Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2011

Marking Scheme

APPLIED MATHEMATICS

Higher Level

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

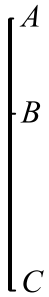
Attempt marks are awarded as follows: 5 (att 2).

2 The marking scheme shows one correct solution to each question.
In many cases there are other equally valid methods.

1. **(a)** A particle is released from rest at A and falls vertically passing two points B and C.

It reaches B after t seconds and takes $\frac{2t}{7}$ seconds to fall from B to C, a distance of 2.45 m.

Find the value of t.



AB

$$s = ut + \tfrac{1}{2}ft^2$$

$$h = 0 + \tfrac{1}{2}gt^2$$ 5

AC

$$s = ut + \tfrac{1}{2}ft^2$$

$$h + 2.45 = 0 + \tfrac{1}{2}g\left(\frac{9t}{7}\right)^2$$ 5

$$\tfrac{1}{2}gt^2 + \tfrac{1}{4}g = 0 + \tfrac{1}{2}g\left(\frac{81t^2}{49}\right)$$ 5

$$2t^2 + 1 = \frac{162t^2}{49}$$

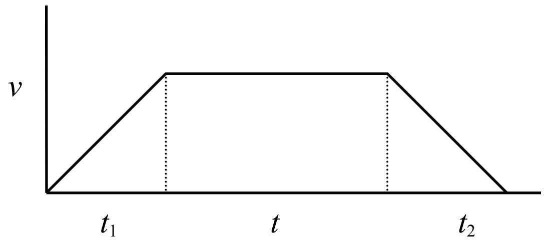
$$64t^2 = 49$$

$$\Rightarrow \quad t = \frac{7}{8}\text{ s}$$ 5 | 20

1. **(b)** A car accelerates uniformly from rest to a speed v in t_1 seconds.
It continues at this constant speed for t seconds and then decelerates uniformly to rest in t_2 seconds.
The average speed for the journey is $\frac{3v}{4}$.

(i) Draw a speed-time graph for the motion of the car.
(ii) Find $t_1 + t_2$ in terms of t.
(iii) If a speed limit of $\frac{2v}{3}$ were to be applied, find in terms of t the least time the journey would have taken, assuming the same acceleration and deceleration as in part (ii).

(i)



5, 5

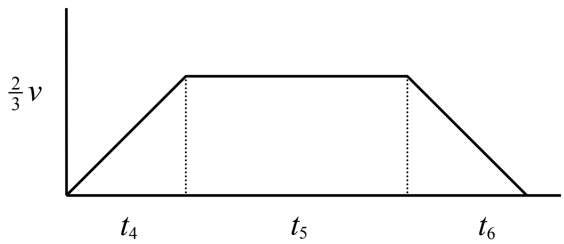
(ii) $$\text{average speed} = \frac{\frac{1}{2}t_1 v + tv + \frac{1}{2}t_2 v}{t_1 + t + t_2}$$

$$\frac{3v}{4} = \frac{\frac{1}{2}t_1 v + tv + \frac{1}{2}t_2 v}{t_1 + t + t_2}$$ 5

$$\frac{3}{4} = \frac{\frac{1}{2}t_1 + t + \frac{1}{2}t_2}{t_1 + t + t_2}$$

$$3t_1 + 3t + 3t_2 = 2t_1 + 4t + 2t_2$$

$$\Rightarrow t_1 + t_2 = t$$ 5



(iii) $$\tfrac{1}{2}t_1 v + tv + \tfrac{1}{2}t_2 v = \tfrac{1}{2}t_4\left(\frac{2v}{3}\right) + t_5\left(\frac{2v}{3}\right) + \tfrac{1}{2}t_6\left(\frac{2v}{3}\right)$$ 5

$$3t_1 v + 6tv + 3t_2 v = 2t_4 v + 4t_5 v + 2t_6 v$$

$$3t_1 + 6t + 3t_2 = 2t_4 + 4t_5 + 2t_6$$

$$9t = 2(t_4 + t_6) + 4t_5$$

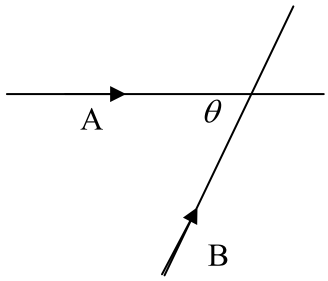
$$t_4 + t_6 = \tfrac{2}{3}t$$

$$9t = 2(\tfrac{2}{3}t) + 4t_5$$

$$\Rightarrow t_5 = \tfrac{23}{12}t$$

$$\Rightarrow t_4 + t_5 + t_6 = \tfrac{2}{3}t + \tfrac{23}{12}t = \tfrac{31}{12}t$$ 5 | 30

2. **(a)** Two cars, A and B, travel along two straight roads which intersect at an angle θ where $\tan\theta = \frac{4}{3}$.



