Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2005*
Matamaitic	*Ardleibhéal*

Marking Scheme	*Leaving Certificate Examination, 2005*
Mathematics	*Higher Level*

Contents

Page

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2005

MATHEMATICS – HIGHER LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 15) marks**	**Att (2, 5)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

1(a) Solve the simultaneous equations:

$$\frac{x}{5}-\frac{y}{4}=0$$

$$3x+\frac{y}{2}=17$$

Part (a) **10 marks** **Att 3**

1(a)

(i) $\frac{x}{5}-\frac{y}{4}=0 \qquad \times 20 \Rightarrow \quad 4x-5y=0$

(ii) $3x+\frac{y}{2}=17 \qquad \times 10 \Rightarrow 30x+5y=170$

$$34x \qquad = 170$$

$$x=5$$

(i) $4x-5y=0$

$4(5) = 5y$

$\Rightarrow y=4$

$x=5$

$y=4$

Blunders (-3)

B1 second variable not found.

Slips (-1)

S1 numerical.

S2 not changing sign in subtraction.

Attempts

A1 no solution.

A2 correct solution by trial and error.

Worthless

W1 values for *x* and *y*.

Part (b)(i) **5 marks** **Att 2**

1(b)(i)

Express $2^{\frac{1}{4}}+2^{\frac{1}{4}}+2^{\frac{1}{4}}+2^{\frac{1}{4}}$ in the form $2^{\frac{p}{q}}$, where $p, q \in \mathbf{Z}$.

Part (b)(i) **5 marks** **Att 2**

1(b)(i) $2^{\frac{1}{4}}+2^{\frac{1}{4}}+2^{\frac{1}{4}}+2^{\frac{1}{4}} = 4\left(2^{\frac{1}{4}}\right) = 2^2\left(2^{\frac{1}{4}}\right) = 2^{\frac{9}{4}}$

Blunders (-3)

B1 indices.

Slips (-1)

S1 not elements of **Z**.

Part (b)(ii) **15 marks** **Att 5**

1(b)(ii)

Let $f(x) = ax^3 + bx^2 + cx + d$.

Show that $(x-t)$ is a factor of $f(x) - f(t)$

Part (b)(ii) **15 marks** **Att 5**

1(b)(ii)

$$
\begin{aligned}
f(x) &= ax^3 + bx^2 + cx + d \\
f(t) &= at^3 + bt^2 + ct + d \\
f(x) - f(t) &= a(x^3 - t^3) + b(x^2 - t^2) + c(x-t) \\
&= a(x-t)(x^2 + tx + t^2) + b(x-t)(x+t) + c(x-t) \\
&= (x-t)[a(x^2 + tx + t^2) + b(x+t) + c]
\end{aligned}
$$

or

1(b)(ii)

$$
\begin{aligned}
f(x) &= ax^3 + bx^2 + cx + d \\
f(t) &= at^3 + bt^2 + ct + d \\
f(x) - f(t) &= ax^3 + bx^2 + cx - at^3 - bt^2 - ct
\end{aligned}
$$

$$
\begin{array}{r|l}
 & ax^2 + (at+b)x + (at^2 + bt + c) \\
\hline
(x-t) & ax^3 + bx^2 + cx - at^3 - bt^2 - ct \\
 & \underline{ax^3 - atx^2} \\
 & (at+b)x^2 \\
 & \underline{(at+b)x^2 - (at+b)tx} \\
 & (at^2 + bt + c)x - at^3 - bt^2 - ct \\
 & \underline{(at^2 + bt + c)x - at^3 - bt^2 - ct} \\
 & 0
\end{array}
$$

* Accept solution by division by $(x-t)$ for full marks.

Blunders (-3)

B1 indices.

B2 factors

Slips (-1)

S1 numerical.

S2 not changing sign when subtracting in division.

Part (c) **20(5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

1(c)	$(x-p)^2$ is a factor of x^3+qx+r Show that $27r^2+4q^3=0$ Express the roots of $3x^2+q=0$ in terms of p.

Factor	**5 marks**	**Att 2**
Values	**5 marks**	**Att 2**
Show	**5 marks**	**Att 2**
Express	**5 marks**	**Att 2**

1(c) (Show)

$$
\begin{array}{r}
x+2p \\
x^2-2px+p^2\overline{)\,x^3+qx+r} \\
\underline{x^3-2px^2+p^2x} \\
2px^2-p^2x+qx+r \\
\underline{2px^2-4p^2x+2p^3} \\
3p^2x+qx+r-2p^3=0
\end{array}
$$

Remainder must $=0$ since $(x-p)^2$ is a factor

$\Rightarrow \quad (3p^2+q)x+(r-2p^3)=(0)x+(0)$

$\Rightarrow \quad$ (i) : $3p^2+q=0 \quad \Rightarrow \quad q=-3p^2$

(ii) : $r-2p^3=0 \quad \Rightarrow \quad r=2p^3$

$$
\begin{aligned}
\therefore \quad 27r^2+4q^3 &= 27(2p^3)^2+4(-3p^2)^3 \\
&= 108p^6-108p^6 \\
&= 0
\end{aligned}
$$

or

If $(x-p)^2$ is a factor of $f(x)$, then let $(x+a)$ be other factor.

$\therefore \quad (x^2-2px+p^2)(x+a)=x^3+qx+r$

$x^3+(-2p+a)x^2+(p^2-2pa)x+p^2(a)=x^3+(0)x^2+(q)x+r$

Equating like to like

(i) $-2p+a=0$ (ii) $p^2-2pa=q$ (iii) $p^2a=r$

(i) $a=2p$ $\quad q=p^2-2p(2p)=-3p^2$ $\quad r=p^2(2p)$ $=2p^3$

$$
\begin{aligned}
27r^2+4q^3 &= 27(2p^3)^2+4(-3p^2)^3 \\
&= 108p^6-108p^6 \\
&= 0
\end{aligned}
$$

(Express)	$3x^2 + q = 0$ $3x^2 = -q$ $3x^2 = -\left(-3p^2\right)$ $3x^2 = 3p^2$ $x^2 = p^2$ $x = \pm p$

Blunders (-3)
B1 indices.
B2 not like to like.
B3 root from equation.
B4 *r* not found, having found *q*.
B5 roots from equation (in "express" part).

Slips (-1)
S1 numerical.
S2 not changing sign in subtraction (division).

Attempts
A1 remainder $\neq$ 0 in division.
A2 any attempt at division.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

2(a) Solve for x: $|x-1|<7$, where $x \in \boldsymbol{R}$

Part (a) **10 marks** **Att 3**

2(a) $|x-1|<7$

$\Rightarrow -7 < x-1 < 7$

$\therefore \quad -7 < x-1$ and $x-1<7$

$-6 < x$ $\qquad x < 8$

$\therefore \quad -6 < x < 8$

or

2(a) $|x-1|<7$

$(x-1)^2 < (7)^2$

$x^2-2x+1<49$

$x^2-2x-48<0$

Solve: $x^2-2x-48=0$

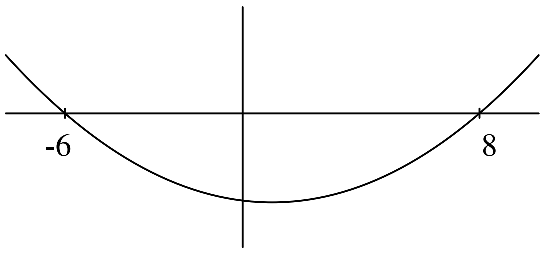
$(x+6)(x-8)=0$

$x+6=0$ or $x-8=0$

$x=-6$ $\qquad x=8$

$+x^2 \quad \Rightarrow$ U-shaped:

$f(x)<0$ when $-6<x<8$



or

2(a) $|x-1|<7$

$(x-1)^2<49$

$x^2-2x-48<0$

$(x-8)(x+6)<0$

case I: $(x-8)>0$ and $(x+6)<0$

$x>8$ and $x<-6$

not possible

case II: $(x-8)<0$ and $(x+6)>0$

$x<8$ and $x>-6$

$\therefore \quad -6<x<8$

Blunders (-3)

B1 upper limit.

B2 lower limit.

B3 expansion of $(x-1)^2$, once only.

B4 inequality sign.
B5 indices.
B6 factors once only.
B7 root formula, once only.
B8 deduction root from factor.
B9 incorrect range.
B10 answer not stated.

Slips (-1)
S1 numerical.

Attempts
A1 one inequality only.
A2 inequality signs ignored.

Part (b) **20(5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

2(b) The cubic equation $4x^3 + 10x^2 - 7x - 3 = 0$ has one integer root and two irrational roots. Express the irrational roots in simplest surd form.

Test	**5 marks**	**Att 2**
Linear Factor	**5 marks**	**Att 2**
Other Factor	**5 marks**	**Att 2**
Roots	**5 marks**	**Att 2**

2(b) $f(x) = 4x^3 + 10x^2 - 7x - 3$

Integral root must be ± 1, ± 3

$f(1): 4 + 10 - 7 - 3 \neq 0$

$f(-1): \quad \neq 0$

$f(3): 108 + 90 - 21 - 3 \neq 0$

$f(-3): -108 + 90 + 21 - 3 = 0$

$\Rightarrow x = -3$ is a root $\qquad \Rightarrow (x + 3)$ is a factor

$$\begin{array}{r|l} & 4x^2 - 2x - 1 \\ x+3 & 4x^3 + 10x^2 - 7x - 3 \\ & \underline{4x^3 + 12x^2} \\ & \quad -2x^2 - 7x \\ & \quad \underline{-2x^2 - 6x} \\ & \qquad -x - 3 \\ & \qquad \underline{-x - 3} \end{array}$$

So, need to solve: $4x^2 - 2x - 1 = 0$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2(4)} = \frac{2 \pm \sqrt{20}}{8} = \frac{2 \pm 2\sqrt{5}}{8} = \frac{1 \pm \sqrt{5}}{4}$$

Irrational roots: $\dfrac{1 + \sqrt{5}}{4}, \dfrac{1 - \sqrt{5}}{4}$

or

2(b) Finds root $x=-3$ as above, and continues as follows:

$x=-3$ is a root $\Rightarrow (x+3)$ is a factor

$\therefore$ other factor $=(4x^2+ax-1)$

$\therefore\ (x+3)(4x^2+ax-1) \quad = 4x^3+10x^2-7x-3$

$4x^3+12x^2+ax^2+3ax-x-3=4x^3+10x^2-7x-3$

$4x^3+(a+12)x^2+(3a-1)x-3=4x^3+10x^2-7x-3$

Equating coefficients:

(i) $a+12=10$ and/or (ii) $(3a-1)=-7$

$a=-2$ $\quad 3a=-6$

$a=-2$

$f(x)=(x+3)(4x^2-2x-1)=0$

Irrational roots: $\dfrac{1\pm\sqrt{5}}{4}$, as above.

Blunders (-3)

B1 indices.

B2 root formula, once only.

B3 not like to like..

B4 deduction factor from root or no factor.

Slips (-1)

S1 numerical.

S2 not changing sign in subtraction (Division).

S3 roots not in simplest form, once only.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

2(c)(i)

Let $f(x)=\dfrac{x^2+k^2}{mx}$, where k and m are constants and $m\neq 0$

(i) Show that $f(km)=f\left(\dfrac{k}{m}\right)$

(ii) a and b are real numbers such that $a\neq 0, b\neq 0$ and $a\neq b$.

Show that if $f(a)=f(b)$, then $ab=k^2$.

(i) $f(km)$	**5 marks**	**Att 2**
$f\left(\frac{k}{m}\right)$	**5 marks**	**Att 2**
(ii) $f(a)=f(b)$	**5 marks**	**Att 2**
ab	**5 marks**	**Att 2**

2(c)

$$f(x)=\frac{x^2+k^2}{mx},\ [\ k\ ,\ m\ \text{ constants}]$$

(i) show that $f(km)=f\left(\frac{k}{m}\right)$

$$f(km)=\frac{(km)^2+k^2}{m(km)}=\frac{k^2(m^2+1)}{k(m^2)}=\frac{k}{m^2}(m^2+1)$$

$$f\left(\frac{k}{m}\right)=\frac{\left(\frac{k}{m}\right)^2+k^2}{m\left(\frac{k}{m}\right)}=\frac{\frac{k^2}{m^2}+k^2}{k}=\frac{k^2+m^2k^2}{m^2k}$$

$$=\frac{k^2(1+m^2)}{k(m^2)}$$

$$=\frac{k}{m^2}(m^2+1)$$

$$\Rightarrow f(km)=f\left(\frac{k}{m}\right)$$

(ii) $f(a)=\dfrac{a^2+k^2}{ma}$

$f(b)=\dfrac{b^2+k^2}{mb}$

$$f(a)=f(b)\qquad\Rightarrow\qquad\frac{a^2+k^2}{ma}=\frac{b^2+k^2}{mb}$$

multiply across by *mab*:

$$b(a^2+k^2)=a(b^2+k^2)$$
$$a^2b+bk^2=ab^2+ak^2$$
$$a^2b-ab^2=ak^2-bh^2$$
$$ab(a-b)=k^2(a-b)$$
$$(a-b)\neq 0\qquad\Rightarrow ab\quad=k^2$$

Blunders (-3)

B1 indices

QUESTION 3

Part (a)	**10 (5, 5)marks**	**Att (2, 2)**
Part (b)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) A^3	**5 marks**	**Att 2**
A^{-1}	**5 marks**	**Att 2**

3(a) Given that $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, show that $A^3 = A^{-1}$.

Part (a) A^3	**5 marks**	**Att 2**
A^{-1}	**5 marks**	**Att 2**

3(a) $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$A^{-1} = \frac{1}{-1-0}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = (-1)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = A$$

$A^3 = A.A.A = A^2.A$

$A^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$A^3 = A^2.A = I.A = A = A^{-1}$

or

$A^{-1} = A$ as above, and:

$A^3 = A.A.A$

$= A^{-1}.A.A$

$= IA$

$= A^{-1}$

or

$A^{-1} = A$ as above, and:

$A^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^3$

$= \begin{pmatrix} (1)^3 & (0)^3 \\ (0)^3 & (-1)^3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= A$

Blunders (-3)

B1 formula inverse.

Slips (-1)

S1 each incorrect element.

S2 numerical.

Part (b)	20(5, 10, 5) marks	Att (2, 3, 2)

3(b) Solve the quadratic equation:

$$2iz^2 + (6+2i)z + (3-6i) = 0, \text{ where } i^2 = -1$$

Part (b)	**Values in formula**	**5 marks**	**Att 2**
	Evaluate $\sqrt{b^2-4ac}$	**10 marks**	**Att 3**
	Roots	**5 marks**	**Att 2**

3(b) Solve: $2iz^2 + (6+2i)z + (3-6i) = 0$

$$z = \frac{-(6+2i) \pm \sqrt{(6+2i)^2 - 4(2i)(3-6i)}}{2(2i)}$$

$$= \frac{-(6+2i) \pm \sqrt{36+24i+4i^2-24i+48i^2}}{4i}$$

$$= \frac{-(6+2i) \pm \sqrt{36-52}}{4i}$$

$$= \frac{-(6+2i) \pm \sqrt{-16}}{4i}$$

$$= \frac{-(6+2i) \pm 4i}{4i}$$

$$= \frac{-6-2i+4i}{4i} \quad \text{or} \quad \frac{-6-2i-4i}{4i}$$

$$= \frac{-6+2i}{4i} \quad \text{or} \quad \frac{-6-6i}{4i}$$

$$= \frac{-3+i}{2i} \quad \text{or} \quad \frac{-3-3i}{2i}$$

$$z_1 = \frac{-3+i}{2i} \cdot \frac{i}{i} = \frac{-3i+i^2}{2i^2} = \frac{-3i-1}{-2} = \frac{1+3i}{2}$$

$$z_2 = \frac{-3-3i}{2i} \cdot \frac{i}{i} = \frac{-3i-3i^2}{2i^2} = \frac{3-3i}{-2} = \frac{-3+3i}{2}$$

Blunders (-3)
B1 indices.
B2 *i*.
B3 expansion $(6+2i)^2$ once only.
B4 root formula, once only

Slips (-1)
S1 numerical.
S2 *i* in denominator

Attempts
A1 3 marks for $\sqrt{a+bi}$ and stops
A2 2 marks for $z = a+bi$ and stops.

