Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2024

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Leaving Certificate Examination 2024

Mathematics

Higher Level

Paper 1

Marking scheme

300 marks

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10 mark scales		0, 4, 10	0, 4, 6, 10	0, 3, 5, 7, 10
15 mark scale			0, 6, 8, 15	0, 4, 6, 8, 15
20 mark scale				0, 8, 11, 13, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error / Misreading	
〰	Horizontal wavy	Error	
P	P		The work presented in the body of the script merits *Partial Credit*
L	L		The work presented in the body of the script merits *Low Partial Credit*
M	M		The work presented in the body of the script merits *Mid Partial Credit*
H	H		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit − 1*
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
⌇	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and 〰 and 〰 appear in the body of the work, then L should be placed in the right margin.

In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

<table>
<tr><th>Q1</th><th>Model Solution –30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$(n-3)^2 = \left(\sqrt{3n+1}\right)^2$
$n^2-6n+9=3n+1$
$n^2-9n+8=0$
$(n-8)(n-1)=0$
$n=8, n=1$

Answer: $n=8$ [as $n=1$ gives $-2=\sqrt{4}$]</td><td>Scale 10D (0, 3, 5, 7, 10)
Note: Low partial credit at most for a linear equation
Low Partial Credit
• Work of merit, for example, indication of squaring
• Trials values of n
Mid Partial Credit
• Fully correct quadratic (2^{nd} line in solution)
High Partial Credit
• Quadratic factorised
• Fully correct substitution into the quadratic formula
• Verifies $n=8$ is a solution
Full Credit -1
• Apply a * for incorrect solution(s) not eliminated.</td></tr>
<tr><td>(b)</td><td>$$\begin{aligned}\frac{4}{2t+1}-\frac{7}{12t} &= \frac{4(12t)-7(2t+1)}{(2t+1)(12t)}\\ &= \frac{48t-14t-7}{(2t+1)(12t)}\\ &= \frac{34t-7}{(2t+1)(12t)}\end{aligned}$$</td><td>Scale 10C (0, 4, 6, 10)
Low Partial Credit:
• Work of merit, for example, identifies the numerator or the common denominator
High Partial Credit
• $\frac{4(12t)-7(2t+1)}{(2t+1)(12t)}$
• $\frac{4(12t)}{(2t+1)(12t)}-\frac{7(2t+1)}{(2t+1)(12t)}$
Full Credit -1
• Numerator simplified correctly with an incorrect common denominator, where the correct common denominator appears earlier in the solution</td></tr>
</table>

Q1	Model Solution –30 Marks	Marking Notes
(c)	**1:** $x + 2y \qquad = 143$ **2x(-2):** $\underline{\quad -2y - 6w = 148}$ **4:** $x \quad - 6w = 291$ **3:** $4x + 5w = 4$ **4x(4):** $\underline{4x - 24w = 1164}$ **5:** $29w = -1160$ So $w = -40$ **4:** $x - 6(-40) = 291$ So $x = 51$ **1:** $51 + 2y = 143$ So $y = 46$	**Scale 10D (0, 3, 5, 7, 10)** Consider solution as involving 3 steps**:** **1.** Arrives at 2 equations in the same 2 variables (one can be a given equation) **2.** Finds 1 equation in 1 variable **3.** Finds 3 variables *Low Partial Credit* • Work of merit, for example, any correct transposition *Mid Partial Credit* • One step correct *High Partial Credit* • Two steps correct

Q2	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $z = \dfrac{-12 \pm \sqrt{(12)^2 - 4(1)(261)}}{2(1)}$ $= \dfrac{-12 \pm \sqrt{-900}}{2}$ $= \dfrac{-12 \pm 30i}{2}$ $= -6 + 15i$ and $-6 - 15i$ Accept as $-6 \pm 15i$ **Method 2** Let $z = x + yi$ (where $y \neq 0$), $(x + yi)^2 + 12(x + yi) + 261 = 0$ $x^2 - y^2 + 2xyi + 12x + 12yi + 261 = 0$ **Im**: $2xy + 12y = 0$ $y(2x + 12) = 0 \;\therefore x = -6$ **Re**: $x^2 - y^2 + 12x + 261 = 0$ $36 - y^2 - 72 + 261 = 0$ $y^2 = 225$ $y = \pm 15$ $z = -6 \pm 15i$ **Method 3** Let $z = x + yi$ Sum of the roots = -12 $x + yi + x - yi = -12$ $2x = -12, \therefore x = -6$ Product of the roots = 261 $(x + yi)(x - yi) = 261$ $x^2 + y^2 - 261 = 0$ $36 + y^2 - 261 = 0$ $y^2 = 225$ $y = \pm 15$ $z = -6 \pm 15i$	**Scale 5D (0, 2, 3, 4, 5)** Note: Award Mid Partial Credit at most if the solutions are not complex **Method 1** *Low Partial Credit:* • Some correct substitution into the quadratic formula *Mid Partial Credit:* • Fully correct substitution into the quadratic formula *High Partial Credit:* • $\frac{-12 \pm 30i}{2}$ • Error(s) in finding the coefficient of i from a fully correct substitution in the quadratic formula, otherwise correct **Method 2** *Low Partial Credit:* • Some correct substitution of $x + yi$ in the in the equation *Mid Partial Credit:* • Equates Reals to zero **and** equates Imaginaries to zero • Finds the x value *High Partial Credit:* • Correct quadratic equation in y **Method 3** *Low Partial Credit:* • Work of merit for example, sum of the roots = -12 *Mid Partial Credit:* • $x + yi + x - yi = -12$ **and** $(x + iy)(x - iy) = 261$ • Finds the x value *High Partial Credit:* • Correct quadratic equation in y

Q2	Model Solution – 30 Marks	Marking Notes
(b)	$r = \sqrt{1^2 + (\sqrt{3})^2} = 2$ $\theta = 300° \ (or - 60°)$ **or** $\theta = \frac{5\pi}{3} \ (or - \frac{\pi}{3})$ $(1+\sqrt{3}i)^9 = [2(\cos 300 + i \sin 300)]^9$ $= 2^9(\cos 9(300) + i \sin 9(300))$ $= 512(-1+0i)$ $= -512$	**Scale 10D (0, 3, 5, 7, 10)** Note: Candidates must engage with polar form or de Moivre's theorem in order to be awarded any credit. Consider the solution as involving 4 steps: **1.** Finds r **2.** Finds θ **3.** Applies de Moivre's Theorem **4.** Finishes (answer in rectangular form) *Low Partial Credit* • Work of merit in finding r or θ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit-1* • Answer in the form $512(-1+0i)$

Q2	Model Solution – 30 Marks	Marking Notes
(c)	**(i)** $u = 4\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 2\sqrt{3} + 2i = 3{\cdot}46 + 2i$ [2DP] **PLOTTED and LABELLED** Or can use radius of 4 and angle of $30°$ **(ii)** w: Argument $= \frac{3\pi}{4}$ u: Argument $= \frac{\pi}{6}$ $$\frac{3\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{12}$$	**Scale 15D (0, 4, 6, 8, 15)** Tolerance for plotting real coordinate: $3 < x \leq 3.5$ Tolerance for plotting imaginary co-ord: $1{\cdot}8 \leq y \leq 2{\cdot}2$ Note: If calculator is in an incorrect mode, then the rectangular form of u must be stated to gain any credit for **(i)** *Low Partial Credit* • Work of merit in calculating or plotting u in (i) • Work of merit in finding any relevant angle in (ii) *Mid Partial Credit* • One part correct • Work of merit in both parts *High Partial Credit* • One part correct and work of merit in the other part *Full Credit -1:* • Apply a * if u is plotted but not labelled • Apply a * for answer in degrees, or answer given as 1.8325 … in part (ii) • Apply a * for calculator in incorrect mode

<table>
<tr><th>Q3</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$\frac{1}{6}\sin 6x + C$</td><td>Scale 5B (0, 2, 5)

Partial Credit
• Some correct integration, for example, $\sin x$

Full Credit –1
• Apply a * if the $+C$ term is missing</td></tr>
<tr><td>(b)
(i)</td><td>Co-ordinates of the point of contact
$f(2) = -21$, so point is $(2, -21)$

Slope of the tangent at $x = 2$
$f'(x) = 6x^2 - 18x + 5$
$f'(2) = -7$ ... slope of tangent

Equation of the tangent at $x = 2$
$y - (-21) = -7(x - 2)$ or equivalent</td><td>Scale 5D (0, 2, 3, 4, 5)
Note: Engagement with Step 3 is required to be awarded credit for Step 4
Consider solution as involving 4 steps:
1. Finds $\boldsymbol{y}$-value at $\boldsymbol{x = 2}$
2. Differentiates $\mathbf{2}\boldsymbol{x}^{\mathbf{3}} - \mathbf{9}\boldsymbol{x}^{\mathbf{2}} + \mathbf{5}\boldsymbol{x} - \mathbf{11}$
3. Finds $\boldsymbol{f}'(\mathbf{2})$
4. Finds the equation of the tangent

