Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2017

Mathematics

Paper 1
Higher Level

Friday 9 June
Afternoon 2:00 – 4:30

300 marks

Examination Number

Centre Stamp

Running Total	

For Examiner					
Q.	Ex.	Adv. Ex.	Q.	Ex.	Adv. Ex.
1			11		
2			12		
3			13		
4			14		
5			15		
6					
7					
8					
9					
10			Total		

Grade

Instructions

There are 15 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1 **(Suggested maximum time: 10 minutes)**

A person's Body Mass Index (BMI) is given by the following formula:

$$\text{BMI} = \frac{w}{h^2}$$

where w is their weight in kg, and h is their height in metres.

(a) **(i)** Geri is an athlete. Her weight is $77{\cdot}5$ kg and her height is $1{\cdot}63$ m.
Work out Geri's **BMI**. Give your answer correct to one decimal place.

(ii) Geri loses some weight during her training.
Her height stays the same. Her new BMI is $24{\cdot}0$.

Work out her new **weight**. Give your answer in kg, correct to one decimal place.

(b) Alex and Jo have the same weight. Alex is 10 cm **taller** than Jo.

Put a tick (✓) in the correct box to show if Alex's BMI is **greater than** Jo's or **less than** Jo's.
Justify your answer fully.

Alex's BMI is:	greater than Jo's	less than Jo's
(tick **one** box only)	❑	❑

Justification:

Question 2 **(Suggested maximum time: 5 minutes)**

A sports shop buys t-shirts for €25 and sells them for €49.

(a) **(i)** Find the **mark up** for the t-shirts (profit as a percentage of cost price).

(ii) Find the **margin** for the t-shirts (profit as a percentage of selling price).
Give your answer correct to the nearest percent.

(b) The shop also sells runners, at a **mark up** of 50%.
Find the **margin** for these runners. Give your answer correct to the nearest percent.

Question 3 **(Suggested maximum time: 5 minutes)**

(a) Write 868 million in the form $a \times 10^n$, where $n \in \mathbb{Z}$ and $1 \leq a < 10, a \in \mathbb{R}$.

(b) During the Apollo-11 mission, it took approximately $1{\cdot}3$ seconds for a radio signal to travel $380\ 000$ km.

Find the **average speed** of the radio signal, in km per minute. Give your answer in the form $a \times 10^n$, where $n \in \mathbb{Z}$, and where $1 \leq a < 10$ is correct to two decimal places.

(c) In 2016, a spacecraft flew around Jupiter, 868 million km from earth.

Find how many minutes it would take a radio signal to travel 868 million km.
Assume that the radio signal would travel at the same speed as your answer to part **(b)**.

Question 4 **(Suggested maximum time: 5 minutes)**

Fruitex and *Juicy* are two drinks.

(a) A shop buys cartons of *Fruitex* from the UK.
In December 2015, the exchange rate was $€1 = £0{\cdot}7241$.
The shop bought *Fruitex* for $£380$.

Find the price of the *Fruitex* in euro (€). Give your answer correct to the nearest cent.

(b) *Fruitex* and *Juicy* are each made from mixing fruit juice and water.
In *Fruitex*, the ratio of fruit juice to water is $3:7$.

(i) Find how many litres of fruit juice are in 20 litres of *Fruitex*.

20 litres of *Fruitex* is mixed with 40 litres of *Juicy*.
In this 60-litre **mixture**, the ratio of fruit juice to water is $7:8$.

(ii) Find the ratio of fruit juice to water in *Juicy*. Give your answer in its simplest form.

Question 5 **(Suggested maximum time: 10 minutes)**

Pete and Maeve are saving to buy an Xbox.

(a) Pete has saved €20 to begin with. He saves a further €12 each week.

(i) Find the **total** amount of money Pete will have saved after 5 weeks.

(ii) Write an expression in n for the **total** amount of money Pete will have saved after $\boldsymbol{n}$ **weeks**.

Pete's total savings after n weeks:

(b) Maeve has saved €15 to begin with. She saves a further €6 each week.
Write an expression in n for the **total** amount of money Maeve will have saved after $\boldsymbol{n}$ **weeks**.