Part (c) **20(5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

3(c) (i) $z = \cos\theta + i\sin\theta$. Use De Moivre's theorem to show that $z^n + \frac{1}{z^n} = 2\cos n\theta$, for $n \in \mathbf{N}$.
(ii) Expand $\left(z + \frac{1}{z}\right)^4$ and hence express $\cos^4\theta$ in terms of $\cos 4\theta$ and $\cos 2\theta$.

Part (c) (i) $\frac{1}{z^n}$	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**
(ii) Expansion	**5 marks**	**Att 2**
Express	**5 marks**	**Att 2**

3(c)(i)

$$z = \cos\theta + i\sin\theta$$
$$z^n = \cos n\theta + i\sin n\theta$$
$$\frac{1}{z^n} = \frac{1}{(\cos\theta + i\sin\theta)^n} = (\cos\theta + i\sin\theta)^{-n}$$
$$= \cos(-n\theta) + i\sin(-n\theta)$$
$$= \cos n\theta - i\sin n\theta$$
$$z^n + \frac{1}{z^n} = (\cos n\theta + i\sin n\theta) + (\cos n\theta - i\sin n\theta)$$
$$= 2\cos n\theta$$

3(c)(ii)

$$z + \frac{1}{z} = (\cos\theta + i\sin\theta) + (\cos\theta + i\sin\theta)^{-1}$$
$$= \cos\theta + i\sin\theta + \cos\theta - i\sin\theta$$
$$= 2\cos\theta$$

$$\left(z + \frac{1}{z}\right)^4 = z^4 + \binom{4}{1}z^3\left(\frac{1}{z}\right) + \binom{4}{2}z^2\left(\frac{1}{z}\right)^2 + \binom{4}{3}z\left(\frac{1}{z}\right)^3 + \left(\frac{1}{z}\right)^4$$
$$(2\cos\theta)^4 = z^4 + 4z^2 + 6 + 4\left(\frac{1}{z^2}\right) + \frac{1}{z^4}$$
$$16\cos^4\theta = \left(z^4 + \frac{1}{z^4}\right) + 4\left(z^2 + \frac{1}{z^2}\right) + 6$$
$$16\cos^4\theta = (2\cos 4\theta) + 4[2\cos 2\theta] + 6$$
$$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$$
$$\cos^4\theta = \frac{1}{16}[2\cos 4\theta + 8\cos 2\theta + 6]$$
$$= \frac{1}{8}[\cos 4\theta + 4\cos 2\theta + 3]$$

Blunders (-3)
B1 statement De Moivre, once only.
B2 application De Moivre.
B3 binomial expansion.
B4 *i*
B5 answer not in required form.
B6 indices.

Slips (-1)
S1 numerical

Worthless
W1 not using De Moivre.
W2 not using "hence" in part (ii).

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (15, 5) marks**	**Att (5, 2)**

Part (a) **10 marks** **Att 3**

4(a) Write the recurring decimal 0.636363…… as an infinite geometric series and hence as a fraction.

Part (a) **10 marks** **Att 3**

4(a)

$$0.6\dot{3} = 0.636363\ldots\ldots\ldots\ldots\ldots$$
$$= 0.63 + 0.0063 + 0.000063 + \ldots\ldots\ldots$$
$$= \frac{63}{100} + \frac{63}{10000} + \frac{63}{1000000} + \ldots\ldots.$$
$$\therefore \quad a = \frac{63}{100} \qquad r = \frac{1}{100}$$
$$S_\infty = \frac{a}{1-r}$$
$$= \frac{\frac{63}{100}}{1-\frac{1}{100}} = \frac{\left(\frac{63}{100}\right)}{\left(\frac{99}{100}\right)} = \frac{63}{99} = \frac{7}{11}$$

Blunders (-3)
B1 indices.
B2 formula for infinite series.
B3 incorrect a .
B4 incorrect r.
B5 not as infinite series.

Slips (-1)
S1 numerical.

Attempts
A1 correct answer with no work or by other method (i.e. not using geometric series).

Part (b)(i)	**10 (5, 5)**	**Att (2, 2)**

4(b)(i) The first three terms in the binomial expansion of $(1+kx)^n$ are $1-21x+189x^2$
Find the value of n and the value of k.

Part(b)(i) equations	**5 marks**	**Att 2**
values	**5 marks**	**Att 2**

4 (b)(i)

$$(1+kx)^n = 1+\binom{n}{1}(kx)+\binom{n}{2}(kx)^2+........$$

$$= 1+(nk)x+\frac{n(n-1)}{2!}\cdot k^2x^2+.........$$

$$= 1+(nk)x+\left[\frac{n(n-1).k^2}{2}\right]x^2+...........$$

$$= 1-21x+189x^2................................$$

[i]: $nk=-21$ [ii]: $\frac{n(n-1).k^2}{2}=189$

[i] $\Rightarrow$ $k=\frac{-21}{n}$

[ii] $\Rightarrow$ $n(n-1)k^2=378$

sub. in: $n(n-1)\left(\frac{-21}{n}\right)^2=378$

$(n^2-n)(441)=378n^2$

$441n^2-441n-378n^2=0$

$63n^2-441n=0$

$63n(n-7)=0$

$\Rightarrow n=0$ or $n=7$ $\therefore$ $n=7 \Rightarrow k=\frac{-21}{7}=-3$

$n=7;\quad k=-3$

* Since must be integers, accept correct values by observation from $nk=-21$, with verification.

Blunders (-3)

B1 errors in binomial expansion, once only.

B2 $\binom{n}{r}$

B3 indices.

B4 not like to like

B5 factors

B6 value from factor.

B7 second value not found, having found first.

Slips (-1)

S1 numerical.

Part (b) (ii) **(5, 5) marks** **Att (2, 2)**

4 (b) (ii)	A sequence is defined by $u_n = (2-n)2^{n-1}$. Show that $u_{n+2} - 4u_{n+1} + 4u_n = 0$, for all $n \in \mathbb{N}$.

Part (b) (ii) Terms simplified **5 marks** **Att 2**
Show **5 marks** **Att 2**

4 (b)(ii)

$$u_n = (2-n)2^{n-1}$$
$$u_{n+1} = [2-(n+1)]2^{(n+1)-1} = (1-n)2^n$$
$$u_{n+2} = [2-(n+2)]2^{(n+2)-1} = (-n)2^{n+1}$$
$$\begin{aligned} u_{n+2} - 4u_{n+1} + 4u_n &= \left(-n.2^{n+1}\right) - 4\left[(1-n)2^n\right] + 4\left[(2-n)2^{n-1}\right] \\ &= -n.2^{n+1} - \left(2^2\right)\left(2^n\right)(1-n) + 2^2\left(2^{n-1}\right)(2-n) \\ &= -n.2^{n+1} - 2^{n+2} + 2n.2^{n+1} + 2.2^{n+1} - n.2^{n+1} \\ &= 2.2^{n+1} - 2^{n+2} \\ &= 2.2^{n+1} - 2.2^{n+1} = 0 \end{aligned}$$

or

4 (b)(ii)

$$u_n = (2-n)2^{n-1} = 2^n - n.2^{n-1} = 2^n - \frac{n}{2}\left(2^n\right)$$
$$u_{n+1} = [2-(n+1)]2^{(n+1)-1} = (1-n)2^n$$
$$u_{n+2} = [2-(n+2)]2^{(n+2)-1} = (-n)2^{n+1} = -2n.2^n$$

Let $a = 2^n$

$$\begin{aligned} \therefore u_{n+2} - 4u_{n+1} + 4u_n &= -2na - 4(1-n)a + 4\left[a - \tfrac{na}{2}\right] \\ &= -2na - 4a + 4na + 4a - 2na \\ &= 0 \end{aligned}$$

Blunders (-3)
B1 indices.
B2 factors.

Slips (-1)
S1 numerical.

Attempts
A1 must do some correct relevant work with indices in “show”.

Part (c)(i)	20(15, 5) marks	Att (5, 2)

4 (c) (i) Show that $\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}}$, where a and b are real numbers.

(ii) The lengths of the sides of a right-angled triangle are a, b and c, where c is the length of the hypotenuse.
Using the result from part **(i)**, or otherwise, show that $a+b \le c\sqrt{2}$.

Part (c)(i)	15 marks	Att 5
Part (c) (ii)	5 marks	Att 2

4(c)(i) $\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}}$

case: $(a+b)$ positive: $\Leftrightarrow \left(\frac{a+b}{2}\right)^2 \le \frac{a^2+b^2}{2}$

$$\frac{a^2+2ab+b^2}{4} \le \frac{a^2+b^2}{2}$$
$$a^2+2ab+b^2 \le 2a^2+2b^2$$
$$0 \le a^2-2ab+b^2$$
$$0 \le (a-b)^2$$

$\Rightarrow$ True when $(a+b)$ positive.

case: $(a+b)$ negative: $(a+b)<0 \Rightarrow \frac{(a+b)}{2}<0$

$$\Rightarrow \frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}}, \quad \text{since } \sqrt{x}>0 \text{ always.}$$

$\Rightarrow$ True when $(a+b)$ negative.

or

4(c)(i) $(a-b)^2 \ge 0$ for all $a,b \in \mathbf{R}$.

$$a^2-2ab+b^2 \ge 0$$
$$(a^2-2ab+b^2)+(a^2+b^2) \ge (a^2+b^2)$$
$$2a^2+2b^2 \ge a^2+2ab+b^2$$
$$2(a^2+b^2) \ge (a+b)^2$$

divide across by 4:

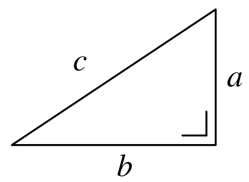
$$\frac{a^2+b^2}{2} \ge \frac{(a+b)^2}{4}$$
$$\frac{a^2+b^2}{2} \ge \left(\frac{a+b}{2}\right)^2$$
$$\sqrt{\frac{a^2+b^2}{2}} \ge \sqrt{\left(\frac{a+b}{2}\right)^2} \ge \frac{a+b}{2}$$

4(c)(ii) From **(i)** above, $\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}} = \sqrt{\frac{c^2}{2}} = \frac{c}{\sqrt{2}}$.

$$\frac{a+b}{2} \le \frac{c}{\sqrt{2}}$$

$$a+b \le \tfrac{2c}{\sqrt{2}}$$

$$a+b \le c\sqrt{2}$$



$a^2 + b^2 = c^2$

Blunders (-3)

B1 indices

B2 inequality sign.

B3 deduction.

B4 *a* and *b* both positive.

B5 expansion $(a-b)^2$.

B6 right angled triangle.

Slips (-1)

S1 numerical.

Worthless

W1 particular values for *a* and *b.*

QUESTION 5

Part (a)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**
Part (b)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**15(5, 5, 5) marks**	**Att (2, 2, 2)**

5(a) Solve for x: $\sqrt{10-x}=4-x$

Part 5(a) Quadratic	**5 marks**	**Att 2**
Factors	**5 marks**	**Att 2**
Solution	**5 marks**	**Att 2**

5(a)

$$\sqrt{10-x}=4-x$$
$$10-x=(4-x)^2$$
$$10-x=16-8x+x^2$$
$$0=x^2-7x+6$$
$$0=(x-1)(x-6)$$
$$\Rightarrow x-1=0 \quad \text{or} \quad x-6=0$$
$$x=1 \qquad x=6$$

Test : $x=1$ L.H.S.: $\sqrt{10-x}=\sqrt{10-1}=\sqrt{9}=3$

R.H.S.: $4-x=4-1=3$ $\therefore$ L.H.S.=R.H.S.

$x=6$ L.H.S.: $\sqrt{10-x}=\sqrt{10-6}=\sqrt{4}=2$

R.H.S.: $4-x=4-6=-2$ $\therefore$ L.H.S.$\neq$ R.H.S.

$\therefore$ 6 is not a solution.

Ans: $x=1$.

Blunders (-3)

B1 indices.
B2 expansion $(4-x)^2$ once only.
B3 factors.
B4 root formula once only.
B5 deduction values from factors.

Slips (-1)

S1 numerical.
S2 excess value.

Attempts

A1 $x=1$ and no other works merits 2 marks.

Part (b)	**15(5, 5, 5) marks**	**Att (2, 2, 2)**

5(b) Prove by induction that $\sum_{r=1}^{n}(3r-2)=\frac{n}{2}(3n-1)$

Part (b) ***P(1)***	**5 marks**	**Att 2**
Assume	**5 marks**	**Att 2**
P(k + 1)	**5 marks**	**Att 2**

5(b) $\sum_{r=1}^{n}(3r-2)=\frac{n}{2}(3n-1)$

Test $n=1: u_1 = 3(1)-2=1$

$\frac{n}{2}(3n-1)=\frac{1}{2}(3-1)=\frac{1}{2}(2)=1$

$\therefore$ True for $n=1$ $P(1)$

Assume true for $n=k$

$S_k = \frac{k}{2}(3k-1)$ $P(k)$

To prove: $S_{k+1}=\frac{(k+1)}{2}[3(k+1)-1]$

$=\frac{k+1}{2}[3k+2]$

$=\frac{1}{2}(k+1)(3k+2)$

Proof: $S_{k+1}=S_k+U_{k+1}$

$=\frac{k}{2}(3k-1)+[3(k+1)-2]$

$=\frac{k}{2}(3k-1)+(3k+1)$

$=\frac{3k^2-k+6k+2}{2}$

$=\frac{1}{2}[3k^2+5k+2]$

$=\frac{1}{2}[(k+1)(3k+2)]$ $P(k+1)$

So, $P(k+1)$ true whenever $P(k)$ true. Since $P(1)$ true, then by induction $P(n)$ true for all positive integers n $(n\in \mathbf{N}, n\geq 1)$.

Blunders (-3)
B1 indices.
B2 $n\neq 1$ (must prove $n=1$ not enough to say true for $n=1$)
B3 factors.

Slips (-1)
S1 numerical.

Part (c)	20(5, 5, 5, 5)marks	Att (2, 2, 2, 2)

5(c) (i) Show that $\dfrac{1}{\log_a b} = \log_b a$, where $a, b > 0$ and $a, b \neq 1$

(ii) Show that $\dfrac{1}{\log_2 c} + \dfrac{1}{\log_3 c} + \dfrac{1}{\log_4 c} + \text{........} + \dfrac{1}{\log_r c} = \dfrac{1}{\log_{r!} c}$, where $c > 0, c \neq 1$.

