Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate 2013

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Coimisiún na Scrúduithe Stáit
State Examination Commission

Scrúdu An Teastais Shóisearaigh

JUNIOR CERTIFICATE EXAMINATION MARKING SCHEME

MATHEMATICS PAPER 1

HIGHER LEVEL

MARKING SCHEME
JUNIOR CERTIFICATE EXAMINATION 2013
MATHEMATICS - HIGHER LEVEL - PAPER 1

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to a candidate's work as follows:
    * Blunders - mathematical errors/omissions   (-3)
    * Slips- numerical errors   (-1)
    * Misreadings   (provided task is not oversimplified)   (-1).

    Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
    * any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
    * if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
    * a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…*etc*.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5·50 may be written as €5,50.

QUESTION 1

Part (a)	15 marks	Att 5
Part (b)	20 marks	Att (2,5)
Part (c)	15 marks	Att (3,2)

Part (a)	15 marks	Att 5

(a)   Adam got 24 marks from a total of 30 marks in a class test.

✍   What percentage mark did Adam get?

(a)	15 marks	Att 5

I $\dfrac{24}{30}$ (or equivalent *e.g.* $\dfrac{12}{15}$)   **5m**

$\dfrac{24}{30} \times 100$ **9m**

$= \dfrac{2400}{30}$ **12m**

$= 80\%$ **15m**

II   6 wrong $= \dfrac{6}{30}$ **5m**

$\dfrac{6}{30} \times 100$ **6m**

$= \dfrac{600}{30}$ **9m**

$= 20\%$ **12m**

$100 - 20$ **12m**

$= 80\%$ **15m**

*   Candidates need not necessarily show all steps for marks

* $\dfrac{24}{30} = \cdot8$ and stops is 9m

Blunders (-3)
B1   Correct answer, no work shown
B2   Decimal error
B3   Incorrect operation
B4   Inversion
B5   Incorrect divisor, but note B4
B6   Incorrect multiplier, but note B4
B7   Fails to calculate final step

Slips (-1)
S1   Numerical errors, to a maximum of 3

Misreadings (-1)
M1   Misreads a number, providing it does not oversimplify the question

Attempts (5 marks)
A1   Mentions 100
A2   Intention to divide
A3   20% only, no work shown
A4 $\dfrac{24}{30}$ $\dfrac{12}{15}$ $\dfrac{8}{10}$ $\dfrac{4}{5}$ or ·8 or equivalent only, no work shown
A5   Some knowledge of percentages
A6   Any relevant work of merit

Worthless (0)
W1   Incorrect answer, no work shown, but note attempts
W2   Irrelevant operation *e.g.* 30 − 24 and stops
W3   Work of no merit

Part (b)   **20 (5,15) marks**   **Att (2,5)**

(i)   Place the following numbers in order, starting with the smallest number:

$$4^{\frac{1}{2}}, \quad 4^{-2}, \quad 2^{0}, \quad 2^{-3}.$$

(ii)   ✍   By rounding correct to the nearest whole number, estimate the value of

$$\frac{7 \cdot 72}{2 \cdot 35} \; + \; (3 \cdot 4)^{2}\left(\frac{8 \cdot 65}{2 \cdot 9} \; - \; \sqrt{1 \cdot 49}\right).$$

Then, evaluate

$$\frac{7 \cdot 72}{2 \cdot 35} \; + \; (3 \cdot 4)^{2}\left(\frac{8 \cdot 65}{2 \cdot 9} \; - \; \sqrt{1 \cdot 49}\right),$$

correct to one decimal place.

(b) (i)   **5 marks**   **Att 2**

$$4^{\frac{1}{2}} \; = \; 2, \quad 4^{-2} \; = \; \frac{1}{16}, \quad 2^{0} \; = \; 1, \quad 2^{-3} \; = \frac{1}{8}. \qquad \textbf{2m}$$

In order, starting with smallest: $4^{-2}, \quad 2^{-3}, \quad 2^{0}, \quad 4^{\frac{1}{2}}$ **5m**

$$= \quad \frac{1}{16}, \quad \frac{1}{8}, \quad 1, \quad 2 \qquad \textbf{5m}$$

$$= \; 0 \cdot 0625, \; 0 \cdot 125, \; 1, \quad 2 \qquad \textbf{5m}$$

---

*   Candidates may decide not to consistently use fractions or decimals.
    Accept correct equivalents in order.
*   No hand symbol, not necessary to show work

Blunders (-3)
B1   Omits a term
B2   Misplaced term
B3   Decimal error
B4   Index error, once if consistent
B5   Mathematical error

Misreading(-1)
M1   Starts with largest number and moves to smallest
M2   Misreads a digit, providing it does not oversimplify the question

Attempts (2 marks)
A1   Finds decimal or fraction or whole number equivalent of one given term
A2   Power of $\frac{1}{2}$ = square root
A3   Some increasing terms
A4   Any relevant step

Worthless (0 marks)
W1   Merely rewrites the original list, with no work of merit
W2   Incorrectly works out term(s) only
W3   No work of merit

(b) (ii)   **15 (5, 10) marks**   **Att (2,3)**

Estimate:

$$\frac{8}{2} + 3^2 \left(\frac{9}{3} - \sqrt{1} \right)$$

$$4 + 9(3 - 1)$$
$$4 + 9(2)$$
$$4 + 18$$
$$= 22$$

Evaluate:

$$\frac{7 \cdot 72}{2 \cdot 35} + (3 \cdot 4)^2 \left(\frac{8 \cdot 65}{2 \cdot 9} - \sqrt{1 \cdot 49} \right) \quad \text{Given}$$

$$3 \cdot 285106383 + 11 \cdot 56(2 \cdot 982758621 - 1 \cdot 220655562)$$
$$3 \cdot 285106383 + 11 \cdot 56 (1 \cdot 762103059)$$
$$3 \cdot 285106383 + 20 \cdot 36991136$$
$$23 \cdot 65501775$$
$$= \quad 23 \cdot 7$$

*   Two parts to mark: **Estimate 5m** and **Evaluate 10m**
*   Consistent errors are penalised once only in **(b)(ii)**

Estimate
Blunders (-3)
B1   Correct answer, no work shown
B2   Precedent error
B3   Mishandles square root
B4   Incorrect squaring
B5   Mathematical error
B6   Calculates first, then rounds

Slips (-1)
S1   Numerical errors to a maximum of 3
S2   Incorrect rounding to a maximum of 3

Misreadings (-1)
M1   Misreads a digit, providing it does not oversimplify the question

Attempts (2 marks)
A1   Some correct rounding
A2   Any correct step without rounding

Worthless (0)
W1   Incorrect answer, no work shown
W2   23·7 or 24 without work
W3   Work of no merit

Evaluate

Blunders (-3)
B1   Correct answer, no work shown
B2   Precedent error
B3   Mishandles square root
B4   Incorrect squaring
B5   Mathematical error
B6   Decimal error

Slips (-1)
S1   Numerical errors to a maximum of 3
S2   Early decimal rounding, once if consistent, otherwise to a maximum of 3
S3   Incorrect or no rounding of final answer

Misreadings (-1)
M1   Misreads a digit, providing it does not oversimplify the question

Attempts (3 marks)
A1   Rounds to whole numbers and continues
A2   Any relevant step *e.g. squaring, multiplying, square root etc.*

Worthless (0)
W1   Incorrect answer, no work shown
W2   Rounds to whole numbers and stops, note A1
W3   Work of no merit

Part (c)   **15 (10,5) marks**   **Att (3,2)**

> Ciara invested €30 000 for three years at 3% per annum compound interest.
>
> **(i)**   ✍   Calculate the amount of the investment at the end of two years.
>
> At the end of two years a sum of money was withdrawn. The money which remained amounted to €12 181·81 at the end of the third year.
>
> **(ii)**   ✍   Calculate the sum of money withdrawn at the end of two years.

(c) (i)   **10 marks**   **Att 3**

I P_1 = €30 000

I_1 = €30 000 $\times \frac{3}{100}$

I_1 = €900

P_2 = €30 000 + €900 = €30 900

I_2 = €30 900 $\times \frac{3}{100}$ (or €900 + €900 $\times \frac{3}{100}$ or €900 $\times \frac{103}{100}$)

I_2 = €927

A = €30 900 + €927

A = €31 827

II P_2 = €30 000 $\times \frac{103}{100}$ = €30 900

A = €30 900 $\times \frac{103}{100}$

A = €31 827

III $A = P(1 + \frac{r}{100})^n$ $F = P(1 + i)^t$

$A = 30\ 000(1 + \frac{3}{100})^2$ $F = 30\ 000(1 + \cdot03)^2$

$A = 30\ 000(1 \cdot 03)^2$ $F = 30000(1\cdot03)^2$

$A = 30\ 000(1 \cdot 0609)$

$A = €31\ 827$

Blunders (-3)
B1   Correct answer, no work shown
B2   Decimal error
B3   Incorrect operation
B4   Inversion
B5   Mathematical error
B6   One error in formula
B7   Precedent error
B8   Incorrect substitution, once only
B9   Index error
B10 Fails to calculate final step

Slips (-1)
S1   Numerical errors to a maximum of 3

Misreadings (-1)
M1   Misreads a digit provided it doesn't oversimplify the question

Attempts (3 marks)
A1   Simple interest (I = €1800 and /or A = €31 800 with work)
A2   Identifies any of the following: P = €30 000, i = ·03, r = 3, t or n = 2
A3   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown (*e.g.* €1800 or €31 800, but note A1)
W2   No work of merit

I	103%	=	€12 181·81
	1%	=	€12181·81 ÷ 103 = €118·27
	100%	=	€118·27 × 100
		=	€11 827
	Withdrawn	→	€31 827 – €11 827
		=	€20 000
II	€12 181·81	=	$P(1 + ·03)^1$
	€12181·81 ÷ 1·03 =		P
	€11 827	=	P
	Withdrawn	→	€31 827 – €11 827
		=	€20 000
III	Amount withdrawn = x		
	€12 181·81	=	$(31\,827 - x) + ·03(31\,827 - x)$
	€12 181·81	=	$31\,827 - x + 954·81 - 0·03\,x$
	€12 181·81	=	$32\,781·81 - 1·03\,x$
	€12 181·81 − € 32 781·81	=	$-1·03x$
	− € 20 600	=	$-1·03x$
	€ 20 600	=	$1·03x$
	€ 20 600 ÷ 1·03	=	x
	€ 20 000	=	Amount withdrawn

*   Use candidate's investment amount from **(c) (i)**

Blunders (-3)
B1   Correct answer, no work shown, note *
B2   Decimal error
B3   Incorrect operation, each time
B4   Inversion
B5   Mathematical error
B6   Incorrect substitution
B7   Fails to complete final step *e.g.* stops at €31 827·00 – €11 827·00 (methods I and II)

Slips (-1)
S1   Numerical slip to a maximum of 3

Misreadings (-1)
M1   Misreads a digit provided it does not oversimplify the question

Attempts (2 marks)
A1   11827 or 118·27 without work
A2   Mentions 103 or 100 + 3
A3   12181·81 divided
A4   Answer **(c)(i) – €**12181·81
A5   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown; note A1
W2   100 mentioned only
W3   No work of merit

QUESTION 2

Part (a)	10 marks	Att 3
Part (b)	25 marks	Att (3,3,2)
Part (c)	15 marks	Att (2,2,2)

Part (a)	10 marks	Att 3

The lengths of two pieces of timber are in a ratio of 5 : 2.
The larger piece measures 250 mm.

✍ Find the length of the shorter piece.

(a)		10 marks	Att 3

I

$$\frac{5}{7} = 250 \qquad \textbf{3m}$$

$250 \div 5 \qquad$ **3m** $\qquad\qquad 250 \times 2$

$\frac{1}{7} = 50 \qquad$ **4m** $\qquad\qquad = 500$

$50 \times 2 \qquad$ **7m** $\qquad\qquad 500 \div 5$

$= 100 \qquad$ **10m** $\qquad\qquad = 100$

→ $\frac{2}{7} = 100$

II

$5 \text{ parts} = 250 \qquad$ **3m**

$1 \text{ part} = 250 \div 5 \qquad$ **3m**

$= 50 \qquad$ **4m**

$2 \text{ parts} = 50 \times 2 \qquad$ **7m**

$\mathbf{=} 100 \qquad$ **10m**

III

$5 : 2 \qquad$ Given

$5 \times 50 : 2 \times 50 \qquad$ **7m**

$250 : 100 \qquad$ **10m**

IV

$$\frac{5}{7} = 250 \qquad \textbf{3m}$$

$250 \div 5 \qquad$ **3m**

$\frac{1}{7} = 50 \qquad$ **4m**

$50 \times 7 = 350 \qquad$ **4m**

$\frac{7}{7} = 350 \qquad$ **7m**

→ $\frac{2}{7} = 100 \qquad$ **10m**

* No need to mention $\frac{5}{7}$ and/or $\frac{1}{7}$ and/or $\frac{2}{7}$ in finding solution

Blunders (-3)
B1　Correct answer, no work shown
B2　$250 \div 7$ and continues (gives an answer of 71·43)
B3　Inversion ($\times \frac{5}{2}$, gives answer 625)
B4　Fails to calculate final step

Slips (-1)
S1　Numerical errors to a maximum of 3

Misreadings (-1)
M1　Misreads a digit providing it does not oversimplify the question

Attempts (3 marks)
A1　$250 \div 5$ or 250×2 and stops
A2　Divides 250 by 5, 2 or 7 only and stops
A3　7 or 350 or 50 with no work
A4　Some relevant step

Worthless (0)
W1　Incorrect answer, no work shown

Part (b)   **25 (10,10,5) marks**   **Att (3,3,2)**

Each week David is paid €14 per hour for the first 35 hours worked

and €21 per hour for any hours worked after that.

(i)   ✍   Last week he worked 38 hours. Calculate his gross pay for last week.

The standard rate of income tax is 20% and the higher rate is 41%.

The standard rate cut-off point is €230 per week and he has a tax credit

of €62 per week.

(ii)   ✍   Calculate David's take-home pay after tax has been deducted.

(iii)   ✍   What percentage of his gross pay is his take-home pay?

Give your answer correct to the nearest whole number.