Low Partial Credit
• Work of merit, for example, some correct differentiation;
Substitutes $x = 2$ in $f(x)$;
Formula for the equation of a line with some relevant substitution

Mid Partial Credit
• 2 steps correct

High Partial Credit
• 3 steps correct</td></tr>
</table>

Q3	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$f'(x) = 6x^2 - 18x + 5$ $f''(x) = 12x - 18$ $f''(x) = 0$ $12x - 18 = 0$ $x = \frac{3}{2}$ **Also accept the following for *Full Credit*:** *x values at local maximum & local minimum* $f'(x) = 0$ $6x^2 - 18x + 5 = 0$ $x_1 = \frac{18 + \sqrt{204}}{12}$ $x_2 = \frac{18 - \sqrt{204}}{12}$ *x co-ordinate of the point of inflection:* $\frac{x_1 + x_2}{2} = \frac{36}{12} \div 2$ $x = \frac{3}{2}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, some correct differentiation of $f(x)$ or $f'(x)$; states $f''(x) = 0$ • Brings down derivative from (i) *High Partial Credit* • Correct $f''(x)$ • Finds x values of local maximum and local minimum
(c)	Slope of $l = \frac{1}{2}$ Lines drawn parallel to l and touching the graph of $p(x)$ at $x \approx 2 \cdot 2$ and $x \approx 6 \cdot 8$	**Scale 15D (0, 4, 6, 8, 15)** *Low Partial Credit* • Work of merit, for example, mentions slope of $l = \frac{1}{2}$ • Draws a line parallel to $l(x)$ • Draws a horizontal line at $y = \frac{1}{2}$ • Relevant work to draw graph of $p'(x)$ • Draws two parallel tangents to $p(x)$ that are **not** parallel to $l(x)$ *Mid Partial Credit* • One tangent drawn correctly *High Partial Credit* • Two tangents drawn correctly • One tangent drawn correctly and corresponding x value estimated correctly • Graphs of $l'(x)$ and $p'(x)$ shown on the diagram

Q4	Model Solution – 30 Marks	Marking Notes
(a)	$f(x+h) = (x+h)^2 - 7(x+h) - 10$ $= x^2 + 2hx + h^2 - 7x - 7h - 10$ $\underline{-f(x) = -x^2 \qquad\qquad + 7x \quad + 10}$ $f(x+h) - f(x) = 2hx + h^2 - 7h$ $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2hx}{h} + \dfrac{h^2}{h} - \dfrac{7h}{h}$ $= 2x + h - 7$ $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = 2x + 0 - 7$ $f'(x) = 2x - 7$	**Scale 10D (0, 3, 5, 7, 10)** Note: No credit if first principles is not used *Low Partial Credit* • Work of merit, for example $f(x+h)$ *Mid Partial Credit* • $f(x+h)$ expanded correctly • $f(x+h) - f(x)$ unsimplified *High Partial Credit* • $f(x+h) - f(x) = 2hx + h^2 - 7h$ • Presents complete RHS without LHS *Full Credit -1* • States $\lim\limits_{h=0}$ or $\lim\limits_{h\to\infty}$ but otherwise correct
(b)	$u = 6x + 1 \qquad v = x^4 + 3$ $\frac{du}{dx} = 6 \qquad \frac{dv}{dx} = 4x^3$ $g'(x) = \frac{(x^4+3)(6)-(6x+1)(4x^3)}{(x^4+3)^2}$ $g'(-2) = \frac{((-2)^4+3)(6)-(6(-2)+1)(4(-2)^3)}{((-2)^4+3)^2}$ $= -\frac{238}{361}$ **OR** $g(x) = (6x+1)(x^4+3)^{-1}$ $u = 6x + 1 \qquad v = (x^4+1)^{-1}$ $\frac{du}{dx} = 6 \qquad \frac{dv}{dx} = -4x^3(x^4+3)^{-2}$ $g'(x) = (6x+1)(-4x^3(x^4+3)^{-2})$ $+ (x^4+3)^{-1}(6)$ $g'(-2) = (6(-2)+1)(-4(-2)^3((-2)^4+3)^{-2})$ $+ ((-2)^4+3)^{-1}(6)$ $= -\frac{238}{361}$	**Scale 10D (0, 3, 5, 7, 10)** Note**:** Award MPC at most if quotient/product rule is not used Consider the solution as involving 4 steps: **1.** Find $\boldsymbol{\frac{du}{dx}}$ **2**. Find $\boldsymbol{\frac{dv}{dx}}$ **3.** Correct substitution in to quotient/product rule **4.** Find $\boldsymbol{g'(-2)}$ *Low Partial Credit* • Work of merit in one step, for example, some correct substitution in to the quotient/product rule *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1* • Answer given as a decimal
(c)	Answer: False Justification: *Any correct justfication, for example, sketch of cubic with local min at (0, 5) and y-values below 5; or states:* $(0, 5)$ is only a local min. There might be points away from $x = 0$ that are below 5.	**Scale 10B (0, 4, 10)** *Partial Credit* • Answer correct • No answer or incorrect answer, but work of merit in justification, for example, any graph with local mimimum indicated

Q5	Model Solution – 30 Marks	Marking Notes
(a)	$(5p-3)-(2p+1)=(6p+7)-(5p-3)$ $3p-4=p+10$ $2p=14$ $p=7$ **OR** $T_n=a+(n-1)d$ $a=2p+1$ and $d=3p-4$ (or $p+10$) $T_n=(2p+1)+(n-1)(3p-4)$ $T_3=2p+1+2(3p-4)$ $=8p-7$ But $T_3=6p+7$ $8p-7=6p+7$ $2p=14$ $p=7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit in establishing equation, for example, indicates T_2-T_1 • $a=2p+1$ • $d=3p-4$ or $d=p+10$ *High Partial Credit* • Correct equation in p established
(b)	**Method 1:** $6r^4=\frac{3}{8}$ $r^4=\frac{3}{48}$ $=\frac{1}{16}$ $r=\pm\frac{1}{2}$ **Method 2:** $\frac{ar^{10}}{ar^6}=\frac{3}{8}\div 6$ $r^4=\frac{3}{48}$ $=\frac{1}{16}$ $r=\pm\frac{1}{2}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit in establishing an equation in r, for example, multiplication by r indicated *High Partial Credit* • $r^4=\frac{3}{48}$ *Full Credit –1:* • Apply a * if only 1 value of r is given
(c)	**(i)** $F_1(x)=2024\,x^{2023}$ $F_2(x)=2024\times 2023\,x^{2022}$ **(ii)** Index $=2024-n$ At $n=2024$, answer is a constant (x^0) So at $n=2025$ answer is 0 Answer: $n=2025$	**Scale 20D (0, 8, 11, 13, 20)** Note: Accept correct answer without work in (ii) *Low Partial Credit* • Work of merit, for example, in (i): F_1 or F_2 correct; in (ii): finds F_3, or indicates that derivative of a constant is 0 *Mid Partial Credit* • (i) correct • Work of merit in both (i) and (ii) *High Partial Credit* • (i) correct and work of merit in (ii) • (ii) correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $h(4) = 0$ $(4)^2 + b(4) - 12 = 0$ $4b = -4$ $b = -1$ **Method 2** $$\begin{array}{r\|l} & x + (b+4) \\ \hline x - 4 & x^2 + bx - 12 \\ & x^2 - 4x \\ \hline & (b+4)x - 12 \\ & (b+4)x - 4(b+4) \\ \hline & -12 + 4(b+4) \end{array}$$ $-12 + 4(b+4) = 0$ $4b = -4$ $b = -1$ **Method 3** $x = 4$ is a root of $h(x)$ Let $x = \alpha$ be the other root $4\alpha = -12$ $\alpha = -3$ $\alpha + 4 = -b$ $-3 + 4 = -b$ $b = -1$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, $x = 4$ • Long division set up • $(-4)^2 + b(-4) - 12 = 0$ *High Partial Credit* • Substitutes $x = 4$ in $h(x) = 0$ • Finds $x + 3$ the other factor of $h(x)$ • $h(4)$ simplified to $4b + 4$ • Finds correct remainder in long division • Finds $x = -3$, the other root of $h(x)$ • $h(-4) = 0$ and finishes correctly
(b)(i)	$f(1{\cdot}2) = e^{9(1\cdot2)}$ $= e^{10\cdot8}$ $= 49020 \cdot 8 \ldots$ $= 4{\cdot}9 \times 10^4 [1\ D.P.]$	**Scale 5B (0, 2, 5,)** Note: If answer is not in the form, $a \times 10^n$, award *Partial Credit* at most *Partial Credit* • Correct substitution *Full Credit -1:* • Apply a * if a is not correct to one decimal place