Part (c) (i)	**5 marks**	**Att 2**
(ii) log$_x$c to a new base	**5 marks**	**Att 2**
log(2.3.4…r)	**5 marks**	**Att 2**
completion	**5 marks**	**Att 2**

5(c)(i) $\log_b a = \dfrac{\log_a a}{\log_a b} = \dfrac{1}{\log_a b}$

5(c)(ii) From (i): $\log_c 2 = \dfrac{1}{\log_2 c}$

Similarly $\log_c 3 = \dfrac{1}{\log_3 c}, \quad \cdots \quad , \log_c r = \dfrac{1}{\log_r c}$

$$\therefore \quad \frac{1}{\log_2 c} + \frac{1}{\log_3 c} + \frac{1}{\log_4 c} + \text{........} + \frac{1}{\log_r c}$$
$$= \log_c 2 + \log_c 3 + \log_c 4 + \text{.......} + \log_c r$$
$$= \log_c (2.3.4\text{..............}r)$$
$$= \log_c (r!)$$
$$= \frac{1}{\log_{r!} c}.$$

or

5(c)(ii) $\log_2 c = \dfrac{\log_{r!} c}{\log_{r!} 2} \Rightarrow \dfrac{1}{\log_2 c} = \dfrac{\log_{r!} 2}{\log_r c}$

Similarly, $\dfrac{1}{\log_3 c} = \dfrac{\log_{r!} 3}{\log_r c}$, etc.

$$\therefore \quad \frac{1}{\log_2 c} + \frac{1}{\log_3 c} + \frac{1}{\log_4 c} + \text{........} + \frac{1}{\log_r c}$$
$$= \frac{\log_{r!} 2}{\log_{r!} c} + \frac{\log_{r!} 3}{\log_{r!} c} + \frac{\log_{r!} 4}{\log_{r!} c} + \text{...........} + \frac{\log_{r!} r}{\log_{r!} c}$$
$$= \frac{\log_{r!} 2 + \log_{r!} 3 + \log_{r!} 4 + \text{...........}\log_{r!} r}{\log_{r!} c}$$
$$= \frac{\log_{r!}(2.3.4\text{.........}r)}{\log_{r!} c}$$
$$= \frac{\log_{r!}(r!)}{\log_{r!} c}$$
$$= \frac{1}{\log_{r!} c}$$

or

5(c)(ii) $\log_2 c = \dfrac{\log_{10} c}{\log_{10} 2}$, $\log_3 c = \dfrac{\log_{10} c}{\log_{10} 3}$, etc.

$$\therefore \quad \frac{1}{\log_2 c} + \frac{1}{\log_3 c} + \frac{1}{\log_4 c} + \;........ + \frac{1}{\log_r c}$$

$$= \frac{\log_{10} 2}{\log_{10} c} + \frac{\log_{10} 3}{\log_{10} c} + \frac{\log_{10} 4}{\log_{10} c} + ... + \frac{\log_{10} r}{\log_{10} c}$$

$$= \frac{\log_{10} 2 + \log_{10} 3 + \log_{10} 4 + ...\log_{10} r}{\log_{10} c}$$

$$= \frac{\log_{10}(2.3.4.........r)}{\log_{10} c}$$

$$= \frac{\log_{10}(r!)}{\log_{10}(c)}$$

$$= \log_c r!$$

$$= \frac{1}{\log_{r!} c}$$

Blunders (-3)

B1 log laws.

B2 factorial.

B3 change of base.

Worthless

W1 no change of base.

QUESTION 6

Part (a)	**10 (5, 5)marks**	**-**
Part (b)	**20 marks**	**-**
Part (c)	**20 (10, 5, 5) marks**	**-**

Note: The marking of Question 6 is not based on slips, blunders and attempts. In the case of each part, descriptions or typical examples of work meriting particular numbers of marks are given. The mark awarded must be one of the marks indicated. For example, in part **(a)(i)**, descriptions are given for work meriting 0, 2 or 5 marks. It is therefore not permissible to award, 1, 3 or 4 marks for this part.

Part (a) **10 (5, 5) marks** **-**

6 (a) Differentiate with respect to x

(i) $(1+7x)^3$ **(ii)** $\sin^{-1}\left(\dfrac{x}{5}\right).$

Part (a) (i) **5 marks** **-**

6(a)(i) $\dfrac{dy}{dx} = 3(1+7x)^2.(7) = 21(1+7x)^2.$

5 marks: correct derivative in any form. (e.g. middle step above is acceptable, as is expansion followed by correct differentiation, unsimplified).
2 marks: differentiates with one or more errors, provided at least some aspect correct.
0 marks: no correct differentiation done. (e.g. integrates or expands the given expression).

Part (a) (ii) **5 marks** **-**

6(a)(ii) $y = \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\left(\frac{x}{a}\right) \Rightarrow a = 5$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}} = \frac{1}{\sqrt{25 - x^2}}$$

or

6(a)(ii) $y = \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}[f(x)]$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - f(x)^2}}.f'(x) = \frac{1}{\sqrt{1 - \left(\frac{x}{5}\right)^2}}.\left(\frac{1}{5}\right)$$

$$= \frac{1}{5\sqrt{\frac{25-x^2}{25}}}$$

$$= \frac{1}{\sqrt{25 - x^2}}$$

or

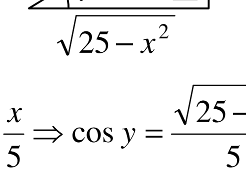
6(a)(ii) $y = \sin^{-1}\left(\frac{x}{5}\right) \;\Rightarrow\; \sin y = \frac{x}{5}$

$$\therefore\ \cos y \tfrac{dy}{dx} = \tfrac{1}{5}$$

$$\therefore\ \tfrac{dy}{dx} = \tfrac{1}{\cos y}\cdot\tfrac{1}{5}$$

$$= \frac{1}{\frac{\sqrt{25-x^2}}{5}\cdot 5}$$

$$= \frac{1}{\sqrt{25 - x^2}}$$



$$\sin y = \frac{x}{5} \Rightarrow \cos y = \frac{\sqrt{25 - x^2}}{5}$$

5 marks: correct derivative in terms of x, simplified or otherwise.
2 marks: differentiates with at least some aspect correct; fails to give answer in terms of x.
0 marks: no correct differentiation done. (e.g. integrates or rearranges the given expression, or gives only the first step in the second method above)

Part (b) **20 marks** -

6 (b) Let $y = \dfrac{1-\cos x}{1+\cos x}$.

Show that $\dfrac{dy}{dx} = t + t^3$, where $t = \tan\dfrac{x}{2}$.

Part (b) **20 marks** -

6(b)(ii)

$$y = \frac{1-\cos x}{1+\cos x} = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{(1+\cos x)(\sin x)-(1-\cos x)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\sin x + \sin x\cos x + \sin x - \sin x\cos x}{(1+\cos x)^2}$$

$$= \frac{2\sin x}{(1+\cos x)^2}$$

$$= \frac{2\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\left(2\cos^2\frac{x}{2}\right)^2}$$

$$= \frac{4\sin\frac{x}{2}\cos\frac{x}{2}}{4\cos^4\frac{x}{2}}$$

$$= \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} \cdot \frac{1}{\cos^2\frac{x}{2}}$$

$$= \tan\tfrac{x}{2}\left(\sec^2\tfrac{x}{2}\right)$$

$$= \tan\tfrac{x}{2}\left(1+\tan^2\tfrac{x}{2}\right)$$

$$= t\left(1+t^2\right)$$

$$= t + t^3$$

or

6(b)(ii)

$$\frac{dy}{dx} = \frac{2\sin x}{(1+\cos x)^2} = \frac{2\left[\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]}{\left[1+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]^2}$$

$$= \frac{\frac{4t}{1+t^2}}{\left[\frac{(1+t^2)+(1-t^2)}{1+t^2}\right]^2}$$

$$= \frac{4t}{(1+t^2)\left[\frac{2}{1+t^2}\right]^2}$$

$$= t\left(1+t^2\right)$$

$$= t + t^3$$

or

6(b)(ii)

$$\frac{dy}{dx}=\frac{2\sin x}{(1+\cos x)^2}=\frac{2\left[\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]}{\left[2\cos^2\frac{x}{2}\right]^2}$$

$$=\frac{4\tan\frac{x}{2}}{\left(1+\tan^2\frac{x}{2}\right).4\cos^2\frac{x}{2}.\cos^2\frac{x}{2}}$$

$$=\frac{4t}{\sec^2\frac{x}{2}.4\frac{1}{\sec^2\frac{x}{2}}.\frac{1}{\sec^2\frac{x}{2}}}$$

$$=t.\left(\sec^2\tfrac{x}{2}\right)$$

$$=t\left(1+\tan^2\tfrac{x}{2}\right)$$

$$=t\left(1+t^2\right)$$

$$=t+t^3$$

or

6(b)(ii)

$$y=\frac{1-\cos x}{1+\cos x}\qquad=\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}\qquad=\frac{\left(1+t^2\right)-\left(1-t^2\right)}{\left(1+t^2\right)+\left(1-t^2\right)}$$

$$y=\frac{2t^2}{2}=t^2$$

$$y=\left(\tan\tfrac{x}{2}\right)^2$$

$$\frac{dy}{dx}=2\left(\tan\tfrac{x}{2}\right)^1.\left(\sec^2\tfrac{x}{2}\right).\tfrac{1}{2}$$

$$=\left(\tan\tfrac{x}{2}\right)\left(1+\tan^2\tfrac{x}{2}\right)$$

$$=t\left(1+t^2\right)$$

$$=t+t^3$$

or

$$y=t^2$$

$$\frac{dy}{dx}=2t\frac{dt}{dx}$$

$$=2t\left[\frac{1}{2}\sec^2\frac{x}{2}\right]$$

$$=2t\left[\frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)\right]$$

$$=t\left(1+t^2\right)$$

$$=t+t^3$$

20 marks: fully correct solution.

17 marks: correct expression for $\frac{dy}{dx}$ in terms of t alone, but not simplified to required form **or** solution with one or two non-critical errors, simplified fully.
[critical error = one that significantly alters the nature or complexity of the task].

14 marks: correct expression for $\frac{dy}{dx}$ in terms of x , simplified **or**
correctly establishes that $y=t^2$ **or** that $\frac{dt}{dx}=\frac{1}{2}\left(1+t^2\right)$

7 marks: correct or almost-correct expression for $\frac{dy}{dx}$ in terms of x **or**
correct expression for $\frac{dt}{dx}$ in terms of x **or**
correct but unsimplified expression for y in terms of t or tan $\frac{x}{2}$

0 marks: no relevant work.

Part (c) 20 (10, 5, 5) marks -

Part (c) (i) 10 marks -

6 (c) The equation of a curve is $y=\frac{x}{x-1}$, where $x \neq 1$.

(i) Show that the curve has no local maximum or local minimum point.

6 (c) (i)

$$y=\frac{x}{x-1}$$

$$\frac{dy}{dx}=\frac{(x-1)(1)-(x)(1)}{(x-1)^2}$$

$$=\frac{x-1-x}{(x-1)^2}$$

$$=\frac{-1}{(x-1)^2}\neq 0$$

$\therefore$ No local max/local min

10 marks: Correct solution, including assertion that derivative $\neq 0$ or <0 or similar conclusion.
7 marks: Correct derivative.
3 marks: Substantial error(s) in differentiation.
0 marks: No relevant work

Part (c) (ii) 5 marks -

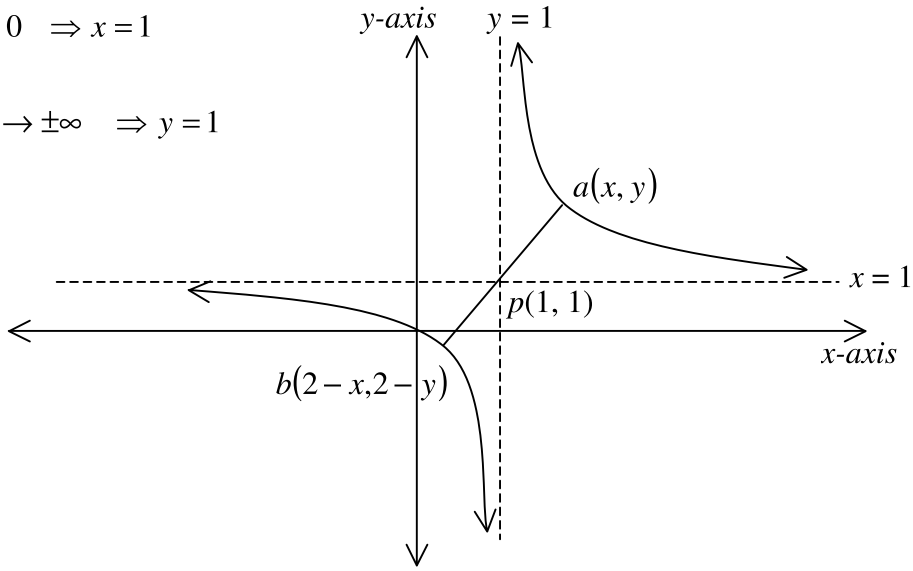
6 (c) (ii) Write down the equations of the asymptotes and hence sketch the curve.

6 (c) (ii)

Vertical asymptote: $x-1=0 \Rightarrow x=1$

Horizontal asymptote:

$$y=\frac{x}{x-1}=\frac{1}{1-\frac{1}{x}}\to 1 \text{ as } x\to\pm\infty \quad \Rightarrow y=1$$



5 marks: Correct solution, (equations of both asymptotes, and sketch).
2 marks: One or two equations correct, or sketch of correct form.
0 marks: No significant work of merit.

Part (c) (iii) **5 marks** **-**

6 (c) (iii)	Show that the curve is its own image under the symmetry in the point of intersection of the asymptotes.

6 (c) (iii) $S_p(a) = b$ *p*: point of intersection of asymptotes = (1, 1).

$a = (x, y) \Rightarrow b = (2-x, 2-y)$

Test to see if $b(2-x, 2-y)$ is on curve $y = \dfrac{x}{x-1}$:

$$(2-y) = \frac{(2-x)}{(2-x)-1}$$

$$2-y = \frac{2-x}{1-x}$$

$$2 - \frac{2-x}{1-x} = y$$

$$\Leftrightarrow y = \frac{2(1-x)-(2-x)}{1-x} = \frac{-x}{1-x}$$

$$\Leftrightarrow y = \frac{x}{x-1}$$ (i.e. *b* is on the curve if and only if *a* is.)

or

6 (c) (iii) $p(1, 1)$: point of intersection of asymptotes

$a\left(x, \dfrac{x}{x-1}\right)$ is on curve $y = \dfrac{x}{x-1}$

$S_p(a) = b \Rightarrow b\left[2-x, 2-\frac{x}{x-1}\right]$

$b\left(2-x, \frac{2(x-1)-x}{x-1}\right)$

$b\left(2-x, \frac{x-2}{x-1}\right)$

Symmetry if $b\left(2-x, \frac{x-2}{x-1}\right) \in y = \frac{x}{x-1}$:

$$\frac{(2-x)}{(2-x)-1} = \frac{2-x}{1-x} = \frac{x-2}{x-1}.$$

5 marks: Fully correct solution.
2 marks: Correctly finds image of general point on the curve, or
Identifies general point on the curve in terms of one variable, ***or***
Fully or partially works a particular case, ***or***
Identifies (1, 1) as the point of intersection of the asymptotes.
0 marks: no relevant work.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 marks**	**Att 3**

7 (a) Find from first principles the derivative of x^2 with respect to *x*.

Part (a)	**10 marks**	**Att 3**

7(a)

$$f(x) = x^2$$
$$f(x+h) = (x+h)^2$$
$$f(x+h) - f(x) = (x^2 + 2hx + h^2) - x^2$$
$$f(x+h) - f(x) = 2hx + h^2$$
$$\frac{f(x+h) - f(x)}{h} = 2x + h$$
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2x$$

or

7(a)

$$y = x^2$$
$$y + \Delta y = (x + \Delta x)^2$$
$$\Delta y = (x + \Delta x)^2 - x^2$$
$$= x^2 + 2x\Delta x + \Delta x^2 - x^2$$
$$= 2x.\Delta x + \Delta x^2$$
$$\frac{\Delta y}{\Delta x} = 2x + \Delta x$$
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x$$

Blunders (-3)

B1 expansion of $(a+b)^2$ once only.