Car A is moving towards the intersection at a uniform speed of 5 m s^{-1}.
Car B is moving towards the intersection at a uniform speed of 10 m s^{-1}.

At a certain instant each car is 100 m from the intersection and approaching the intersection.

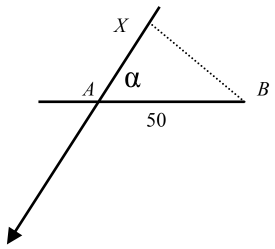
Find **(i)** the velocity of A relative to B
(ii) the shortest distance between the cars.

(i)

$$\vec{V}_A = 5\vec{i} + 0\vec{j}$$
$$\vec{V}_B = 6\vec{i} + 8\vec{j}$$
$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$ 5
$$= -\vec{i} - 8\vec{j}$$ 5

magnitude $= \sqrt{65}$ m s^{-1} 5

direction = West $\tan^{-1} 8$ South 5

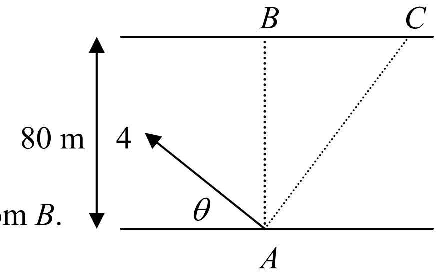


(ii)

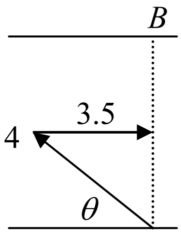
$$|BX| = 50\sin\alpha$$ 5
$$= 50\left(\frac{8}{\sqrt{65}}\right)$$
$$= 49.6 \text{ m}$$ 5

30

2 **(b)** A woman can row a boat at 4 m s^{-1} in still water.
She rows across a river 80 m wide.
The river flows at a constant speed of $3.5\ \text{m s}^{-1}$ parallel to the straight banks.
She wishes to land between *B* and *C*.
The point *B* is directly across from the starting point *A* and the point *C* is $20\sqrt{3}$ m downstream from *B*.

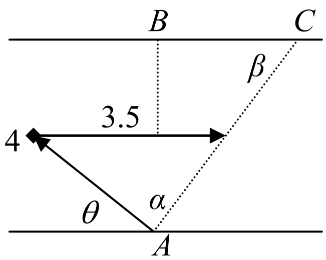


If θ is the direction she takes, find the range of values of θ if she lands between *B* and *C*.



$$\cos\theta = \frac{3.5}{4}$$ 5

$$\theta = 28.955°$$ 5



$$\tan\beta = \frac{80}{20\sqrt{3}}$$ 5

$$\beta = 66.59°$$

$$\frac{\sin\alpha}{3.5} = \frac{\sin\beta}{4}$$

$$\sin\alpha = 0.8029$$

$$\alpha = 53.41°$$

$$\theta = 180 - 66.59 - 53.41$$
$$= 60°$$

$$28.955 \le \theta \le 60$$ 5

20

3. (a) A particle is projected from a point P on horizontal ground.
The speed of projection is $35\ \text{m s}^{-1}$ at an angle $\tan^{-1} 2$ to the horizontal.
The particle strikes a target whose position vector relative to P is $x\vec{i} + 50\vec{j}$.

Find **(i)** the value of x
(ii) a second angle of projection so that the particle strikes the target.