(b) (i)   **10 marks**   **Att 3**

Gross pay for week:	€14 × 35	and/or	€21 × 3	**3m**
	€490	and/or	€63	**4m**
	€490	+	€63	**7m**
	=	€553		**10m**

Blunders (-3)
B1   Correct answer, no work shown
B2   Decimal error
B3   Incorrect operation
B4   Mathematical error
B5   Fails to calculate final step

Slips (-1)
S1   Numerical error to a maximum of 3

Misreadings (-1)
M1   Misreads a digit providing it does not oversimplify the question

Attempts (3 marks)
A1   €14 × 35 and/or €21 × 3 only
A2   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   No work of merit

(b) (ii)   **10 marks**   **Att 3**

Tax:	€230 @ 20%	=	€46	**3m** ↕
	€323 @ 41%	=	€132·43	**4m** ↕ Interchangeable
Gross tax:		=	€46 + €132·43	
		=	€178·43	**4m**
Tax credit:		=	€62	Given
Tax paid:		=	€178·43 − €62	**4m**
		=	€116·43	**7m**
Take home pay:		=	€553 − €116·43	**7m**
		=	€436·57	**10m**

*   Use candidate's gross pay figure from **(b)(i)**

Blunders (-3)
B1   Correct answer, no work shown, note *
B2   Decimal error
B3   Incorrect operation
B4   Inversion (%)
B5   Mathematical error
B6   Mishandles tax credits
B7   €323 @ 20% and/or €230 @ 41%
B8   Fails to complete final step. Stops at €116·43 or €553 − €116·43

Slips (-1)
S1   Numerical error to a maximum of 3

Misreadings (-1)
M1   Uses 21%
M2   Uses 40%
M3   Misreads a digit, providing it does not oversimplify the question

Attempts (3 marks)
A1   553 − 230 or 323 or indication of subtraction and stops
A2   Gets 20%
A3   Gets 41%
A4   Mentions 100
A5   Shows some knowledge of tax credits *e.g.* writes "Tax payable = total tax minus tax credits"
A6   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   553 + 230 or €783 and stops
W3   No work of merit

(b) (iii)   **5 marks**   **Att 2**

Take home pay as a percentage:	
$\frac{436 \cdot 57}{553} \times 100$	**2m**
$= \cdot 789 \ \times 100$ or $0 \cdot 789457504 \ \times 100$	**2m**
$= \ \ 78 \cdot 9\%$ or $78 \cdot 9457504$	**4m**
$= \ \ 79\%.$	**5m**

*   Accept candidate's figures from **(b)(i)** and **(b)(ii)**

Blunders (-3)
B1   Correct answer no work shown, note *
B2   Decimal error
B3   Does not multiply by 100
B4   Inversion
B5   Mathematical error
B6   Incorrect gross pay or take home pay used, but note M1
B7   Stops at $\cdot 789 \times 100$ or $\cdot 789457504 \times 100$

Slips (-1)
S1   Fails to round or rounds incorrectly
S2   Numerical errors to a maximum of 3

Misreadings (-1)
M1   Misreads a digit providing it does not oversimplify the question

Attempts (2 marks)
A1 $\frac{436 \cdot 57}{553}$ or $\cdot 79$ (or candidate's equivalent) and stops
A2 $\frac{436 \cdot 57}{553} \times 100$ (or candidate's equivalent) and stops.
A3   Mentions 100
A4   Some relevant step

Worthless (0)
W1   Incorrect answer no work shown; note attempts
W2 $7 \cdot 9$ or $\cdot 7$ without work
W3   No work of merit

A survey was carried out in a class to find which of the films A, B or C
the students had seen.
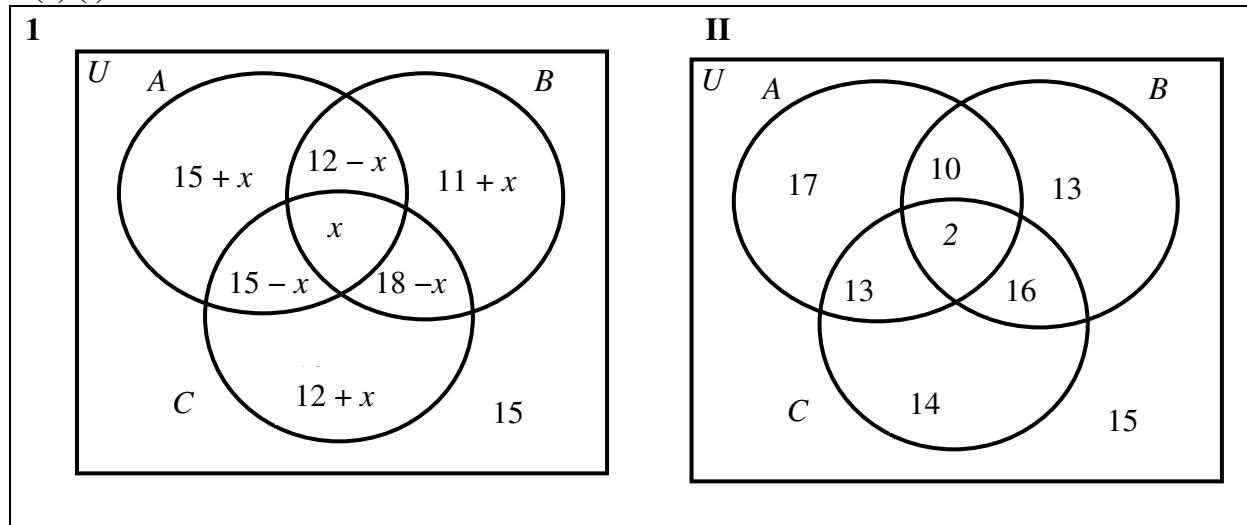The following data was collected:
42% saw film A   41% saw film B   45% saw film C
12% saw both A and B 18% saw both B and C 15% saw both A and C
15% saw none of these films.

(i)   ✍   Represent this information on a Venn diagram.

(ii)   ✍   What percentage of the students in the class saw all three films?

(iii)   ✍   What percentage of the students in the class saw two or more of the films?

(c) (i)   **5 marks**   **Att 2**

1

U A B

$15 + x$ $12 - x$ $11 + x$

x

$15 - x$ $18 - x$

C $12 + x$ 15

II

U A B

17 10 13

2

13 16

C 14 15

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*   The Venn diagram can be considered as work
*   Follow candidate's work closely
*   Circles may be positioned differently

Blunders (-3)
B1   Each incorrect or missing term to a maximum of 3 (8 terms to check)

Slips (-1)
S1   Universal box not drawn

Misreadings (-1)
M1   Misreads a digit, providing it does not oversimplify the question
Attempts (2 marks)
A1   Any correct entry
A2   Arbitrary value of x selected
A3   Three intersecting circles
A4   Mentions x
A5   Any relevant step
A6   Oversimplification

Worthless (0)
W1   Two intersecting circles only with/without Universal box; note A1
W2   Universal box only
W3   No work of merit

(c) (ii)	5 marks		Att 2
I	$15 + x + 12 - x + 11 + x + 15 - x + x + 18 - x + 12 + x + 15 = 100$	**2m**	
	$98 + x = 100$	**2m**	
	$x = 2\%$	**5m**	
II	Reads from Diagram **II**		
	$x = 2\%$	**5m**	

* Accept candidate's values from **(c)(i)**

Blunders (-3)
B1 Correct answer, no work shown, Method 1
B2 # *U* not equal to 100
B3 Missing element from previous work in forming equation
B4 Transposition error
B5 Mathematical error
B6 Adds unlike terms

Slips (-1)
S1 Numerical error, to a maximum of 3

Misreadings (-1)
M1 Misreads a digit providing it does not oversimplify

Attempts (2 marks)
A1 Any correct term in forming equation and stops
A2 Any effort to combine terms from Venn Diagram
A3 Any correct step

Worthless (0)
W1 Incorrect answer, no work shown
W2 No work of merit

(c) (iii)   **5 marks**   **Att 2**

Two or more films:

I

$(12 - x) + (15 - x) + (18 - x) + (x)$ **2m**

$(12 - 2) + (15 - 2) + (18 - 2) + (2)$

$10 + 13 + 16 + 2$

$= \quad 41\%$ **5m**

II

$(12 - x) + (15 - x) + (18 - x) + (x)$ **2m**

$12 - x + 15 - x + 18 - x + x$

$12 - x + 15 - x + 18 - x + x$

$45 - 2x$

$45 - 2(2)$

$45 - 4$

41% **5m**

III

$10 + 13 + 16 + 2$ **2m**

$= 41\%$ **5m**

---

*   Accept candidate's expressions from **(c)(i)**

*   Accept candidate's value of x from (**c**)(**ii**)

Blunders (-3)

B1   Correct answer, no work shown, but note both *

B2   Omits a term

B3   Includes a non relevant term

B4   Uses incorrect x value

B5   Precedent error

Slips (-1)

S1   Numerical errors to a maximum of 3

Misreadings (-1)

M1   Misreads a digit, proving it does not oversimplify the question

Attempts (2 marks)

A1   Imports $12 - x$ and/ or $15 - x$ and/or $18 - x$ and/or x or candidate's equivalent from **(c)(i)**

A2   Imports $x = 2\%$ or candidate's equivalent from **(c)(ii)**

A3   Some correct substitution

A4   Any relevant step

Worthless (0)

W1   Incorrect answer, no work shown

W2   No work of merit

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att (3,2,2)**
Part (c)	**20 marks**	**Att (2,5)**

Part (a)	**10 marks**	**Att 3**

> ✎ Solve for x:
>
> $$3x - [5 - (x - 3)] = 6.$$

(a)		**10 marks**	**Att 3**

I

$$3x - 5 + x - 3 = 6 \quad \textbf{3m}$$
$$4x - 8 = 6 \quad \textbf{4m}$$
$$4x = 6 + 8$$
$$4x = 14 \quad \textbf{7m}$$
$$x = 14 \div 4$$
$$x = \frac{14}{4} \, \textbf{(9m)} \quad \frac{7}{2} \;\text{or}\; 3\frac{1}{2} \;\text{or}\; 3\cdot5 \quad \textbf{10m}$$

II

$$3x - [5 - x + 3] = 6 \quad \textbf{3m}$$
$$3x - [8 - x] = 6 \quad \textbf{3m}$$
$$3x - 8 + x = 6$$
$$4x - 8 = 6 \quad \textbf{4m}$$
$$4x = 6 + 8$$
$$4x = 14 \quad \textbf{7m}$$
$$x = 14 \div 4$$
$$x = \frac{14}{4} \, \textbf{(9m)} \quad \frac{7}{2} \;\text{or}\; 3\frac{1}{2} \;\text{or}\; 3\cdot5 \quad \textbf{10m}$$

* $x = 3\cdot5$ **fully verified** is 10m

Blunders (-3)
B1 Correct answer, no work shown
B2 Decimal error
B3 Incorrect operation
B4 Inversion
B5 Mathematical error
B6 Transposition error
B7 Distribution error
B8 Omits a term
B9 Sign error
B10 Fails to complete final step

Slips (-1)

S1 $x = \dfrac{14}{4}$ and stops
S2 Numerical errors to a maximum of 3

Misreadings (-1)
M1 Misreads a digit providing it does not oversimplify the question

Attempts (3 marks)
A1   Removes brackets
A2   Correct transposition
A3   Effort to isolate *x*
A4   Some relevant step

Worthless (0)
W1   Incorrect answer, no work
W2   No work of merit

Part (b)   **20 (10,5,5) marks**   **Att (3,2,2)**

(i)   ✍   Find the largest possible value of *n* such that

$$5n + 48 > 8n - 6, \ n \in \mathbb{N}.$$

(ii)   *x* represents an even number. Explain why $x + 2$ is the next even number.

(iii)   If one third of the smaller even number is subtracted from half of the larger even number the result is 8.

✍   Find the value of *x*.

(b) (i)   **10 marks**   **Att 3**

$5n - 8n$	$>$	$-6 - 48$	**3m**	$48 + 6 > 8n - 5n$		
$-3n$	$>$	-54	**4m**			
$3n$	$<$	54	**7m**	54	$>$	$3n$
n	$<$	18	**9m**	18	$>$	n
→ n	$=$	17	**10m**	n	$=$	17

Blunders (-3)
B1   Correct answer, no work shown
B2   Sign error
B3   Mishandles inequality
B4   Inversion *e.g.* $\frac{3}{54}$ or 0·05555; S1 may also apply
B5   Mathematical error
B6   Transposition error

Slips (-1)
S1   Does not or cannot deduce correct value of *n*

Misreadings (-1)
M1   Misreads a digit, provided it does not oversimplify the question

Attempts (3 marks)
A1   Treats as equation and gets $n = 18$ and stops
A2   Tests at least 3 values for *n*
A3   Some relevant step

Worthless (0)
W1   Incorrect answer, no work shown

(b) (ii)   **5 marks**   **Att 2**

Two reasons: <u>next</u> and <u>even</u>
- ► two is the smallest even number   **4m**

                                     &

- ► when two even numbers are added, the result is an even number or divisible by 2 *etc*
                                                     5m

---

*   Two reasons = 5m, one reason = 4m
*   **Note** for candidates answering through the medium of Irish:
        There is an error in the translation of "even number" in the Irish version of the paper.
        Therefore any candidate who has difficulty with part (ii) or part (iii) because of this
        error should be awarded full marks.

Attempts (2 marks)
A1   Some understanding of even numbers *e.g.* lists 2, 4, 6

Worthless (0)
W1   No understanding demonstrated
W2   No work of merit

(b) (iii)   **5 marks**   **Att 2**

$$\frac{x+2}{2} - \frac{x}{3} = 8 \qquad \text{2m}$$

$$\frac{3(x+2) - 2(x)}{(2)(3)} = 8$$

$$\frac{3x+6-2x}{6} = 8$$

$$3x + 6 - 2x = 6\,(8)$$
$$x + 6 = 48$$
$$x = 48 - 6 \qquad \text{2m}$$
$$x = 42 \qquad \text{5m}$$

*   Correct trial and error fully verified is 5m

Blunders (-3)
B1   Correct answer, no work shown
B2   Sign error in setting up equation
B3   Expression not equal to 8
B4   Incorrect/mishandles denominator(s)
B5   Incorrect/ mishandles numerator(s)
B6   Transposition error
B7   Distribution error
B8   Mathematical error
B9   Fails to complete final step

Slips (-1)
S1   Numerical errors, to a maximum of 3

Misreadings (-1)
M1   Misreads a digit, providing it does not oversimplify the question

Attempts (2 marks)
A1   Uses $= 8$
A2   Uses $\frac{1}{3}$ and/or $\frac{1}{2}$
A3   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   No work of merit

Part (c)   **20 (5,15) marks**   **Att (2,5)**

(i) ✍ Given that $a^2 = \dfrac{bc^2 + a + c}{b}$,

       show that $b = \dfrac{1}{a-c}$.