Q6	Model Solution – 30 Marks	Marking Notes
(b) (ii)	**Method 1** $\ln\sqrt{x} = 3{\cdot}5$ ... *LPC* $\sqrt{x} = e^{3\cdot 5}$ ... *HPC* $x = (e^{3\cdot 5})^2 = e^7$ **Method 2** $\ln\sqrt{x} = 3{\cdot}5$ ... *LPC* $\frac{1}{2}\ln x = 3{\cdot}5$ ... *HPC* $\ln x = 7$ $x = e^7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Sets up log equation *High Partial Credit* • Method 1: correct equation in x without logs [line 2] • Method 2: correct equation in $\ln x$ [line 2]
(b) (iii)	$g(f(x)) = \ln\sqrt{e^{9x}}$ $= \ln(e^{9x})^{\frac{1}{2}}$ $= \ln e^{\frac{9x}{2}}$ $= \frac{9x}{2}$ or $4{\cdot}5x$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Some correct substitution into composite function • Some correct substitution into $f(g(x))$ *High Partial Credit:* • Writes $\ln\sqrt{e^{9x}}$ as $\ln(e^{9x})^{\frac{1}{2}}$ or equivalent • $f(g(x)) = x^{\frac{9}{2}}$

Q7	Model Solution – 50 Marks	Marking Notes
(a)	20% of $40000 = 8000$ 40% of $14000 = 5600$ Gross tax $= 13600$ $\text{Net Pay} = 54000 - (13600 - 1775)$ $= 54000 - 11\,825$ $= [€]42\,175$	**Scale 10D (0, 3, 5, 7, 10)** Consider the solution as involving 3 steps **1.** Finds Gross Tax **2.** Subtracts Gross Tax from Gross Income **3.** Deals with Tax Credit Note: 3 may happen before 2, that is, Tax Credit may be subtracted from Gross Tax, or added to Gross Income (because Tax Credit is less than Gross Tax) *Low Partial Credit* • Work of merit in one step *Mid Partial Credit* • One step correct *High Partial Credit* • Two steps correct

<table>
<tr><th>Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>
(i)
$\frac{1647 \cdot 75}{1 \cdot 00279}$, $\frac{1647 \cdot 75}{(1 \cdot 00279)^2}$, and $\frac{1647 \cdot 75}{(1 \cdot 00279)^3}$

(ii)
Method 1:
$\frac{1647 \cdot 75}{1 \cdot 00279} + \frac{1647 \cdot 75}{(1 \cdot 00279)^2} + \cdots \frac{1647 \cdot 75}{(1 \cdot 00279)^{300}}$
$S_{\boldsymbol{n}} = \frac{\frac{1647 \cdot 75}{1 \cdot 00279}\left[1 - \left(\frac{1}{1 \cdot 00279}\right)^{300}\right]}{1 - \frac{1}{1 \cdot 00279}}$
$= €334562.61 \ldots.$
$= €334563$ [nearest euro]

Method 2:
$1647 \cdot 75 = P\frac{0 \cdot 00279(1 \cdot 00279)^{300}}{(1 \cdot 00279)^{300} - 1}$
$P = \frac{1647 \cdot 75[(1 \cdot 00279)^{300} - 1]}{0 \cdot 00279(1 \cdot 00279)^{300}}$
$= €334\ 562.61$
$= €334563$ [nearest euro]
</td><td>
Scale 15D (0, 4, 6, 8, 15)
Note1: Step 3 is not given if there is more than one error in substitution in Step 2
Note2: If work of merit is awarded in (i), then the same work of merit cannot get credit in (ii). For example, if 0.00279 is awarded WOM in (i), then 0.00279 cannot be awarded WOM in (ii).
Note 3: It is acceptable for the solution to part (ii) to appear in the answer box for part (i)
Note 4: Accept solutions where two monthly repayments of €$\mathbf{1647 \cdot 75}$ are being made, that is all correct answers will be doubled
3 Steps
1. Present values of the 1st three monthly repayments
2. Fully correct substitution into geometric/amortisation formula
3. Finds sum of money borrowed
Low Partial Credit
• Work of merit in either part, for example, in (i) writes 0.279% as a decimal; in (ii) some correct substitution into relevant formula
Mid Partial Credit
• 1 step correct
• Work of merit in both parts
High Partial Credit
• 2 steps correct
Full Credit-1
• Repayments made at the start of each month, otherwise correct
• Rounded incorrectly or no rounding, otherwise correct
• (i) not written in the correct form, otherwise correct
</td></tr>
</table>

Q7	Model Solution – 50 Marks	Marking Notes
(c) **(i)**	$$\frac{dF}{dt} = 5000(0 \cdot 04)e^{0 \cdot 04t}$$ $$= 200e^{0 \cdot 04t}$$ $t = 3 \cdot 5$: $$\frac{dF}{dt} = 5000(0 \cdot 04)e^{0 \cdot 04(3 \cdot 5)}$$ $$= 200e^{0 \cdot 14}$$ $$= 230 \cdot 05$$ $= 230$ [nearest € per year]	**Scale 10C (0, 4, 6, 10)** Note: $F'(t) = 5000e^{0.04t}$ – Award LPC at most *Low Partial Credit* • Work of merit in differentiation, for example, $F'(t) = ae^{0.04t}$ *High Partial Credit* • Differentiation fully correct • $F'(t) = \frac{5000e^{0.04t}}{0.04}$ and finishes correctly *Full Credit -1* • Apply a * for incorrect rounding or no rounding
(c) **(ii)**	$$\frac{1}{5}\int_0^5 (5000e^{0 \cdot 04t})dt$$ $$= \frac{1}{5}(5000)\left[\frac{e^{0 \cdot 04t}}{0 \cdot 04}\right]_0^5$$ $$= 1000\left(\frac{e^{0 \cdot 04(5)}}{0 \cdot 04} - \frac{e^{0 \cdot 04(0)}}{0 \cdot 04}\right)$$ $= €5535 \cdot 06..$ $= €5535$ [nearest euro]	**Scale 10D (0, 3, 5, 7, 10)** Note1: Indication of integration is required to be awarded any credit Note2: If $\frac{1}{5}$ is omitted, treat step 1 as not fully correct, but all other steps can be accepted as correct **1.** $\frac{1}{5}\left[\int_0^5 \boldsymbol{F(t)dt}\right]$ **2.** Integrates correctly **3.** Subs in limits **4.** Evaluates correctly *Low Partial Credit* • Work of merit, for example, integration indicated *Mid Partial* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1* Apply a * for incorrect rounding or no rounding

Q7	Model Solution – 50 Marks	Marking Notes
(c) (iii)	$e^{0\cdot 04} = 1\cdot 04081...$ AER$= 4\cdot 08\%$ [2DP] **OR** $F(0) = 5000e^{0.04(0)}$ $= 5000$ $F(1) = 5000e^{0.04(1)}$ $= 5204.053\ ...$ $F(1) - F(0) = 204.053\ ...$ $\text{AER} = \dfrac{204.053\ ...}{5000} \times 100$ $= 4\cdot 08\%$ [2DP] **OR** $\dfrac{5204\cdot 053\ ...}{5000} = 1\cdot 04081\ ...$ $\text{AER} = 4\cdot 08\%$ [2DP]	**Scale 5B (0, 2, 5)** *Partial Credit* • Finds any one term in $F(t)$ • Identifies $e^{0\cdot 04}$ as the compounding factor *Full Credit -1:* • Answer given as 0.04, with work

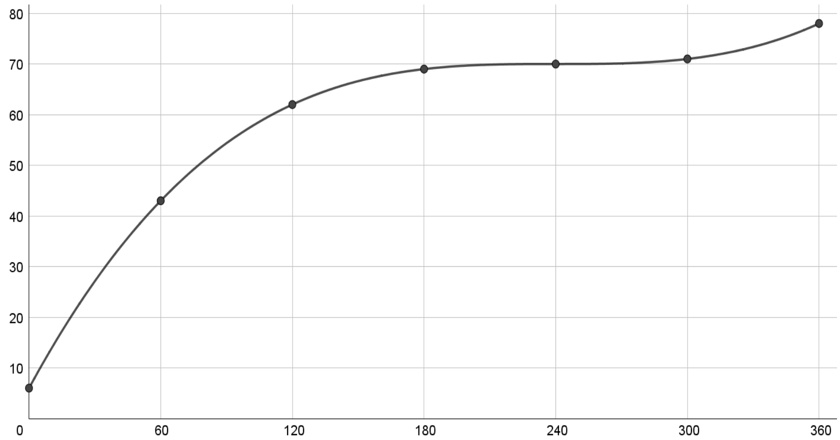
Q8	Model Solution – 50 Marks	Marking Notes

(a)

(i)

x	0	60	120	180	240	300	360
$T(x)$	6	*43*	62	69	70	71	*78*

(ii)



Scale 10D (0, 3, 5, 7, 10)

Solution consists of 13 parts:

- 5 values in table
- 7 points plotted
- Points joined appropriately (not with line segments)