(i)

$$35\cos\alpha . t = x$$

$$t = \frac{x}{7\sqrt{5}} \qquad 5$$

$$35\sin\alpha . t - 4.9t^2 = 50 \qquad 5$$

$$35\left(\frac{2}{\sqrt{5}}\right)\left(\frac{x}{7\sqrt{5}}\right) - 4.9\left(\frac{x}{7\sqrt{5}}\right)^2 = 50$$

$$x^2 - 100x + 2500 = 0$$

$$x = 50 \qquad 5$$

(ii)

$$35\cos\alpha . t = 50$$

$$t = \frac{10}{7\cos\alpha}$$

$$35\sin\alpha . t - 4.9t^2 = 50$$

$$35\sin\alpha . \left(\frac{10}{7\cos\alpha}\right) - 4.9\left(\frac{10}{7\cos\alpha}\right)^2 = 50 \qquad 5$$

$$50\tan\alpha - 10\left(1 + \tan^2\alpha\right) = 50$$

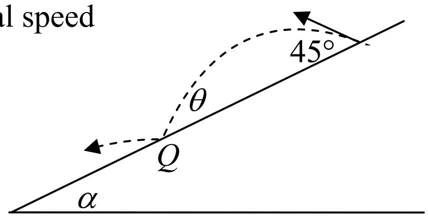
$$\tan^2\alpha - 5\tan\alpha + 6 = 0$$

$$(\tan\alpha - 2)(\tan\alpha - 3) = 0$$

$$\tan\alpha = 3$$

$$\alpha = 71.6^\circ \qquad 5 \qquad 25$$

3 (b) A plane is inclined at an angle α to the horizontal.
A particle is projected down the plane with initial speed of 10 m s^{-1} at an angle 45° to the inclined plane.
The plane of projection is vertical and contains the line of greatest slope.



The particle strikes the plane at Q with a landing angle θ where $\tan\theta = \frac{1}{4}$.

(i) Find the value of α.

(ii) If the magnitude of the rebound velocity at Q is $5\sqrt{33}$, find the value of e, the coefficient of restitution.

(i)

$$r_j = 0$$

$$0 = 10\sin 45.t - \tfrac{1}{2} g\cos\alpha.t^2$$

$$\Rightarrow \quad t = \frac{10\sqrt{2}}{g\cos\alpha}$$ 5

$$v_i = 10\cos 45 + g\sin\alpha\left(\frac{10\sqrt{2}}{g\cos\alpha}\right)$$ 5

$$= 5\sqrt{2} + 10\sqrt{2}\tan\alpha$$

$$v_j = 10\sin 45 - g\cos\alpha\left(\frac{10\sqrt{2}}{g\cos\alpha}\right)$$ 5

$$= -5\sqrt{2}$$

$$\tan\theta = \frac{-v_j}{v_i}$$

$$\frac{1}{4} = \frac{5\sqrt{2}}{5\sqrt{2} + 10\sqrt{2}\tan\alpha}$$

$$\tan\alpha = 1.5 \quad \Rightarrow \quad \alpha = 56.3°$$ 5

(ii)

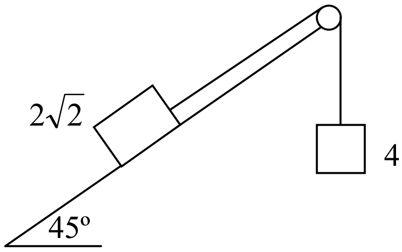
$$v_i = 20\sqrt{2}$$

$$v_j = 5e\sqrt{2}$$

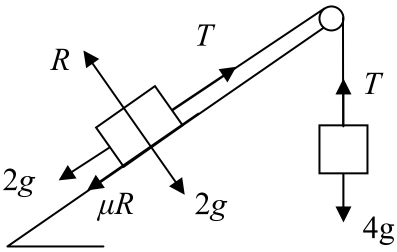
$$5\sqrt{33} = \sqrt{\left(20\sqrt{2}\right)^2 + \left(5e\sqrt{2}\right)^2}$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$ 5 | 25

4. (a) A block of mass $2\sqrt{2}$ kg rests on a rough plane inclined at 45º to the horizontal. It is connected by a light inextensible string which passes over a smooth, light, fixed pulley to a particle of mass 4 kg which hangs freely under gravity. The coefficient of friction between the block and the plane is $\frac{1}{4}$.



Find the acceleration of the 4 kg mass.



