2022. M32 2022L020A1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2022

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE – AFTERNOON, 2:00 TO 4:30

Five questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 9·8 m s^{-2}.

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

Diagrams are generally not drawn to scale.

Do not hand this up.

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1. **(a)** A train takes 40 minutes to travel from rest at station A to rest at station B. The distance between the stations is 20 km. The train left station A at 10:00. At 10:15 the speed of the train was 32 km h^{-1} and at 10:30 the speed was 48 km h^{-1}.

The speed of 48 km h^{-1} was maintained until the brakes were applied, causing a uniform deceleration which brought the train to rest at B.

During the first and second 15-minute intervals the accelerations were constant.

(i) Draw a speed-time graph of the motion.

(ii) Find the time taken for the first 16 km.

(iii) Find the deceleration of the train.

(b) A ball *E* is thrown vertically upwards with a speed of 42 m s^{-1}.

T (< 8) seconds later another ball, *F*, is thrown vertically upwards from the same point with the same initial speed.

(i) Find where ball *E* is after 5 s and the total distance it has travelled in this time.

(ii) Prove that when *E* and *F* collide, they will each be travelling with speed $\frac{1}{2}gT$.

2. **(a)** A ship is travelling at 22 km h^{-1} in a direction west 30° north. A boat sets out to intercept the ship from a point 25 km south of the ship.

The speed of the boat is 55 km h^{-1}.

Find

(i) the direction the boat should steer

(ii) the time, to the nearest minute, that it takes the boat to intercept the ship

(iii) the distance between the boat and the ship 10 minutes before they meet.

(b) A woman can swim at u m s^{-1} in still water. In a river she can cover a distance d m against the current in time t_1 and the same distance with the current in time t_2. The current flows parallel to the straight banks at v m s^{-1}.

(i) Show that $v = \frac{d(t_1 - t_2)}{2t_1 t_2}$.

The width of the river is d m and $v < u$.

(ii) Find, in terms of t_1 and t_2, the time taken by the woman to cross the river by the shortest path.

3. **(a)** A particle is projected out to sea from a point *P* on a cliff to hit a target 60 m horizontally from *P* and 60 m vertically below *P*.

The velocity of projection is $14\sqrt{3}$ m s^{-1} at an angle α to the horizontal.

Find

(i) the two possible values of α

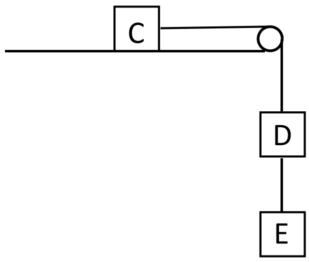
(ii) the times of flight.

(b) A particle is projected up a plane with speed *u* m s^{-1} at an angle β to the plane. The plane is inclined at 30° to the horizontal.

The plane of projection is vertical and contains the line of greatest slope.

Find the greatest range up the plane in terms of *u*.

4. **(a)** A block C of mass 6*m* rests on a rough horizontal table.



It is connected by a light inextensible string which passes over a smooth fixed pulley at the edge of the table to a block D of mass 3*m*. D is connected by another light inextensible string to a block E of mass 2*m*, as shown in the diagram.

The coefficient of friction between C and the table is $\frac{1}{3}$.

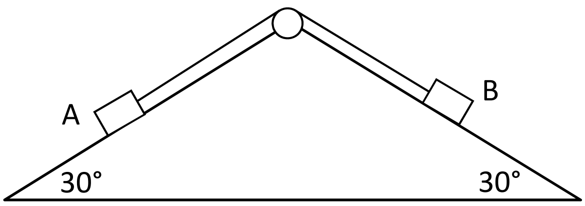
The system is released from rest.

(i) Show on separate diagrams the forces acting on each block.

(ii) Find the acceleration of C.

(iii) Find the tension in each string.

(b) Particles A and B of masses *m* and 2*m* are connected by a light inextensible string which passes over a pulley at the top of a wedge, one particle resting on each of the faces, which are smooth.



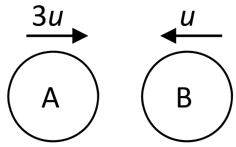
Each of the inclined faces of the wedge makes an angle of 30° with the horizontal.

The wedge of mass 3*m* rests on a smooth horizontal table.

The system is released from rest.

Find the acceleration of the wedge.

5. **(a)** A smooth sphere A of mass $2m$, moving with speed $3u$ on a smooth horizontal table collides directly with a smooth sphere B of mass m, moving in the opposite direction with speed u.



The coefficient of restitution between A and B is e.

Find, in terms of u and e,

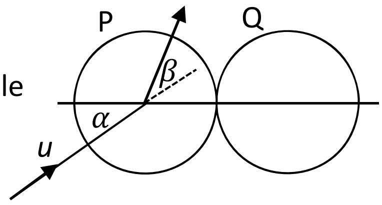
(i) the speed of each sphere after the collision

(ii) the magnitude of the impulse imparted to B due to the collision.

The loss of the kinetic energy due to the collision is $kmu^2(1 - e^2)$.

(iii) Find the value of k.

(b) A smooth sphere P has mass m and speed u. It collides obliquely with a smooth sphere Q, of mass m, which is at rest. Before the collision, the direction of P makes an angle α with the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is $\frac{1}{3}$.

During the impact the direction of motion of P is turned through an angle β.

Show that $\tan\beta = \frac{2\tan\alpha}{1+3\tan^2\alpha}$.