Maeve's total savings after n weeks:

(c) Pete will give **one quarter** of his savings to buy the Xbox.
Maeve will give **two thirds** of her savings to buy the Xbox.
The Xbox costs €200.

After how many weeks will they have enough money saved to buy the Xbox?

Question 6 **(Suggested maximum time: 15 minutes)**

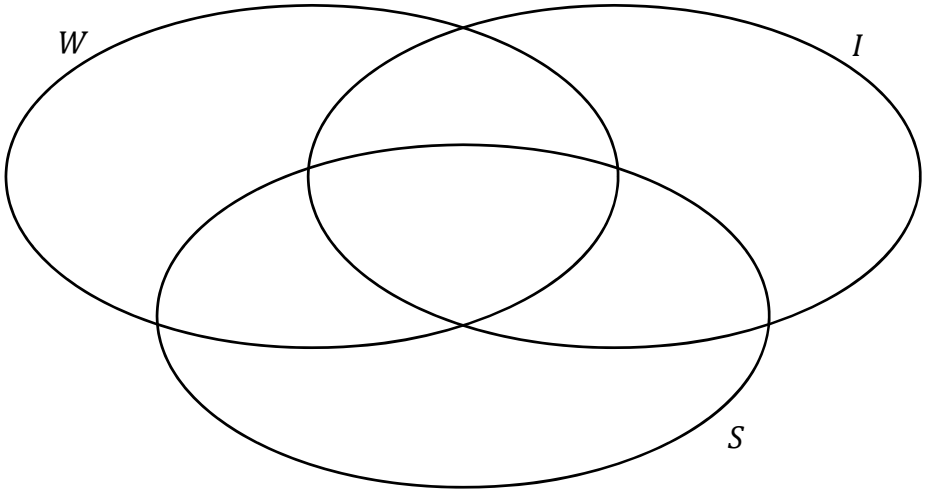
Each of the students in sixth year in a particular school has WhatsApp (W), Instagram (I), or Snapchat (S). The numbers who have each app are as follows:

36 students have WhatsApp
40 students have Instagram
54 students have Snapchat

14 students have WhatsApp and Instagram
24 students have Instagram and Snapchat
x students have WhatsApp and Snapchat, but **not** Instagram

8 students have all three apps.

(a) Use this information to fill in the Venn diagram below, in terms of x.



(b) There are 80 students in total in sixth year in the school.
Find the value of x.

(c) The table below shows four statements. Each statement is written in mathematical notation and in English. **Complete** the table.

	Mathematical notation	**English**
Statement 1	$8 = \#(W \cap I \cap S)$	
Statement 2		24 students have Instagram and Snapchat
Statement 3	$10 = \#(I \backslash (W \cup S))$	
Statement 4		More students have Snapchat than have WhatsApp