(ii) ✍ If $a = 1\dfrac{1}{2}$ and $c = 2\dfrac{1}{3}$, find the value of b.

(c) (i)   **5 marks**   **Att 2**

I $ba^2 = bc^2 + a + c$ **2m**

         $ba^2 - bc^2 = a + c$

         $b(a^2 - c^2) = a + c$

         $b = \dfrac{a+c}{a^2-c^2}$

         $b = \dfrac{a+c}{(a+c)(a-c)}$

         $b = \dfrac{1}{a-c}$ **5m**

Blunders (-3)
B1   Error in cross multiplying
B2   Distribution error
B3   Transposition error
B4   Inversion
B5   Does not or cannot isolate b by factorising
B6   Error in establishing factors of $a^2 - c^2$
B7   Does not or cannot cancel terms
B8   Mathematical error

Misreadings (-1)
M1   Misreads, provided it does not oversimplify the question

Attempts (2 marks)
A1   Cross multiplies and stops
A2   Oversimplifies
A3   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   No work of merit

(c) (ii)   **15 marks**   **Att 5**

I $b = \dfrac{1}{1\frac{1}{2} - 2\frac{1}{3}}$ **9m**

$b = \dfrac{1}{\frac{3}{2} - \frac{7}{3}} = \dfrac{1}{\frac{9}{6} - \frac{14}{6}} = \dfrac{1}{\frac{-5}{6}}$ **12m**

$= -\dfrac{6}{5} \ [\text{or} -1\frac{1}{5}]$ **15m**

II $a^2 = \dfrac{bc^2 + a + c}{b}$ $\left(\frac{3}{2}\right)^2 = \dfrac{b\ (\frac{7}{3})^2 + (\frac{3}{2}) + (\frac{7}{3})}{b}$ **9m**

$\dfrac{9b}{4} = b\left(\dfrac{49}{9}\right) + \dfrac{3}{2} + \dfrac{7}{3}$ $\rightarrow$ $\dfrac{9b}{4} - \dfrac{49b}{9} = \dfrac{9 + 14}{6}$

$\dfrac{81b - 196b}{36} = \dfrac{23}{6}$ $\rightarrow$ $\dfrac{-115b}{36} = \dfrac{23}{6}$

$-115(6)\ b = 23\ (36)$

$b = \dfrac{-138}{115}$ **14m**

$= \dfrac{-6}{5}$ **15m**

Blunders (-3)
B1   Correct answer, no work shown
B2   Sign error
B3   Mathematical error
B4   Mishandles numerator/denominator
B5   Incorrect substitution, note S3
B6   Does not substitute into equations given in **(c)(i)**

Slips (-1)
S1   Numerical error, to a maximum of 3
S2   Uses $c = 2 \cdot 3$
S3   Swaps a and c when substituting
S4 $b = \dfrac{-138}{115}$ or equivalent and stops

Misreadings (-1)
M1   Misreads a digit, providing it does not affect final answer

Attempts (5 marks)
A1   Some correct substitution
A2   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2 $\dfrac{3}{2}, \dfrac{7}{3}$, 1·5, 2·3 or 2·33 and stops
W3   No work of merit

QUESTION 4

Part (a)	10 marks	Att 3
Part (b)	20 marks	Att (2,3,2)
Part (c)	20 marks	Att (2,2,2,2)

Part (a)	10 marks	Att 3

✍   Express in its simplest form:

$$\frac{5-x}{5} + \frac{x-4}{4}.$$

(a)	10 marks	Att 3

$$\frac{4(5-x)+5(x-4)}{20}$$

$$= \frac{20-4x+5x-20}{20} \qquad \textbf{7m}$$

$$= \frac{x}{20} \qquad \textbf{10m}$$

Blunders (-3)
B1   Correct answer, no work shown
B2   Incorrect common denominator
B3   Mishandles common denominator
B4   Mishandles numerator
B5   Distribution error
B6   Mathematical error
B7   Fails to combine like terms in final answer

Slips (-1)
S1   Numerical slips to a maximum of 3

Misreadings (-1)
M1   Misreads a digit, provided it does not oversimplify the question
M2   Reads middle sign as minus and continues

Attempts (3 marks)
A1   Correct common denominator and stops
A2   No denominator used
A3   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   Adds across to get $\frac{1}{9}$ or similar
W3   No work of merit

Part (b)   **20 (5,10,5) marks**   **Att (2,3,2)**

(i)   Factorise $8x^2 - 12x$.

(ii)   Factorise $4x^2 - 12x + 9$.

(iii)   ✍   Simplify $\dfrac{8x^2 - 12x}{4x^2 - 12x + 9}$.

(b) (i)   **5 marks**   **Att 2**

$4x\,(2x - 3)$

*   Accept correct answer for full marks. No work required, no ✍
*   Note minus solution $-4x(-2x + 3)$ for 5 marks:
*   Accept with or without brackets for 5 marks**:**
        $(4x)\ and\ (2x - 3)$

        $(4x)\ or\ (2x - 3)$

        $(4x)\ ,\ (2x - 3)$
*   Quadratic equation formula method is subject to slips and blunders

Blunders (-3)
B1   An incorrect factor
B2   Not fully factorised *e.g.* $2x\,(4x - 6)$
B3   Mathematical error *e.g.* $2x \pm (4x - 6)$

Attempts (2 marks)
A1   Indication of common factor "4" or "2x" *etc*. or underlining *x* terms and stops
A2   Some correct factoring
A3   Any relevant step

Worthless (0)
W1   Work of no merit
W2   (   )(   )

(b) (ii)   **10 marks**   **Att 3**

I

$$4x^2 - 12x + 9$$

$$4x^2 - 6x - 6x + 9$$

$$2x(2x-3) - 3(2x-3)$$

$$(2x-3)(2x-3)$$

$$\text{or } (2x-3)^2$$

II

$$4x^2 - 12x + 9$$

$2x \qquad -3$

$2x \qquad -3$

$$(2x-3)(2x-3)$$

$$\text{or } (2x-3)^2$$

---

*   Accept correct answer for full marks. No work required, no ✍

*   Note minus solution $(-2x+3)(-2x+3)$ for 10 marks

*   Accept with or without brackets for 10 marks

          $(2x-3)$ *and* $(2x-3)$

          $(2x-3)$ *or* $(2x-3)$

          $(2x-3)$, $(2x-3)$

*   Quadratic equation formula is subject to slips and blunders. $x = \frac{3}{2}$ and stops is 3m

Blunders (-3)

B1   Incorrect factors of $4x^2$. B3 may also apply

B2   Incorrect factors of 9. B3 may also apply

B3   Factors leading to an incorrect middle term

B4   Each incorrect common factor and continues (applies to guide number method)

B5   Correct cross method but factors not written

Slips (-1)

S1   Numerical errors to a maximum of 3

Misreadings (-1)

M1   Misreading, provided it does not oversimplify the question

Attempts (3 marks)

A1   Quadratic equation formula with some correct substitution.

A2   Attempts to find factors of 9 or 4 or $4x^2$ or x^2

A3 $\sqrt{9}$ or $\sqrt{4}$

A4   Identifies the guide number (36) and stops

A5   Any relevant step

Worthless (0)

W1 $9 \div 2 = 4{\cdot}5$ and stops

W2   States $4 \div 2 = 2$

W3   No work of merit

(b) (iii)   **5 marks**   **Att 2**

$\dfrac{4x(2x-3)}{(2x-3)(2x-3)}$	**2m**
$= \dfrac{4x}{(2x-3)}$	**5m**

*   Accept candidate's answers from **(b)(i)** and **(b)(ii)**

Blunders (-3)
B1   Correct answer, no work shown, note *
B2   Mathematical error
B3   Does not or cannot cancel
B4   Fails to complete final step

Attempts (2 marks)
A1   Attempt to use answer(s) from **(b)(i)** and/or **(b)(ii)**
A2   Some effort at factorising
A3   Sets up division
A4   Any relevant step

Worthless (0)
W1   No work of merit.

Part (c)   **20 (5,5,5,5) marks**   **Att (2,2,2,2)**

A teacher checks out the cost of calculators for her students on two websites, C and D.

On website C, for €480, she can get a class set of calculators, one for each student.

On website D, for the same price, she can get 4 extra calculators.

If x represents the number of students in her class,

(i)   write an expression in x for the cost per calculator on website C

and an expression in x for the cost per calculator on website D.

The cost per calculator on website D is €6 cheaper than the cost per calculator on website C.

(ii)   ✍   Use this information to form an equation in x and solve it to find

the number of students in the class.

(c) (i)   **10(5,5) marks**   **Att (2,2)**

Cost on website C	$=$	$\dfrac{480}{x}$	**5m**
Cost on website D	$=$	$\dfrac{480}{x+4}$	**5m**

*   Accept correct answers for full marks. No work required, no ✍

*   2 expressions: **Cost on website C** and **Cost on website D** to mark separately: 5m each

<u>Cost on website C</u>

Blunders (-3)

B1   Inversion $\dfrac{x}{480}$

Attempts (2 marks)

A1   Any combination of x and 480
A2   Division indicated
A3   Any relevant step

Worthless (0 marks)

W1   No work of merit

<u>Cost on website D</u>

Blunders (-3)

B1   Inversion $\dfrac{x+4}{480}$ Do not penalise again, if already penalised above

B2   Uses $\dfrac{480}{x} \pm 4$

Attempts (2 marks)

A1   Any combination of x with 4 and/or 480
A2   Division indicated
A3   Any relevant step

Worthless (0 marks)

W1   No work of merit

(c) (ii) **10(5,5) marks** **Att (2,2)**

Equation: $\dfrac{480}{x} - \dfrac{480}{x+4} = 6$ or equivalent equation **5m**

$x(x+4)\ \dfrac{480}{x} - x(x+4)\ \dfrac{480}{x+4} = 6\,x(x+4)$

$(x+4)\,480 - (x)\,480 = 6(x)(x+4)$

$480\,x + 1920 - 480\,x = 6x^2 + 24\,x$

$1920 = 6x^2 + 24\,x$

$6x^2 + 24\,x - 1920 = 0$

$x^2 + 4\,x - 320 = 0$

$(x-16)(x+20) = 0$

$x = 16 \qquad x = -20$ **4m**

$\rightarrow \quad x = 16$ **5m**

* Mark in 2 parts: 5m for establishing an **equation** and 5m for **solving**
* Writing correct equation considered work but for solution $x = 16$ algebraic work required to earn full marks.
* Accept candidate's answers from previous work but note A1

Equation

Blunders (-3)

B1 Sign error
B2 Mathematical error

Attempts (2 marks)

A1 Attempt to form equation in x

Worthless (0)

W1 Work of no merit

Solving

Blunders (-3)

B1 Correct answer, no work shown
B2 Incorrect factors
B3 Sign error
B4 Mathematical error
B5 Error in using quadratic
B6 Transposition error

Slips (-1)

S1 Numerical errors to a maximum of 3
S2 Fails to conclude

Misreadings (-1)

M1 Misreads a digit, provided it does not oversimplify the question

Attempts (2 marks)

A1 Solves or attempts to solve linear equation is 2 marks at most
A2 Correct trial and error, fully verified
A3 Any relevant step

Worthless (0)

W1 Incorrect answer, no work shown
W2 $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with no relevant substitution
W3 No work of merit

QUESTION 5

Part (a)	15 marks	Att 5
Part (b)	20 marks	Att (5,2)
Part (c)	15 marks	Att (2,2,2)

Part (a)	15 marks	Att 5

$g(x) = \sqrt{5x-2}$, $x \in \mathbb{N}$. Find $g(2)$.

Give your answer in the in the form $a\sqrt{a}$, $a \in \mathbb{N}$.

(a)			15 marks	Att 5

$g(2)$	$=$	$\sqrt{5(2)-2}$	**5m**	
	$=$	$\sqrt{10-2}$	**9m**	
	$=$	$\sqrt{8}$	**12m**	
	$=$	$2\sqrt{2}$	**15m**	

Blunders (-3)
B1   Correct answer, no work shown
B2   Solves $\sqrt{5x-2} = 2$ ($x = \frac{6}{5}$, also incurs B7)
B3   Multiplication error
B4   Sign error
B5   Mathematical error
B6   Square root error
B7   Does not present answer in required form

Slips (-1)
S1   Numerical error to a maximum of 3

Misreadings (-1)
M1   Misreads a digit, provided it does not oversimplify the question

Attempts (5 marks)
A1   States $x = 2$ and stops
A2   Substitutes $x = 2$ and stops
A3   Any relevant step

Worthless (0)
W1   Squares or tries to square $\sqrt{5x-2}$
W2   Incorrect answer, no work shown
W3   No work of merit

(i)   ✍   Solve the equation $x^2 = 3x + 2$.
Give your answers correct to two decimal places.

(ii)   ✍   Hence, or otherwise, find two values for p for which $p = 3\sqrt{p} + 2$.
Give your answers correct to one decimal place.