6. **(a)** A particle moves on a straight line with simple harmonic motion about point O as centre. Its displacement from O at any time t is x.

At time $t = 0$ the particle passes through a point H at a distance of 3 cm from O, moving away from O. The particle next passes through H at time $t = 4$ s, moving towards O, and it passes through H for a third time after a further 12 s.

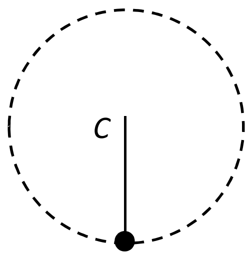
(i) Find the period of the motion.

(ii) Show that $x = A\sin(\omega t + \varepsilon)$, where A, ω and ε are constants, satisfies the differential equation

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

(iii) Find the values of A, ω and ε for the particle.

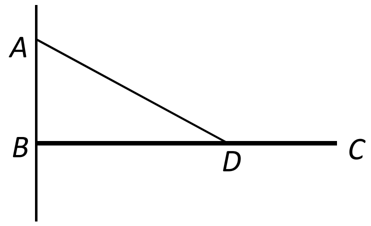
(b) A particle is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point C. The particle moves in a vertical circle.



The greatest and least tensions in the string are $3T$ and T, respectively.

Find the speed of the particle at the lowest point.

7. **(a)** A uniform rod *BC* of length 3 m, has a mass of 20 kg. The end *B*, about which the rod can turn freely, is attached to a vertical wall. The rod is kept in a horizontal position by a rope attached to a point *D* on the rod and to a point *A* of the wall vertically above *B*, as shown in the diagram.



$|AB| = h$ m and $|BD| = 2$ m.

(i) Prove that the tension in the rope is $\frac{147\sqrt{h^2+4}}{h}$.

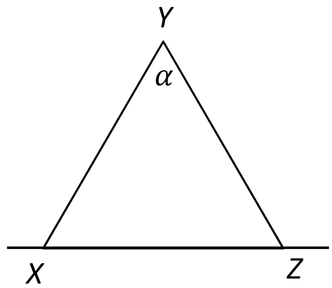
(ii) If the tension in the rope cannot exceed 245 N, show that $h \geq 1.5$.

(b) Two uniform rods *XY* and *YZ* of equal length and of weights $2W$ and W respectively are smoothly hinged at *Y*.

The rods are at rest in a vertical plane with ends *X* and *Z* on a rough horizontal plane.

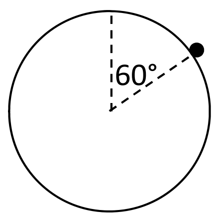
$|\angle XYZ| = \alpha$.

If the coefficient of friction is $\frac{\sqrt{3}}{5}$, find the maximum value of α such that the rods remain at rest.



8. **(a)** Prove that the moment of inertia of a uniform disc, of mass m and radius r about an axis through its centre, perpendicular to its plane, is $\frac{1}{2}mr^2$.

(b) A uniform disc of mass $4m$ and radius 20 cm is free to turn about a horizontal axis through its centre perpendicular to its plane.



A particle of mass *m* is attached to the edge of the disc.

Motion starts from the position in which the radius to the particle makes an angle of 60° with the upward vertical.

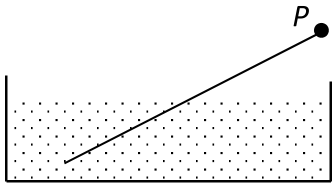
(i) Find the angular velocity of the disc when the particle is at its lowest point.

(ii) Find the angular displacement of the particle when the angular velocity of the disc is 5 rad s^{-1} for the first time.

9. **(a)** When placed in liquid A, a uniform solid cylinder floats upright with $\frac{2}{3}$ of its volume immersed in the liquid.

When placed in liquid B, the uniform solid cylinder floats upright with $\frac{4}{5}$ of its volume immersed in the liquid.

What fraction of the cylinder's volume is immersed when the cylinder floats upright in a uniform mixture of equal volumes of liquid A and liquid B?



(b) A uniform rod, of length ℓ and weight W, is freely hinged at the point P.

The rod is free to move about a horizontal axis through P.
The other end of the rod is immersed in a liquid of density ρ.
The density of the rod is $s\rho$ ($s < 1$).

The rod is in equilibrium and is inclined as shown in the diagram.
The length of the immersed part of the rod is $x\ell$.

(i) Find x in terms of s.

(ii) If the reaction at the hinge is $\frac{1}{6}W$ upwards, find the value of s.

10. **(a)** A particle moves in a horizontal line such that its speed v at time t is given by the differential equation

$$\frac{dv}{dt} = 5 - 8e^{-t}.$$

(i) Given that $v = 2$ when $t = 0$, find an expression for v in terms of t.

(ii) Find the minimum value of v.

(iii) Find the distance travelled by the particle before it attains its minimum speed.

(b) The rate of decay at any instant of a radioactive substance is proportional to the amount of the substance remaining at that instant. The initial amount of the radioactive substance is N and the amount remaining after time t (hours) is x.

(i) Prove that $x = Ne^{-kt}$, where k is a constant.

(ii) If the initial amount N was reduced to $\frac{N}{3}$ in 14 hours, find the value of k.

(iii) If the amount remaining is reduced from $\frac{N}{3}$ to $\frac{N}{4}$ in t hours, find the value of t.

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Leaving Certificate Examination – Higher Level

Applied Mathematics

Friday, 24 June
Afternoon, 2:00 – 4:30