B2 indices.

B3 no limit shown or implied, or no indication $\to 0$.

B4 $x.\Delta x = \Delta x^2$

Worthless

W1 not from 1st principles.

Part (b) (i)	**10 (5, 5) marks**	**Att (2, 2)**

7 (b) (i) The parametric equations of a curve are:

$$x = 8 + \ln t^2$$

$$y = \ln(2 + t^2), \text{ where } t > 0.$$

Find $\frac{dy}{dx}$ in terms of *t* and calculate its value at $t = \sqrt{2}$.

Part (b)(i) $\frac{dx}{dt}, \frac{dy}{dt}$	**5 marks**	**Att 2**
value	**5 marks**	**Att 2**

7 (b) (i)

$$x = 8 + \ln t^2 \qquad\qquad y = \ln(2 + t^2),$$

$$x = 8 + 2\ln t \qquad\qquad \frac{dy}{dt} = \frac{1}{2 + t^2}.2t$$

$$\frac{dx}{dt} = 2\left(\frac{1}{t}\right) = \frac{2}{t} \qquad\qquad = \frac{2t}{2 + t^2}$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{2t}{2+t^2}\right)}{\left(\frac{2}{t}\right)} = \frac{t^2}{2 + t^2}$$

At $t = \sqrt{2}$: $\quad t^2 = 2 \quad \Rightarrow \frac{dy}{dx} = \frac{t^2}{2 + t^2} = \frac{2}{2 + 2} = \frac{1}{2}$

* $f'(x)$ must be expressed as a function of *t* for second 5 marks.

Blunders (-3)

B1 differentiation.
B2 logs.
B3 indices
B4 definition of $\frac{dy}{dx}$
B5 incorrect value or no value.

Attempts

A1 error in differentiation formula.

Worthless

W1 integration.
W2 no differentiation.

Part (b) (ii)	10 (5, 5) marks	Att (2, 2)

7 (b) (ii)	Find the slope of the tangent to the curve $xy^2 + y = 6$ at the point (1, 2).

Part (b)(ii) Differentiation	5 marks	Att 2
Slope	5 marks	Att 2

7 (b) (ii)

$$xy^2 + y = 6$$

$$\left(x.2y\frac{dy}{dx} + y^2\right) + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2xy+1) = -y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{2xy+1}$$

At $p(1, 2)$ $\quad x = 1$ and $y = 2$

$$m = \frac{dy}{dx} = \frac{-(2)^2}{2(1)(2)+1} = \frac{-4}{5}$$

Blunders (-3)

B1 differentiation.

B2 indices.

B3 incorrect value of *x* or no value of *x*.

B4 incorrect value of *y* or no value of *y*.

Slips (-1)

S1 numerical.

Attempts

A1 error in differentiation formula.

A2 $\frac{dy}{dx} = 2xy\frac{dy}{dx} + y^2 + \frac{dy}{dx}$ and uses all three $\left(\frac{dy}{dx}\right)$ terms.

Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

7 (c) (i) Write down a quadratic equation whose roots are $\pm\sqrt{k}$.

(ii) Hence use the Newton-Raphson method to show that the rule

$$u_{n+1} = \frac{(u_n)^2 + k}{2u_n}$$

can be used to find increasingly accurate approximations for $\sqrt{k}$.

(iii) Using the above rule and taking $\frac{3}{2}$ as the first approximation for $\sqrt{3}$, find the third approximation, as a fraction.

Part (c) (i)	**5 marks**	**Att 2**
(ii) Newton-Raphson	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**
(iii)	**5 marks**	**Att 2**

7(c) (i) Roots $\pm\sqrt{k} \Rightarrow$ Equation: $x^2 - k = 0$.

7(c)(ii) Equation: $x^2 = k$ or $x^2 - k = 0$, so let $f(x) = x^2 - k$.

$$\therefore \quad f(u_n) = u_n^{\ 2} - k$$
$$f'(u_n) = 2u_n$$

Newton-Raphson:
$$u_{n+1} = u_n - \frac{f(u_n)}{f'(u_n)}$$
$$= u_n - \frac{u_n^{\ 2} - k}{2u_n}$$
$$= \frac{2u_n^{\ 2} - (u_n^{\ 2} - k)}{2u_n}$$
$$u_{n+1} = \frac{u_n^{\ 2} + k}{2u_n}$$

Hence the given rule is the Newton-Raphson method applied to $f(x) = x^2 - k$. Thus it can be used with a suitable initial value to find increasingly accurate approximations for $\sqrt{k}$.

7(c)(iii) $u_2 = \frac{u_1^{\ 2} + k}{2u_1}$ $\qquad k = 3; \quad u_1 = \frac{3}{2}$

$$u_2 = \frac{\left(\frac{3}{2}\right)^2 + 3}{2\left(\frac{3}{2}\right)} = \frac{\frac{9}{4} + 3}{3} = \frac{21}{12} = \frac{7}{4}$$
$$u_3 = \frac{(u_2)^2 + k}{2u_2} = \frac{\left(\frac{7}{4}\right)^2 + 3}{2\left(\frac{7}{4}\right)} = \frac{\frac{49}{16} + 3}{\frac{7}{2}} = \frac{\left(\frac{97}{16}\right)}{\left(\frac{7}{2}\right)} = \frac{97}{56}$$

Blunders (-3)
B1 equation
B2 Newton-Raphson formula; apply once only to second 5 marks in **(ii)** or to 5 marks in **(iii)**.
B3 differentiation.
B4 indices.
B5 $k \neq 3$.
B6 $U_1 \neq \frac{3}{2}$, once only
B7 U_3 not found.

Slips (-1)
S1 numerical.
S2 not as fraction.

Misreadings (-1)
M1 takes “above rule” in **c(iii)** to mean “Newton-Raphson method” and uses this in **(iii)**.

QUESTION 8

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

8 (a) Find **(i)** $\int (2+x^3)dx$ **(ii)** $\int e^{3x}\,dx.$

Part (i)	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**

8 (a) **(i)** $\int (2+x^3)dx = 2x + \frac{x^4}{4} + c$

(ii) $\int e^{3x}\,dx = \frac{e^{3x}}{3} + c$

* If c shown once, then no penalty

Blunders (-3)

B1 integration.

B2 no 'c' (Penalise 1st integration)

B3 indices.

Attempts

A1 anything + c.

Worthless

W1 differentiation instead of integration.

Part (b)	**20 (10, 10) marks**	**Att (3, 3)**

8 (b) (i) Evaluate $\int_1^4 \frac{2x+1}{x^2+x+1}\,dx$.

(ii) Evaluate $\int_0^{\frac{\pi}{8}} \sin^2 2\theta\, d\theta.$

Part (b) (i)	**10 marks**	**Att 3**
(ii)	**10 marks**	**Att 3**

8(b)(i) $\int_1^4 \frac{2x+1}{x^2+x+1}\,dx$

Let $u = x^2+x+1$

$\frac{du}{dx} = 2x+1$

$du = (2x+1)dx$

$= \int \frac{(2x+1)dx}{(x^2+x+1)}$

$= \int \frac{du}{u} \quad = \ln u$

$= \ln(x^2+x+1)\,\Big]_1^4 = \ln(16+4+1) - \ln(1+1+1) \quad = \ln\frac{21}{3} \quad = \ln 7$

8(b)(ii) $\displaystyle\int_0^{\frac{\pi}{8}} \sin^2 2\theta \, d\theta. = \frac{1}{2}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{8}}$ [Tables page 42]

$$= \frac{1}{2}\left[\left(\frac{\pi}{8} - \frac{\sin\frac{4\pi}{8}}{4}\right) - (0-0)\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{8} - \frac{1}{4}\right)$$

$$= \frac{\pi}{16} - \frac{1}{8}$$

or

8(b)(ii) $\displaystyle\int_0^{\frac{\pi}{8}} \sin^2 2\theta \, d\theta$

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\Rightarrow \sin^2 2\theta = \frac{1}{2}(1-\cos 4\theta)$$

$$= \int \frac{1}{2}(1-\cos 4\theta)\,d\theta$$

$$= \frac{1}{2}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{8}}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{8} - \frac{\sin\frac{4\pi}{8}}{4}\right) - (0-0)\right] \quad = \frac{1}{2}\left(\frac{\pi}{8} - \frac{1}{4}\right) \quad = \frac{\pi}{16} - \frac{1}{8}$$

Blunders (-3)
B1 integration.
B2 indices.
B3 limits.
B4 no limits.
B5 incorrect order in applying limits.
B6 not calculating substituted limits.
B7 not changing limits.
B8 differentiation.
B9 trig formula.

Slips (-1)
S1 numerical.
S2 trig value.

Worthless
W1 differentiation instead of integration except where other work merits attempt.

Note: Incorrect substitution and unable to finish yields attempt at most.
Note: (-3) is maximum deduction when evaluating limits

Note: In **8(b)(ii)**, do not penalise $\frac{\pi}{16} = 11.25^\circ$, etc.

Part (c)(i) **10 marks** **Att 3**

8 (c) (i) Evaluate $\displaystyle\int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx$.

Part (c)(i) **10 marks** **Att 3**

8 (c) (i) $\displaystyle\int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx$

$$3+2x-x^2$$
$$=4-\left(x^2-2x+1\right)$$
$$=(2)^2-(x-1)^2$$

$$\int \frac{dx}{\sqrt{3+2x-x^2}} = \int \frac{dx}{\sqrt{2^2-(x-1)^2}}$$

$$\int \frac{du}{\sqrt{2^2-u^2}} \qquad \left[\text{Let } u = x-1 \Rightarrow \frac{du}{dx}=1 \Rightarrow du = dx\right]$$

$$=\sin^{-1}\left(\frac{u}{2}\right)$$

$$=\sin^{-1}\left(\tfrac{x-1}{2}\right)\Big]_1^2$$

$$=\sin^{-1}\left(\frac{1}{2}\right)-\sin^{-1}(0) \quad = \frac{\pi}{6}-0 \quad = \frac{\pi}{6}$$

or

8 (c) (i) $\displaystyle\int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx$

$$3+2x-x^2$$
$$=4-\left(1-2x+x^2\right)$$
$$=(2)^2-(1-x)^2$$

$$\int \frac{dx}{\sqrt{3+2x-x^2}} = \int \frac{dx}{\sqrt{2^2-(1-x)^2}}$$

$$\int \frac{-dw}{\sqrt{2^2-w^2}} \qquad \left[\text{Let } w = 1-x \Rightarrow \frac{dw}{dx}=-1 \Rightarrow -dw = dx\right]$$

$$=-\sin^{-1}\left(\frac{w}{2}\right)$$

$$=-\sin^{-1}\left(\tfrac{1-x}{2}\right)\Big]_1^2$$

$$=-\left[\sin^{-1}\left(\frac{-1}{2}\right)-\sin^{-1}(0)\right] \quad = -\left[\frac{-\pi}{6}-0\right] \quad = \frac{\pi}{6}$$

Blunders (-3)
B1 integration
B2 completing square once only.
B3 limits
B4 no limits
B5 incorrect order in applying limits
B6 not calculating substituted limits
B7 not changing limits.
B8 differentiation.

Slips (-1)
S1 numerical
S2 trig value.

Worthless:
W1 no effort at completing square
W2 differentiation instead of integration except where other work merits attempt.
W3 puts $u = 3 + 2x - x^2$

Note: Incorrect substitution and unable to finish yields attempt at most.
Note: (-3) is maximum deduction when evaluating limits

Part (c) (ii) **10 marks** **Att 3**

8 (c) (ii) Use integration methods to derive a formula for the volume of a cone.

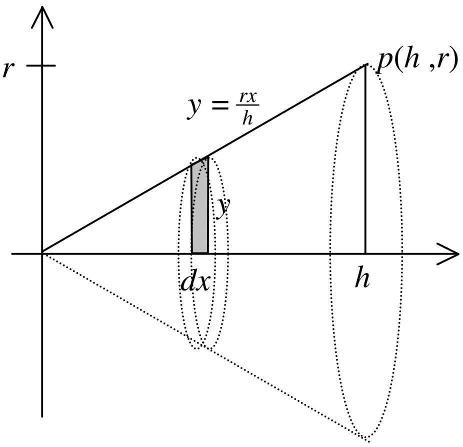
Part (c)(ii) **10 marks** **Att 3**

8 (c) (ii) Vol of cone, with height $= h$, and base-radius $= r$

Equation *op*: slope $= \frac{r}{h}$; through (0, 0) $\Rightarrow \quad y = \frac{r}{h}(x)$

$$V = \int_0^h \pi y^2 dx \quad = \pi \int_0^h \left(\frac{rx}{h}\right)^2 dx \quad = \frac{\pi r^2}{h^2} \int_0^h x^2 dx$$

$$= \frac{\pi r^2}{3h^2}\left[x^3\right]_0^h \quad = \frac{\pi r^2}{3h^2}\left[h^3 - 0\right] \quad = \frac{1}{3}\pi r^2 h$$



Blunders (-3)
B1 integration
B2 slope of line.
B3 equation of line.
B4 volume formula provided it is quadratic
B5 limits
B6 no limits.
B7 incorrect order in applying limits.
B8 indices.

Slips (-1)
S1 numerical

Attempts
A1 uses $v = \pi y$

Worthless
W1 differentiation instead of integration.

Note: (-3) is maximum deduction when evaluating limits.

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2005

MATHEMATICS – HIGHER LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

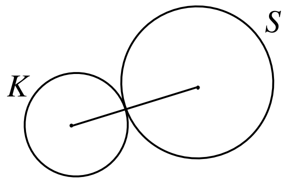
QUESTION 1

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a)	**15 marks**	**Att 5**

1 (a)

Circles S and K touch externally.
Circle S has centre (8, 5) and radius 6.
Circle K has centre (2, −3).
Calculate the radius of K.



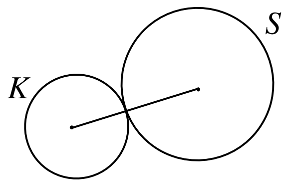
Radius of *K*	**15 marks**	**Att 5**

1 (a)

$a\,(8,5)$ and $b\,(2,-3)$.

$|\,ab\,| = \sqrt{(8-2)^2 + (5+3)^2} = \sqrt{36+64} = 10.$

But $r + 6 = 10. \quad \therefore \quad r(\text{radius } K) = 4.$



Blunders (–3)
B1 Error in distance formula.

Slips (–1)
S1 Arithmetic error.

Attempts (5 marks)
A1 Distance between centres.
A2 Correct condition for circles touching externally.

Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (b) (i)	**10 marks (5, 5)**	**Att (2, 2)**

1(b) (i) Prove that the equation of the tangent to the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) is $xx_1 + yy_1 = r^2$.

Slope of tangent	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1(b) (i)

Equation of tangent T: $y - y_1 = m(x - x_1)$.

Slope of normal $op = \dfrac{y_1 - 0}{x_1 - 0} = \dfrac{y_1}{x_1}$.

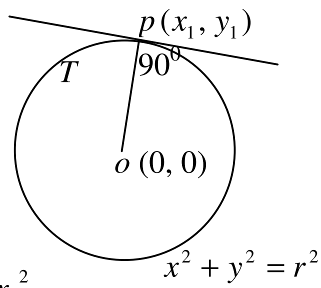
$\therefore$Slope of T at point $p(x_1, y_1) = -\dfrac{x_1}{y_1}$.

Equation of T: $y - y_1 = \dfrac{-x_1}{y_1}(x - x_1) \Rightarrow yy_1 - y_1^{\,2} = -xx_1 + x_1^{\,2}$

$xx_1 + yy_1 = x_1^{\,2} + y_1^{\,2}$.