(b) (i)   **15 marks**   **Att 5**

$$x^2 - 3x - 2 = 0 \qquad \textbf{5m}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, \quad b = -3, \quad c = -2$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 + 8}}{2}$$

$$x = \frac{3 \pm \sqrt{17}}{2} \qquad \textbf{12m}$$

$$x = \frac{3 \pm 4{\cdot}123105626}{2}$$

$$x = \frac{7{\cdot}123105626}{2} \qquad\qquad x = \frac{-1{\cdot}123105626}{2}$$

$$x = 3{\cdot}561552813 \qquad\qquad x = -0{\cdot}561552812$$

$$x = 3{\cdot}56 \text{ or } -0{\cdot}56 \qquad \textbf{15m}$$

*   One error in writing formula is a blunder; two errors in writing formula, with some correct substitution, is Attempt 5m

Blunders (-3)
B1   Correct answer, no work shown
B2   Decimal error
B3   Incorrect operation
B4   Transposition error
B5   Mathematical error
B6   Error in squaring
B7   Error in formula
B8   Substitution error
B9   Sign error
B10   Does not find $\sqrt{17}$ or is unable to find square root of number
B11   Incorrect division
B12   One solution only with work, S2 may apply

Slips (-1)
S1   Numerical errors to a maximum of 3
S2   Fails to round or rounds incorrectly, each time

Misreadings (-1)

M1 Misreads a digit provided it does not oversimplify the question

Attempts (5 marks)

A1 Any correct substitution
A2 Identifies $a = 1$ and/or $b = -3$ and/or $c = -2$
A3 Correct transposition
A4 Any relevant step
A5 Oversimplification, solves $x^2 + 3x + 2 = 0$
A6 One solution, *e.g.* 3·56 or similar, without work

Worthless (0)

W1 Incorrect answer, no work shown

W2 $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and stops

W3 No work of merit

(b) (ii)   **5 marks**   **Att 2**

Take $\sqrt{p}$	$=$	x		**2m**
p	$=$	x^2		**2m**
p	$=$	$3{\cdot}56^2$	[or $(-0{\cdot}56)^2$]	**2m**
p	$=$	$12{\cdot}6736$	[or $0{\cdot}3136$]	**4m**
p	$=$	$12{\cdot}7$	[or $0{\cdot}3$]	**5m**

* $12{\cdot}7$ is the only valid solution to $p = 3\sqrt{p} + 2$

   As two solutions were asked for, do not penalise candidates if both solutions are offered.

*   Accept candidate's answers from **(b)(i)**

Blunders (-3)
B1   Correct answer, no work shown, note 2nd *
B2   Squaring error
B3   Mathematical error

Slips (-1)
S1   Answer not given to one decimal place
S2   Incorrect or no rounding

Misreadings (-1)
M1   Misreads a digit, providing it does not oversimplify the question

Attempts (2 marks)
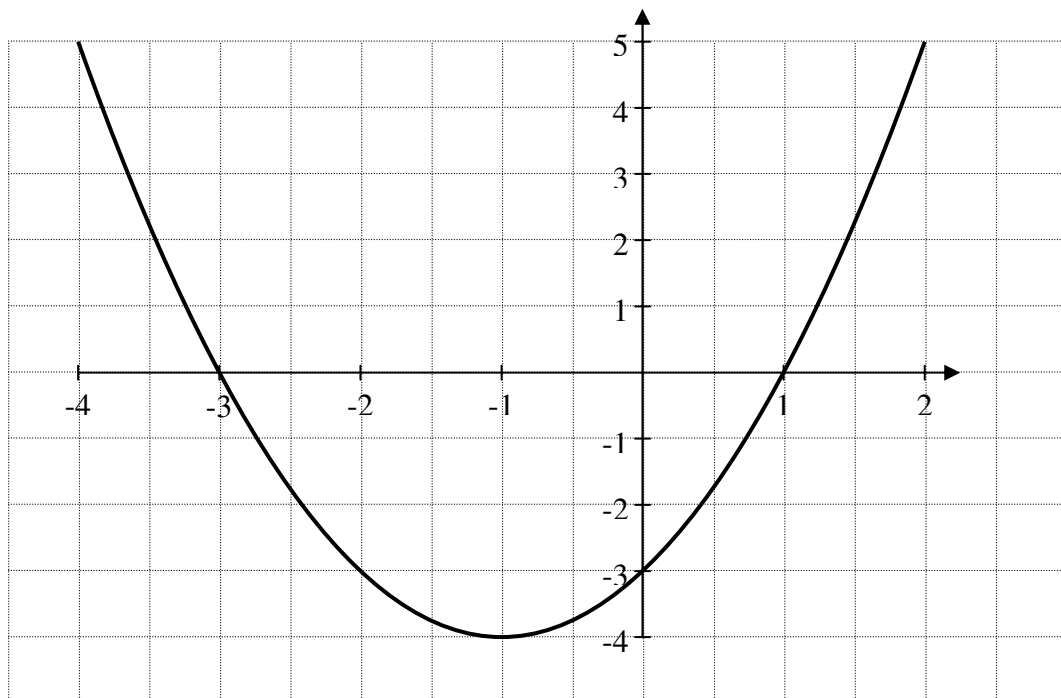A1   Imports answer(s) from **(b)(i)**
A2   Squaring
A3   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   No work of merit

The diagram below shows part of the graph of the function

$$f : x \rightarrow x^2 + bx + c, \text{ where } x \in \mathbb{R} \text{ and } b, c \in \mathbb{Z}.$$

(i) The graph cuts the x axis at $(-3, 0)$ and $(1, 0)$, as shown in the diagram.

     ✎ Calculate the value of b and the value of c.

(ii) The graph has a minimum point at $(-1, -4)$.

Write down the equation of the axis of symmetry of the function

in the form $x = k$, where $k \in \mathbb{Z}$.

(iii) ✎ Show that $f(x + 1) = x^2 + 4x$.

(c)(i)   **5m**   **Att 2**

I

Roots: $x \quad = \quad -3$ and $x = 1$

           $f(x) \quad = \quad (x + 3)(x - 1) = 0$

           $f(x) \quad = \quad x^2 + 2x - 3 = 0$

           $f(x) \quad = \quad x^2 + bx - c = 0$

           $\rightarrow \qquad\qquad b = 2$ and $c = -3.$

II $f : x \rightarrow x^2 + bx + c$

    $(-3, 0) \quad 0 = (-3)^2 + bx + c$

            $(-3)^2 + b(-3) + c = 0$

            $9 - 3b + c = 0$

            $-3b + c = -9$

    $f : x \rightarrow x^2 + bx + c$

    $(1, 0) \qquad 0 = (1)^2 + b(1) + c$

            $(1)^2 + b + c = 0$

            $1 + b + c = 0$

            $b + c = -1$

$$-3\,b + c = -9$$
$$b + c = -1$$

$$3\,b - c = 9$$
$$\underline{b + c = -1}$$
$$4\,b \quad = \quad 8$$
$$b \quad = \quad 8 \div 4$$
$$b \quad = \quad 2$$

$$b + c = -1$$
$$2 + c = -1$$
$$c = -1 - 2$$
$$c = -3$$

III

Graph cuts y axis at $(0, -3)$

$$f : x \rightarrow x^2 + bx + c$$
$$(0)^2 + b\,(0) + c = -3$$
$$c = -3$$

$(-3, 0)$
$$0 = (-3)^2 + b\,(-3) - 3$$
$$(-3)^2 + b\,(-3) - 3 = 0$$
$$9 - 3\,b - 3 = 0$$
$$-3\,b + 6 = 0$$
$$-3\,b = 0 - 6$$
$$-3\,b = -6$$
$$3\,b = 6$$
$$b = 6 \div 3$$
$$b = 2$$

or

$(1, 0)$
$$(1)^2 + b\,(1) - 3 = 0$$
$$1 + b - 3 = 0$$
$$b - 2 = 0$$
$$b = 2$$

*   States $c = -3$ OR $b = 2$ and stops, no work shown, is 4m
*   Other points may be used in Methods **II** or **III** *e.g.* $(-4, 5), (-2, -3), (0, -3) (2, 5)$

Blunders (-3)
B1   Correct answer, no work shown
B2   Sign error
B3   Incorrect substitution
B4   Inversion
B5   Mathematical error
B6   Squaring error
B7   Transposition error
B8   Solves for one unknown only
B9   Distribution error

Slips (-1)
S1   Numerical error, to a maximum of 3
S2   Fails to correctly identify b and/or c in Method 1

Misreadings (-1)
M1   Misreads a digit, providing it does not affect final answer

Attempts (2 marks)
A1   Some correct substitution
A2   Incorrect value created for one unknown and used to find second unknown, but note *
A3   States $x = -3$ or $x = 1$ and stops
A4   One or two simplified equations and stops
A5   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   No work of merit

(c) (ii)	**5 marks**	**Att 2**

Axis of symmetry: $x = -1$

*   Check graph

Blunders (-3)

Slips (-1)
S1 $x + 1 = 0$
S2 -1 only, omits $x =$

Attempts (2 marks)
A1 $x = -4$
A2   Draws axis of symmetry on graph, equation not written
A3 $x + 1$
A4   States any point on the axis of symmetry *e.g.*$(-1, 3)$
A5   Work of merit

Worthless (0)
W1   Incorrect answer with no work
W2   Draws horizontal line on graph
W3   No work of merit

(c) (iii)   **5 marks**   **Att 2**

$f(x) = x^2 + 2x - 3$	**2m**
$f(x+1) = (x+1)^2 + 2(x+1) - 3$	**2m**
$\quad = x^2 + 2x + 1 + 2x + 2 - 3$	**2m**
$\quad = x^2 + 4x$	**5m**

*   Accept candidate's values of b and c from **(c)(i)**

*   If candidate did not have correct b and c value(s) in **(c)(i)** and now used the correct values without showing work, apply Blunder (-3)

Blunders (-3)
B1   No expansion of $(x+1)^2$ shown
B2   Squaring error
B3   Incorrect or incomplete substitution
B4   Distribution error
B5   Not in correct form
B6   Mathematical error

Slips (-1)
S1   Numerical errors to a max of -3
S2   Invalid conclusion

Misreadings (-1)
M1   Misreads a digit, providing it does not oversimplify the question

Attempts (2 marks)
A1   Some correct substitution
A2   Some correct squaring or multiplication
A3   Any relevant step

Worthless (0)
W1   No work of merit

QUESTION 6

Part (a)	5 marks	Att 2
Part (b)	35 marks	Att (2,5,5)
Part (c)	10 marks	Att (2,2)

Part (a)	5 marks	Att 2

A rectangular site, with one side facing a road,
is to be fenced off.
The side facing the road, which does not require fencing,
is l m in length.
The sides perpendicular to the road are x m in length.
The length of fencing that will be used to enclose
the rest of the site is 140 m.

Road

l m

x m
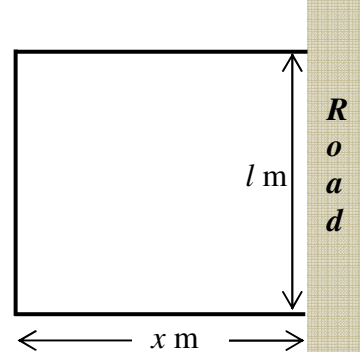
(a) ✍ Write an expression, in terms of x, for the length (l) of the side facing the road.

(a)	5 marks	Att 2

2 widths + 1 length	**2m**
Fence = 2 widths + 1 length	
2 widths + 1 length = 140	
$2x + l = 140$	**2m**
$l = 140 - 2x$	**5m**

Blunders (-3)
B1 Correct answer, no work shown
B2 Incorrect operation
B3 Transposition error
B4 Uses $2l + 2x = 140$ and continues correctly
B5 Mathematical error

Slips (-1)
S1 x not used. Answer given as: length = 140 − 2 widths
S2 w used instead of x
S3 $2l + x = 140 \rightarrow l = \frac{140 - x}{2}$ or $l = 70 - \frac{x}{2}$

Misreadings (-1)
M1 Expresses x in terms l to give $x = \frac{140 - l}{2}$ or similar
M2 Misreads a digit, providing it does not oversimplify the question

Attempts (2 marks)
A1 Any effort to add length and width
A2 Width = *l*
A3 Length = *x*
A4 Perimeter formula only
A5 Some relevant use of *x* or *l* or 140
A6 Any relevant step

Worthless (0)
W1 Incorrect answer, no work shown
W2 Area formula
W3 Reproduced diagram
W4 No work of merit

Part (b)   **35 (5,15,15) marks**   **Att (2,5,5)**

> **(i)**   ✍   Show that the area of the site, in m^2, is $-2x^2 + 140x$.
>
> **(ii)** Let *f* be the function $f : x \rightarrow -2x^2 + 140x$.
>
> ✍   Evaluate $f(x)$ when $x = 0, 10, 20, 30, 40, 50, 60, 70$.
>
> Hence, draw the graph of *f* for $0 \leq x \leq 70$, $x \in \mathbb{R}$.

(b)(i)   **5 marks**   **Att 2**

Area of the site	=	length × width	**2m**
	=	$l \times x$	**2m**
	=	$(140 - 2x)(x)$	**2m**
	=	$-2x^2 + 140x$	**5m**

*   Accept value of *l* from **(a)**

Blunders (-3)
B1 Mathematical error
B2 Fails to finish

Slip(-1)
S1 Invalid conclusion

Misreadings (-1)
M1 Misreads a digit, providing it does not oversimplify the question

Attempts (2 marks)
A1 Area formula
A2 Any relevant step

Worthless (0)
W1 Incorrect answer, no work shown
W2 Copies diagram
W3 Work of no merit

(b) (ii) Values   **15 marks**   **Att 5**

1

$f : x \rightarrow -2x^2 + 140x$ Given

x	0	10	20	30	40	50	60	70
$f(x)$	0	1200	2000	2400	2400	2000	1200	0

11 $f : x \rightarrow -2x^2 + 140x$

$f(0)$	$\rightarrow -2(0)^2 + 140(0) \rightarrow -2(0) + 140(0) = 0 + 0$		$= 0$
$f(10)$	$\rightarrow -2(10)^2 + 140(10) \rightarrow -2(100) + 1400 = -200 + 1400$		$= 1200$
$f(20)$	$\rightarrow -2(20)^2 + 140(20) \rightarrow -2(400) + 2800 = -800 + 2800$		$= 2000$
$f(30)$	$\rightarrow -2(30)^2 + 140(30) \rightarrow -2(900) + 4200 = -1800 + 4200$		$= 2400$
$f(40)$	$\rightarrow -2(40)^2 + 140(40) \rightarrow -2(1600) + 5600 = -3200 + 5600$		$= 2400$
$f(50)$	$\rightarrow -2(50)^2 + 140(50) \rightarrow -2(2500) + 7000 = -5000 + 7000$		$= 2000$
$f(60)$	$\rightarrow -2(60)^2 + 140(60) \rightarrow -2(3600) + 8400 = -7200 + 8400$		$= 1200$
$f(70)$	$\rightarrow -2(70)^2 + 140(70) \rightarrow -2(4900) + 9800 = -9800 + 9800$		$= 0$

III

x	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**
$-2x^2$	0	-200	-800	-1800	-3200	-5000	-7200	-9800
$+140x$	0	$+1400$	$+2800$	$+4200$	$+5600$	$+7000$	$+8400$	$+9800$
$f(x)$	**0**	**1200**	**2000**	**2400**	**2400**	**2000**	**1200**	**0**

*   If values do not appear here, check graph. If graph is correct, marks awarded are part (i) 12m and part (ii) 15 m.