Low Partial Credit
- Work of merit in finding one value of T
- 1 part correct

Mid Partial Credit
- 7 parts correct

High Partial Credit
- 10 parts correct

Full Credit-1
- 12 parts correct
- Correct graph with no table entries

<table>
<tr><th>Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>Max $= 21 + 19(1) = 40$
Min $= 21 - 19(1) = 2$
OR
$S'(t)$
$= -(19)\left(\frac{2\pi}{365}\right)\sin\frac{2\pi t}{365}$
$S'(t) = 0$
$-(19)\left(\frac{2\pi}{365}\right)\sin\frac{2\pi t}{365} = 0$
$\frac{2\pi t}{365} = 0$
$\Rightarrow t = 0$
$\text{Max} = S(0)$
$= 40$
$\frac{2\pi t}{365} = \pi$
$\Rightarrow t = \frac{365}{2}$
$\text{Min} = S\left(\frac{365}{2}\right)$
$= 2$</td><td>Scale 5C (0, 2, 3, 5)
Note: Accept correct answer without supporting work
Low Partial Credit
• Work of merit, for example, mentions max value of $cosA = 1$
• Indicates $+19$ or -19
• Correct indication of 21 or 19 on the graph
• Some correct differentiation of $S(t)$
High Partial Credit
• Max or min correct
• Finds the values of t for which $S'(t) = 0$
• Indicates ± 19
Full Credit -1
• Answers given as 2 and 40, but doesn't indicate which is maximum and which is minimum
• Maximum $=$ 2 and Minimum $= 40$</td></tr>
<tr><td>(c)</td><td>$C(t) = S(t)$
$S(t) = S(t) - 2.4 + 0.03t$
$-2 \cdot 4 + 0 \cdot 03t = 0$
$0 \cdot 03t = 2 \cdot 4$
$t = 80$ [days]</td><td>Scale 10C (0, 4, 6, 10)
Low Partial Credit
• Work of merit in setting up equation
High Partial Credit
• $-2 \cdot 4 + 0 \cdot 03t = 0$</td></tr>
<tr><td>(d)</td><td>Graph L
Any valid justification, for example: $-2 \cdot 4 + 0 \cdot 03t$ is linear with a positive slope and so it will make the trigonometric function $S(t)$ go upwards over time</td><td>Scale 15C (0, 6, 8, 15)
Low Partial Credit
• Correct graph selected
• Work of merit in justification, for example, connects function type with graphs
High Partial Credit
• Justification for the graph increasing over time, but doesn't justify the wave nature of the graph
• Justification for the wave nature of the graph but doesn't justify the increase over time</td></tr>
</table>

Q8	Model Solution – 50 Marks	Marking Notes
(e)	$C'(t) = 0{\cdot}03 - \frac{38\pi}{365}\sin\left(\frac{2\pi t}{365}\right) = 0$ $0{\cdot}03 = \frac{38\pi}{365}\sin\left(\frac{2\pi t}{365}\right)$ $\frac{0\cdot 03(365)}{38\pi} = \sin\left(\frac{2\pi t}{365}\right)$ $\sin^{-1}\left(\frac{0\cdot 03(365)}{38\pi}\right) = \frac{2\pi t}{365}$ $0\cdot 09185 = \frac{2\pi t}{365}$ $t = 5\cdot 336 = 5$ [days] [$\in \mathbb{N}$]	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, sets up equation *High Partial Credit* • $\frac{0{\cdot}03(365)}{38\pi} = \sin\left(\frac{2\pi t}{365}\right)$ *Full Credit -1* • Apply a * for calculator in incorrect mode

Q9	Model Solution – 50 Marks	Marking Notes
(a)	$C = \frac{\pi(4)^2}{3}(3(13) - 4)$ $= \frac{560\pi}{3}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Some correct substitution into the given formula *High Partial Credit* • Fully correct substitution
(b)	**(i)** $C = \frac{\pi(y)^2}{3}(3(8) - y) = 36\pi y$ $\left[\frac{y^2}{3}(24 - y) = 36y\right]$ $\frac{y}{3}(24 - y) = 36$ **(ii)** $\frac{y}{3}(24 - y) = 36$ $24y - y^2 = 108$ $y^2 - 24y + 108 = 0$ $(y - 6)(y - 18) = 0$ $y = 6, \;\; y = 18$ Ans: $y = 6$ [as $y \leq 8$]	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • Work of merit in one part, for example, some relevant substitution in C *Mid Partial Credit* • Part **(i)** correct • Part **(ii)** correct • Work of merit in both parts *High Partial Credit* • Part **(i)** correct and work of merit in part **(ii)** • Part **(ii)** correct and work of merit in part **(i)** *Full Credit -1* • $y = 18$ not eliminated
(c)	3 litres = 3000 cm^3 $\frac{\pi}{12}x^3 = 3000$ $x^3 = \frac{3000(12)}{\pi}$ $x = \sqrt[3]{\frac{36000}{\pi}} = 22 \cdot 5$ [1 DP]	**Scale 15C (0, 6, 8, 15)** *Low Partial Credit* • Work of merit in setting up the equation, states 3 litres = 3000 cm^3 *High Partial Credit* • Finds x^3 • Incorrect volume used but finishes correctly *Full Credit-1* • Apply a * for incorrect rounding or no rounding

Q9	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{dV}{dt} = 450$ $V = \frac{\pi}{12}x^3$ $\frac{dV}{dx} = \frac{\pi}{4}x^2$ $\frac{dx}{dV} = \frac{4}{\pi x^2}$ $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ $\frac{dx}{dt} = \frac{4}{\pi x^2} \times 450 = \frac{1800}{\pi x^2}$ At $x = 20$ $\frac{dx}{dt} = \frac{1800}{\pi(20)^2} = 1 \cdot 4$ [cm/sec]	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • States a relevant derivative • Work of merit in finding a relevant derivative • Some correct differentiation *Mid Partial Credit* • Any **two** of the following: ○ $\frac{dV}{dt} = 450$ ○ $\frac{dV}{dx} = \frac{\pi}{4}x^2$ ○ $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ or equivalent *High Partial Credit* • Finds $\frac{dx}{dt} = \frac{4}{\pi x^2} \times 450$ *Full Credit -1:* • Incorrect rounding or no rounding
(e)	$S = \pi r\sqrt{r^2 + h^2}$ $\frac{S}{\pi r} = \sqrt{r^2 + h^2}$ $\frac{S^2}{\pi^2 r^2} = r^2 + h^2$ $\frac{S^2}{\pi^2 r^2} - r^2 = h^2$ $\frac{S^2 - \pi^2 r^4}{\pi^2 r^2} = h^2$ $\sqrt{\frac{S^2 - \pi^2 r^4}{\pi^2 r^2}} = h$ $\frac{\sqrt{S^2 - \pi^2 r^4}}{\pi r} = h$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Any work of merit, for example, squares both sides or divides both sides by πr *High Partial Credit* • Writes h^2 in terms of S, π and r.

Q10	Model Solution – 50 Marks	Marking Notes
(a)(i)	$W(15) = 0 \cdot 667(15) + 1 \cdot 5(15)^2 - 0 \cdot 025(15)^3$ $= 263.13$ $= 263$ [mm] [$\in \mathbb{N}$]	**Scale 5B (0, 2, 5)** Accept correct answer without work *Partial Credit* • Some correct substitution *Full Credit -1* • Incorrect rounding or no rounding
(a)(ii)	$W'(x) = 0 \cdot 667 + 3x - 0 \cdot 075x^2$	**Scale 5B (0, 2, 5)** *Partial Credit* • Any term correctly differentiated
(b)	$1 \cdot 1 + 2 \cdot 73x - 0 \cdot 078x^2 > 24$ $0.078x^2 - 2.73x + 22 \cdot 9 < 0$ Roots: $x = \frac{2 \cdot 73 \pm \sqrt{(2 \cdot 73)^2 - 4(0.078)(22 \cdot 9)}}{2(0 \cdot 078)}$ $= 21 \cdot 06$ or $13 \cdot 94$ So $14 \leq x \leq 21$	**Scale 10D (0, 3, 5, 7, 10)** Note: For a linear inequality, award low partial credit at most Note: Accept set of natural numbers listed: $\{14, 15, 16, 17, 18, 19, 20, 21\}$ Note: Accept correct solution set for *Full Credit* if 13 and *14* **and** 21 and 22 have been trialled (so end points have been established) *Low Partial Credit* • Inequality correctly formulated • Trials one value of x *Mid Partial Credit* • Quadratic formula fully substituted • Trials 13 and 14 • Trials 21 and 22 *High Partial Credit* • Roots of quadratic found • Trials 13 and 14 **and** 21 and 22 • Solves $P'(x) > 0$ correctly *Full Credit -1:* • Incorrect rounding or no rounding