But $(x_1, y_1) \in x^2 + y^2 = r^2 \quad \Rightarrow \quad x_1^{\,2} + y_1^{\,2} = r^2$.

$\therefore$Equation of tangent T: $xx_1 + yy_1 = r^2$.



or

1(b) (i) $x^2 + y^2 = r^2 \Rightarrow 2x + 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{-x}{y}$.

Slope of tangent T at point $p(x_1, y_1) = \dfrac{-x_1}{y_1}$.

Equation of T: $y - y_1 = \dfrac{-x_1}{y_1}(x - x_1) \Rightarrow yy_1 - y_1{}^2 = -xx_1 + x_1{}^2$

$xx_1 + yy_1 = x_1{}^2 + y_1{}^2$.

But $(x_1, y_1) \in x^2 + y^2 = r^2 \Rightarrow x_1{}^2 + y_1{}^2 = r^2$.

$\therefore$ Equation of tangent T: $xx_1 + yy_1 = r^2$.

Blunders (–3)

B1 Incorrect sign in slope formula.
B2 Slope formula inverted.
B3 Incorrect perpendicular slope.
B4 Error in differentiation.
B5 Fails to show that $x_1{}^2 + y_1{}^2 = r^2$.

Slips (–1)

S1 Arithmetic error.

Attempts (2, 2 marks)

A1 Correct slope of normal.
A2 Correct differentiation.
A3 Correct substitution into tangent formula and stops.
A4 Stops at $xx_1 + yy_1 = x_1{}^2 + y_1{}^2$.

Part (b) (ii)	**10 marks**	**Att 3**

1 (b) (ii) Hence, or otherwise, find the two values of b such that the line $5x + by = 169$ is a tangent to the circle $x^2 + y^2 = 169$.

Values of *b*	**10 marks**	**Att 3**

1 (b) (ii) By part (i) the line $5x + by = 169$ is a tangent to the circle $x^2 + y^2 = 169$ at the point (5, b).

But (5, b) $\in x^2 + y^2 = 169 \Rightarrow 25 + b^2 = 169$.

$b^2 = 144 \Rightarrow b = \pm 12$.

or

1 (b) (ii) Perpendicular distance from centre of circle to tangent $5x + by = 169$ equals radius.

$\left| \dfrac{5(0) + b(0) - 169}{\sqrt{25 + b^2}} \right| = 13 \Rightarrow |-169| = 13\sqrt{25 + b^2}$

$\sqrt{25 + b^2} = 13 \Rightarrow 25 + b^2 = 169 \Rightarrow b^2 = 144. \quad \therefore b = \pm 12.$

Blunders (–3)
B1 Error in solving for b other than slip.
B2 Only one correct value of b given.
B3 Incorrect radius.

Slips (–1)
S1 Arithmetic error.

Attempts (3 marks)
A1 $(5, b)$ point of tangency.
A2 Perpendicular distance formula with substitution.

Part (c)	**15 marks (5, 5, 5)**	**Att (2, 2, 2)**

1 (c) A circle passes through the points (7, 2) and (7, 10).
The line $x = -1$ is a tangent to the circle.
Find the equation of the circle.

Two equations in g, f and c	**5 marks**	**Att 2**
Value of f	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1 (c) Circle: $x^2 + y^2 + 2gx + 2fy + c = 0.$

$(7,2) \in C \Rightarrow 49 + 4 + 14g + 4f + c = 0 \quad \Rightarrow 14g + 4f + c = -53$

$(7,10) \in C \Rightarrow 49 + 100 + 14g + 20f + c = 0 \Rightarrow 14g + 20f + c = -149$

$\therefore 16f = -96 \Rightarrow f = -6.$

$x + 1 = 0$ is a tangent.

$\therefore$ Perpendicular distance from $(-g, -f)$ to $x + 1 = 0$ equals radius.

$$\therefore \left|\frac{-g+1}{1}\right| = \sqrt{g^2 + 36 - c} \Rightarrow g^2 - 2g + 1 = g^2 + 36 - c \Rightarrow 2g - c = -35.$$

But $14g + 4f + c = -53 \Rightarrow 14g + c = -29.$ But $2g - c = -35.$

$\Rightarrow 16g = -64. \therefore g = -4$ and $c = 27.$

$\therefore$ Circle : $x^2 + y^2 - 8x - 12y + 27 = 0.$

or

***y* value of centre**	**5 marks**	**Att 2**
'Quadratic' in *x*	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1 (c) $a(7, 2)$ and $b(7,10)$. $\therefore$ Mid-point of $[ab]$ is $(7, 6)$.
Equation of mediator of chord $[ab]$ is $y = 6$.
Centre point of circle is $c(x, 6)$.
As $x = -1$ is a tangent then point of tangency is $d(-1, 6)$.

$|cd|^2 = |ca|^2 \Rightarrow (x+1)^2 = (x-7)^2 + 16.$

$\therefore x^2 + 2x + 1 = x^2 - 14x + 49 + 16 \Rightarrow 16x = 64 \Rightarrow x = 4.$

$\therefore$ Centre is $(4, 6)$ and radius $= 5$

$\therefore$ Equation of circle is $(x-4)^2 + (y-6)^2 = 25.$

Blunders (–3)
B1 Error in mid-point formula.
B2 Error in perpendicular distance formula.
B3 Error in radius formula.
B4 Circle equation formula error.

Slips (–1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 One equation in f, g and c.
A2 Mid-point of [ab].
A3 Attempt at solving simultaneous equations.
A4 $|ca|$, $|cb|$ or $|cd|$ found.
A5 Distance from centre to tangent with substitution.
A6 Attempt at solving quadratic for x.
A7 Value of third unknown.
A8 Length of radius.

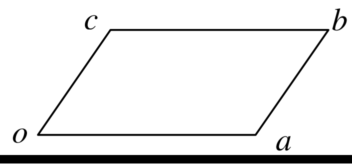
QUESTION 2

Part (a)	**5 marks**	**-**
Part (b)	**25 (10, 5, 10) marks**	**-**
Part (c)	**20 (15, 5) marks**	**-**

Note: The marking of Question 2 is not based on slips, blunders and attempts. In the case of each part, descriptions or typical examples of work meriting particular numbers of marks are given. The mark awarded must be one of the marks indicated. For example, in part **(a) (i),** descriptions are given for work meriting 2, 4 or 5 marks. It is therefore not permissible to award, 1 or 3 marks for this part.

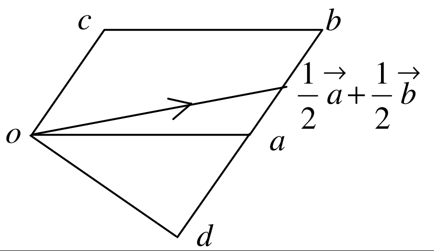
Part (a) **5 marks** **-**

2 (a) Copy the parallelogram *oabc* into your answerbook. Showing your work, construct the point *d* such that $\vec{d} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} - \vec{c}$, where *o* is the origin.



Point *d* **5 marks** **-**

2 (a)



* Accept any labelled parallelogram with vertices *o*, *a*, *b*, *c*.

5 marks: point *d* shown in correct position in diagram. Point *d* need not be joined to origin.

4 marks: Correct work with one error or omission e.g. $\frac{1}{2}\left(\vec{a} + \vec{b}\right)$ or $\frac{1}{2}\vec{a} - \vec{c}$ or $\frac{1}{2}\vec{b} - \vec{c}$ correctly on diagram.

2 marks: One correct significant step e.g. $\frac{1}{2}\vec{a}$ or $\frac{1}{2}\vec{b}$ or $-\vec{c}$ or $\left(\vec{a} + \vec{b}\right)$ correctly shown on diagram.

0 marks: No significant work of merit.

Part (b) **25 (10, 5, 10) marks** **-**

Part (b) (i) **15 (10, 5) marks** **-**

2 (b) (i) $\vec{p} = 3\vec{i} + 4\vec{j}$. $\vec{q}$ is the unit vector in the direction of $\vec{p}$.

(i) Express $\vec{q}$ and $\vec{q}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

Express $\vec{q}$ **10 marks** **-**

2 (b)(i) $\vec{q} = \frac{\vec{p}}{\left|\vec{p}\right|} = \frac{3\vec{i} + 4\vec{j}}{\sqrt{9+16}} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$.

10 marks: Correct solution for $\vec{q}$, simplified or otherwise.

7 marks: Calculates $\left|\vec{p}\right|$ correctly but does not give unit vector or writes $\dfrac{\vec{p}}{\left|\vec{p}\right|}$ and stops or divides $3\vec{i}+4\vec{j}$ by any number.

3 marks: Unit vector expressed as $\dfrac{a\vec{i}+b\vec{j}}{\sqrt{a^2+b^2}}$.

0 marks: No significant work of merit.

Express $\vec{q}^{\perp}$ **5 marks** -

2 (b) (i)

$\vec{q}^{\perp}=-\frac{4}{5}\vec{i}+\frac{3}{5}\vec{j}$, or equivalent from candidates $\vec{q}$.

5 marks: Fully correct answer.

2 marks: Gives $\vec{q}^{\perp}=\frac{4}{5}\vec{i}-\frac{3}{5}\vec{j}$ as solution, or equivalent from candidates $\vec{q}$.

0 marks: Any other answer.

Part (b) (ii) **10 marks** -

2 (b) (ii) Express $11\vec{i}-2\vec{j}$ in the form $k\vec{q}+l\vec{q}^{\perp}$, where $k, l \in \mathbf{R}$.

Express **10 marks** -

2 (b) (ii)

$k\vec{q}+l\vec{q}^{\perp}=11\vec{i}-2\vec{j}$.

$k\left(\frac{3}{5}\vec{i}+\frac{4}{5}\vec{j}\right)+l\left(-\frac{4}{5}\vec{i}+\frac{3}{5}\vec{j}\right)=11\vec{i}-2\vec{j}$.

$\left(\frac{3}{5}k-\frac{4}{5}l\right)\vec{i}+\left(\frac{4}{5}k+\frac{3}{5}l\right)\vec{j}=11\vec{i}-2\vec{j}$.

$\therefore 3k-4l=55$ and $4k+3l=-10$.

$9k-12l=165$

$16k+12l=-40$

$25k=125 \quad \therefore k=5$. But $3(5)-4l=55 \Rightarrow l=-10$.

$\therefore 11\vec{i}-2\vec{j}=5\vec{q}-10\vec{q}^{\perp}$.

10 marks: Correct k and l found.

7 marks: Solves for k and/or for l with minor error(s).

3 marks: One equation in k and l allowing for minor error(s).

0 marks: No significant work of merit.

Part (c) **20 marks (15, 5)** -

Part (c) (i) **15 marks** -

2 (c)	$\vec{u} = \vec{i} + 5\vec{j}$ and $\vec{v} = 4\vec{i} + 4\vec{j}$.
	(i) Find $\cos\angle uov$, where o is the origin.

2 (c) (i)

$$\cos\angle uov = \frac{\left(\vec{i}+5\vec{j}\right)\left(4\vec{i}+4\vec{j}\right)}{\left|\left(\vec{i}+5\vec{j}\right)\right|.\left|\left(4\vec{i}+4\vec{j}\right)\right|} = \frac{4+20}{\sqrt{26}\sqrt{32}} = \frac{24}{8\sqrt{13}} = \frac{3}{\sqrt{13}}.$$

15 marks: $\cos\angle uov$ expressed as fraction of real numbers, simplified or otherwise.

10 marks: Correctly evaluates $\vec{u}\,.\,\vec{v}$ and either $\left|\vec{u}\right|$ or $\left|\vec{v}\right|$ allowing for minor error(s).

5 marks: Correctly evaluates $\left|\vec{u}\right|$ or $\left|\vec{v}\right|$ or $\vec{u}\,.\,\vec{v}$.

0 marks: No significant work of merit.

Part (c) (ii) **5 marks** -

2 (c) (ii)	$\vec{r} = (1-k)\vec{u} + k\,\vec{v}$, where $k \in \mathbf{R}$ and $k \neq 0$.
	Find the value of k for which $\left|\angle uov\right| = \left|\angle vor\right|$.

2 (c) (ii) $\vec{r} = (1-k)\left(\vec{i}+5\vec{j}\right) + k\left(4\vec{i}+4\vec{j}\right) = (1+3k)\vec{i} + (5-k)\vec{j}.$

$$\cos\angle vor = \frac{\left(4\vec{i}+4\vec{j}\right)\left[(1+3k)\vec{i}+(5-k)\vec{j}\right]}{\sqrt{32}\sqrt{(1+3k)^2+(5-k)^2}} = \frac{3}{\sqrt{13}}.$$

$$\therefore\ \frac{4+12k+20-4k}{4\sqrt{2}\sqrt{26-4k+10k^2}} = \frac{3}{\sqrt{13}}$$

$$\sqrt{13}(24+8k) = 12\sqrt{2}\sqrt{26-4k+10k^2}$$

$$\sqrt{13}(6+2k) = 3\sqrt{2}\sqrt{26-4k+10k^2}$$

$$568+312k+52k^2 = 468-72k+180k^2 \ \Rightarrow\ 128k^2-384=0$$

$$\therefore\ k^2-3k=0 \ \Rightarrow\ k-3=0 \text{ as } k\neq 0.\ \therefore\ k=3.$$

5 marks: Fully correct solution.

4 marks: Complete solution with minor error(s).

3 marks: Correct or substantially correct equation in k (without $\vec{i}$ and $\vec{j}$).

2 marks: $\vec{r}$ expressed in the form $a\,\vec{i} + b\,\vec{j}$, allowing for minor error(s).

0 marks: No significant work of merit.

QUESTION 3

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

Part (a)	**15 marks**	**Att 5**

3 (a) The line $L_1 : 3x - 2y + 7 = 0$ and the line $L_2 : 5x + y + 3 = 0$ intersect at the point p. Find the equation of the line through p perpendicular to L_2.

Equation of line	**15 marks**	**Att 5**

3 (a)

$$3x - 2y + 7 = 0 \Rightarrow 3x - 2y = -7$$

$$5x + y + 3 = 0 \Rightarrow \underline{10x + 2y = -6}$$

$$13x = -13 \quad \Rightarrow \quad x = -1. \ \therefore y = 2. \quad p(-1, 2).$$

$$L_2 : y = -5x - 3 \Rightarrow \text{slope } L_2 = -5. \ \therefore \text{perpendicular slope} = m = \frac{1}{5}.$$

$$\text{Equation of line}: y - 2 = \frac{1}{5}(x + 1) \Rightarrow x - 5y + 11 = 0.$$

or

3 (a)

$$\text{Required line}: 3x - 2y + 7 + \lambda(5x + y + 3) = 0.$$

$$\therefore x(3 + 5\lambda) + y(\lambda - 2) + (7 + 3\lambda) = 0$$

$$\text{Slope} = \frac{3 + 5\lambda}{2 - \lambda}.$$

$$L_2 : y = -5x - 3 \Rightarrow \text{slope } L_2 = -5. \ \therefore \text{Slope of required line} = \frac{1}{5}.$$

$$\frac{3 + 5\lambda}{2 - \lambda} = \frac{1}{5} \Rightarrow 15 + 25\lambda = 2 - \lambda \Rightarrow 26\lambda = -13. \ \therefore \lambda = -\frac{1}{2}.$$

$$\therefore \frac{1}{2}x - \frac{5}{2}y + \frac{11}{2} = 0 \Rightarrow \text{Required line}: x - 5y + 11 = 0.$$

Blunders (–3)
B1 Error in slope of L_2 other than slip.
B2 Incorrect perpendicular slope.

Slips (–1)
S1 Arithmetic error.