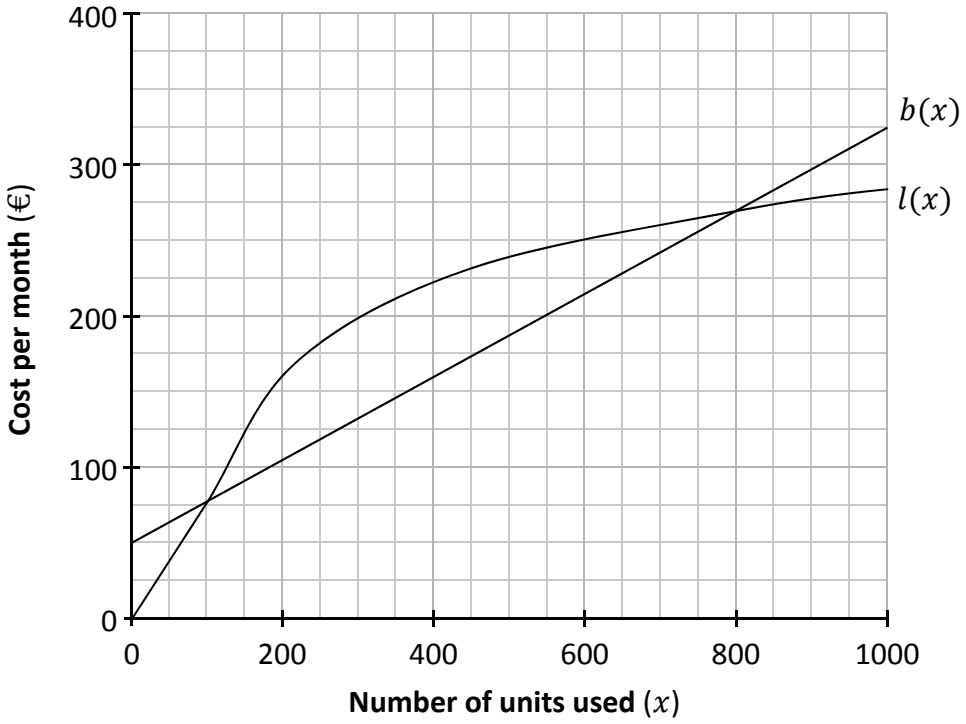
Question 7 **(Suggested maximum time: 5 minutes)**

(a) Give an example of two non-empty sets A and B for which $A \cap B = A$.

(b) Give an example of two non-empty sets P and Q for which $P \backslash Q = P$.

Question 8 **(Suggested maximum time: 15 minutes)**

A school can get its electricity from one of two companies, *Buzz* or *Lecky*. The graphs below show the cost of the electricity per month from each company, if the school uses x units of electricity. The cost from *Buzz* is $b(x)$, and the cost from *Lecky* is $l(x)$.



One of the companies charges a fixed fee each month, plus a fee for each unit of electricity used.

(a) State which company charges **no** fixed fee.
Give a reason for your answer, based on the graph.

Answer:

Reason:

(b) Write down the **domain** and the **range** of the function $b(x)$, as shown on the diagram.

Domain $=$ Range $=$

(c) **(i)** Use the graphs to estimate the set of values of $x \in \mathbb{R}$ for which

$$b(x) < l(x).$$

(ii) **Explain** what your answer to part **(c)(i)** means about the cost of electricity from *Buzz* and *Lecky*.

(d) **(i)** Find the **slope** of the graph of $b(x)$.

(ii) **Explain** what your answer to part **(d)(i)** means about the cost of electricity from *Buzz*.

Question 9 **(Suggested maximum time: 15 minutes)**

(a) Solve the equation $x^2 - 2x - 4 = 0$. Give your answers in the form $a \pm \sqrt{b}$, where $a, b \in \mathbb{N}$.

(b) Given that $\left(\sqrt{d}\right)^2 = d$, multiply out and simplify $\left(c + \sqrt{d}\right)^2$.

(c) The table below shows whether each of the given numbers is an element of the natural numbers ($\mathbb{N}$), the integers ($\mathbb{Z}$), the rational numbers ($\mathbb{Q}$), and the irrational numbers ($\mathbb{R}\backslash\mathbb{Q}$).

Complete the table by writing "**Yes**" or "**No**" into each box.
One row is already done for you: 16 is an element of $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$, but not of $\mathbb{R}\backslash\mathbb{Q}$.

		$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}\backslash\mathbb{Q}$
Number	16	Yes	Yes	Yes	No
	$\sqrt{6}$				
	$\frac{2}{3}$				
	-4				

Question 10 **(Suggested maximum time: 5 minutes)**

Write each of the following in the form 2^n, where $n \in \mathbb{Q}$.

(a) $2^3 \times 2^5 \times 2^{10}$

(b) 8^{25}

(c) $\sqrt{8}$

Question 11 **(Suggested maximum time: 5 minutes)**

(a) In a particular **linear** sequence, the second term is 40 and the sixth term is 116.
Fill in the boxes below to show the rest of the first six terms of this sequence.