5

$$4g - T = 4f$$ 5

$$T - 2g - \mu R = 2\sqrt{2}f$$ 5

$$T - 2g - \tfrac{1}{4}(2g) = 2\sqrt{2}f$$

$$4g - 2g - \frac{1}{2}g = \left(4 + 2\sqrt{2}\right)f$$

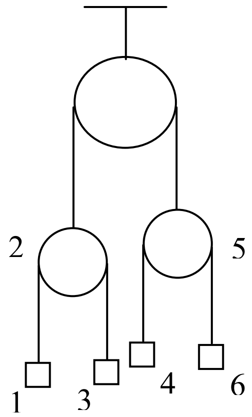
$$\frac{3g}{2} = \left(4 + 2\sqrt{2}\right)f$$

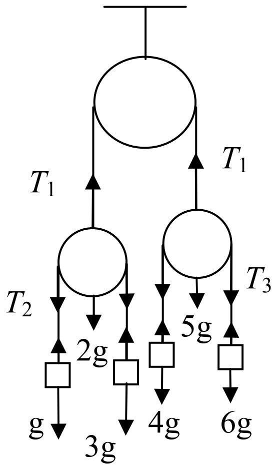
$$f = \frac{3g}{2\left(4 + 2\sqrt{2}\right)}$$

$$\Rightarrow \quad f = 2.15 \ \text{m s}^{-2}$$ 5

20

4 **(b)** A smooth pulley, of mass 2 kg, is connected by a light inextensible string passing over a smooth light fixed pulley to a smooth pulley of mass 5 kg.
Two particles of masses 1 kg and 3 kg are connected by a light inextensible string passing over the 2 kg pulley.
Two particles of masses 4 kg and 6 kg are connected by a light inextensible string passing over the 5 kg pulley.
Find the tension in each string, when the system is released from rest.





5

$$6g - T_3 = 6(c+a)$$
$$T_3 - 4g = 4(c-a) \quad \Rightarrow \quad 24g - 5T_3 = 24a$$ 5

$$3g - T_2 = 3(b-a)$$ 5
$$T_2 - g = (b+a) \quad \Rightarrow \quad 6g - 4T_2 = -6a$$ 5

$$T_1 - 2T_2 - 2g = 2a$$
$$2T_3 + 5g - T_1 = 5a \quad \Rightarrow \quad 2T_3 - 2T_2 + 3g = 7a$$ 5

$$2\left\{\frac{24g-24a}{5}\right\} - 2\left\{\frac{6g+6a}{4}\right\} + 3g = 7a$$

$$\Rightarrow \quad a = 4.8 \text{ m s}^{-2}$$

$$\left.\begin{aligned} T_1 &= 73 \text{ N} \\ T_2 &= 21.9 \text{ N} \\ T_3 &= 24 \text{ N} \end{aligned}\right\}$$ 5

30

5. **(a)** A smooth sphere P, of mass $2m$ kg, moving with speed u m s^{-1} collides directly with a smooth sphere Q, of mass $3m$ kg, moving in the opposite direction with speed u m s^{-1}.
The coefficient of restitution between the spheres is e and $0 < e < 1$.

(i) Show that P will rebound for all values of e.

(ii) For what range of values of e will Q rebound?

(i) PCM $\quad 2m(u) + 3m(-u) = 2mv_1 + 3mv_2$ — 5

NEL $\quad v_1 - v_2 = -e(u+u)$ — 5

$$\left.\begin{aligned} v_1 &= \frac{-u(1+6e)}{5} \\ v_2 &= \frac{u(-1+4e)}{5} \end{aligned}\right\}$$ — 5

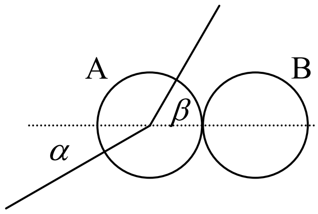
$$v_1 = \frac{-u(1+6e)}{5}$$
$$< 0 \quad \forall \ e, \quad 0 < e < 1$$ — 5

(ii)
$$v_2 = \frac{u(-1+4e)}{5} > 0$$
$$4e > 1$$
$$e > \frac{1}{4}$$

$$\Rightarrow \ \frac{1}{4} < e < 1$$ — 5

25

5 **(b)** A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B which is at rest.
The direction of motion of A before and after impact makes angles α and β respectively with the line of centres at the instant of impact.