Attempts (5 marks)
A1 x or y coordinate of point p.
A2 Correct slope of L_2.
A3 Correct perpendicular slope.

Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (b) (i)	**10 marks**	**Att 3**

3 (b) (i) The line K passes through the point (–4, 6) and has slope m, where $m > 0$.

Write down the equation of K in terms of m.

Equation of *K*	**10 marks**	**Att 3**

3 (b) (i)

$$y - 6 = m(x + 4).$$

Blunders (–3)

B1 Error in equation line formula.

Slips (–1)

S1 Arithmetic error.

Attempts (3 marks)

A1 Equation of line with some substitution.

Part (b) (ii)	**5 marks**	**Att 2**

3 (b) (ii) Find, in terms of m, the co-ordinates of the points where K intersects the axes.

Co-ordinates	**5 marks**	**Att 2**

3 (b) (ii)

$$y - 6 = m(x + 4) \Rightarrow mx - y + 6 + 4m = 0.$$

Cuts *x-axis* at $p(x, 0)$. $mx = -6 - 4m \Rightarrow x = \dfrac{-6 - 4m}{m}$. $p\left(\dfrac{-6 - 4m}{m}, 0\right)$.

Cuts *y-axis* at $q(0, y)$. $y = 6 + 4m$. $q(0, 6 + 4m)$.

Blunders (–3)

B1 Equation of axes incorrect.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 One correct coordinate.

Part (b) (iii) **5 marks** **Att 2**

3 (b) (iii)	The area of the triangle formed by *K*, the *x*-axis and the *y*-axis is 54 square units. Find the possible values of *m*.

Values of *m* **5 marks** **Att 2**

(b) (iii)

Area triangle *opq* = 54 square units.

Area triangle $opq = \frac{1}{2}\left|x_1 y_2 - x_2 y_1\right|.$

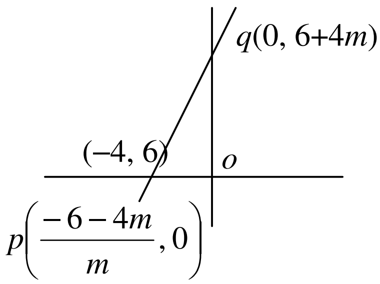
$\therefore \ \frac{1}{2}\left|(0)(0) - \left(\frac{-6-4m}{m}\right)(6+4m)\right| = 54.$

$(6+4m)(6+4m) = 108m.$

$\therefore 16m^2 + 48m + 36 = 108m \Rightarrow 16m^2 - 60m + 36 = 0$

$4m^2 - 15m + 9 = 0 \Rightarrow (4m-3)(m-3) = 0.$

$\therefore m = \frac{3}{4}$ or $m = 3.$



Blunders (–3)
B1 Error in triangle area formula.
B2 Error in factors or quadratic formula.
B3 Misuse of modulus in formula.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Triangle area formula with some substitution.
A2 Quadratic in *m*.

Part (c) **15 (10, 5) marks** **Att (3, 2)**

Part (c) (i) **10 marks** **Att 3**

3 (c) (i) f is the transformation $(x, y) \to (x', y')$, where $x' = 3x - y$ and $y' = x + 2y$.

(i) Prove that f maps every pair of parallel lines to a pair of parallel lines. You may assume that f maps every line to a line.

Prove **10marks** **Att 3**

3(c)(i)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$$

Let L have equation: $ax + by + c = 0$, and M: $ax + by + d = 0$.

$\therefore\ f(L): \frac{a}{7}(-x' + 3y') + \frac{b}{7}(2x' + y') + c = 0 \Rightarrow f(L): (-a + 2b)x' + (3a + b)y' + 7c = 0$

and $f(M): \frac{a}{7}(-x' + 3y') + \frac{b}{7}(2x' + y') + d = 0 \Rightarrow f(L): (-a + 2b)x' + (3a + b)y' + 7d = 0$

So, $f(L) \parallel f(M)$, since $(-a + 2b)(3a + b) = (3a + b)(-a + 2b)$, [i.e. $a_1b_2 = a_2b_1$]

or

3 (c) (i)

$L: y = mx + c$ and $M: y = mx + k$ are two parallel lines.

$x' = 3x - y \Rightarrow 2x' = 6x - 2y$

$y' = x + 2y \Rightarrow y' = x + 2y. \therefore 2x' + y' = 7x \Rightarrow x = \frac{1}{7}(2x' + y')$.

But $y' = x + 2y \Rightarrow y' = \frac{1}{7}(2x' + y') + 2y \Rightarrow y = \frac{1}{7}(-x' + 3y')$.

$\therefore f(L): \frac{1}{7}(-x' + 3y') = \frac{m}{7}(2x' + y') + c \Rightarrow f(L): -x' + 3y' = 2mx' + my' + 7c$.

$\therefore f(L): (3 - m)y' = (1 + 2m)x' + 7c \Rightarrow f(L): y' = \left(\frac{1 + 2m}{3 - m}\right)x' + \frac{7c}{3 - m}$.

Similarly $f(M): y' = \left(\frac{1 + 2m}{3 - m}\right)x' + \frac{7k}{3 - m}$.

Both lines have same slope, $\frac{1 + 2m}{3 - m}$, $\therefore$ parallel.

or

Let L and M pass through p and q respectively and both be in the direction $\vec{m}$.

$\therefore\ L = \vec{p} + t\vec{m}$ and $M = \vec{q} + t\vec{m}$, where $t \in \mathbf{R}$

$\therefore f(L) = f(\vec{p} + t\vec{m}) = f(\vec{p}) + tf(\vec{m})$ and $f(M) = f(\vec{q} + t\vec{m}) = f(\vec{q}) + tf(\vec{m})$

$\therefore f(L)$ and $f(M)$ are both lines in the direction of $f(\vec{m})$, and hence are parallel.

* Note: second method above fails to deal with the case where L and M are vertical, or where they have slope 3. Do not penalise this.

Blunders (–3)

B1 Error in determining slope other than slip.
B2 Incorrect matrix or matrix multiplication.
B3 Failure to establish image lines parallel.

Slips (–1)

S1 Arithmetic error.

Attempts (3 marks)

A1 Expressing x or y in term of primes.
A2 correct matrix for f.
A3 Finds image of one line and stops.

Part (c) (ii)	**5 marks**	**Att 2**

3 (c) (ii) $oabc$ is a parallelogram, where $[ob]$ is a diagonal and o is the origin. Given that $f(c)=(-1,9)$, find the slope of ab.

Slope *ab*	**5 marks**	**Att 2**

3 (c) (ii) $f(c)=(-1,9)$. $x=\frac{1}{7}(2x'+y')$ and $y=\frac{1}{7}(-x'+3y')$.

$\therefore x=1$ and $y=4 \Rightarrow c(1,4)$.

Slope $oc=4 \Rightarrow$ slope $ab=4$ as ab is parallel to oc.

or

3 (c)(ii)

$$\text{Matrix } f=\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix} \Rightarrow \begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}^{-1}\begin{pmatrix}-1\\ 9\end{pmatrix}=\frac{1}{7}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}\begin{pmatrix}-1\\ 9\end{pmatrix}=\left(\frac{1}{7}\right)\begin{pmatrix}7\\ 28\end{pmatrix}=\begin{pmatrix}1\\ 4\end{pmatrix}=c.$$

$\therefore$ Slope $oc=4 \Rightarrow$ slope $ab=4$.

or

3 (c) (ii) $f(c)=(-1,9)$. $x'=3x-y$ and $y'=x+2y$.

$3x-y=-1 \Rightarrow 6x-2y=-2$

$x+2y=9 \Rightarrow \quad x+2y=9$

$7x=7 \Rightarrow x=1$ and hence $y=4$.

$\therefore c(1,4)$ and slope $oc=4$.

But ab is parallel to $oc \Rightarrow$ slope $ab=4$.

Blunders (–3)

B1 Slope oc and stops.
B2 Incorrect matrix.
B3 Incorrect matrix multiplication other than slip.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Two simultaneous equations.
A2 Correct point c and stops.

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 marks** **Att 3**

4 (a) Evaluate $\lim\limits_{\theta \to 0} \dfrac{\sin 4\theta}{3\theta}$.

* Accept correct answer without work. If candidate's answer is correct, ignore the work.

Evaluate **10 marks** **Att 3**

4 (a)

$$\lim_{\theta \to 0} \frac{\sin 4\theta}{3\theta} = \lim_{\theta \to 0}\left(\frac{\frac{\sin 4\theta}{4\theta} \times 4\theta}{3\theta} \right) = \lim_{\theta \to 0}\left(\frac{\sin 4\theta}{4\theta} \right) \times \frac{4}{3} = \frac{4}{3}.$$

or $f(\theta) = \sin 4\theta$ and $g(\theta) = 3\theta$. $\therefore \lim\limits_{\theta \to 0} \dfrac{f(\theta)}{g(\theta)} = \dfrac{f'(0)}{g'(0)} = \dfrac{4\cos(0)}{3} = \dfrac{4}{3}$.

Blunders (–3)

B1 $\sin 4\theta = 4\sin\theta$.

B2 Error in differentiation.

Slips (–1)

S1 Arithmetic error.

Attempts (3 marks)

A1 Has $\dfrac{\sin 4\theta}{4\theta}$ in solution.

A2 Correct differentiation.

Part (b) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

Part (b) (i) **10 marks** **Att 3**

4 (b) (i) Using $\cos 2A = \cos^2 A - \sin^2 A$, or otherwise,

prove $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$.

Prove **10 marks** **Att 3**

4 (b) (i)

$$\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A)$$

$$\therefore\ 2\cos^2 A = 1 + \cos 2A \ \Rightarrow\ \cos^2 A = \frac{1}{2}(1 + \cos 2A).$$

Blunders (–3)
B1 Error in cos2A formula.
B2 Error in $\sin^2$A formula.

Slips (–1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Correct substitution for cos2A.
A2 $\text{Sin}^2\text{A} = 1 - \cos^2\text{A}$.

Part (b) (ii) **10 (5, 5) marks** **Att (2, 2)**

4 (b) (ii)	Hence, or otherwise, solve the equation $1+\cos 2x = \cos x$, where $0^\circ \le x \le 360^\circ$.

Quadratic in Cosx **5 marks** **Att 2**
Solution for x **5 marks** **Att 2**

4 (b) (ii)

$$1+\cos 2x = \cos x \Rightarrow 2\cos^2 x = \cos x.$$

$$\cos x(2\cos x - 1) = 0 \Rightarrow \cos x = 0 \text{ or } \cos x = \frac{1}{2}.$$

$$\therefore\ x = 90^\circ, 270^\circ \text{ or } x = 60^\circ, 300^\circ. \quad \therefore \text{solution} = \{60^\circ, 90^\circ, 270^\circ, 300^\circ\}.$$

Blunders (–3)
B1 Incorrect substitution for 1+cos2x or cos2x.
B2 Error in factors.
B3 Each incorrect solution or missing solution.

Slips (–1)
S1 Arithmetic error.

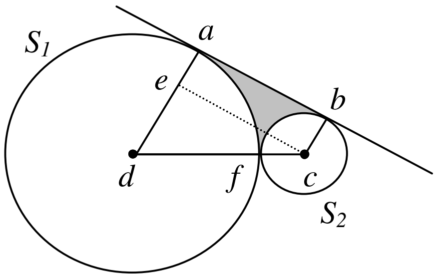
Attempts (2, 2 marks)
A1 $\cos 2x = \cos^2 x - \sin^2 x$.
A2 Correct factors.
A3 One correct solution.

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

Part (c) (i) **15 (10, 5) marks** **Att (3, 2)**

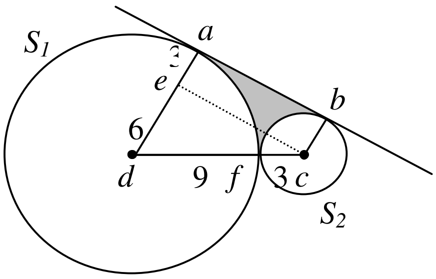
4 (c) (i) S_1 is a circle of radius 9 cm and S_2 is a circle of radius 3 cm.
S_1 and S_2 touch externally at *f*.
A common tangent touches S_1 at point *a* and S_2 at *b*.

(i) Find the area of the quadrilateral *abcd*.
Give your answer in surd form.



Find |ec| **10 marks** **Att 3**
Area quadrilateral abcd **5 marks** **Att 2**

4 (c) (i)



$|ec|^2 = |dc|^2 - |de|^2 \Rightarrow |ec|^2 = 144 - 36 = 108. \therefore |ec| = \sqrt{108} = 6\sqrt{3}$. But $|ec| = |ab|$.

Area of the quadrilateral *abcd* $= \frac{1}{2}|ab|\left[|ad| + |bc|\right] = \frac{1}{2}\left(6\sqrt{3}\right)\left[9+3\right] = 36\sqrt{3}\text{ cm}^2$.

or

4 (c) (i)

Area of quadrilateral *abcd* = triangle *dce* + rectangle *ecba*

$= \frac{1}{2}(6)\left(6\sqrt{3}\right) + 3\left(6\sqrt{3}\right) = 36\sqrt{3}$.

Blunders (–3)
B1 Incorrect application of Pythagoras.
B2 Error in area formula.

Slips (–1)
S1 Arithmetic error.

Attempts (3, 2 marks)
A1 Correct length of $|dc|$ or $|de|$.
A2 Area of triangle *dce* or rectangle *ecba* correct.
A3 Area formula for trapezium *abcd* with some substitution.

Part (c) (ii) **5 marks** **Att 2**

4 (c) (ii)	Find the area of the shaded region, which is bounded by [*ab*] and the minor arcs *af* and *bf*.

Area of shaded region **5 marks** **Att 2**

4 (c) (ii)

$$\cos|\angle edc| = \frac{6}{12} = \frac{1}{2} \Rightarrow |\angle edc| = 60^{\text{o}}. \quad \therefore |\angle bcf| = 30^{\text{o}} + 90^{\text{o}} = 120^{0}.$$

$$\text{Area of sector } adf = \frac{1}{2}r^2\theta = \frac{1}{2}(81)\left(\frac{\pi}{3}\right) = \frac{27\pi}{2}.$$

$$\text{Area of sector } bcf = \frac{1}{2}r^2\theta = \frac{1}{2}(9)\left(\frac{2\pi}{3}\right) = 3\pi.$$

$$\therefore \text{ Area of shaded region } = 36\sqrt{3} - \frac{27\pi}{2} - 3\pi = 36\sqrt{3} - \frac{33\pi}{2}.$$

Blunders (–3)

B1 Error in sector area formula.
B2 Finds area of both sectors but fails to finish.
B3 Incorrect conversion from degree to radians.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 $|\angle edc| = 60^0$ or $|\angle ecd| = 30^0$ or $|\angle bcf| = 120^0$.

A2 $\cos\angle edc = \frac{6}{12}$ or $\sin\angle ecd = \frac{6}{12}$.

QUESTION 5

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (15, 5) marks**	**Att (5, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att 5**

5(a) The area of an equilateral triangle is $4\sqrt{3}\text{ cm}^2$.
Find the length of a side of the triangle.

Length of side **15 marks** **Att 5**

5 (a) Area of triangle $= \frac{1}{2}ab\text{Sin}C$, where $a = b$ and $|\angle C| = \frac{\pi}{3}$.

$\therefore \frac{1}{2}a^2\sin\frac{\pi}{3} = 4\sqrt{3} \Rightarrow \frac{1}{2}a^2\frac{\sqrt{3}}{2} = 4\sqrt{3}.$

$\therefore a^2 = 16 \Rightarrow a = 4.$ Length of side $= 4$ cm.