$\square$, 40 , $\square$, $\square$, $\square$, 116

(b) Orla is asked to write down a **quadratic** sequence. She writes down the following:

$$5, \quad 6, \quad 9, \quad 14, \quad 22, \quad 30, \quad 41$$

Exactly **one** of the terms in Orla's sequence is incorrect.

Write down the correct quadratic sequence in the spaces below.
You may only change **one** of the terms in Orla's sequence.

Correct Sequence: $\square$, $\square$, $\square$, $\square$, $\square$, $\square$, $\square$

Question 12 **(Suggested maximum time: 20 minutes)**

(a) Factorise $n^2 - 11n + 18$.

(b) Factorise fully $wy - y - 1 + w$.

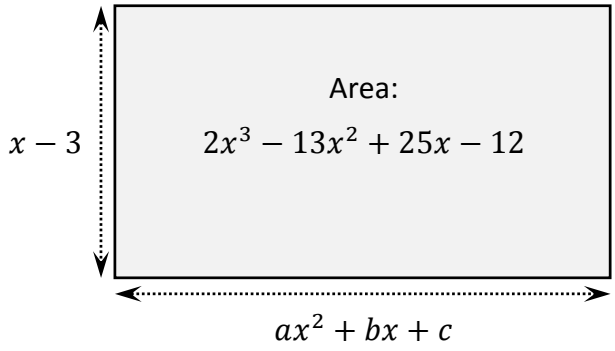
(c) Find the value of $\frac{5}{3x-2} - \frac{7}{6x-12}$, when $x = 4$.

This question continues on the next page.

(d) Use factorisation to simplify $\frac{4e^2-9}{2e^2+3e-9}$.

(e) A rectangle has sides of length $x-3$ units and ax^2+bx+c units, where $a, b, c \in \mathbb{Z}$.
The **area** of the rectangle is $2x^3-13x^2+25x-12$ square units.

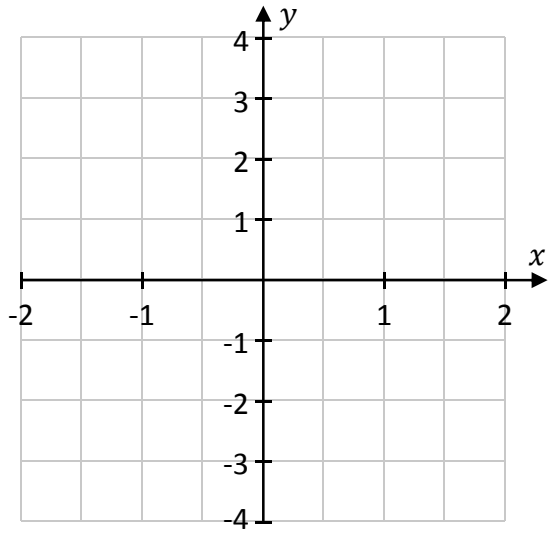
Find the value of a, the value of b, and the value of c.



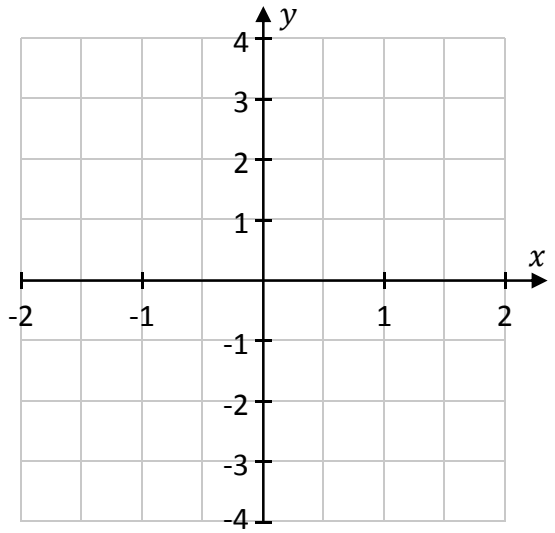
Question 13 **(Suggested maximum time: 10 minutes)**

Draw each of the following three functions in the domain $-2 \leq x \leq 2$, for $x \in \mathbb{R}$.

