Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION, 2011

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

FRIDAY, 10 JUNE – AFTERNOON, 2.00 to 4.30

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Attempt **ALL** questions.

Each question carries 50 marks.

Graph paper may be obtained from the superintendent.

The symbol ✍ indicates that supporting work <u>must</u> be shown to obtain full marks.

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1. **(a)** Peter and Anne share a lotto prize in the ratio $3\frac{1}{2}$ to $2\frac{1}{2}$.

Peter's share is €35 000.

✍ What is the total prize fund?

(b) **(i)** The diameters of Venus and Saturn are $1\cdot21\times10^{4}\,\text{km}$ and $1\cdot21\times10^{5}\,\text{km}$.

✍ What is the difference between the diameters of the two planets?

Give your answer in the form of $a\times10^{n}$ where $n \in \mathbb{Z}$ and $1 \le a < 10$.

(ii) ✍ Write $\dfrac{\sqrt{3}\times27}{3^{2}}$ in the form of 3^{n} where $n \in \mathbb{Q}$.

(c) **(i)** ✍ By rounding to the nearest whole number estimate the value of

$$\frac{\sqrt{(7\cdot17)^{2}+14\cdot59}}{8\cdot29-1\cdot64\times2\cdot23}\ .$$

Then evaluate $\dfrac{\sqrt{(7\cdot17)^{2}+14\cdot59}}{8\cdot29-1\cdot64\times2\cdot23}$, correct to one decimal place.

(ii) Úna and Conor were travelling to South Africa.
They bought 5760 rand in the bank.
The bank charged them €630, which included a 5%
service charge.

✍ What was the value of the euro in rand (the exchange rate) on that day?

2. **(a)** A computer salesperson is paid an annual salary of €30 000.

He is also paid a commission of 4% on sales.

Last year the salesperson earned €38 000.

✍ Calculate the value of the sales.

(b) Aoife is single and earned €40 000 last year. Aoife's tax credits are listed below.

Single Person Tax Credit	€1830
PAYE Tax Credit	€1830
Rent Allowance Tax Credit	€400
Trade Union Payment Tax Credit	€70

(i) ✍ Calculate Aoife's total tax credits.

The standard rate cut-off point for a single person was €36 400.

The standard rate of income tax was 20% and the higher rate was 41%.

(ii) ✍ Calculate the tax paid by Aoife on her income.

Aoife also had to pay a 2% income levy on her gross income.

(iii) ✍ Calculate Aoife's net income after all deductions had been made.

(c) U is the universal set and P and Q are two subsets of U.

$\# U = 30,\ \# P = 16$ and $\# Q = 6$.

(i) ✍ Find with the aid of a Venn diagram the minimum value of $\# (P \cup Q)'$.

(ii) ✍ Find with the aid of a Venn diagram the maximum value of $\# (P \cup Q)'$.

$\# U = u,\ \# P = p,\ \# Q = q$ and $\# (P \cup Q)' = x$.

(iii) ✍ Show with the aid of a Venn diagram, that if $p > q$ and x is a maximum, then $u = p + x$.

3. **(a)** ✍ Given that $t^2 - s = r$, express t in terms of r and s.

(b) **(i)** ✍ Divide $3x^2 + 5x - 28$ by $x + 4$.

(ii) ✍ Solve the equation $\dfrac{4x+2}{5} - \dfrac{6-x}{3} = -5$.

(c) A car park can accommodate cars and mini-buses.
On a particular day there were x cars and y mini-buses
in the car park, giving a total of 520 vehicles.
The parking area for a car is 7 m^2 and the parking area
for a mini-bus is 12 m^2.
On that day a total area of 3840 m^2 was occupied by cars and mini-buses.

(i) Write down two equations to represent the above information.

(ii) ✍ Solve these equations to find the number of cars and the number of
mini-buses in the car park on that day.

There is a flat rate charge per day for parking.
The flat rate for mini-buses is 3 times that for cars. On that day €3000 was taken in.

(iii) ✍ What is the flat rate for cars?

4. **(a)** ✎ Graph on the number line the solution set of

$$-2x + 1 > -7, \ x \in \mathbb{N}.$$

(b) **(i)** Factorise $x^2 - 1$.

