{W} = 1000\left(x^2 \times 10^{-4} \times 0.14\right)g$ 5

$T + 120 = 280$

$T = 160 \quad \mathrm{N}$ 5

(iii) $\rho\left(x^2 \times 0.14\right)g = 120$ 5

$$\rho = \frac{120}{(0.20408 \times 0.14)g}$$

$= 428.6 \text{ kg m}^{-3}$ 5 30

10. **(a)** A particle moving in a straight line experiences a retardation of $0.7v^3$ m s^{-2}, where v m s^{-1} is its speed.

It takes 0.04 seconds to reduce its speed from an initial value of 200 m s^{-1} to v_1 m s^{-1}.

Find **(i)** the value of v_1

(ii) the distance travelled during this 0.04 seconds.

(i)

$$\frac{dv}{dt} = -0{\cdot}7v^3$$

$$-\int_{200}^{v_1} \frac{dv}{v^3} = 0{\cdot}7\int_0^{0.04} dt \qquad 5$$

$$\left[\frac{1}{2v^2}\right]_{200}^{v_1} = 0{\cdot}7\left[t\right]_0^{0.04} \qquad 5$$

$$\frac{1}{2v_1^2} - \frac{1}{2\times 200^2} = 0{\cdot}028$$

$$v_1 = 4{\cdot}225 \text{ m s}^{-1} \qquad 5$$

(ii)

$$v\frac{dv}{ds} = -0{\cdot}7v^3$$

$$-\int_{200}^{4{\cdot}225} \frac{dv}{v^2} = 0{\cdot}7\int_0^{s} ds \qquad 5$$

$$\left[\frac{1}{v}\right]_{200}^{4{\cdot}225} = 0{\cdot}7s \qquad 5$$

$$\frac{1}{4{\cdot}225} - \frac{1}{200} = 0{\cdot}7s$$

$$s = 0{\cdot}33 \text{ m} \qquad 5$$

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10. **(b)** A particle moves in a straight line with an acceleration (2*t* – 3) m s^{-2} at time *t* seconds. At time $t = 0$ the particle has velocity of 2 m s^{-1} and displacement of 1 m relative to a fixed point *O* on the line. Find

(i) the times when the particle changes direction

(ii) an expression for the displacement of the particle from *O* at time *t*

(iii) the total distance travelled in the first 2 seconds.

(i)

$$\frac{dv}{dt} = 2t - 3$$

$$[v]_2^v = [t^2 - 3t]_0^t$$

$$v - 2 = t^2 - 3t$$

$$v = t^2 - 3t + 2 \qquad 5$$

$$v = 0 \Rightarrow t = 1, \quad t = 2 \qquad 5$$

(ii)

$$\frac{ds}{dt} = t^2 - 3t + 2$$

$$[s]_1^s = \left[\tfrac{1}{3}t^3 - \tfrac{3}{2}t^2 + 2t\right]_0^t$$

$$s - 1 = \tfrac{1}{3}t^3 - \tfrac{3}{2}t^2 + 2t$$

$$s = \tfrac{1}{3}t^3 - \tfrac{3}{2}t^2 + 2t + 1 \qquad 5$$

(iii)

$$t = 1 \quad \Rightarrow \quad s = \frac{11}{6}$$

$$t = 2 \quad \Rightarrow \quad s = \frac{5}{3}$$

$$x = \left(\frac{11}{6} - 1\right) + \left(\frac{11}{6} - \frac{5}{3}\right)$$

$$\Rightarrow x = 1 \text{ m.} \qquad 5$$

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