The coefficient of restitution between the spheres is e.

(i) If $\tan\alpha = k\tan\beta$, find k, in terms of e.
(ii) If the magnitude of the impulse imparted to each sphere due to the collision is $\frac{7}{8}mu\cos\alpha$, find the value of e.

(i)	PCM	$m(u\cos\alpha) + m(0) = mv_1 + mv_2$	5
	NEL	$v_1 - v_2 = -e\,(u\cos\alpha - 0)$	5
		$v_1 = \dfrac{u\cos\alpha(1-e)}{2}$	
		$v_2 = \dfrac{u\cos\alpha(1+e)}{2}$	5
		$\tan\beta = \dfrac{u\sin\alpha}{v_1}$	
		$= \dfrac{2u\sin\alpha}{u\cos\alpha(1-e)}$	
		$= \dfrac{2\tan\alpha}{1-e}$	
		$\tan\beta = \dfrac{2k\tan\beta}{1-e}$	
		$1-e = 2k$	
		$\Rightarrow k = \dfrac{1-e}{2}$	5
(ii)		$I = mv_2 - m(0)$	
		$\dfrac{7}{8}mu\cos\alpha = \dfrac{1}{2}mu\cos\alpha(1+e)$	
		$e = \dfrac{3}{4}$	5
			25

6. **(a)** The distance, x, of a particle from a fixed point, O, is given by

$$x = a\sin(\omega t + \varepsilon)$$

where a, ω and ε are positive constants.

(i) Show that the motion of the particle is simple harmonic.

A particle moving with simple harmonic motion starts from a point 1 m from the centre of the motion with a speed of 9.6 m s^{-1} and an acceleration of 16 m s^{-2}.

(ii) Calculate a, ω and ε.

(i)

$$x = a\sin(\omega t + \varepsilon)$$

$$\dot{x} = a\omega\cos(\omega t + \varepsilon)$$ 5

$$\ddot{x} = -a\omega^2\sin(\omega t + \varepsilon)$$

$$= -\omega^2 x$$ 5

(ii)

$$\ddot{x} = \omega^2 x$$

$$16 = \omega^2(1)$$

$$\Rightarrow \omega = 4\ \text{rad s}^{-1}$$ 5

$$\dot{x} = a\omega\cos(\omega t + \varepsilon)$$

$$9.6 = \text{a}(4)\cos\varepsilon$$

$$\Rightarrow a\cos\varepsilon = 2.4$$

$$x = a\sin(\omega t + \varepsilon)$$

$$1 = a\sin\varepsilon$$

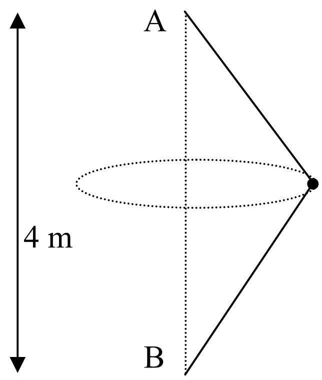
$$\frac{a\sin\varepsilon}{a\cos\varepsilon} = \frac{1}{2.4}$$

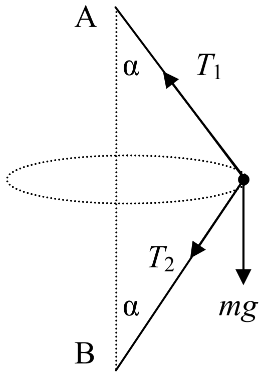
$$\tan\varepsilon = \frac{5}{12} \Rightarrow \varepsilon = 0.395\ \text{rad}$$ 5

$$a = \frac{1}{\sin 0.395} = 2.6\ \text{m}$$ 5 | 25

6 **(b)** A and B are two fixed pegs.
A is 4 m vertically above B.
A mass m kg, connected to A and B by two light inextensible strings of equal length, ℓ, is describing a horizontal circle with uniform angular velocity ω.