Q10	Model Solution – 50 Marks	Marking Notes
(c)(i)	$s=\int_0^1 2x-x^2\,dx$ $=x^2-\frac{1}{3}x^3\Big\vert_{x=0}^{x=1}$ $=\left((1)^2-\frac{1}{3}(1)^3\right)-\left((0)^2-\frac{1}{3}(0)^3\right)$ $=\frac{2}{3}$ $c=\int_0^1 x^2\,dx$ $=\frac{1}{3}x^3\Big\vert_{x=0}^{x=1}$ $=\left(\frac{1}{3}(1)^3\right)-\left(\frac{1}{3}(0)^3\right)$ $=\frac{1}{3}$ $s-c=\frac{2}{3}-\frac{1}{3}$ $=\frac{1}{3}$ $\text{Area}=2(s-c)$ $=2\left(\frac{1}{3}\right)$ $=\frac{2}{3}\,[\text{square units}]$ **OR** $2\int_0^1 (s-c)\,dx=2\int_0^1 (2x-x^2-x^2)\,dx$ $=2\int_0^1 (2x-2x^2)\,dx$ $=2\left(x^2-\frac{2x^3}{3}\right)\Big\vert_{x=0}^{x=1}$ $=2\left(1^2-\frac{2(1^3)}{3}-(0)\right)$ $=\frac{2}{3}\,[\text{square units}]$	**Scale 5D (0, 2, 3, 4, 5)** **3 steps**: 1. Integrate $\boldsymbol{s}$ and $\boldsymbol{c}$ 2. Combine 3. Evaluate with limits They may do these in a different order, e.g., combine $\boldsymbol{s}$ and $\boldsymbol{c}$ and then integrate this function *Low Partial Credit* • Integration indicated • Some correct integration *Mid Partial Credit* • 1 Step correct • 1 relevant area calculated *High Partial Credit* • 2 relevant areas calculated correctly • Step 1 correct and one relevant area calculated • 2 steps correct *Full Credit-1* • Finds the correct area in the 1st quadrant, but does not double the answer

Q10	Model Solution – 50 Marks	Marking Notes
(c)(ii)	$k(x) = s(-x)$ ... s of the image of x under axial symmetry in the y-axis So $k(x) = 2(-x) - (-x)^2 = -2x - x^2$ $b = -2$ and $c = 0$ **OR** $(0, 0) \in k$ $k(0) = 0$ $c = 0$ $(-1, 1) \in k$ $k(-1) = 1$ $b = -2$	**Scale 15C (0, 6, 8, 15)** *Low Partial Credit* • Some work of merit, for example references $s(-x)$ *High Partial Credit* • Either b or c correct *Full Credit -1* • Apply a * if $k(x)$ is correct but b and c are not explicitly identified.
(d)	Answer: Option 1 Option 1: $0{\cdot}9p - r$ Option 2: $0{\cdot}9(p - r)$ **OR** $0{\cdot}9p - 0{\cdot}9r$	**Scale 10D (0, 3, 5, 7, 10)** Note: Checking for particular values of p and r is given at most *Low Partial Credit* *Low Partial Credit* • Answer correct • Work of merit in writing one option in terms of p and r • Work of merit in evaluating answer for one set of values of p and r *Mid Partial Credit* • One option correctly written in terms of p and r *High Partial Credit* • Both options in terms of p and r, but incorrect or no option picked

Leaving Certificate Examination 2024

Mathematics

Higher Level

Paper 2

Marking scheme

300 marks

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales	0,5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10 mark scales			0, 4, 6, 10	0, 3, 5, 7, 10
15 mark scale			0, 6, 8, 15	0, 4, 6, 8, 15
20 mark scale				0, 8, 11, 13, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error / Misreading	
	Horizontal wavy	Error	
P	P		The work presented in the body of the script merits *Partial Credit*
L	L		The work presented in the body of the script merits *Low Partial Credit*
M	M		The work presented in the body of the script merits *Mid Partial Credit*
H	H		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit – 1*
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and and appear in the body of the work, then L should be placed in the right margin.

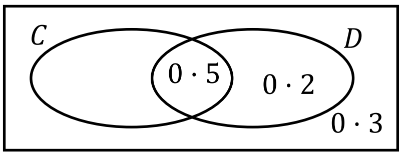
In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	**(i)** Mode $= 34$ so $\boldsymbol{a = 4}$ **(ii)** Range $= 49 \ldots 69 - 20 = 49$ So $\boldsymbol{b = 0}$ and $\mathbf{c} = \mathbf{9}$ **(iii)** Median $= \frac{"4d"+45}{2} = 43{\cdot}5$ $"4d" + 45 = 87$ $“4d” = 42$ So $\boldsymbol{d = 2}$	**Scale 15D (0, 4, 6, 8, 15)** Note: solution requires 4 values: $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \textbf{and } \boldsymbol{d}$ Full credit for correct answers without work. *Low Partial Credit:* • Work of merit in finding one value, for example, indicates median is between $"4d"$ and 45 *Mid Partial Credit* • Two values correct • One value correct and work of merit in finding two other values *High Partial Credit:* • Three values correct • Two values correct and work of merit in finding the other two values *Full Credit –1* • Apply a * if both digits are given instead of just second digit in parts (ii) and (iii), once only. Do not penalise thereafter
(b) (c)	**(b)** *A valid description, for example*: Everyone improved but the stronger swimmers improved to a greater extent than the weaker swimmers. **(c)** $r = 0{\cdot}91451 \ldots = 0{\cdot}9145$ [4 D.P.]	**Scale 15C (0, 6, 8, 15)** In **(b)**, needs to mention both increase for these students and non-linearity of increase (for example, “the better swimmers improved more”) to be considered correct. *Low Partial Credit:* • Work of merit in (b), for example, mentions that swimmers improved, or reference to non-linearity *High Partial Credit* • (b) or (c) correct *Full Credit –1* • Apply a * for incorrect rounding in (c) (incorrect to 4 DP, or correct to a different number of DP)

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$(0)(0{\cdot}3) + (2)(0{\cdot}4) + (x - 10)(0{\cdot}28) + (x)(0{\cdot}02) = 10$ $0{\cdot}8 + 0{\cdot}28x - 2{\cdot}8 + 0{\cdot}02x = 10$ $0{\cdot}3x = 12$ $x = 40$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, some correct term in $E(x)$ *High Partial Credit:* • Fully correct equation
(b)	$P(A \cap B) = 0$ $P(A \cup B) = 0{\cdot}5$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, correct relevant formula, such as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; draws Venn diagram with some relevant use of $0{\cdot}1$ and/or $0{\cdot}4$
(c)	To maximise $(C \cup D)'$ need to max $(C \cap D)$  Max $P[(C \cup D)'] = 0 \cdot 3$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, Venn diagram with some relevant use of $0{\cdot}5$ and/or $0{\cdot}7$ • States that $C \cap D$ needs to be maximised or that $C \cup D$ needs to be minimised • Indicates that one set is a subset of the other *High Partial Credit:* • $C \cap D = 0.5$
(d)	If it is raining I am more likely to wear a coat than if it isn't	**Scale 5A (0, 5)**

Q3	Model Solution – 30 Marks	Marking Notes
(a)	Area of $ABC = \frac{1}{2}(10)(13)\sin 110 = 61{\cdot}08..$ Area of $ABCD = 2(61{\cdot}08 \ldots)$ $= 122{\cdot}16 \ldots$ $= 122$ [cm^2] [nearest cm^2] **OR** Area of $ABCD = (10)(13)\sin 110$ $= 122{\cdot}16 \ldots$ $= 122$ [cm^2] [nearest cm^2]	**Scale 15C (0, 6, 8, 15)** *Low Partial Credit:* • Work of merit, for example, triangle formula with some correct substitution; formula ah with some correct substitution or further work; formula $ab \sin C$ with some correct substitution; a correctly labelled diagram *High Partial Credit:* • Fully correct substitution into parallelogram formula • Fully correct substitution in area of triangle formula *Full Credit-1:* • No rounding or incorrect rounding • Calculator in incorrect mode
(b)	Reference angle in 1st and 4th quadrants $\cos^{-1}\frac{\sqrt{3}}{2} = 30°$ or $330°$ $2X = 30°, 330°, 390°, 690°$ $X = 15°, 165°, 195°, 345°$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit in finding reference angle *Mid Partial Credit:* • One correct value of X found • Two correct values of $2X$ found *High Partial Credit:* • Two correct values of X found • All 4 correct values of $2X$ found but fails to find X
(c)	$\frac{\sin\theta}{45} = \frac{\sin 25}{15\sqrt{3}}$ $\sin\theta = \frac{45(0.4226)}{15\sqrt{3}} = 0.73196$ $\sin\theta = 0.73196 \ldots\ 47.05°$ angle in 1st and 2nd quadrants $\theta = 47.05°$ or $\theta = 132.95°$ $\theta = 47°$ or $\theta = 133°$ [nearest degree]	**Scale 10D (0, 3, 5, 7, 10)** Accept correct answer without units (degrees) *Low Partial Credit:* • Work of merit, for example, some correct substitution into the Sine rule, or correctly labelled diagram *Mid Partial Credit:* • Sine rule with fully correct substitution *High Partial Credit:* • One correct angle found *Full Credit-1* • Apply a * if not rounded correctly

<table>
<tr><th>Q4</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>Correct construction</td><td>Scale 15D (0, 4, 6, 8, 15)
Low Partial Credit:
• Work of merit in finding the midpoint of any side, for example, measures the length of one side
Mid Partial Credit:
• Midpoints of 2 sides identified (construction/measurements shown)
• One median drawn with no construction/measurements shown
High Partial Credit:
• One median drawn (construction/measurements shown)
• 2 medians with no construction</td></tr>
<tr><td>(b)</td><td>Any valid proof, for example:

Construction:
Draw a line through B parallel to DF.
Where this cuts AD and CF, label X and Y, respectively.