*   Middle lines of table do not have to be shown

*   Consistent errors in rows/columns attract a maximum deduction of minus 3

*   Candidates may choose not to use a table

Blunders (-3)

B1   Error in calculating $-2x^2$, once only, note A3. Methods **II** and **III**

B2   Error in calculating $+140x$, once only. Methods **II** and **III**

B3   Mathematical error in $f(x)$ row Methods **II** and **III**

B4   Each incorrect value, without work. Method **I**

B5   Each missing $f(x)$ value

B6   If a table is used, adds in domain row when evaluating $f(x)$ value, once if consistent

B7   Mathematical errors in calculation, once if consistent

Slips (-1)
S1   Numerical errors to a maximum of 3

Misreadings (-1)
M1   Misreads a digit, provided it does not oversimplify the question

Attempts (5 marks)
A1   Any correct value for *f(x)*
A2   Some correct substitution
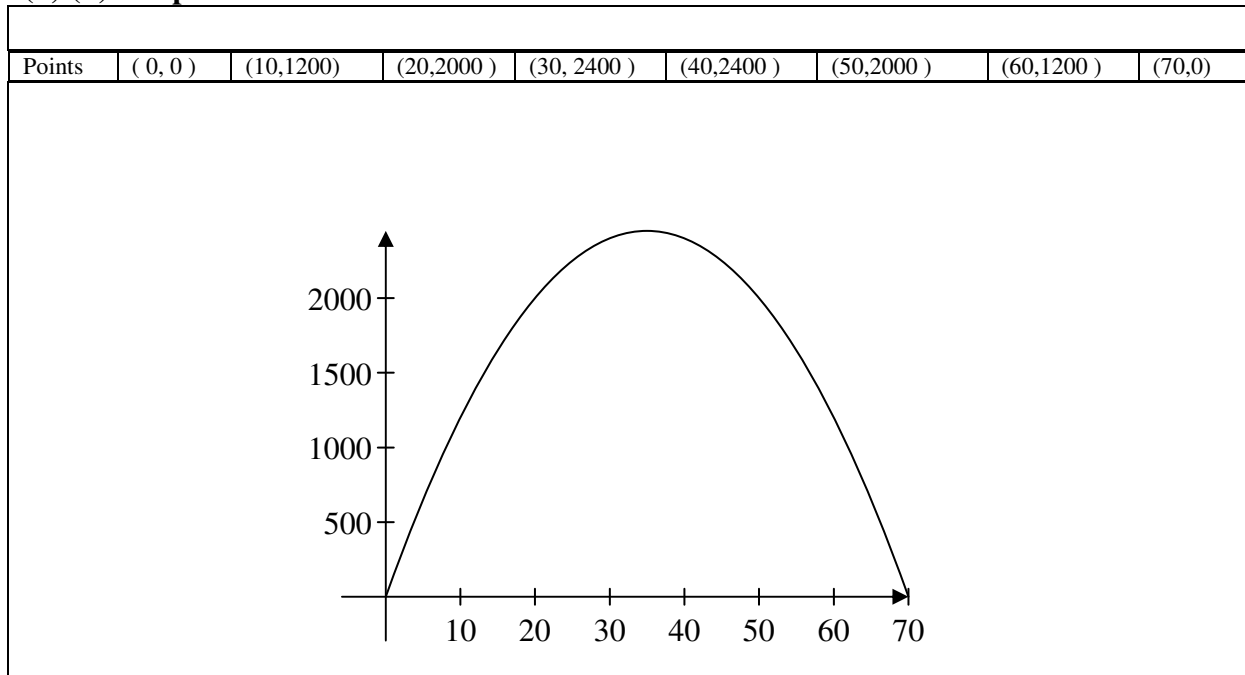A3   Oversimplification, linear
A4   Some relevant step

Worthless (0)
W1   Table with no relevant work
W2   Work of no merit

(b) (ii) Graph   **15 marks**   **Att 5**

Points	(0, 0)	(10,1200)	(20,2000)	(30, 2400)	(40,2400)	(50,2000)	(60,1200)	(70,0)

*   Graph constitutes work in this question
*   Use candidate's *f(x)* values from **(b)(i)**

Blunders (-3)
B1   Incorrect scaling, apply once
B2   Points not joined to form curve or joined in incorrect order, once only
B3   Graph not extended to include full domain, once only
B4   Reversed co-ordinates plotted against non reversed axes or similar, once only

Slips (-1)
S1   Each point plotted incorrectly, 8 points to check, must have one correct or attempt only
S2   Each missing point 8 points to check, must have one correct or attempt only

Attempts (5 marks)
A1   Draws axes, with some indication of scaling
A2   Some effort to plot a point
A3   Linear graph
A4   Any relevant step

Worthless (0)
W1   Work of no merit

Part (c)   **10 (5,5) marks**   **Att (2,2)**

> Use your graph from part **(b)** to estimate:
>
> **(i)**   ✍   the maximum possible area of the site
>
> **(ii)**   ✍   the area of the site when the road frontage (*l*) is 30 m long.

(c) (i)   **5 marks**   **Att 2**

Maximum possible area of the site: c. 2450

*   Answer of 2450 must be clearly stated for full marks
*   Accept value from candidate's graph in **(b)(ii)**
*   Tolerance of 50

Blunders (-3)
B1   Reads from incorrect axis
B2 $f(35) = -2(35)^2 + 140(35) = -2(1225) + 140(35) = -2450 + 4900 = 2450$, graph not used
B3   Indicated on graph, but no statement of value of maximum possible area
B4   Indicated correctly on graph but incorrect statement of value of maximum possible area
B5   Outside of tolerance

Slips (-1)
S1   Writes maximum point instead of maximum value

Attempts (2 marks)
A1   Indicates maximum on graph
A2   Indication of $x = 35$
A3   States maximum is highest point or turning point without any further work of merit
A4   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   No work of merit

(c) (ii)   **5 marks**   **Att 2**

> When $l = 30$
>
> . $l = 140 - 2x$ **2m**
>
> $30 = 140 - 2x$
>
> $30 - 140 = -2x$
>
> $-110 = -2x$
>
> $110 = 2x$
>
> $110 \div 2 = x$
>
> $x = 55$
>
> $\rightarrow$ Area:
>
> $f(55)$ from graph $= 1650$ **5m**

*   Graph must be used to find area
*   Answer of 1650 must be clearly stated for full marks
*   Accept value from candidate's graph in **(b)(ii)**
*   Tolerance of 50

Blunders (-3)
B1   Correct answer, no work shown
B2   Mathematical error
B3   Incorrect substitution
B4   Transposition error
B5   Sign error
B6 $f(55) = -2(55)^2 + 140(55) = -2(3025) + 140(55) = -6050 + 7700 = 1650$, graph not used
B7   Indicated on graph, but no statement of value of area
B8   Indicated correctly on graph but incorrect statement of value of area
B9   Finds $f(x)$ from graph, where $x \neq 55$
B10   Outside of tolerance
B11   Reads from incorrect axis
B12   Begins with $x = 55$, but reads $f(x)$ value before line touches graph
B13 $x = 55$ with work and stops

Slips (-1)
S1   Numerical error, to a maximum of 3
S2   Writes co-ordinates instead of y value

Misreadings (-1)
M1   Misreads a digit, provided it does not oversimplify equation

Attempts (2 marks)
A1   Imports l value from **(a)**
A2   Some correct substitution
A3   Indicates 55 on graph
A4   States $x = 55$
A5   Any relevant step

Worthless (0)
W1   Incorrect answer, no work shown
W2   Area = length × width only
W3   No work of merit

Coimisiún na Scrúduithe Stáit
State Examination Commission

Scrúdu An Teastais Shóisearaigh

JUNIOR CERTIFICATE EXAMINATION MARKING SCHEME
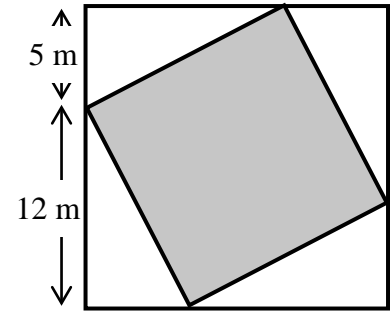
MATHEMATICS (PROJECT MATHS – PHASE 1) PAPER 2

HIGHER LEVEL

Question 1   **20 marks**

Liam's garden is in the shape of a square. It has four equal right-angled triangular lawns and a smaller square patio in the centre, as shown.

5 m

12 m

(a) Find the length of the hypotenuse of one of the right angled triangular lawns.

$$h^2 = a^2 + b^2$$
$$\text{Length} = \sqrt{12^2 + 5^2} \quad = \quad 13 \text{ m}$$

(b) Find the area of one of the triangular lawns.

$$\text{Area} = \frac{1}{2}(12)(5) \quad = \quad 30 \text{ m}^2$$

(c) Find the area of the square patio in the middle.

$$\text{Area of patio} \quad = \quad 13^2 \quad = \quad 169 \text{ m}^2$$

or

$$\text{Area of patio} \quad = \quad 17^2 - (4 \times 30) \quad = \quad 169 \text{ m}^2$$

(d) The patio is to be paved with rectangular flagstones of length 80cm and width 50cm. Calculate the number of flagstones Liam needs to buy to cover the patio, allowing an extra 20% for waste.

$$\text{Area of flagstone} \quad = \quad 0{\cdot}8 \times 0{\cdot}5 \quad = \quad 0{\cdot}4 \text{ m}^2$$
$$\text{Number of flagstones} \quad = \quad \frac{169}{0.4} \quad = \quad 422{\cdot}5 \qquad (\text{or } 423)$$

Extra 20% $= \quad 422{\cdot}5 \times 0{\cdot}2 = 84{\cdot}5$ **or**   120% $= \quad 422{\cdot}5 \times 1{\cdot}2$
Total number of flagstones $= \quad 507$

or

Extra 20% $= \quad 423 \times 0{\cdot}2 = 84{\cdot}6$ **or**   120% $= \quad 423 \times 1{\cdot}2$
Total number of flagstones $= \quad 507{\cdot}6$

or

Area to cover $= \quad 169 \times 1{\cdot}2 \quad = \quad 202{\cdot}8$

$$\text{Total number of flagstones} \quad = \quad \frac{202{\cdot}8}{0{\cdot}4} = 507$$

Question 2   30 marks

The ages of the 30 people who took part in an aerobics class are as follows:

18	24	32	37	9	13	22	41	51	49
15	42	37	58	48	53	27	54	42	24
33	48	56	17	61	37	63	45	20	39

The ages of the 30 people who took part in a swimming class are as follows:

16	22	29	7	36	45	12	38	52	13
33	41	24	35	51	8	47	22	14	24
42	62	15	24	23	31	53	36	48	18

(a) Represent this data on a back-to-back stem-and-leaf diagram.

			Aerobics class					Swimming class						
						9	**0**	7	8					
		8	7	5	3	1	**1**	2	3	4	5	6	8	
	7	4	4	2	0	2	**2**	2	2	3	4	4	4	9
9	7	7	7	3	2	3	**3**	1	3	5	6	6	8	
9	8	8	5	2	2	1	**4**	1	2	5	7	8		
	8	6	4	3	1	5	**5**	1	2	3				
				3	1	6	**6**	2						
								Key: **1 | 5** means **15**						

(b) Use your diagram to identify the median in each case.

Aerobic median: $\dfrac{37 + 39}{2}$ =   38

Swimming median: $\dfrac{29 + 31}{2}$ =   30

(c) What other measure of central tendency could have been used when examining this data?

Mean or Mode

(d) Based on the data make one observation about the ages of the two groups.

An older age group take Aerobics class
or
A younger age group take Swimming class
or
Similar

Question 3   **15 marks**

The arrows represent the different routes that a skier can take when skiing down a mountain.
The circles on the diagram represent different points on the routes.

(a) When leaving any particular point on the mountain a skier is equally likely to choose any of the
available routes from that point. Fill in the boxes in the diagram which represent the probability
that the skier will take that route.
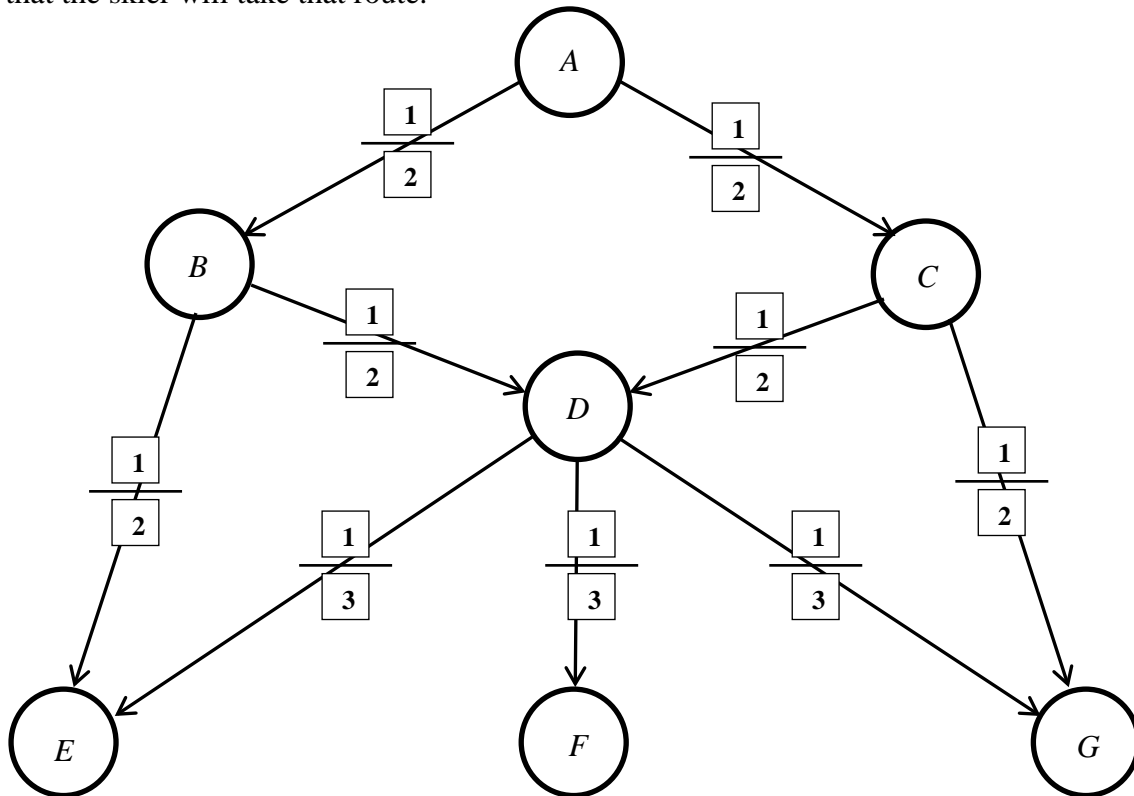
(b) **(i)** If the skier starts at point A, in how many different ways can the skier reach the point E?