Blunders (–3)
B1 Error in triangle area formula.
B2 Incorrect evaluation of $\sin 60^o$.
B3 $\sin 60^o$ in decimal form.

Slips (–1)
S1 Arithmetic error.

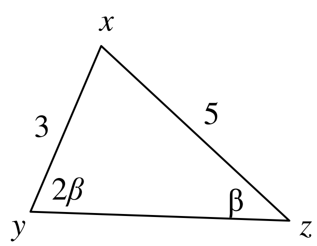
Attempts (5 marks)
A1 Triangle area formula with substitution.

Part (b) **20 (15, 5) marks** **Att (5, 2)**

Part (b) (i) **15 marks** **Att 5**

5 (b) (i) In the triangle *xyz*, $|\angle xyz| = 2\beta$ and $|\angle xzy| = \beta$.
$|xy| = 3$ and $|xz| = 5$.

(i) Use this information to express $\sin 2\beta$ in the form $\frac{a}{b}\sin\beta$, where $a, b \in$ **N**.



Express **15 marks** **Att 5**

5 (i)

$$\frac{\sin 2\beta}{5} = \frac{\sin\beta}{3} \Rightarrow \sin 2\beta = \frac{5}{3}\sin\beta.$$

Blunders (–3)
B1 Error in substitution into Sine rule.

Slips (–1)
S1 Arithmetic error.

Attempts (5 marks)
A1 $\dfrac{3}{\sin\beta}$ or $\dfrac{5}{\sin 2\beta}$.

Part (b) (ii)	**5 marks**	**Att 2**

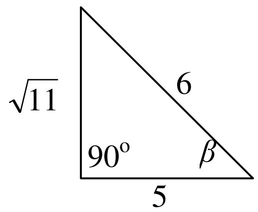
5 (b) (ii) Hence express $\tan\beta$ in the form $\dfrac{\sqrt{c}}{d}$, where $c, d \in \mathbf{N}$.

Express tanβ	**5 marks**	**Att 2**

5 (b) (ii)

$$\sin 2\beta = \frac{5}{3}\sin\beta \;\Rightarrow\; 2\sin\beta\cos\beta = \frac{5}{3}\sin\beta.$$

$$\therefore \cos\beta = \frac{5}{6} \;\Rightarrow\; \tan\beta = \frac{\sqrt{11}}{5}.$$



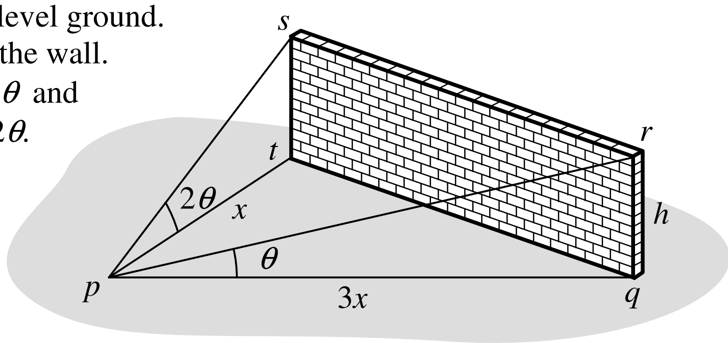
Blunders (–3)
B1 Error in $\sin 2\beta$ formula.
B2 Incorrect ratio of sides for $\cos\beta$ or $\tan\beta$.
B3 Incorrect application of Pythagoras.
B4 $\cos\beta = \dfrac{5}{6}$ and stops.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Equation in β.

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

5 (c) *pqrs* is a vertical wall of height h on level ground.
p is a point on the ground in front of the wall.
The angle of elevation of r from p is θ and the angle of elevation of s from p is 2θ.
$|pq| = 3|pt|$.
Find θ.



Tanθ or Tan2θ in terms of h and x	**5 marks**	**Att 2**
Equation in tan3θ or tan2θ	**5 marks**	**Att 2**
Find θ	**5 marks**	**Att 2**

5 (c)

$$\tan\theta = \frac{h}{3x} \Rightarrow h = 3x\tan\theta. \text{ Also } \tan 2\theta = \frac{h}{x} \Rightarrow h = x\tan 2\theta.$$

$$\therefore 3x\tan\theta = x\tan 2\theta \Rightarrow 3\tan\theta = \frac{2\tan\theta}{1-\tan^2\theta} \Rightarrow 3t\left(1-t^2\right) = 2t, \text{ where } t = \tan\theta.$$

$$\therefore 3t - 3t^3 = 2t \Rightarrow 3t^3 - t = 0. \quad t\left(3t^2-1\right) = 0 \Rightarrow t^2 = \frac{1}{3}, t \neq 0.$$

$$\therefore t = \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}.$$

Blunders (–3)
B1 Incorrect ratio of sides for tan.
B2 Error in tan2θ formula.
B3 Incorrect factors.
B4 Incorrect value for θ.

Slips (–1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 Tanθ or tan2θ expressed as ratio of sides.
A2 Tan2θ expressed in terms of tanθ.
A3 Correct value for $\tan^2\theta$.

QUESTION 6

Part (a)	**10 (5, 5) marks**	**Att (-, 2)**
Part (b)	**25 (5, 5, 5, 5, 5) marks**	**Att (2, 2, 2, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (-, 2)**

Part (a) (i) **5 marks** **Hit/Miss**

6 (a) (i) How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, if

(i) the three digits are all different

6 (a) (i) Answer $= {}^5P_3 = 5 \times 4 \times 3 = 60$.

Part (a) (ii) **5 marks** **Att 2**

6 (a) (ii) How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, if

(ii) the three digits are all the same?

6 (a) (ii) Answer $= 5 \times 1 \times 1 = 5$.

Blunders (–3)
B1 5×5×1.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 5×5×5.

Part (b)	25 (5, 5, 5, 5, 5) marks	Att (2, 2, 2, 2, 2)

Part (b) (i)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

6 (b) (i) Solve the difference equation $u_{n+2} - 4u_{n+1} - 8u_n = 0$, where $n \geq 0$, given that $u_0 = 0$ and $u_1 = 2$.

Characteristic equation	**5 marks**	**Att 2**
Characteristic roots	**5 marks**	**Att 2**
Simultaneous equations	**5 marks**	**Att 2**
Solution	**5 marks**	**Att 2**

6 (b) (i)

$$u_{n+2} - 4u_{n+1} - 8u_n = 0 \;\Rightarrow\; x^2 - 4x - 8 = 0.$$

$$\therefore\; x = \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}.$$

$$u_n = k\left(2 + 2\sqrt{3}\right)^n + l\left(2 - 2\sqrt{3}\right)^n.$$

$$u_o = 0 \;\Rightarrow\; k + l = 0.\; l = -k.$$

$$u_1 = 2 \;\Rightarrow\; k\left(2 + 2\sqrt{3}\right) + l\left(2 - 2\sqrt{3}\right) = 2$$

$$\therefore\; k\left(2 + 2\sqrt{3}\right) - k\left(2 - 2\sqrt{3}\right) = 2 \;\Rightarrow\; 2k + 2k\sqrt{3} - 2k + 2k\sqrt{3} = 2$$

$$\therefore\; 4k\sqrt{3} = 2 \;\Rightarrow\; k = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}.\;\; \therefore\; l = -\frac{\sqrt{3}}{6}.$$

$$\therefore\; u_n = \frac{\sqrt{3}}{6}\left(2 + 2\sqrt{3}\right)^n - \frac{\sqrt{3}}{6}\left(2 - 2\sqrt{3}\right)^n.$$

Blunders (–3)
B1 Error in characteristic equation.
B2 Error in quadratic formula.
B3 Incorrect use of initial conditions.

Slips (–1)
S1 Arithmetic error.

Attempts (2, 2, 2, 2 marks)
A1 An equation in *k* and *l*.
A2 Correct value for *k* or *l*.

Part (b) (ii) **5 marks** **Att 2**

6 (b) (ii)	Verify that your solution gives the correct value for u_2.

Verify **5 marks** **Att 2**

6 (b) (ii)

$u_2 - 4u_1 - 8u_0 = 0$. But $u_1 = 2$ and $u_0 = 0$.

$\therefore u_2 = 8 + 0 = 8$.

But $u_2 = \frac{\sqrt{3}}{6}\left(2+2\sqrt{3}\right)^2 - \frac{\sqrt{3}}{6}\left(2-2\sqrt{3}\right)^2 = \frac{\sqrt{3}}{6}\left(4+8\sqrt{3}+12-4+8\sqrt{3}-12\right)$

$u_2 = \frac{\sqrt{3}}{6}\left(16\sqrt{3}\right) = 8$. $\therefore$ Verified.

or

6 (b) (ii)

$u_n = \frac{\sqrt{3}}{6}\left(2+2\sqrt{3}\right)^n - \frac{\sqrt{3}}{6}\left(2-2\sqrt{3}\right)^n$.

$\therefore u_2 = \frac{\sqrt{3}}{6}\left(2+2\sqrt{3}\right)^2 - \frac{\sqrt{3}}{6}\left(2-2\sqrt{3}\right)^2$.

$u_2 = \frac{\sqrt{3}}{6}\left(4+8\sqrt{3}+12-4+8\sqrt{3}-12\right) \Rightarrow u_2 = \frac{\sqrt{3}}{6}\left(16\sqrt{3}\right) \Rightarrow u_2 = 8$.

Substituting $u_o = 0$, $u_1 = 2$ and $u_2 = 8$ into $u_{n+2} - 4u_{n+1} - 8u_n$,

gives $8 - 4(2) - 0 = 0$. $\therefore$ Verified.

Blunders (–3)

B1 Error in calculating u_2 other than slip.

B2 Finds u_2 but fails to verify.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct value for $u_{2.}$

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

Part (c) (i) **5 marks** **Att 2**

6 (c) (i)	Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards. **(i)** Find the probability that the card numbered 8 is not drawn.

Probability **5 marks** **Att 2**

6 (c) (i)	Total outcomes (choose three cards from nine): ${}^{9}C_3 = 84$. Outcomes of interest (choose three from the eight allowed): ${}^{8}C_3 = 56$. $\therefore$ Probability $= \frac{56}{84} = \frac{2}{3}$.

or

6 (c) (i)	(first card not 8) *and* (second card not 8) *and* (third card not 8) $\Rightarrow$ Probability $= \frac{8}{9} \times \frac{7}{8} \times \frac{6}{7} = \frac{2}{3}$.

Blunders (–3)

B1 Incorrect number of possible outcomes.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct number of possible outcomes.

A2 Correct number of favourable outcomes.

Part (c) (ii) **5 marks** **Att 2**

6 (c) (ii)	Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards. **(ii)** Find the probability that all three cards drawn have odd numbers.

Probability **5 marks** **Att 2**

6 (c) (ii)	Outcomes of interest (choose three from the five odd-numbered): ${}^{5}C_3 = 10$. $\therefore$ Probability $= \frac{10}{84} = \frac{5}{42}$.

or

6 (c) (ii)	(first card odd) *and* (second card odd) *and* (third card odd) $\therefore$ Probability $= \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{60}{504} = \frac{5}{42}$.

Blunders (–3)

B1 Incorrect number of possible outcomes.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct number of possible outcomes.

A2 Correct number of favourable outcomes.

Part (c) (iii) **5 marks** **Att 2**

6 (c) (iii)	Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards. **(iii)** Find the probability that the sum of the numbers on the cards drawn is greater than the sum of the numbers on the cards not drawn.

Probability **5 marks** **Att 2**

6 (c) (iii) Outcomes of interest:

Sum of all the cards numbered 1 to 9 is 45.

$\therefore$ Sum of three drawn cards must be ≥ 23, (i.e. more than half of total).

Sum of cards 7, 8, 9 $= 24$

Sum of cards 6, 8, 9 $= 23$

No other possibilities.

$\therefore$ Only two possible favourable outcomes.

$\therefore$ Probability $= \dfrac{2}{84} = \dfrac{1}{42}$.

Blunders (–3)

B1 Incorrect number of possible outcomes.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct number of favourable outcomes.

A2 Correct number of possible outcomes.

A3 One correct element properly identified e.g. 9+8+7=24 > 21.

QUESTION 7

Part (a)	**10 (5, 5) marks**	**Att (-, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, -, 2)**

Part (a) **10 (5, 5) marks** **Att (-, 2)**

Part (a) (i) **5 marks** **Hit/Miss**

7 (a) (i) How many different groups of four can be selected from five boys and six girls?

7 (a) (i) Choose four from eleven $\Rightarrow$ answer $= {}^{11}C_4 = 330$.

Part (a) (ii) **5 marks** **Att 2**

7 (a) (ii) How many of these groups consist of two boys and two girls?

7 (a) (ii) Choose two from five *and* choose two from six $\Rightarrow$ answer $= {}^5C_2 \times {}^6C_2 = 10 \times 15 = 150$.

Blunders (–3)

B1 ${}^5C_2 + {}^6C_2$.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 ${}^5C_2$ or ${}^6C_2$.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

Part (b) (i) **5 marks** **Att 2**

7 (b) (i) There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that

(i) the four discs are blue

Part (b) (i) **5 marks** **Att 2**

7 (b) (i) Total outcomes (choose four discs from sixteen): ${}^{16}C_4 = 1820$.

Outcomes of interest (choose four of the five blue): ${}^5C_4 = 5$.

$\therefore$ Probability $= \dfrac{5}{1820} = \dfrac{1}{364}$.

or

7 (b) (i) (first blue) *and* (second blue) *and* (third blue) *and* (fourth blue)

$\therefore$ Probability $= \dfrac{5}{16} \times \dfrac{4}{15} \times \dfrac{3}{14} \times \dfrac{2}{13} = \dfrac{120}{43680} = \dfrac{1}{364}$.

Blunders (–3)

B1 Incorrect number of possible outcomes.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 Correct number of possible outcomes.

A2 Correct number of favourable outcomes.

Part (b) (ii) **5 marks** **Att 2**

7 (b) (ii)	There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that **(ii)** the four discs are the same colour

Probability **5 marks** **Att 2**

Part (b) (ii) Outcomes of interest: (four blue *or* four red): ${}^5C_4 + {}^6C_4 = 5 + 15 = 20$.

$\therefore$ Probability $\dfrac{20}{1820} = \dfrac{1}{91}$.

or

7 (b) (ii) Probability = P(4 blue) + P(4 red)

$$= \left(\frac{5}{16}\times\frac{4}{15}\times\frac{3}{14}\times\frac{2}{13}\right) + \left(\frac{6}{16}\times\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}\right) = \frac{120+360}{43680} = \frac{480}{43680} = \frac{1}{91}.$$

Blunders (–3)
B1 Incorrect number of possible outcomes.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.
A3 P (4 red) correct.

Part (b) (iii) **5 marks** **Att 2**

7 (b) (iii)	There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that **(iii)** all four discs are different in colour

Probability **5 marks** **Att 2**

7 (b) (iii) Outcomes of interest:

one blue *and* one green *and* one red *and* one yellow: ${}^5C_1 \times {}^3C_1 \times {}^6C_1 \times {}^2C_1 = 180$.

$\therefore$ Probability = $\dfrac{180}{1820} = \dfrac{9}{91}$.

or

7 (b) (iii) (first blue) *and* (second green) *and* (third red) *and* (fourth yellow) or any permutation;

$\therefore$ Probability $= \dfrac{5}{16}\times\dfrac{3}{15}\times\dfrac{6}{14}\times\dfrac{2}{13}\times 4! = \dfrac{4320}{43680} = \dfrac{9}{91}$.

Blunders (–3)
B1 Incorrect number of possible outcomes.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.

Part (b) (iv) **5 marks** **Att 2**

7 (b) (iv)	There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that **(iv)** two of the discs are blue and two are not blue?