(ii) ✎ Factorise fully $ax - 3 - a + 3x$.

(iii) Factorise $6x^2 + x - 35$.

(c) The new Lansdowne Road stadium has seating capacity for 200 journalists.
It was decided initially that this seating would be in x rows of equal value.

(i) Write, in terms of x, the number of seats per row required to accommodate the 200 journalists.

During the construction it was decided to have 3 fewer rows to accommodate the 200 journalists.

(ii) Write, in terms of x, the number of seats per row now required.

It was found that 15 extra seats per row were required compared to the initial plan.

(iii) ✎ Write an equation using the above information and solve for x.

5. **(a)** ✍ Given that $f(x) = 3x - 4$ and that $f(k) = 11$, find the value of k.

(b) Let f be the function $f : x \rightarrow 7x - x^2$.

✍ Draw the graph of f for $0 \le x \le 7, \ x \in \mathbb{R}$.

(c) The formula for the height, y metres, of a golf ball above ground level x seconds after it is hit, is given by $7x - x^2$.
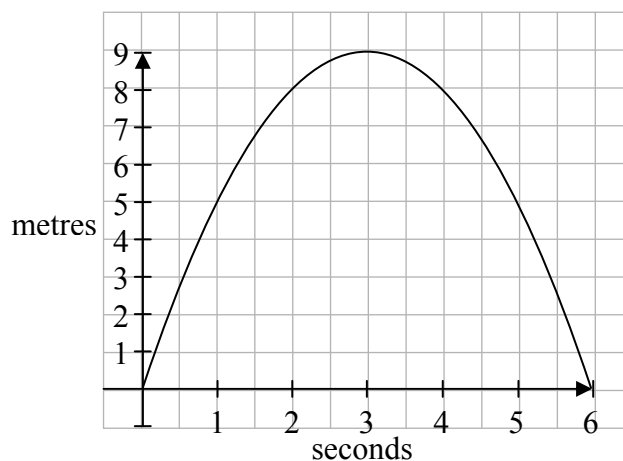
Use your graph from part **(b)**:

(i) ✍ to find the maximum height reached by the golf ball

(ii) ✍ to estimate the number of seconds the golf ball was

more than 2 metres above the ground.

The graph below represents the flight of another golf ball.

The flight of the golf ball is given by the formula $ax - x^2, \ x \in \mathbb{R}$.

(iii) ✍ Find the value of a.

6. **(a)** ✍ When $a = \dfrac{1}{4}$, find the value of $\dfrac{a+5}{3} - \dfrac{a+4}{2}$.

(b) **(i)** ✍ Express in its simplest form:
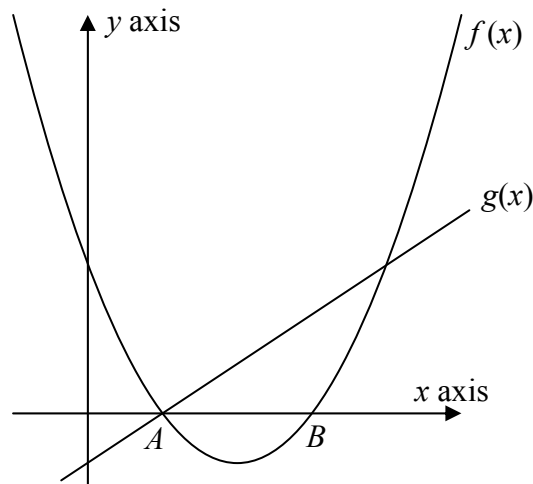
$$\frac{4}{x-1} - \frac{5}{x+2}.$$

(ii) ✍ Hence, or otherwise, solve the equation:

$$\frac{4}{x-1} - \frac{5}{x+2} = \frac{3}{2},$$

giving your answers correct to one decimal place.

(c) The diagram below shows part of the graphs of the functions

$$f(x) = x^2 - 4x + 3 \text{ and } g(x) = x + k.$$

The graph of $f(x)$ cuts the x axis at A and B.

The graphs of $f(x)$ and $g(x)$ intersect at A.

(i) ✍ Find the coordinates of A and the coordinates of B.

(ii) ✍ Find the value of k.

(iii) ✍ Verify that $f(x)$ and $g(x)$ intersect also at the point $(4, 3)$.

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