Proof:
$|AB| = |BC|$ ... given
$|\angle XAB| = |\angle BCY|$ ... alternate
$|\angle XBA| = |\angle CBY|$ ... vertically opposite
So $\Delta XBA \equiv \Delta YBC$ ... ASA

Therefore $|BX| = |BY|$ ... congruency

But $DEBX$ is a parallelogram, as the opposite sides are parallel.
So $|BX| = |DE|$ ... opposite sides of a parallelogram

Similarly, $|BY| = |EF|$.

So $|DE| = |EF|$.</td><td>Scale 15D (0, 4, 6, 8, 15)
Low Partial Credit:
• Work of merit, for example, correct construction indicated, identifies parallelograms, reference to congruency
Mid Partial Credit:
• Construction, and 2 correct steps in establishing congruency of ΔXBA and ΔYBC (no justifications)
High Partial Credit:
• Construction, and 3 correct steps (including correct relevant side) in establishing congruency of ΔXBA and ΔYBC, with at least one justification
• Fully correct logical statements, but no justifications</td></tr>
</table>

Q5	Model Solution – 30 Marks	Marking Notes
(a) (i)	$x^2 + y^2 + 4x - 6y + 5 = 0$ $g = 2, \quad f = -3$ Centre $= (-2, 3)$ $r = \sqrt{g^2 + f^2 - c} = \sqrt{(2)^2 + (-3)^2 - 5}$ $= 2\sqrt{2}$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit:* • Work of merit in finding centre or radius, for example, identifies g, f or c *Mid Partial Credit* • Centre correct • Radius correct *High Partial Credit:* • One of centre or radius correct and work of merit in finding the other
(a) (ii)	Centre of $c = (2, -1)$ Radius of $c = \sqrt{72} = 6\sqrt{2}$ Distance between centres $= \sqrt{(2-(-2))^2 + (-1-3)^2}$ $= \sqrt{16+16}$ $= 4\sqrt{2}$ Difference between radii $= 6\sqrt{2} - 2\sqrt{2}$ $= 4\sqrt{2}$ $4\sqrt{2} = 4\sqrt{2}$ Therefore, circles touch internally	**Scale 10D (0, 3, 5, 7, 10)** Consider the solution as having 4 steps before the conclusion: **1.** Find centre of circle $\boldsymbol{c}$ **2.** Find radius of circle $\boldsymbol{c}$ **3.** Find distance between the centres of circle $\boldsymbol{c}$ and circle $\boldsymbol{s}$ **4.** Find difference in radii *Low Partial Credit:* • Work of merit in any one of the 4 steps *Mid Partial Credit* • 2 correct steps *High Partial Credit:* • 3 correct steps *Full Credit –1* • Apply a * if there is no conclusion or an incorrect conclusion

Q5	Model Solution – 30 Marks	Marking Notes
(b)	Centre = $(9, k)$ So distance from $(7,10)$ to $(9, k)$ = distance from $(12,8)$ to $(9, k)$ $\sqrt{(9-7)^2+(k-10)^2} = \sqrt{(12-9)^2+(8-k)^2}$ $4 + k^2 - 20k + 100 = 9 + 64 - 16k + k^2$ $4k = 31$ $k = \frac{31}{4}$ So, centre= $(9, \frac{31}{4})$ Radius = $\sqrt{(9-7)^2+\left(\frac{31}{4}-10\right)^2} = \frac{\sqrt{145}}{4}$ Eqn: $(x-9)^2+\left(y-\frac{31}{4}\right)^2 = \frac{145}{16}$ **OR** Centre = $(-g, -f) = (9, k)$ $\therefore$ $g = -9 \ldots \boldsymbol{Eqn\ A}$ $(7,10)$: $7^2+10^2+2g(7)+2f(10)+c=0 \ldots \boldsymbol{Eqn\ B}$ So: $20f + c = -23$ $(12,8)$: $12^2+8^2+2g(12)+2f(8)+c=0 \ldots \boldsymbol{Eqn\ C}$ So: $16f + c = 8$ From **B** and **C:** $f = \frac{-31}{4}$, $c = 132$ Eqn: $x^2+y^2-18x-\frac{31}{2}y+132=0$ **OR** **Method 3** Consider the chord joining $(7,10)$ and $(12,8)$ Midpoint $=(\frac{19}{2}, 9)$, Slope $= -\frac{2}{5}$ Therefore Eq. of line perpendicular to chord passing through the point $(\frac{19}{2}, 9)$: $y - 9 = \frac{5}{2}(x - \frac{19}{2})$ $(9, k) \in y - 9 = \frac{5}{2}(x - \frac{19}{2})$ $k - 9 = \frac{5}{2}(9 - \frac{19}{2})$ $k = \frac{31}{4}$ So, centre= $(9, \frac{31}{4})$ Radius = $\sqrt{(9-7)^2+\left(\frac{31}{4}-10\right)^2} = \frac{\sqrt{145}}{4}$ Eqn: $(x-9)^2+\left(y-\frac{31}{4}\right)^2 = \frac{145}{16}$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit:* • Work of merit in finding the centre/ radius • Some substitution into general equation of a circle • Work of merit in finding midpoint or slope of chord joining $(7, 10)$ to $(12, 8)$ *Mid Partial Credit* • $\sqrt{(9-7)^2+(k-10)^2}$ and $\sqrt{(12-9)^2+(8-k)^2}$ • Two of $\boldsymbol{Eqns\ A, B, or\ C}$ • Finds equation of the line perpendicular to the chord joining $(7, 10)$ to $(12, 8)$ *High Partial Credit:* • Finds $k = \frac{31}{4}$ (Method 1 or 3) • Two independent equations in f and c (Method 2)

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$(1, 13) \rightarrow +5$ and $-2 \rightarrow (6, 11)$ $(6, 11) \rightarrow +3(5)$ and $3(-2) \rightarrow B(21, 5)$ **OR** $(1, 13) \rightarrow +5$ and $-2 \rightarrow (6, 11)$ $(1, 13) \rightarrow +4(5)$ and $4(-2) \rightarrow B(21, 5)$ **OR** $B = (x_2, y_2),\ m{:}n = 1{:}3$ $(6, 11) = \left(\frac{1(x_2)+3(1)}{1+3}, \frac{1(y_2)+3(13)}{1+3}\right)$ $6 = \frac{x_2+3}{4}$ and $11 = \frac{y_2+39}{4}$ $x_2 = 21$ and $y_2 = 5$ $B = (21, 5)$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit, for example, identifies one correct part of translation; some correct substitution into formula; relevant diagram with labelling *Mid Partial Credit:* • Initial translation correct • Fully correct substitution into formula • Treats C as midpoint, otherwise correct. *High Partial Credit:* • Identifies trebling/quadrupling of initial translation • $6 = \frac{x_2+3}{4}$ and $11 = \frac{y_2+39}{4}$, or equivalent • x_2 correct or y_2 correct
(b)	$\frac{4}{3}x - y - 11 = 0$ Perpendicular distance: $= \frac{\left\|\frac{4}{3}(5) + (-1)(-2) - 11\right\|}{\sqrt{\left(\frac{4}{3}\right)^2 + (-1)^2}}$ $= \frac{\|-7\|}{5}$ $= 1{\cdot}4$ [units]	**Scale 10C (0, 4, 6, 10)** Consider the solution as having 3 steps: **1.** Rewrite the equation in the form $\boldsymbol{ax + by + c = 0}$ **2.** Substitute into formula **3.** Evaluate *Low Partial Credit:* • Work of merit in one step, for example, some correct transposition *High Partial Credit:* • 2 steps correct *Full Credit –1:* • Apply a * if the modulus is omitted

Q6	Model Solution – 30 Marks	Marking Notes
(c)	**(i)** $\binom{16}{2} = 120$ [pairs] Also accept $16 \times 15 = 240$ [pairs], i.e. where a pair is considered as being ordered **(ii)** $\binom{4}{2} \times 4 = 24$ [unordered pairs that give horizontal lines] So probability $= \frac{24}{120}$ **or** $\frac{1}{5}$ **OR** $(4 \times 3) \times 4 = 48$ [ordered pairs that give horizontal lines] So probability $= \frac{48}{240}$ **or** $\frac{1}{5}$	**Scale 15D (0, 4, 6, 8, 15)** Accept solutions where pairs are considered unordered or ordered. Note that the answer in **(c)(ii)** should be the same regardless. If pairs are treated as ordered in one part and unordered in the other, award *HPC* at most. **Note**: Accept for full credit correct answers without work *Low Partial Credit:* • Work of merit, for example, in (i), uses 16 in calculation, or, in (ii), gets some number divided by 120 or 240, as appropriate; or draws horizontal lines joining given points on diagram *Mid Partial Credit* • One part correct • Work of merit in both parts. Note that WOM in the diagram can only count as WOM in one part *High Partial Credit* • One part correct **and** work of merit in the other part