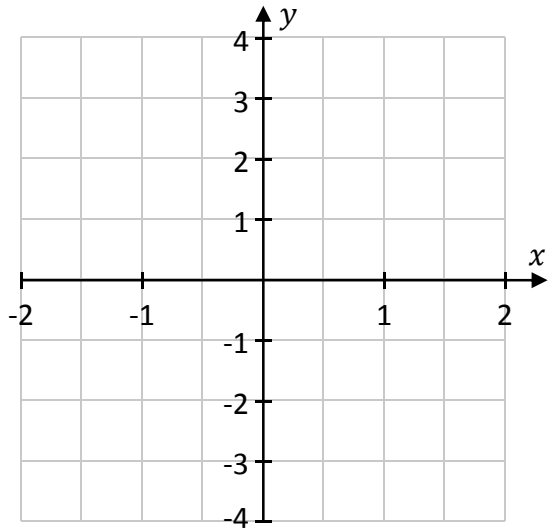
Function: $y = x - 1$



Function: $y = 2 - x^2$



Function: $y = 2^x$



Question 14 **(Suggested maximum time: 10 minutes)**

The table below shows some information about regular polygons.
These are shapes where all of the angles are the same size.

Number of angles in the polygon	**Part (a)** **Sum** of the angles	**Part (c)** **Size of each** angle
3	$180°$	$60°$
4	$360°$	
5		
6		

(a) The **sum** of the angles increases in a **linear** pattern.

Complete the column in the table above showing the sum of the angles in each of these shapes.

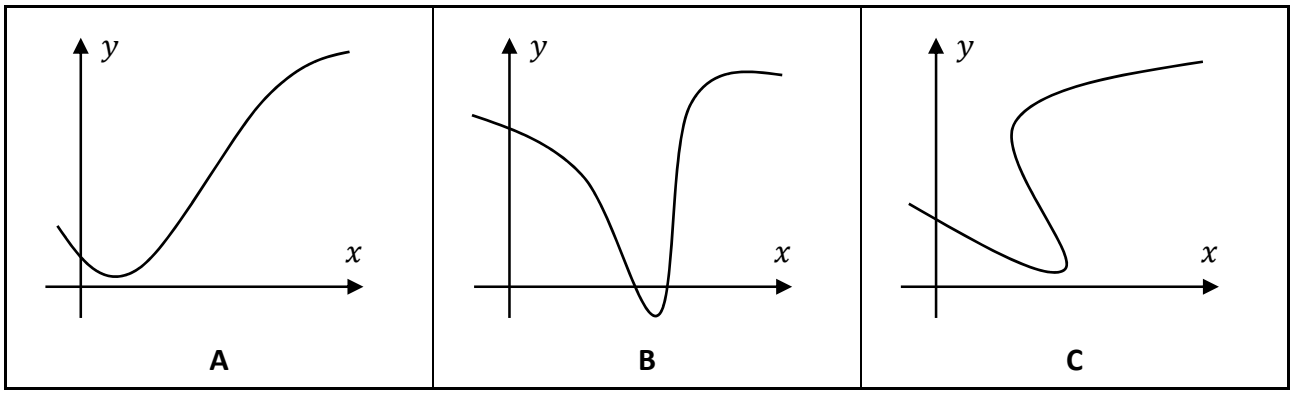
(b) Find a **formula** for the **sum** of the angles in a regular polygon with n angles.
Remember that these values follow a linear pattern.

(c) Complete the column in the table above showing the **size of each** angle in each of these shapes. Remember that, in each polygon, all of the angles are the same size.

(d) Find a **formula** for the **size of each** angle in a regular polygon with n angles.

Question 15 **(Suggested maximum time: 5 minutes)**

The three curves **A**, **B**, and **C** are shown in the co-ordinate diagrams below.
Two of the curves show a function of x.



Put a tick (✓) in the correct box to show which curve does **not** show a function of x.
Give a reason for your answer.

Curve which is **not** a function of x:	**A**	**B**	**C**
(tick **one** box only)	❑	❑	❑

Reason:

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