Find the value of ω if the ratio of the tensions in the two strings is 11: 9.





$$T_1 \sin\alpha + T_2 \sin\alpha = m(\ell \sin\alpha)\omega^2$$
$$T_1 + T_2 = m\ell\omega^2 \qquad 5$$

$$\tfrac{11}{9}T_2 + T_2 = m\ell\omega^2$$
$$T_2 = \tfrac{9}{20}m\ell\omega^2 \qquad 5$$

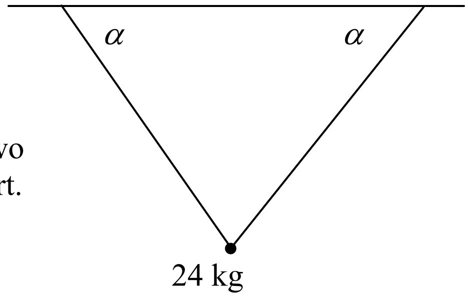
$$T_1 \cos\alpha - T_2 \cos\alpha = mg$$
$$T_1 - T_2 = \frac{mg}{\cos\alpha} = \tfrac{1}{2}mg\ell \qquad 5$$

$$\tfrac{11}{9}T_2 - T_2 = \tfrac{1}{2}mg\ell$$
$$T_2 = \tfrac{9}{4}mg\ell \qquad 5$$

$$\tfrac{9}{20}m\ell\omega^2 = \tfrac{9}{4}mg\ell$$
$$\omega^2 = 49$$
$$\omega = 7 \text{ rad s}^{-1} \qquad 5$$

25

7. **(a)** A particle of mass 24 kg is attached to two light elastic strings, each of natural length 33 cm and elastic constant k.

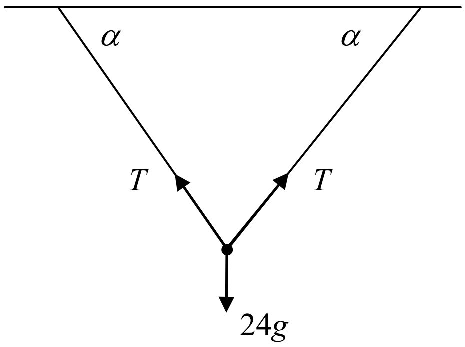


The other ends of the strings are attached to two points on the same horizontal level 64 cm apart.

Each string makes an angle α with the horizontal, where $\tan\alpha = \dfrac{3}{4}$.

(i) Show that the extension of each string is 7 cm.

(ii) Find the value of k.



(i)

$$\cos\alpha = \frac{32}{33+x}$$ 5

$$\frac{4}{5} = \frac{32}{33+x}$$

$$x = 7\text{ cm}$$ 5

(ii)

$$2T\sin\alpha = 24g$$ 5

$$2T\left(\frac{3}{5}\right) = 24g$$

$$T = 20g$$ 5

$$T = kx$$

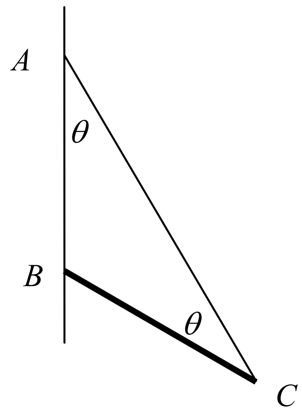
$$20g = k(0.07)$$

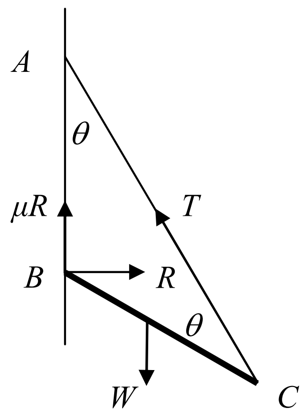
$$\Rightarrow k = 2800\ \text{N m}^{-1}$$ 5 | 25

7. (b) A uniform rod *BC*, of length 2*p* and weight *W*, rests in equilibrium with *B* in contact with a rough vertical wall.
One end of a light inextensible string is fixed to a point *A* on the wall vertically above *B*, the other end is attached to *C*.

The coefficient of friction between the rod and the wall is μ.

