Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2008

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

FRIDAY, 6 JUNE – MORNING, 9:30 to 12:00

Attempt **SIX QUESTIONS** (50 marks each).

WARNING: **Marks will be lost if all necessary work is not clearly shown.**

Answers should include the appropriate units of measurement, where relevant.

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1. **(a)** Simplify fully $\frac{x^2+4}{x^2-4}-\frac{x}{x+2}$.

(b) Given that one of the roots is an integer, solve the equation

$$6x^3-29x^2+36x-9=0.$$

(c) Two of the roots of the equation $ax^3+bx^2+cx+d=0$ are p and $-p$.
Show that $bc=ad$.

2. **(a)** Express $x^2+10x+32$ in the form $(x+a)^2+b$.

(b) α and β are the roots of the equation $x^2-7x+1=0$.

(i) Find the value of $\alpha^2+\beta^2$.

(ii) Find the value of $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$.

(c) Show that if a and b are non-zero real numbers, then the value of

$\frac{a}{b}+\frac{b}{a}$ can never lie between –2 and 2.

Hint: consider the case where a and b have the same sign separately from the case where a and b have opposite sign.

3. **(a)** Let A be the matrix $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$.

Find the matrix B, such that $AB = \begin{pmatrix} 4 & 6 \\ 3 & 2 \end{pmatrix}$.

(b) **(i)** Let $z = \dfrac{5}{2+i} - 1$, where $i^2 = -1$.

Express z in the form $a + bi$ and plot it on an Argand diagram.

(ii) Use De Moivre's theorem to evaluate z^6.

(c) Prove, by induction, that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \text{ for } n \in \mathbf{N}.$$

4. **(a)** $2 + \frac{2}{3} + \frac{2}{9} + \ldots\ldots..$ is a geometric series.

Find the sum to infinity of the series.

(b) Given that $u_n = 2\left(-\frac{1}{2}\right)^n - 2$ for all $n \in \mathbf{N}$,

(i) write down u_{n+1} and u_{n+2}

(ii) show that $2u_{n+2} - u_{n+1} - u_n = 0$.

(c) **(i)** Write down an expression in n for the sum $1 + 2 + 3 + \ldots\ldots\ldots + n$

and an expression in n for the sum $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2$.

(ii) Find, in terms of n, the sum $\sum_{r=1}^{n} \left(6r^2 + 2r + 5 + 2^r\right)$.

5. **(a)** Find the range of values of x that satisfy the inequality

$$x^2 - 3x - 10 \leq 0.$$

(b) **(i)** Solve the equation

$$2^{x^2} = 8^{2x+9}.$$

(ii) Solve the equation

$$\log_e(2x+3) + \log_e(x-2) = 2\log_e(x+4).$$

(c) Show that there are no natural numbers n and r for which

$\binom{n}{r-1}, \binom{n}{r}$ and $\binom{n}{r+1}$ are consecutive terms in a geometric sequence.

6. **(a)** Differentiate $\sqrt{x^3}$ with respect to x.

(b) Let $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

Show that $\dfrac{dy}{dx} = \dfrac{4}{\left(e^x + e^{-x}\right)^2}$.

(c) The function $f(x) = 2x^3 + 3x^2 + bx + c$ has a local maximum at $x = -2$.

(i) Find the value of b.

(ii) Find the range of values of c for which $f(x) = 0$ has three distinct real roots.

7. **(a)** Differentiate $2x + \sin 2x$ with respect to x.

(b) The equation of a curve is $5x^2 + 5y^2 + 6xy = 16$.

(i) Find $\frac{dy}{dx}$ in terms of x and y.

(ii) $(1, 1)$ and $(2, -2)$ are two points on the curve.
Show that the tangents at these points are perpendicular to each other.

(c) Let $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

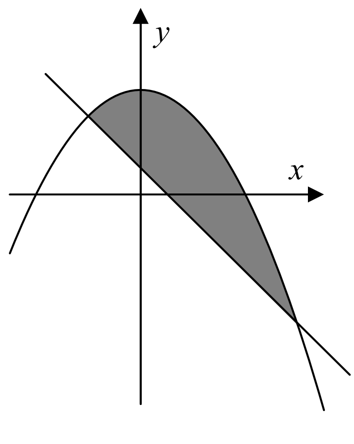
Find $\frac{dy}{dx}$ and express it in the form $\frac{a}{a + x^b}$, where $a, b \in \mathbf{N}$.

8. **(a)** Find $\int (2x + \cos 3x)\,dx$.

(b) Evaluate **(i)** $\int_0^1 3x^2 e^{x^3}\,dx$ **(ii)** $\int_2^4 \frac{2x^3}{x^2 - 1}\,dx$.

(c) The diagram shows the curve $y = 4 - x^2$ and the line $2x + y - 1 = 0$.

Calculate the area of the shaded region enclosed by the curve and the line.



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