1.   A → B → E
2.   A → B → D → E   } 3 ways
3.   A → C → D → E

(ii) If the skier starts at point A, find the probability that the skier will reach the point E.

$$1. \quad \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$2. \quad \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$$

$$3. \quad \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$$

$$\text{Probability} = \frac{1}{4} + \frac{1}{12} + \frac{1}{12} = \frac{5}{12}$$

Question 4

10 marks

A football strip consists of a shirt, shorts and socks.
Aspen United has two shirts, blue and green, from which to select. They also can select from three different colours of shorts and five different colours of socks, including red in each case.

(a) Calculate how many different strips Aspen United can have.

Different strips: $2 \times 3 \times 5 = 30$

(b) Willow Celtic plays in an all red strip. When Aspen United plays Willow Celtic, Aspen United are not allowed to use their red shorts or their red socks. Calculate how many different strips Aspen United can have when they play Willow Celtic.

Different strips: $2 \times 2 \times 4 = 16$

Question 5

25 marks

In total 7150 second level school students from 216 schools completed the 2011/2012 phase 11*CensusAtSchool* questionnaire. The questionnaire contained a question relating to where students keep their mobile phones while sleeping.

Phone location while sleeping – Female

In another room 8%
Other 2%
Under my pillow 35%
In my bedroom 55%

Phone location while sleeping – Male

In another room 10%
Other 3%
Under my pillow 23%
In my bedroom 64%
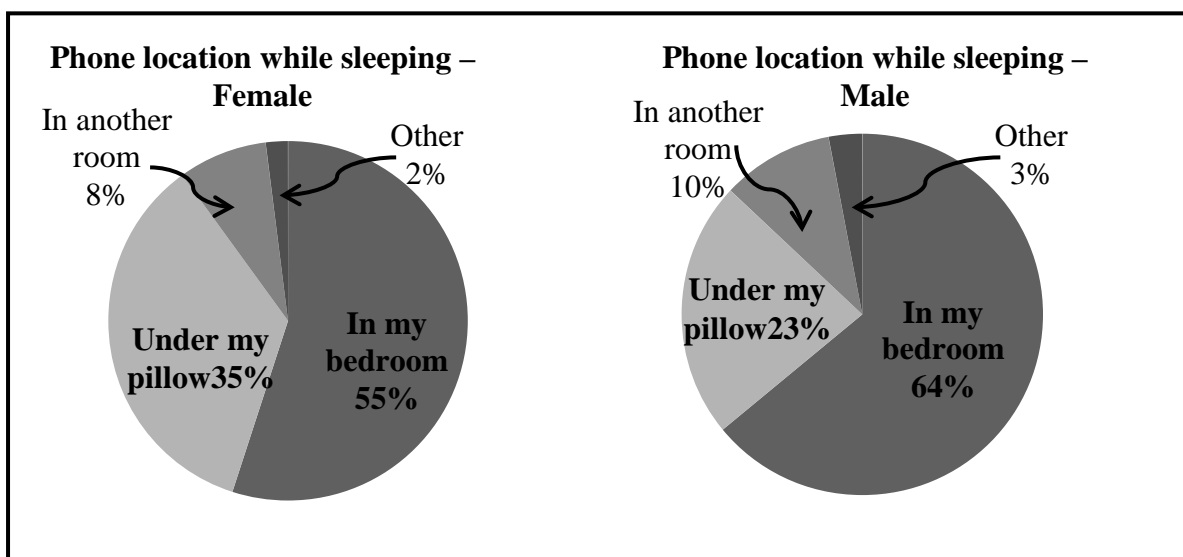
(a) Given that this question was answered by 4171 girls and 2979 boys, calculate how many female students kept their mobile phones under their pillows.

Girls – phone under pillow $\quad = \quad$ 35% of 4171

$$= \quad 4171 \times 0{\cdot}35$$

$$= \quad 1459{\cdot}85$$

$$= \quad 1460 \qquad \text{(or } 1459{\cdot}85 \text{ or } 1459)$$

(b) Calculate the overall percentage of students who kept their mobile phones under their pillows.

Total number of students $\quad = \quad$ 7150

Boys – phone under pillow $\quad = \quad$ 23% of 2979

$$= \quad 685{\cdot}17$$

$$= \quad 685 \qquad \text{(or } 685{\cdot}17 \text{ or } 686)$$

Total $\quad = \quad 1460 + 685 \quad = \quad 2145 \qquad \text{(or } 2145{\cdot}02)$

Percentage $\quad = \quad \dfrac{2145}{7150} \times 100 \qquad \left(\text{or } \dfrac{2145{\cdot}02}{7150} \times 100\right)$

$$= \quad 30\% \qquad \text{(or } 30{\cdot}0002\%).$$

(c) A new pie chart is to be drawn showing the mobile phone location for all students. Calculate the measure of the angle that would represent the students who kept their mobile phones under their pillows.

Angle $\quad = \quad$ 30% of 360°
$\qquad\quad = \quad 360 \times 0{\cdot}3$
$\qquad\quad = \quad 108° \qquad \text{(or } 108{\cdot}00072)°$

Question 6   **30 marks**

The salaries, in €, of the different employees working in a call centre are listed below.

22000	16500	38000	26500	15000	21000	15500	46000
42000	9500	32000	27000	33000	36000	24000	37000
65000	37000	24500	23500	28000	52000	33000	25000
23000	16500	35000	25000	33000	20000	19500	16000

(a) Use this data to complete the grouped frequency table below.

Salary (€1000)	0 – 10	10 –20	20 –30	30–40	40 –50	50 –60	60 –70
No. of Employees	1	6	12	9	2	1	1

[Note: 10–20 means €10000 or more but less than €20000, etc.]

(b) Using mid-interval values find the mean salary of the employees.

The mid - interval values are 5000, 15000, 25000, 35000, 45000, 55000, 65000

Mean =
$$\frac{(5000 \times 1) + (15\,000 \times 6) + (25\,000 \times 12) + (35\,000 \times 9) + (45\,000 \times 2) + (55\,000 \times 1) + (65\,000 \times 1)}{32}$$

$$= \frac{5000 + 90000 + 300000 + 315000 + 90000 + 55000 + 65000}{32}$$

$$= \frac{920000}{32}$$

$$= €28,750$$

(c) **(i)** Outline another method which could have been used to calculate the mean salary.

Add up all the individual salaries and divide by 32.

(ii) Which method is more accurate? Explain your answer.

Answer	Adding up individual salaries and dividing by 32

Reason	This gives the actual mean as estimates (mid-intervals) are not used.

Question 7   **20 marks**

In a survey, 54 people were asked which political party they had voted for in the last three elections.
The results are as follows:

      30 had voted for the Conservatives
      22 had voted for the Liberals
      22 had voted for the Republicans

      12 had voted for the Conservatives and for the Liberals
      9 had voted for the Liberals and for the Republicans
      8 had voted for the Conservatives and for the Republicans

         5 had voted for all three parties.

(a) Represent the information in a Venn diagram.
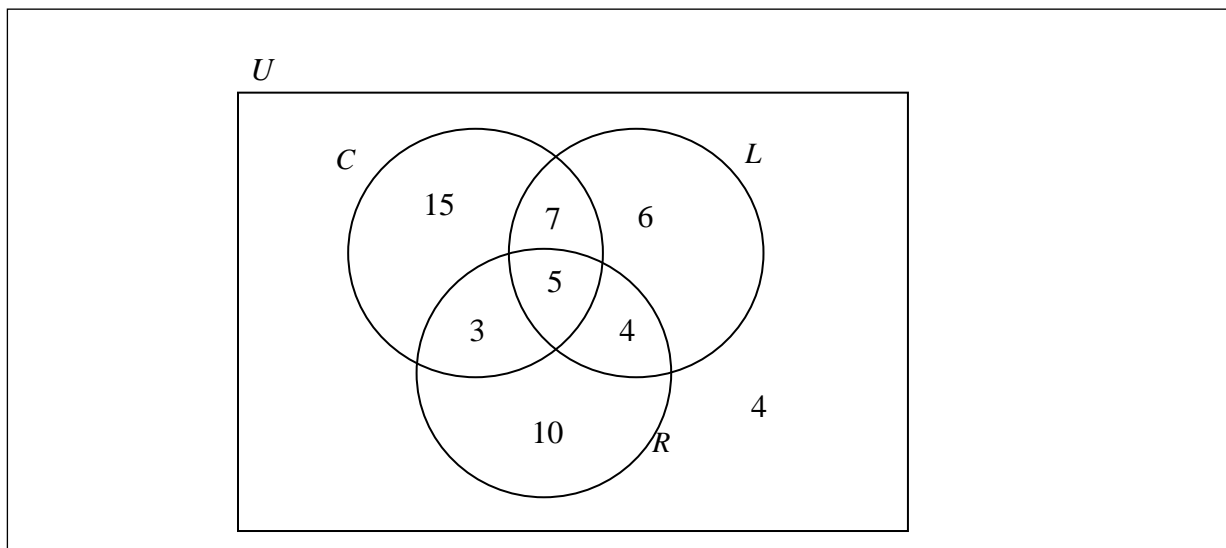
(b) If one person is chosen at random, what is the probability that the person chosen did not vote in
any of the three elections?

Probability person did not vote $= \quad \dfrac{4}{54} \quad \text{or} \quad \dfrac{2}{27}$

(c) If one person is chosen at random, what is the probability that the person chosen voted for at
least two different parties?

Probability person voted for at least two parties $= \quad \dfrac{3+5+7+4}{54} = \dfrac{19}{54}$

(d) If one person is chosen at random, what is the probability that the person chosen voted for the
same party in all three elections?

Probability person voted for the same party $= \quad \dfrac{15+6+10}{54} = \dfrac{31}{54}$

Question 8
10 marks

Construct the bisector of the $\angle ABC$ below, using only a compass and straight edge.
Show all construction work.
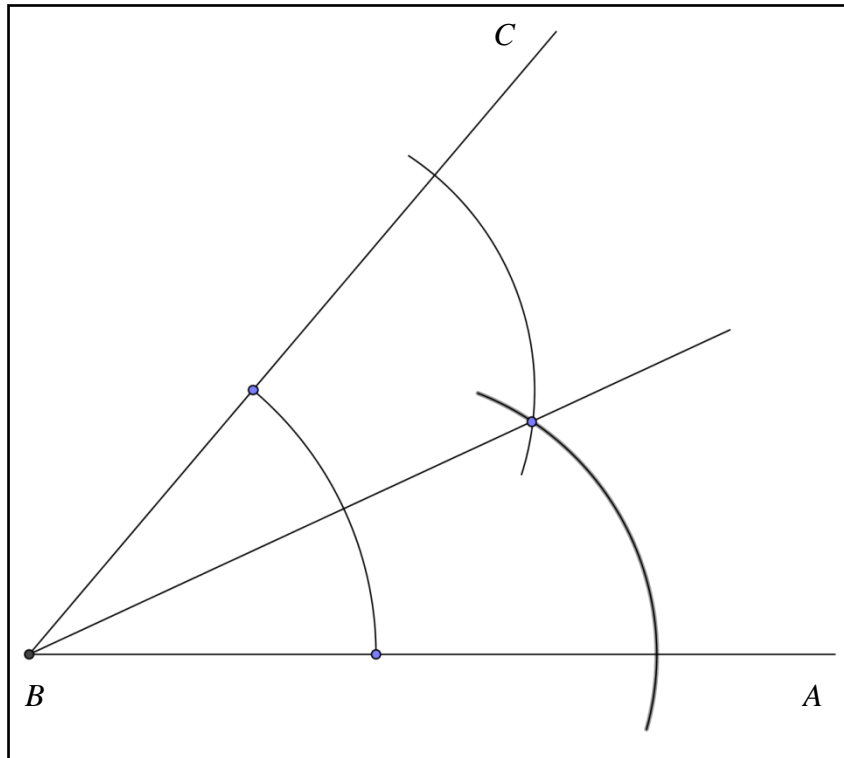
Question 9
15 marks

If l_1, l_2 and l_3 are parallel lines, find the measure of the angles α, β and γ.
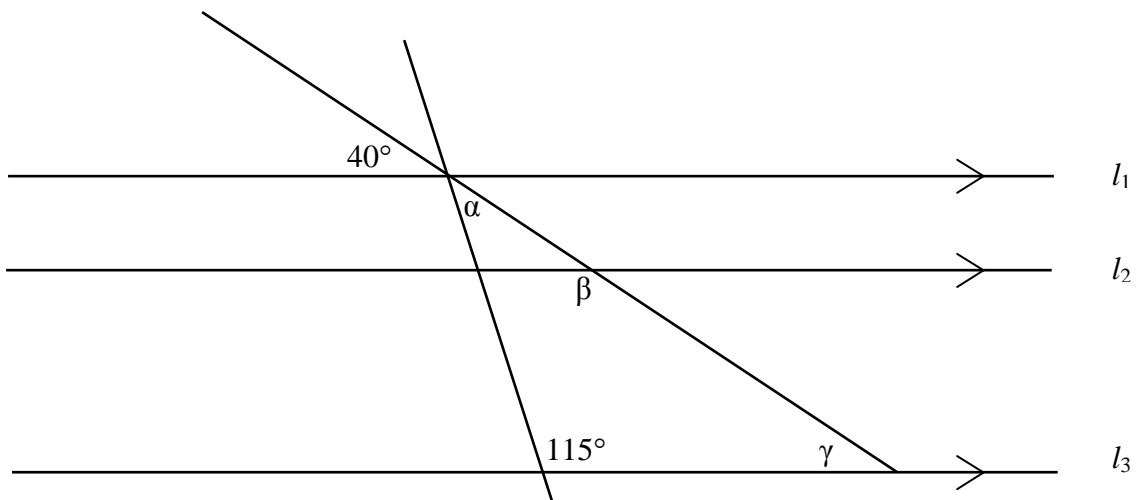
α	=	$180 - (115 + 40)$	=	$25°$
β	=	$180 - 40$	=	$140°$
γ	=	$40°$		

Question 10   **20 marks**

In the triangle ABC, $|AB| = 2$ and $|BC| = 1$.