If $|\angle CAB| = |\angle BCA| = \theta$, prove that $\mu \geq \tan\theta$.





$$T\sin\theta(2p) = W\sin 2\theta(p)$$
$$T\sin\theta(2p) = W2\sin\theta\cos\theta(p)$$
$$T = W\cos\theta \qquad 5$$

$$R = T\sin\theta \qquad 5$$

$$\mu R + T\cos\theta = W \qquad 5$$
$$\mu T\sin\theta + T\cos\theta = \frac{T}{\cos\theta}$$
$$\mu\sin\theta = \frac{1}{\cos\theta} - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta} \qquad 5$$
$$\mu\sin\theta = \sin\theta\tan\theta$$
$$\mu = \tan\theta$$

$$\Rightarrow \mu \geq \tan\theta \qquad 5$$

25

8. **(a)** Prove that the moment of inertia of a uniform square lamina of mass m and side 2ℓ about an axis through its centre parallel to one of its sides is $\frac{1}{3}m\ell^2$.

Let M = mass per unit area

$$\text{mass of element} = M\{2\ell\, dx\}$$

$$\text{moment of inertia of the element} = M\{2\ell\, dx\}x^2 \qquad 5$$

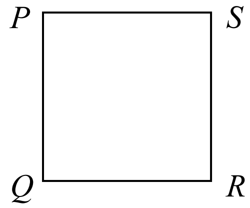
$$\text{moment of inertia of the lamina} = 2\ell M \int_{-\ell}^{\ell} x^2\, dx \qquad 5$$

$$= 2\ell M\left[\frac{x^3}{3}\right]_{-\ell}^{\ell} \qquad 5$$

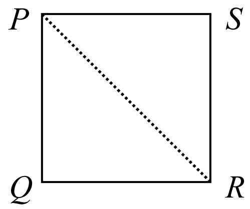
$$= 4M\frac{\ell^4}{3}$$

$$= \tfrac{1}{3}m\ell^2 \qquad 5 \qquad 20$$

8. **(b)** A square lamina *PQRS*, of side 60 cm and mass *m*, can turn freely about a horizontal axis through *P* perpendicular to the plane of the lamina.
The lamina is released from rest when *PS* is horizontal.
(i) Find the angular velocity of the lamina when *PR* is vertical.



A mass *m* is attached to the lamina at *R*. The compound pendulum is set in motion.
(ii) Find the period of small oscillations of the compound pendulum and hence, or otherwise, find the length of the equivalent simple pendulum.



(i)

$$\text{Gain in KE} = \text{Loss in PE}$$

$$\tfrac{1}{2}I\omega^2 = mgh$$

$$\tfrac{1}{2}\left\{\tfrac{4}{3}m(0.3)^2 + \tfrac{4}{3}m(0.3)^2\right\}\omega^2 = mg\left(0.3\sqrt{2} - 0.3\right)$$ 5, 5

$$\omega^2 = \frac{g\left(\sqrt{2}-1\right)}{0.4} = 10.1482$$

$$\omega = 3.19 \ \text{rad s}^{-1}$$ 5

(ii)

$$I = \tfrac{8}{3}m(0.3)^2 + m\left(0.6\sqrt{2}\right)^2$$
$$= 0.96m$$
$$Mgh = mg\left(0.3\sqrt{2}\right) + mg\left(0.6\sqrt{2}\right)$$
$$= 0.9\sqrt{2}\, mg$$ 5

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$
$$= 2\pi\sqrt{\frac{0.96m}{0.9\sqrt{2}\, mg}}$$
$$= 1.74 \text{ s}$$ 5

$$2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.96}{0.9\sqrt{2}\, g}}$$

$$\Rightarrow L = \frac{0.96}{0.9\sqrt{2}} = 0.75 \text{ m}$$ 5

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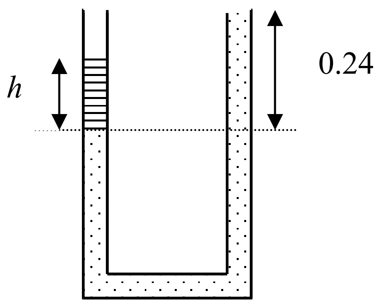
9. **(a)** A U-tube of cross-sectional area of 0.15 cm^2 contains oil of relative density 0.8.

The surface of the oil is 12 cm from the top of both branches of the U-tube.