Q7	Model Solution – 50 Marks	Marking Notes
(a) **(i)**	$z = \dfrac{50 - 48{\cdot}2}{10{\cdot}6} = 0{\cdot}1698$ $P(z < 0{\cdot}17) = 0{\cdot}5675\ or\ 56{\cdot}75\%$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula, relevant diagram drawn, indicates μ or σ *High Partial Credit:* • Finds z-score $\left(\frac{50-48{\cdot}2}{10{\cdot}6}\right)$
(a)(ii)	Oldest 10% ∴ 90% less than 'A' years old So, $z = 1{\cdot}28$ (or $1{\cdot}29$) $1{\cdot}28 = \dfrac{A - 48{\cdot}2}{10{\cdot}6}$ $A = 61{\cdot}768$ Using $z = 1{\cdot}29, A = 61{\cdot}874$ $A = 62$ [nearest whole number]	**Scale (0, 4, 6, 10)** Consider solution as requiring 3 steps: **1.** Finds z-value **2.** Writes equation in A **3.** Finds value of A *Low Partial Credit:* • Work of merit, for example, mentions 90%, or finds z-value of 10% *High Partial Credit:* • 2 correct steps *Full Credit –1* • Apply a * for incorrect or no rounding
(b) **(i)**	2 successes: $\binom{6}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 = \frac{768}{3125} = 0{\cdot}24576$	**Scale 5C (0, 2, 3, 5)** **Note:** No penalty applies if answer is rounded to either 3 or 4 decimal places *Low Partial Credit:* • WOM, for example, mentions $\frac{4}{5}$ or $\binom{6}{2}$; or P(success) $= \frac{1}{5}$ *High Partial Credit* • $\binom{6}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4$ • $\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4$ evaluated

Q7	Model Solution – 50 Marks	Marking Notes
(b)(ii)	$\binom{n}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^n = 0\cdot 0047$ $0{\cdot}8^n = 0{\cdot}0047$ $n = \frac{ln\, 0{\cdot}0047}{ln\, 0{\cdot}8}$ $n = 24 \cdot 021..$ $n = 24$	**Scale 5C (0, 2, 3, 5*)*** Accept $n = 24$ if it is indicated that $0 \cdot 8^{24} = 0 \cdot 0047$ … *for Full Credit* *Low Partial Credit:* • Work of merit, for example, mentions $\binom{n}{0}$, $\left(\frac{1}{5}\right)^0$ or $\frac{4}{5}$ *High Partial Credit* • Fully correct equation *Full Credit -1* • Apply a * for answer not rounded correctly
(c)	Booked: $(0{\cdot}45)\left(\frac{1}{3}\right) + (0{\cdot}55)\left(\frac{2}{5}\right) = \frac{37}{100}$ Booked on new system: $(0{\cdot}55)\left(\frac{2}{5}\right) = \frac{11}{50}$ Probability $= \frac{\left(\frac{11}{50}\right)}{\left(\frac{37}{100}\right)} = \frac{22}{37} = 59\%$ $[\in \mathbb{N}]$	**Scale 15D (0, 4, 6, 8, 15)** *Low Partial Credit:* • Work of merit, for example, one correct relevant multiplication • Tree diagram with some correct value(s) *Mid Partial Credit:* • $\frac{37}{100}$ or $\frac{11}{50}$ (or equivalent) *High Partial Credit* • $\frac{37}{100}$ and $\frac{11}{50}$ *Full Credit -1* • Apply a * for answer not in correct form

Q7	Model Solution – 50 Marks	Marking Notes
(d)	**Null Hypothesis**: $p = 0{\cdot}75$ *or similar* (for example, "in 2024, PK Hotels were rated the best hotel chain in Europe by 75% of their customers") **Alternative Hypothesis**: $P \neq 0{\cdot}75$ *or similar* **Calculations**: Conf Int: $0{\cdot}765 \pm \frac{1}{\sqrt{1000}} = 0{\cdot}765 \pm 0{\cdot}0316\ldots$ So $0{\cdot}7334 < p < 0{\cdot}7966$ **Conclusion**: Fail to reject H_0 **or** there is not enough evidence to conclude that this percentage has changed. Accept: the percentage has not changed, *or similar* **Reason**: $0{\cdot}75$ is within the interval Note: accept the margin of error based on $1{\cdot}96\sqrt{\frac{(0{\cdot}75)(0{\cdot}25)}{1000}} = 0{\cdot}0268\ldots$ **or** $1{\cdot}96\sqrt{\frac{(0{\cdot}765)(0{\cdot}235)}{1000}} = 0{\cdot}0262\ldots$	**Scale 10D (0, 3, 5, 7, 10)** Note: 4 steps to check: **1.** Hypotheses **2.** Calculations **3.** Conclusion **4.** Reason Reason may be included with Calculations (or Conclusion), and must be based on relevant calculations – if either a confidence interval or test statistic is not found, a conclusion cannot be drawn If the null or alternative hypothesis is incorrect, then Step 1 is not considered correct. In this case, mark Steps 2 and 4 as if the null and alternative hypothesis had been given. However, Step 3 (conclusion) cannot be considered correct, so award *Mid Partial Credit* at most. *Low Partial Credit:* • Work of merit in one step *Mid Partial Credit:* • Two steps correct *High Partial Credit* • Three steps correct

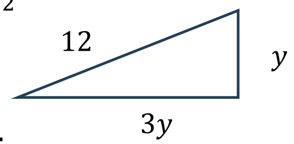
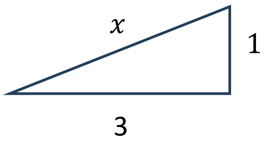
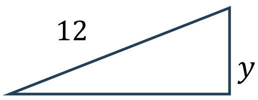
Q8	Model Solution – 50 Marks	Marking Notes
(a)	One dimension: 15 [cm] Other dimension: $2\pi r = 2\pi(5) = 31{\cdot}41 \ldots = 31{\cdot}4$ [cm] [1 DP] Dimensions can be presented in either order.	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit in finding length, for example, circumference formula • Width correct • $2\pi rh = 150\pi$ *High Partial Credit* • Width correct and work of merit in finding length *Full Credit-1* • Apply a * if dimensions are not rounded correctly to one decimal place where appropriate
(b)	$r^2 = 6^2 + 11^2$ $r = \sqrt{36 + 121} = \sqrt{157}$ Vol $= \frac{4}{3}\pi\left(\sqrt{157}\right)^3$ $= 8240{\cdot}20 \ldots$ $= 8240{\cdot}2$ [cm^3] [1 DP]	**Scale 5D (0, 2, 3, 4, 5)** **4 steps:** **1.** Sets up Pythagoras **2.** Finds $\boldsymbol{r}$ **3.** Subs into volume formula **4.** Evaluates volume *Low Partial Credit:* • Work of merit, for example, radius of cylinder$=$ 6cm *Mid Partial Credit* • 2 steps correct • 1 step correct and work of merit in 2 other steps *High Partial Credit* • 3 steps correct

Q8	Model Solution – 50 Marks	Marking Notes
(c) **(i)** **(ii)**	**(i)** *Any valid reason given, for example:* Opposite angles in a cyclic quadrilateral sum to 180° **OR** Angle at the circumference standing on the diameter is 90° **(ii)** $\|\angle CEB\| = \|\angle DEB\|$ ... both 90° $\|\angle ECB\| = 90 - \|\angle EDB\|$ $\|\angle DBE\| = 90 - \|\angle EDB\|$ So $\|\angle ECB\| = \|\angle DBE\|$ So $\|\angle CBE\| = \|\angle EDB\|$ ... angles in Δ add to 180° **OR** $\|\angle CEB\| = \|\angle DEB\|$ ... both 90° $\|\angle CAB\| = \|\angle CBE\|$ ... isosceles ΔACB $\|\angle CAB\| = \|\angle CDB\|$ ... both standing on arc CB So $\|\angle CBE\| = \|\angle CDB\|$ So $\|\angle EBD\| = \|\angle ECB\|$ ... angles in Δ add to 180°	**Scale 20D (0, 8, 11, 13, 20)** Much of the work for **(ii)** may be presented on the diagram. *Low Partial Credit:* • Part **(i)** correct • Work of merit in part (ii), for example, identifies 2 equal angles *Mid Partial Credit* • Part **(i)** correct **and** one pair of corresponding angles in triangles BCE and DBE identified in Part **(ii)** • Two pairs of corresponding angles in triangles BCE and DBE identified in Part **(ii)** *High Partial Credit* • **(ii)** correct (with justification for $\|\angle ECB\| = \|\angle DBE\|$ **or** $\|\angle CBE\| = \|\angle EDB\|$) • **(i)** correct **and** two pairs of corresponding angles in triangles BCE and DBE identified in Part **(ii)**
(c)(iii)	$\frac{r}{20-h} = \frac{h}{r}$ $r^2 = h(20-h)$ $r^2 = 20h - h^2$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit in setting up ratio, for example $\|ED\| = 20 - h$ *High Partial Credit* • $\frac{r}{20-h} = \frac{h}{r}$ *Full Credit –1* • $r^2 = h(20-h)$
(c)(iv)	V$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(20h - h^2)h$ $= \frac{1}{3}\pi(20h^2 - h^3)$ $\frac{dV}{dh} = \frac{1}{3}\pi(40h - 3h^2)$ $\frac{1}{3}\pi(40h - 3h^2) = 0$ at local max/min $40h - 3h^2 = 0$ $h(40 - 3h) = 0$	**Scale 10C (0, 4, 6, 10)** Consider solution involving 3 steps: **1:** Express V in terms of π and h **2:** Find $\frac{dV}{dh}$ **3:** Solve for h, when $\frac{dV}{dh} = 0$ *Low Partial Credit:* • Work of merit, for example, some correct differentiation or substitution *High Partial Credit* • 2 steps correct