(a) Find $|AC|$, giving your answer in surd form.

$$h^2 = a^2 + b^2$$
$$2^2 = |AC|^2 + 1^2$$
$$\Rightarrow |AC| \quad = \quad \sqrt{2^2 - 1^2} \quad = \quad \sqrt{3}$$
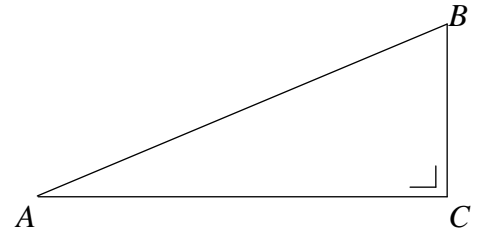
(b) Write $\cos \angle BAC$ and hence find $\left| \angle BAC \right|$.

$$\cos \angle BAC \quad = \quad \frac{\sqrt{3}}{2}$$
$$\left| \angle BAC \right| \quad = \quad 30°$$

(c) Sketch a right angled isosceles triangle in which the equal sides are 1unit each and use it to write $\cos 45°$ in surd form.

$$\text{Hypotenuse} \quad = \quad \sqrt{1^2 + 1^2} \quad = \quad \sqrt{2}$$
$$\cos 45° \quad = \quad \frac{1}{\sqrt{2}}$$
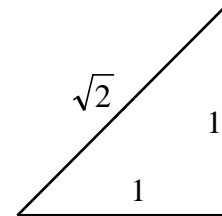
(d) Show that $\cos 75° \neq \cos 45° + \cos 30°$.

$$\cos 75° \quad = \quad \frac{\sqrt{6} - \sqrt{2}}{4} \quad = \quad 0{\cdot}2588$$
$$\cos 45° + \cos 30° \quad = \quad \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \quad = \quad 0{\cdot}7071 + 0{\cdot}8660 \; = 1{\cdot}5731$$
$$(0{\cdot}2588 \neq 1{\cdot}5731)$$

Question 11   **10 marks**

The two triangles shown are similar. Find the value of x.

$$\frac{x}{6} = \frac{25}{15}$$
$$\Rightarrow \quad x \quad = \quad 10$$
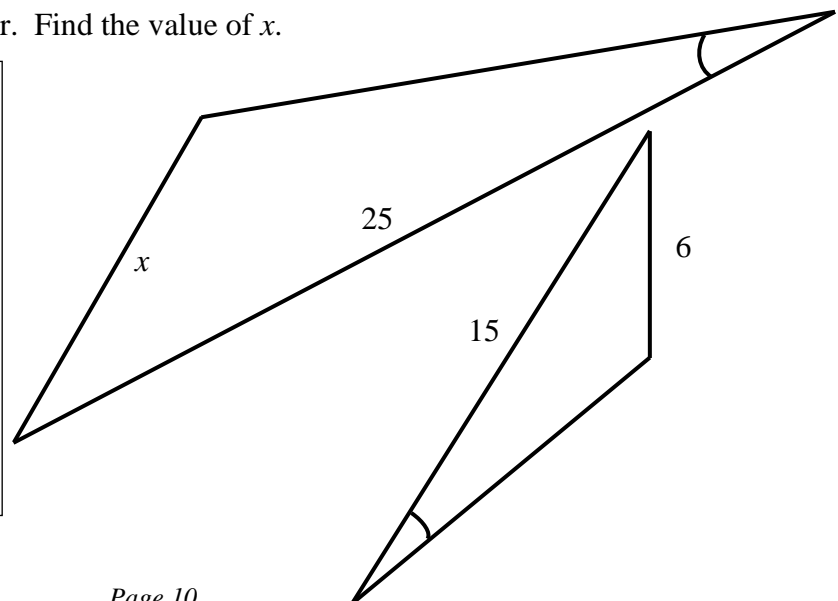
Question 12 45 marks
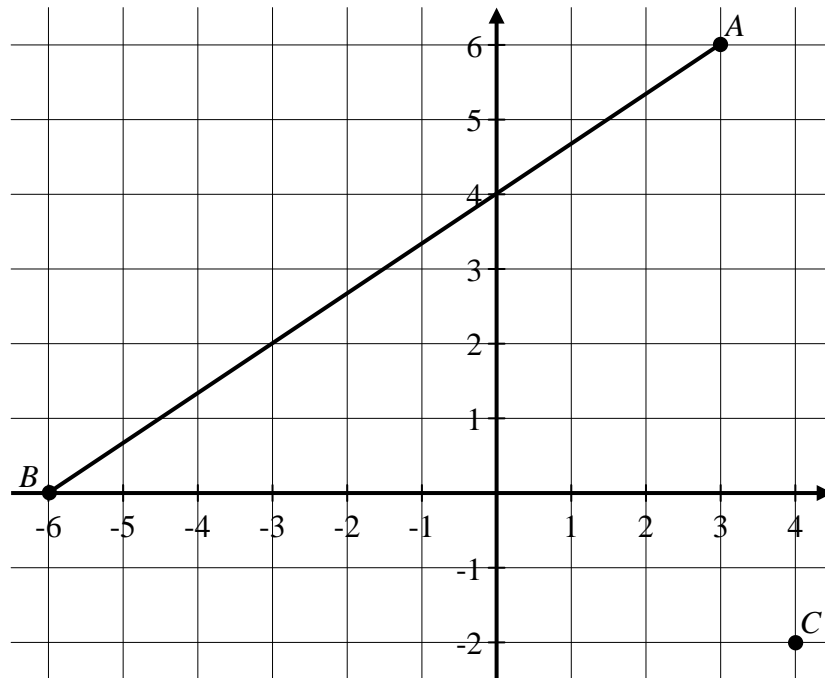
(a) Write the coordinates of *A*, *B* and *C*.

$A(\ 3\ ,\ \ 6\ )$ $B(-6\ ,\ \ 0)$ $C(\ 4\ ,-2\ )$

(b) Find the co-ordinates of *D*, the mid-point of [*AB*].

$$D\ \ =\ \ \left(\frac{3-6}{2},\frac{6+0}{2}\right)\ \ =\ \ \left(-\frac{3}{2},3\right)$$

(c) Find the equation of the line *AB*.

Slope *AB* $=\ \ \dfrac{0-6}{-6-3}\ \ =\ \ \dfrac{2}{3}$

Equation *AB*: $y-0=\dfrac{2}{3}(x+6)$ **or** $y-6=\dfrac{2}{3}(x-3)$

or

$y=\dfrac{2}{3}x+4$

$2x-3y+12\ =\ 0$

(d) Find the equation of the line through *C*, perpendicular to *AB*.

Perpendicular slope $= -\dfrac{3}{2}$

Line through *C*: $y + 2 = -\dfrac{3}{2}(x - 4)$

$3x + 2y - 8 = 0$

or

The line is of the form $3x + 2y + c = 0$

$(4, -2)$: $3(4) + 2(-2) + c = 0 \Rightarrow c = -8$

$3x + 2y - 8 = 0$

(e) Let *E* be the point where this perpendicular line through *C* intersects *AB*.
Calculate the coordinates of the point *E*.

E the point of intersection of two lines

$2x - 3y + 12 = 0$ (i)
$3x + 2y - 8 = 0$ (ii)

$2 \times$ (i) $4x - 6y = -24$ **or** $y = \dfrac{2x + 12}{3}$

$+ 3 \times$ (ii) $9x + 6y = 24$ $\Rightarrow 3x + 2\left(\dfrac{2x + 12}{3}\right) - 8 = 0$

$\Rightarrow 9x + 4x + 24 - 24 = 0$

$\Rightarrow \quad x = 0$ and $y = 4$

(f) Which is the shorter distance, $|CD|$ or $|CE|$? Find this distance.

$|CD| = \sqrt{\left(4 + \dfrac{3}{2}\right)^2 + (-2 - 3)^2} = \sqrt{55 \cdot 25}$ or   7·433

$|CE| = \sqrt{(4 - 0)^2 + (-2 - 4)^2} = \sqrt{52}$ or   7·211
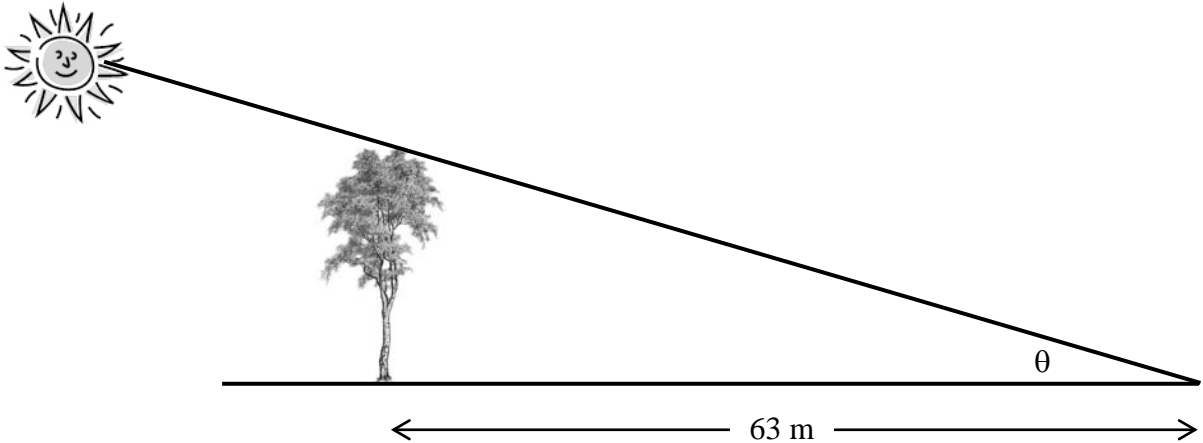
| *CE* | is the shorter distance

or

|CE| (is the perpendicular distance and therefore is the shorter distance.)

$|CE| = \sqrt{(4 - 0)^2 + (-2 - 4)^2} = \sqrt{52}$ or   7·211

Question 13   **10 marks**
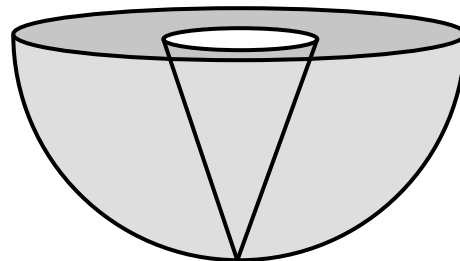
A tree 32 m high casts a shadow 63m long. Calculate θ, the angle of elevation of the sun.
Give your answer in degrees and minutes (correct to the nearest minute).

$$\tan\theta \quad = \quad \frac{32}{63} \qquad\qquad \textbf{or} \qquad \tan\alpha \quad = \quad \frac{63}{32}$$

$\Rightarrow \theta$	=	26·9277		$\Rightarrow \alpha$	=	63·0723
$\Rightarrow \theta$	=	26°55'39·64"		$\Rightarrow \theta$	=	90 − 63·0723
	=	26°56'			=	26·9277
					=	26°55'39·64"
					=	26°56'

Question 14   **20 marks**

A solid metal hemisphere has a radius of 12 cm.

(a)   Calculate the volume of the hemisphere. Give your answer in terms of π.

$$\text{Volume of hemisphere} \quad = \quad \frac{2}{3}\pi r^3$$

$$= \quad \frac{2}{3} \times \pi \times 12^3$$

$$= \quad 1152\pi \ \text{cm}^3$$

(b) A solid cone of radius 4 cm and height 12 cm is cut from the hemisphere.
Calculate the volume of the cone. Give your answer in terms of π.

$$\text{Volume of cone} \quad = \quad \frac{1}{3}\pi r^2 h$$

$$= \quad \frac{1}{3} \times \pi \times 4^2 \times 12$$

$$= \quad 64\pi \text{ cm}^3$$

(c) The remaining metal in the hemisphere is melted down and recast into cones of the same dimensions as the cone above. How many cones can be formed from the remaining metal?

$$\text{Remaining metal} \quad = \quad 1152\pi - 64\pi$$

$$= \quad 1088\pi$$

$$\text{Volume of cone} \quad = \quad 64\pi$$

$$\text{Number of cones} \quad = \quad \frac{1088\pi}{64\pi}$$

$$= \quad 17$$

or

$$\text{Number of cones} \quad = \quad \frac{1152\pi}{64\pi} - 1$$

$$= \quad 18 - 1$$

$$= \quad 17$$

Question 15   **20 marks**

The dimensions of two solid cylinders are shown in the diagrams below.
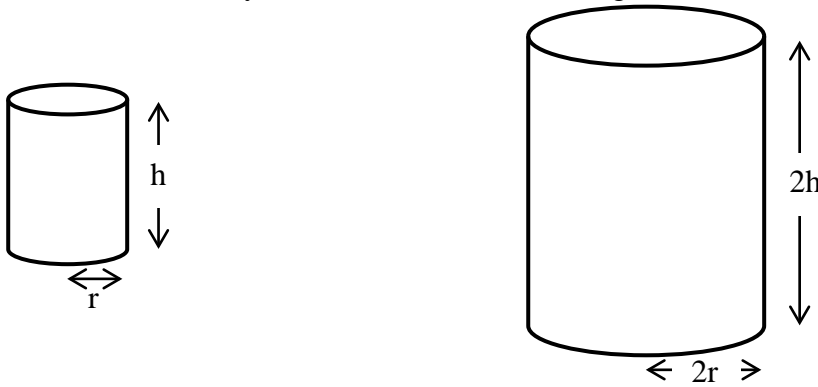
(a) Calculate the ratio of the curved surface area of the smaller cylinder to the curved surface area of the larger cylinder.

Curved surface area of a cylinder	$=$	$2\pi rh$
Curved surface area of small cylinder	$=$	$2 \times \pi \times r \times h$
	$=$	$2\pi rh$
Curved surface area of large cylinder	$=$	$2 \times \pi \times (2r) \times (2h)$
	$=$	$8\pi rh$
Ratio	$=$	$2\pi rh : 8\pi rh$
	$=$	$1 : 4$

(b) Calculate the ratio of the volume of the smaller cylinder to the volume of the larger cylinder.