What volume of water can be poured into one of the branches before the oil overflows in the other branch?



$$1000gh = 800g(0.24)$$ 5

$$h = 0.192 \text{ m}$$ 5

$$\text{Volume} = h\,A$$
$$= 0.192\left(0.15 \times 10^{-4}\right)$$ 5

$$= 2.88 \times 10^{-6} \text{ m}^3$$ 5 20

9 **(b)** A uniform solid cylinder floats upright with $\frac{1}{3}$ of its axis immersed when placed in liquid A.

When placed in liquid B, the uniform solid cylinder floats upright with $\frac{3}{5}$ of its axis immersed.

What fraction of the cylinder's axis is immersed when the cylinder floats upright in a uniform mixture of equal volumes of the two liquids?

A

$$B_A = W$$

$$\frac{\frac{1}{3}Ws_A}{s} = W$$

$$\Rightarrow s_A = 3s \qquad 5$$

B

$$B_B = W$$

$$\frac{\frac{3}{5}Ws_B}{s} = W$$

$$\Rightarrow s_B = \tfrac{5}{3}s \qquad 5$$

A + B

$$B_M = W$$

$$\frac{yWs_M}{s} = W$$

$$\Rightarrow s_M = \tfrac{1}{y}s \qquad 5$$

$$s_A V + s_B V = s_M (2V) \qquad 5$$

$$s_A + s_B = 2s_M$$

$$3s + \tfrac{5}{3}s = \tfrac{2}{y}s \qquad 5$$

$$\tfrac{14}{3}s = \tfrac{2}{y}s$$

$$\Rightarrow y = \frac{3}{7} \qquad 5$$

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10. **(a)** If

$$x^2 \frac{dy}{dx} - xy = 7y$$

and $y = 1$ when $x = 1$, find the value of y when $x = 2$.

$$x^2 \frac{dy}{dx} = xy + 7y$$

$$\frac{dy}{dx} = \frac{y(x+7)}{x^2}$$

$$\int \frac{1}{y}\, dy = \int \frac{x+7}{x^2}\, dx \qquad 5$$

$$\int \frac{1}{y}\, dy = \int \left(\frac{1}{x} + \frac{7}{x^2} \right) dx$$

$$\ln y = \ln x - \frac{7}{x} + C \qquad 5$$

$$y = 1,\ x = 1$$

$$\Rightarrow C = 7 \qquad 5$$

$$\ln y = \ln x - \frac{7}{x} + 7$$

$$\ln y = \ln 2 - \frac{7}{2} + 7$$

$$= 4.1931$$

$$\Rightarrow y = e^{4.1931} = 66.23 \qquad 5 \qquad \boxed{20}$$

10 **(b)** A particle travelling in a straight line has a deceleration of

$$\frac{v^2}{400}+16 \quad \text{m s}^{-2}$$

where v is its speed at any time t.

If its initial speed is 40 m s^{-1}, find

(i) the distance travelled before it comes to rest
(ii) the average speed of the particle during the motion.

(i)

$$v\frac{dv}{dx} = -\left(\frac{v^2}{400}+16\right)$$

$$= -\left(\frac{v^2+80^2}{400}\right)$$

$$\int_{40}^{0} \frac{v}{v^2+80^2}\,dv = -\frac{1}{400}\int_{0}^{x} dx \qquad 5$$

$$\left[\tfrac{1}{2}\ln\left(v^2+80^2\right)\right]_{40}^{0} = \left[-\frac{x}{400}\right]_{0}^{x} \qquad 5$$

$$\tfrac{1}{2}\ln\left(\frac{40^2+80^2}{80^2}\right) = \frac{x}{400}$$

$$x = 200\ln\left(\frac{5}{4}\right)$$

$$x = 44.63\text{ m} \qquad 5$$

(ii)

$$\frac{dv}{dt} = -\left(\frac{v^2+80^2}{400}\right)$$

$$\int_{40}^{0} \frac{1}{v^2+80^2}\,dv = -\frac{1}{400}\int_{0}^{t} dt$$

$$\left[\tfrac{1}{80}\tan^{-1}\left(\frac{v}{80}\right)\right]_{40}^{0} = \left[-\frac{t}{400}\right]_{0}^{t} \qquad 5$$

$$t = 5\tan^{-1}\left(\frac{1}{2}\right)$$

$$= 2.32 \qquad 5$$

$$\text{average speed} = \frac{44.63}{2.32} = 19.24\text{ m s}^{-1} \qquad 5$$

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