Q8	Model Solution – 50 Marks	Marking Notes
	$h = 0,\ h = \frac{40}{3}$ Answer: $h = \frac{40}{3}$ [cm] [as $h = 0$ gives $V = 0$]	• One step correct and work of merit in other 2 steps

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$(x-1)^2+(y-17)^2=144$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, some correct substitution into equation of circle; or indication of radius on the diagram
(a)(ii)	$(a-1)^2+(8-17)^2=144$ $(a-1)^2=63$ $a-1=\sqrt{63}$ (as $a>0$) $=1+\sqrt{63}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, some correct substitution into equation *High Partial Credit:* • $a^2-2a-62=0$ • $(a-1)^2=63$ *Full Credit-1* • Apply a * if $1-\sqrt{63}$ is also presented as a solution
(a)(iii)	$\lvert PC\rvert=\sqrt{(10-1)^2+(6-17)^2}=\sqrt{202}$ $\lvert PC\rvert$ −radius $=\sqrt{202}-12=2{\cdot}2126\ldots$ Shortest distance= $2{\cdot}2126\ldots\times 100$ $=221{\cdot}26\ldots$ $=221$ [m] [nearest m]	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, some correct substitution into distance/perpendicular distance formula; finds slope of CP; *High Partial Credit:* • $\lvert PC\rvert$ −radius $=\sqrt{202}-12$ or equivalent *Full Credit* -1 • Apply a * for no rounding or incorrect rounding
(b)	$x=13$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, some indication that equation is of form $x=$ constant, or draws in relevant tangent or normal

Q9	Model Solution – 50 Marks	Marking Notes
(c)	**Step 1**: Slope of $w = \frac{1}{3}$ **Step 2**: $\perp$ slope $= -3$ **Step 3**: $y - 6 = -3(x - 10)$ **Step 4**: $3x + y = 36$ $3x + y = 36 \quad (\times 3)$ $\underline{x - 3y = 9}$ $9x + 3y = 108$ $\underline{x - 3y = 9}$ $10x = 117$ $x = 11{\cdot}7$ **Step 5**: $11{\cdot}7 - 3y = 9$ $y = 0{\cdot}9$ $(11{\cdot}7, 0{\cdot}9)$	**Scale 10D (0, 3, 5, 7, 10)** Consider the solution involving 5 steps: **1.** Find the slope of $\boldsymbol{w}$ **2.** Find the perpendicular slope **3.** Find the equation of the line through $\boldsymbol{P}$ that is perpendicular to $\boldsymbol{w}$ **4.** Find one co-ordinate of the point of intersection of the two lines **5.** Find the other co-ordinate of the point of intersection of the two lines **Note**: $w: x - 3y = 9$, therefore eq. of $\perp$ $line\ is\ 3x + y + c = 0$ (Step 1 & Step 2) *Low Partial Credit:* • Work of merit, for example, $-3y = -x + 9$ *Mid Partial Credit* • 2 steps correct and work of merit in another step *High Partial Credit* • 4 steps correct

Q9	Model Solution – 50 Marks	Marking Notes
(d)	$y^2 + (3y)^2 = (12)^2$ $10y^2 = 144$  $y = \sqrt{14{\cdot}4} = 3{\cdot}8 \ldots$ $x = 9 + 3\sqrt{14{\cdot}4} = 20{\cdot}4 \ldots$ $(20{\cdot}4, 3{\cdot}8)$ [1 DP] **OR** $x^2 = 3^2 + 1^2$ $x^2 = 10$ $x = \sqrt{10}$  $1200\text{m} = 12 \text{ units}$ Factor increase $= \frac{12}{\sqrt{10}} = 3 \cdot 794..$ $x = 3 \times 3 \cdot 794 + 9 = 20.38..$ $y = 3.794..$ $(20{\cdot}4, 3{\cdot}8)$ [1 DP] **OR**  Slope of line $w = \frac{1}{3}$ Angle between line w and the $x - axis =$ $\tan^{-1}\frac{1}{3} = 18.435$ Then, $\sin 18.435 = \frac{y}{12}$ $y = 3.794 \ldots$ $x = 3.794 \times 3 + 9 = 20.384 \ldots.$ $(20{\cdot}4, 3{\cdot}8)$ [1 DP]	**Scale 15D (0, 4, 6, 8, 15)** *Low Partial Credit:* • Work of merit in identifying inputs for Pythagoras; slope of $w = \frac{1}{3}$; $1200\text{m} = 12 \text{ units}$ *Mid Partial Credit:* • $y^2 + (3y)^2 = (12)^2$ or equivalent • Finds angle between line w and the positive direction of the $x - axis$ *High Partial Credit* • Finds x or y value *Full Credit-1* • Apply a * for no rounding or incorrect rounding, once only

Q10	Model Solution – 50 Marks	Marking Notes
(a)(i)	$\lvert OB\rvert = \lvert ON\rvert = 100 + 20 = 120$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, states$\lvert OB\rvert = \lvert ON\rvert$
(a)(ii)	$\cos\angle BOT = \frac{90}{120}$ $\angle BOT = \cos^{-1}\frac{90}{120} = 41{\cdot}40\ldots° = 41{\cdot}4°$ [1 DP]	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, some correct substitution into a trigonometric ratio *High Partial Credit* • $\cos\angle BOT = \frac{90}{120}$ or equivalent
(a) (iii)	$\lvert\angle BOB'\rvert = 180 - 2(41{\cdot}4) = 97{\cdot}2°$ $\sin 41{\cdot}4 = \dfrac{20}{\lvert OA\rvert}$ $\lvert OA\rvert = \dfrac{20}{\sin 41{\cdot}4} = 30{\cdot}24\ldots$ Area $ABB'A'$ =Area BOB' −Area AOA' $= \dfrac{97{\cdot}2}{360}\pi(120)^2 - \dfrac{97{\cdot}2}{360}\pi(30{\cdot}24)^2$ $= 11438{\cdot}84\ldots = 11\,439$ [cm^2] [$\in \mathbb{N}$]	**Scale 10D (0, 3, 5, 7, 10)** Consider solution as involving 4 steps: **1:** Finds $\lvert OA\rvert$ **2:** Finds $\lvert\angle BOB'\rvert$ **3:** Finds area of one sector (either BOB' or AOA') **4:** Finds area of second sector and finishes *Low Partial Credit:* • Work of merit in one step, for example, finds $\lvert\angle AOM\rvert$ *Mid Partial Credit* • One step correct and work of merit in another step *High Partial Credit* • Three steps correct

Q10	Model Solution – 50 Marks	Marking Notes
(b)	**Method 1:** $180^2 = x^2 + x^2 - 2x^2 \cos 105$ $180^2 = x^2(2 - 2\cos 105)$ $x = \dfrac{180}{\sqrt{2 - 2\cos 105}} = 113.44 \ldots$ $= 113{\cdot}4$ [cm] [1 DP] **OR** **Method 2:** $\lvert \angle OE'E \rvert = \dfrac{180 - 105}{2} = 37{\cdot}5°$ $x^2 = x^2 + 180^2 - 2(x)(180)\cos 37{\cdot}5$ $2(x)(180)\cos 37{\cdot}5 = 180^2$ $x = \frac{32400}{360 \cos 37{\cdot}5} = 113.44\ldots$ $= 113{\cdot}4$ [cm] [1 DP]	**Scale 10D (0, 3, 5, 7, 10)** Consider solution as involving 3 steps: **Method 1:** **1.** Substitutes into Cosine Rule **2.** Isolates x^2 **3.** Evaluates x **OR** **Method 2:** **1.** Finds angle 37.5° **2.** Substitutes into Cosine Rule **3.** Evaluates x Note: Award *Mid Partial Credit* at most if the Cosine Rule is not used to find x *Low Partial Credit:* • Work of merit, for example, some correct substitution into Cosine Rule, or correctly indicates x on the diagram *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct
(c)(i)	**(i)** $3^5 = 243$	**Scale 5B (0, 2, 5)** Accept correct answer without work • Work of merit, for example, lists some correct patterns

Q10	Model Solution – 50 Marks	Marking Notes
(c) **(ii)** **(iii)**	**(ii)** $1 \times 3^3 \times 2 = 54$ **(iii)** $3 \times 2 \times 2 \times 2 \times 2 = 48$	**Scale 10D (0, 3, 5, 7, 10)** Accept correct answer(s) without work *Low Partial Credit:* • Work of merit in one part, for example, lists some correct patterns *Mid Partial Credit* • One part correct • Work of merit in both parts *High Partial Credit* • One part correct and work of merit in the other part