Volume of a cylinder	$=$	$\pi r^2 h$
Volume of small cylinder	$=$	$\pi \times r^2 \times h$
	$=$	$\pi r^2 h$
Volume of large cylinder	$=$	$\pi \times (2r)^2 \times (2h)$
	$=$	$8\pi r^2 h$
Ratio	$=$	$\pi r^2 h : 8\pi r^2 h$
	$=$	$1 : 8$

Structure of the marking scheme (Questions 1 – 15)

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C
No of categories	2	3	4
2 mark scale	0, 2	0, 1, 2	
5 mark scale	0, 5	0, 4, 5	0, 2, 3, 5
10 mark scale	0, 10	0, 5, 10	0, 3, 7, 10
15 mark scale	0, 15	0, 7, 15	0, 7, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk.

Thus, for example, Scale 10C* indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Question 1 (20)

(a) 5C*

(b) 5C*

(c) 5C*

(d) 5C

Question 2 (30)

(a) 15C*

(b) 5C

(c) 5A

(d) 5A

Question 3 (15)

(a) 5C

(b)(i) 5C

(b)(ii) 5C

Question 4 (10)

(a) 5A

(b) 5A

Question 5 (25)

(a) 5C

(b) 15C

(c) 5C*

Question 6 (30)

(a) 15C

(b) 10C

(c) 5C

Question 7 (20)

(a) 5C

(b) 5A

(c) 5B

(d) 5B

Question 8 (10)

10A

Question 9 (15)

15C *

Question 10 (20)

(a) 5C

(b) 5C*

(c) 5C

(d) 5C

Question 11 (10)

10C

Question 12 (45)

(a) 5C

(b) 5C

(c) 10C

(d) 10C

(e) 10C

(f) 5C

Question 13 (10)

10C*

Question 14 (20)

(a) 5C*

(b) 5C*

(c) 10C

Question 15 (20)

(a) 10C

(b) 10C

Detailed Marking Notes

The * for units or rounding off is to be applied only if answer is fully correct. The * is to be applied once only per question.

Question 1

(a) Scale 5C*

High Partial Credit

- $\sqrt{12^2 + 5^2}$

Low Partial Credit

- Some correct substitution into Pythagoras' theorem formula
- Correct formula from tables

Note: Apply * if no units

(b) Scale 5C*

High Partial Credit

- $\frac{1}{2}(5)(12)$

- $4 \times \frac{1}{2}(5)(12) = 120$

Low Partial Credit

- Area of a triangle $= \frac{1}{2}\left(\text{base} \times \text{perpendicular height}\right)$

- $2(5)(12)$
- Correct formula from tables

Note: Apply * if no units

(c) Scale 5C*

High Partial Credit
- 13^2
- $17^2 - 4 \times 30$

Low Partial Credit
- 17^2
- 4×30
- Area $= l^2$ or $l \times w$

Note: Accept candidate's answers from parts (a) and (b)
Note: Apply * if no units

(d) Scale 5C

   High Partial Credit

- $\dfrac{169}{0\cdot4} = 422\cdot5$

- $422\cdot5 \times 0\cdot2 = 84\cdot5$

- $\dfrac{202\cdot8}{0\cdot4}$

   Low Partial Credit

- 80×50
- $0\cdot4$ or 4000
- $0\cdot4 \times 1\cdot20$ or $0\cdot4 \times 0\cdot2$
- $\dfrac{169}{0\cdot4}$ or $\dfrac{169}{4000}$ or $422\cdot5$
- Incorrect conversion of units
- Correct answer with no work

Note: Accept candidate's answers from previous parts

Question 2

(a) Scale 15C*

   High Partial Credit

- 16 + ages placed correctly on both aerobics class and swimming class.

   Low Partial Credit

- 1 to 15 ages placed correctly on either side

  Note**:** It is not necessary to order the stem and leaf
  Note**:** Apply * if the key is missing or sides switched

(b) Scale 5C

   High Partial Credit

- 37 to 39 **and** 29 to 31 given

   Low Partial Credit

- One median only correct
- 37 or 39 or 29 or 31 given from a correctly ordered list

(c) Scale 5A

(d) Scale 5A

Question 3

(a) Scale 5C
  High Partial Credit
- 6, 7 or 8 probabilities correct

  Low Partial Credit
- 1 to 5 probabilities correct

(b)(i) Scale 5C
  High Partial Credit
- 2 paths correctly identified

  Low Partial Credit
- 1 path correctly identified
- More than 3 paths listed with at least 1 correct

(b) (ii) Scale 5C
  High Partial Credit

- The three probabilities $\dfrac{1}{4}$, $\dfrac{1}{12}$, $\dfrac{1}{12}$ given
- Probability $= \dfrac{1}{4} + \dfrac{1}{12} + \dfrac{1}{12}$
- Error in one probability with addition correctly completed

  Low Partial Credit
- Probability for 1 or 2 paths correct

Question 4
  (a) Scale 5A

  (b) Scale 5A

Question 5
   (a) Scale 5C
      High Partial Credit
- $4171 \times 0{\cdot}35$

      Low Partial Credit
- 35%
- $0{\cdot}35$

   (b) Scale 15C
      High Partial Credit
- $\dfrac{2145}{7150} \times 100 \qquad \left(\text{or } \dfrac{2145{\cdot}02}{7150} \times 100 \right)$

      Low Partial Credit
- 685   (or 685·17 or 686)

      Note: Accept candidate's answer from part (a)

   (c) Scale 5C*
      High Partial Credit
- $360 \times 0{\cdot}3$
- $1 \text{ student} = \dfrac{360}{7150} = 0{\cdot}05^{\circ}$

      Low Partial Credit
- 30% of 360
- Indication that the answer is a fraction of 360
- Identifies 7150 as 360

      Note: Accept candidate's answer from part (b)
      Note: Apply * if units missing

Question 6

    (a) Scale 15C

        High Partial Credit

- Entry ± 2 in not more than 2 frequencies.

        Low Partial Credit

- Some correct entry in table

    (b) Scale 10C

        High Partial Credit

- Answer = 28·75

        Low Partial Credit

- Error in numerator - e.g. incorrect mid-interval values or no mid-interval values
- Error in denominator
- Effort at mid-interval values
- Sum of frequencies indicated

    (c) Scale 5C

        High Partial Credit

- Correct method and correct answer. No reason given.

        Low partial Credit

- Correct method only.

Question 7

    (a) Scale 5C

        High Partial Credit

- 2 – 7 correct entries.

        Low Partial Credit

- 1 correct entry

    (b) Scale 5A

Note: Accept value from candidate's Venn diagram except where negative values used
Note: Answer must be consistent with candidate's Venn diagram

(c) Scale 5B
Partial Credit

- $\dfrac{3+5+7+4}{54}$
- Not more than one element incorrect or omitted

Note: Accept values from candidate's Venn diagram unless probability > 1
Note: Answer must be consistent with candidate's Venn diagram

(d) Scale 5B
Partial Credit

- $\dfrac{15+6+10}{54}$
- Not more than one element incorrect or omitted

Note: Accept values from candidate's Venn diagram unless probability > 1
Note: Accept value from candidate's Venn diagram except where negative values used
Note: Answer must be consistent with candidate's Venn diagram

Question 8
Scale 10A

Note: Accept tolerance of $\pm 2^{\circ}$
Note: All constructions must be shown

Question 9
Scale 15C*
High Partial Credit

- 2 angles correct

Low Partial Credit

- 1 angle correct
- Some correct work on alternate / corresponding/vertically opposite angles
- Some correct work on the angles in a straight line adding to 180° or $\beta + \chi = 180^{\circ}$
- Some correct work on the angles of a triangle adding to 180°

Note: Apply * if units missing

Question 10
 (a) Scale 5C
   High Partial Credit
   - $\sqrt{2^2 - 1^2}$
   - Answer not in surd form

   Low Partial Credit
   - Some correct substitution into Pythagoras' theorem formula
   - $|\angle BAC| = 30^\circ$ or $|\angle ABC| = 60^\circ$
   - Correct formula from tables

 (b) Scale 5C*
   High Partial Credit
   - $\cos \angle BAC = \dfrac{\sqrt{3}}{2}$

   Note: Accept candidate's answer from part (a) if cos ratio < 1
   Note: Apply * if units missing

   Low Partial Credit
   - Some correct use of cos ratio
   - Indication that $\cos = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
   - $|\angle BAC|$ found without use of cos ratio

   Note: Accept candidate's answer from part (a) if cos ratio ≥ 1

 (c) Scale 5C
   High Partial Credit
   - Sketch drawn and $\sqrt{2}$ found or sketch drawn and $\sqrt{2}$ shown on diagram
   - Answer not in surd form

   Low Partial Credit
   - Correct sketch drawn
   - $\cos 45^\circ = \dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$ from tables/calculator
   - Correct sketch drawn and $\cos 45^\circ = 0{\cdot}7071$ from calculator
   - Indication that $\cos = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
   - Some correct substitution into Pythagoras' theorem formula

(d) Scale 5C
High Partial Credit
- Both correct but not in forms in which they can be easily compared
- Calculator in incorrect mode

Low Partial Credit
- cos 75° found correctly
- cos 45° and/ or cos 30° found or transferred from part (b) and part(c)

Question 11
Scale 10C
High Partial Credit
- $\dfrac{x}{6} = \dfrac{25}{15}$ or equivalent

Low Partial Credit
- Corresponding sides/angles identified
- One correct relevant ratio
- Indication that corresponding sides are proportional

Question 12
(a) Scale 5C
High Partial Credit
- 2 points correct

Low Partial Credit
- 1 point correct
- All 3 reversed (y, x)

(b) Scale 5C
High Partial Credit
- Correct substitution into midpoint formula
- Both x and y reversed in substitution

Low Partial Credit
- Some correct substitution into midpoint formula
- One ordinate correct
- Coordinates reversed
- Correct formula from tables

Note: Correct answer written in part (b) merits full marks.
Note: Accept candidate's points from part (a)

(c) Scale 10C
High Partial Credit
- Error in slope but continues correctly
- Correct slope with 1 incorrect substitution into a line formula
- Correct slope but both x and y reversed in substitution

Low Partial Credit
- Some correct substitution into slope formula
- Slope found
- Some correct substitution into line formula
- Indication that slope $= \dfrac{\text{rise}}{\text{run}}$
- Correct formula from tables

Note: It is not necessary to write the equation in the form $ax + by + c = 0$ for full marks.

(d) Scale 10C
High Partial Credit
- Error in slope but continues correctly
- Correct slope with 1 incorrect substitution into a line formula
- Correct slope but both x and y reversed in substitution

Low Partial Credit
- Some correct substitution into line formula
- Slope correct
- Indication that the product of the slopes of perpendicular lines is -1
- $3x + 2y + c = 0$
- Correct formula from tables

Note: It is not necessary to write the equation in the form $ax + by + c = 0$ for full marks

(e) Scale 10C
High Partial Credit
- One (correct) ordinate only found
- 1 error in simplifying equations in parts (c) or (d) but solved correctly

Low Partial Credit
- More than 1 error in simplifying equations from parts (c) and (d) in this part and continues with some correct work
- Equations from parts (c) **and** (d) correctly simplified in this part
- Point E correctly read from graph. Graph must have perpendicular line correctly drawn from C - otherwise 0 marks
- Equations from parts (c) and (d) correctly simplified but continues incorrectly

Note: Accept candidate's equations from parts (c) and (d)

(f) Scale 5C

High Partial Credit

- $\left| CD \right|$ and $\left| CE \right|$ both found but no indication of shorter distance

Low Partial Credit

- Some correct substitution into distance formula
- $\left| CD \right|$ only found
- States $\left| CE \right|$ (or perpendicular distance) is the shorter but $\left| CE \right|$ not found
- $\left| CE \right|$ found but not identified
- Correct formula from tables

Question 13

Scale 10C*

High Partial Credit

- $\theta = 26 \cdot 9277$
- Calculator in incorrect mode

Low Partial Credit

- $\tan\theta = \dfrac{32}{63}$
- Correct relevant trigonometric formula
- Indication that $\tan = \dfrac{\text{opposite}}{\text{adjacent}}$

Note: Apply * if answer not correctly rounded to the nearest minute and/or no units

Question 14

(a) Scale 5C*

High Partial Credit

- Volume of sphere found
- $\dfrac{2}{3} \times \pi \times 1728$

Low Partial Credit

- Correct or some correct substitution into volume of sphere/hemisphere formula
- Volume of hemisphere = ½ (Volume of sphere)
- Incorrect relevant formula used
- Correct formula from tables

Note: Apply * if answer not written in terms of π and/or no units

(b) Scale 5C*

High Partial Credit

- $\frac{1}{3} \times \pi \times 16 \times 12$

Low Partial Credit

- Correct or some correct substitution into volume of cone formula
- Incorrect relevant formula used
- Correct formula from tables

Note: Apply * if answer not written in terms of π and/or no units(if not applied in part (a))

(c) Scale 10C

High Partial Credit

- $\frac{1152\pi}{64\pi} = 18$
- $\frac{1088\pi}{64\pi}$

Low Partial Credit

- Subtraction of candidate's volumes
- Indication of division by 64π

Note: Accept candidate's values from previous parts provided answer in part (a) is greater than answer in part (b) – otherwise 0 marks

Question 15

(a) Scale 10C

High Partial Credit

- Both 2πrh and 8πrh written
- Values substituted for r and h and correct ratio found

Low Partial Credit

- Some correct substitution into curved surface area of cylinder formula
- $2 \times \pi \times r \times h$ and /or $2 \times \pi \times 2r \times 2h$ written
- Incorrect relevant formula used
- Values substituted for r and h and ratio incorrect
- Correct answer without work
- Correct formula from tables

(b) Scale 10C
   High Partial Credit
   - Both $\pi r^2 h$ and $8\pi r^2 h$ written
   - Values substituted for r and h and correct ratio found

   Low Partial Credit
   - Some correct substitution into volume of cylinder formula
   - $\pi \times r^2 \times h$ and/or $\pi \times (2r)^2 \times 2h$ written
   - Values substituted for r and h and ratio incorrect
   - Incorrect relevant formula used
   - Correct answer without work
   - Correct formula from tables