Probability **5 marks** **Att 2**

7 (b) (iv)	Of interest: (choose two of five blue *and* two of remaining eleven) ${}^{5}C_2 \times {}^{11}C_2 = 550$. $\therefore$ Probability $= \frac{550}{1820} = \frac{55}{182}$.

or

7 (b) (iv)	(first blue) *and* (second blue) *and* (third not blue) *and* (fourth not blue), *or* any permutation thereof; $\therefore$ Probability $= \frac{5}{16} \times \frac{4}{15} \times \frac{11}{14} \times \frac{10}{13} \times \frac{4!}{2!.2!} = \frac{52800}{174720} = \frac{55}{182}$.

Blunders (–3)
B1 Incorrect number of possible outcomes.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Correct number of possible outcomes.
A2 Correct number of favourable outcomes.
A3 P (two blue) correct.
A4 P (two are not blue) correct.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, -, 2)**

Part (c) (i) **10 (5, 5) marks** **Att (2, 2)**

7 (c) (i)	On 1st September 2003 the mean age of the first-year students in a school is 12.4 years and the standard deviation is 0.6 years. One year later all of these students have moved into second year and no other students have joined them. **(i)** State the mean and the standard deviation of the ages of these students on 1st September 2004. Give a reason for each answer.

Mean **5 marks** **Att 2**
Standard deviation **5 marks** **Att 2**

(c) (i)	Mean = 13.4 years. As all the students are one year older, the mean is increased by one. Standard deviation = 0.6 years. The spread of ages in the group is still the same. **or** As they are each one year older and the mean is increased by one, each deviation from the mean is unchanged, and hence so is the standard deviation.

Blunders (–3)
B1 Reason for new mean not given or incorrect reason.
B2 Reason for new standard deviation not given or incorrect reason.

Slips (–1)
S1 Arithmetic error.

Attempts (2, 2 marks)
A1 Correct new mean.
A2 Correct new standard deviation.

Part (c) (ii) **5 marks** **Hit/Miss**

7 (c) (ii) A new group of first-year students begin on 1st September 2004. This group has a similar age distribution and is of a similar size to the first-year group of September 2003.

(ii) State the mean age of the combined group of the first-year and second-year students on 1st September 2004.

Combined mean **5 marks** **Hit/Miss**

Part (c) (ii)

$$\text{Mean} \approx \frac{12.4+13.4}{2} = 12.9 \text{ years.}$$

Part (c) (iii) **5 marks** **Att 2**

7 (c) (iii) State whether the standard deviation of the ages of this combined group is less than, equal to, or greater than 0.6 years. Give a reason for your answer.

State & reason **5 marks** **Att 2**

7 (c) (iii) Standard deviation > 0.6 years. There is a greater spread of ages in the combined group than in a single year group. [**or**: Data more spread out.]

Blunders (–3)
B1 Incorrect reason given.
B2 No reason given.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 States greater than 0.6 years.

[Aside: the actual value is approximately 0.8; this is not required.]

QUESTION 8

Part (a)	**15 marks**	**Att 5**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **15 marks** **Att 5**

8 (a) Use integration by parts to find $\int x^2 \ln x dx$.

Integration by parts **15 marks** **Att 5**

8 (a) $\int x^2 \ln x dx = uv - \int v du.$

$u = \ln x \Rightarrow du = \frac{1}{x}dx. \quad dv = x^2 dx \Rightarrow v = \int x^2 dx = \frac{1}{3}x^3.$

$\therefore \int x^2 \ln x dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3\left(\frac{1}{x}\right)dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + \text{constant}.$

Blunders (–3)
B1 Incorrect differentiation or integration.
B2 Constant of integration omitted.
B3 Incorrect 'parts' formula.

Slips (–1)
S1 Arithmetic error.

Attempts (5 marks)
A1 Correct assigning to parts formula.
A2 Correct differentiation or integration.

Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (b) (i)	**10 marks**	**Att 3**

8 (b) (i) Derive the Maclaurin series for $f(x) = \ln(1+x)$ up to and including the term containing x^3.

Maclaurin series	**10 marks**	**Att 3**

8 (b) (i) $f(x) = f(0) + \dfrac{f'(0)x}{1!} + \dfrac{f''(0)x^2}{2!} + \dfrac{f'''(0)x^3}{3!} + ...$

$f(x) = \ln(1+x) \quad \Rightarrow \quad f(0) = \ln 1 = 0.$

$f'(x) = \dfrac{1}{1+x} = (1+x)^{-1} \quad \Rightarrow \quad f'(0) = 1.$

$f''(x) = -1(1+x)^{-2} \quad \Rightarrow \quad f''(0) = -1.$

$f'''(x) = 2(1+x)^{-3} \quad \Rightarrow \quad f'''(0) = 2.$

$\therefore f(x) = \ln(1+x) = 0 + x - \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 - .... = x - \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 - ....$

Blunders (–3)

B1 Incorrect differentiation.

B2 Incorrect evaluation of $f^{(n)}(0)$.

B3 Each term not derived.

B4 Error in Maclaurin series.

Slips (–1)

S1 Arithmetic error.

Attempts (3 marks)

A1 Correct expansion of $\ln(1+x)$ given but not derived.

A2 $f(0)$ correct.

A3 Any one correct term derived.

Part (b) (ii)	**5 marks**	**Att 2**

8 (b) (ii) Use those terms to find an approximation for $\ln\dfrac{11}{10}$.

Find approximation	**5 marks**	**Att 2**

8 (b) (ii) $\ln\dfrac{11}{10} = \ln\left(1+\dfrac{1}{10}\right) = \dfrac{1}{10} - \dfrac{1}{200} + \dfrac{1}{3000} = \dfrac{300-15+1}{3000} = \dfrac{286}{3000} = \dfrac{143}{1500}.$

Blunders (–3)

B1 Error in simplification other than slip.

Slips (–1)

S1 Arithmetic error.

Attempts (2 marks)

A1 $\dfrac{11}{10} = 1 + \dfrac{1}{10}.$

A2 Correct value for x.

Part (b) (iii) **5 marks** **Att 2**

8 (b)(iii)	Write down the general term of the series $f(x)$ and hence show that the series converges for $-1 < x < 1$.

General term/converges **5 marks** **Att 2**

(b) (iii) General term $= u_n = \dfrac{(-1)^{n+1} x^n}{n}$. $\quad \therefore\ u_{n+1} = \dfrac{(-1)^{n+2} x^{n+1}}{n+1}$

$$\therefore \underset{n\to\infty}{\text{Limit}}\left|\frac{u_{n+1}}{u_n}\right| = \underset{n\to\infty}{\text{Limit}}\left|\frac{(-1)^{n+2} x^{n+1}}{n+1} \times \frac{n}{(-1)^{n+1} x^n}\right| = \underset{n\to\infty}{\text{Limit}}\left|\frac{(-1)xn}{n+1}\right| = \underset{n\to\infty}{\text{Limit}}\left|\frac{x}{1+\frac{1}{n}}\right| = |x|.$$

Series converges when $|x| < 1 \ \Rightarrow\ -1 < x < 1$.

Blunders (–3)
B1 Incorrect power in general term.
B2 (–1) omitted from general term.
B3 Error in u_{n+1}.
B4 Error in evaluating limit other than slip.
B5 Evaluates limit as $|x|$ and stops.

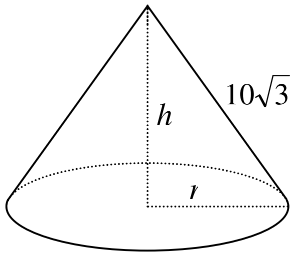
Slips (–1)
S1 Arithmetic error.

Attempts (2marks)
A1 Power of x correct.
A2 Denominator correct.
A3 u_{n+1} correct, given that u_n is not worthless.
A4 Correct substitution into ratio test and fails to finish.

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**

8 (c) A cone has radius r cm, vertical height h cm and slant height $10\sqrt{3}$ cm.

Find the value of h for which the volume is a maximum.



Volume in terms of *h* or *r*	**5 marks**	**Att 2**
Correct differentiation	**5 marks**	**Att 2**
Value of *h*	**5 marks**	**Att 2**

8 (c)

$h^2 + r^2 = 300 \;\Rightarrow\; r^2 = 300 - h^2.$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h\left(300 - h^2\right)$

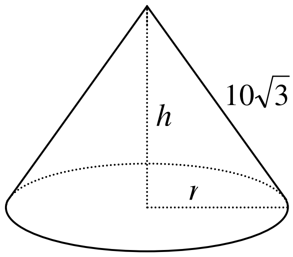
$\therefore V = \frac{1}{3}\pi\left(300h - h^3\right).$

$\frac{dV}{dh} = \frac{1}{3}\pi\left(300 - 3h^2\right) = 0$ for maximum volume.

$\therefore\; 300 - 3h^2 = 0 \;\Rightarrow\; h = 10,$ (since $h > 0$).

$\frac{d^2V}{dx^2} = -2\pi h < 0$ for $h = 10$.

$\therefore h = 10$ cm gives maximum volume.



* $\frac{d^2V}{dh^2} < 0$, for $h = 10$ cm not required.

Blunders (–3)

B1 Incorrect application of Pythagoras.
B2 Error in differentiation.
B3 Error in solving for h or r, other than slip.

Slips (–1)

S1 Arithmetic error.
S2 Correct value for r, but value of h not given.

Attempts (2, 2, 2 marks)

A1 $h^2 + r^2 = 300.$
A2 Some part of differentiation correct.
A3 $\frac{dV}{dh} = 0$, given that candidate's work is not worthless.

QUESTION 9

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a)	**10 marks**	**Att 3**

9 (a) z is a random variable with standard normal distribution. Find $P(1 < z < 2)$.

9 (a) $P(1 < z < 2) = 0.9772 - 0.8413 = 0.1359.$

Blunders (–3)
B1 $P(z \leq 1)$ or $P(z < 2)$ incorrect.

Slips (–1)
S1 Arithmetic error.

Attempts (3 marks)
A1 $P(z \leq 1)$ or $P(z < 2)$ correct.

Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (b) (i)	**10 marks**	**Att 3**

9 (b) (i) During a match John takes a number of penalty shots. The shots are independent of each other and his probability of scoring with each shot is $\frac{4}{5}$.

(i) Find the probability that John misses each of his first four penalty shots.

Probability	**10 marks**	**Att 3**

9 (b) (i) Probability $= \left(\frac{1}{5}\right)^4 = \frac{1}{625}$ or ${}^4C_4\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^4 = \frac{1}{625}$.

Blunders (–3)
B1 Error in binomial.
B2 Incorrect q.

Slips (–1)
S1 Arithmetic error.

Attempts (3 marks)
A1 Correct q.

Part (b) (ii) **5 marks** **Att 2**

9 (b) (ii) Find the probability that John scores exactly three of his first four penalty shots.

Probability **5 marks** **Att 2**

9 (b) (ii) Probability $= {}^{4}C_{3}\left(\frac{4}{5}\right)^{3}\left(\frac{1}{5}\right) = \frac{256}{625}$.

Blunders (–3)
B1 Error in binomial.
B2 Incorrect q.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 $\left(\frac{4}{5}\right)^{3}.\left(\frac{1}{5}\right)$.

Part (b) (iii) **5 marks** **Att 2**

9 (b) (iii) If John takes ten penalty shots during the match, find the probability that he scores at least eight of them.

Probability **5 marks** **Att 2**

9 (b) (iii) P (scores at least eight) = P(scores eight) + P(scores nine) + P(scores ten).

$$= {}^{10}C_{8}\left(\frac{4}{5}\right)^{8}\left(\frac{1}{5}\right)^{2} + {}^{10}C_{9}\left(\frac{4}{5}\right)^{9}\left(\frac{1}{5}\right)^{1} + {}^{10}C_{10}\left(\frac{4}{5}\right)^{10}\left(\frac{1}{5}\right)^{0}$$

$$= \frac{2949120 + 2621440 + 1048576}{9765625} = \frac{6619136}{9765625} \quad (\approx 0.678).$$

Blunders (–3)
B1 Error in binomial.
B2 Omits one essential probability.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 Finds one correct probability.
A2 Probability = P (scoring eight) + P (scoring nine) + P (scoring ten).

Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

9 (c) A survey was carried out to find the weekly rental costs of holiday apartments in certain country. A random sample of 400 apartments was taken. The mean of the sample was €320 and the standard deviation was €50.

Form a 95% confidence interval for the mean weekly rental costs of holiday apartments in that country.

Correct standard error	**10 marks**	**Att 3**
Correct confidence interval	**5 marks**	**Att 2**
Final solution	**5 marks**	**Att 2**

9 (c) $\bar{x} = 320.$ $\sigma = 50.$ $n = 400.$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{20} = 2.5.$$

The 95% confidence interval is

$$\left[\bar{x} - 1.96(\sigma_{\bar{x}}), \bar{x} + 1.96(\sigma_{\bar{x}})\right]$$

$$= \left[320 - 1.96(2.5), 320 + 1.96(2.5)\right] = [€315.10, €324.90]$$

Blunders (–3)

B1 Error in standard error of mean.
B2 Error in confidence interval.
B3 Answer not simplified.

Slips (–1)

S1 Arithmetic error.

Attempts (3, 2, 2 marks)

A1 Standard error of mean with some substitution.
A2 Correct confidence with substitution.

QUESTION 10

Part (a)	**15 (10, 5) marks**	**Att (3, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a)	**15 (10, 5) marks**	**Att (3, 2)**

10 (a)

Show that {0, 2, 4} forms a group under addition modulo 6. You may assume associativity.

Show closure	**10 marks**	**Att 3**
Identity and inverses	**5 marks**	**Att 2**

10 (a) (i)

$+_{\text{mod } 6}$	0	2	4
0	0	2	4
2	2	4	0
4	4	0	2

Closed: No new element.
Identity = 0.
Inverses: $0^{-1} = 0,\ 2^{-1} = 4,\ 4^{-1} = 2.$
$\therefore$ Group.

Blunders (–3)
B1 Identity not given.
B2 Inverses not stated.

Slips (–1)
S1 Arithmetic error.
S2 each inverse not given.

Attempts (3, 2 marks)
A1 Incomplete Cayley table or error in Cayley table.
A2 Identity given.
A3 One inverse given.

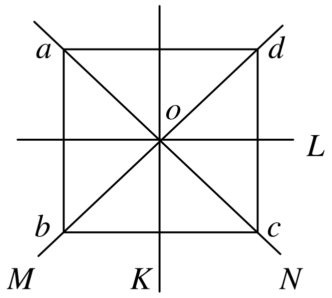
Part (b) **20 (10, 10) marks** **Att (3, 3)**

Part (b) (i) **10 marks** **Att 3**

10 (b) (i)

R_{90^o} and S_M are elements of D_4, the dihedral group of a square.

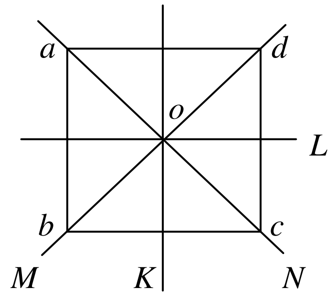
(i) List the other elements of the group.



List elements **10 marks** **Att 3**

10 (b) (i)

$R_{0^o}, R_{180^o}, R_{270^o}, S_N, S_L, S_K$.



Blunders (–3)

B1 Each incorrect element.

B2 Each missing element.

Slips (–1)

S1 Arithmetic error.

Attempts (3 marks)

A1 One correct element.

Part (b) (ii) **10 marks** **Att 3**

10 (b) (ii)

Find $C(S_M)$, the centralizer of S_M.

Find centralizer **10 marks** **Att 3**

10 (b) (ii)

$C(S_M) = R_{0^0}, S_M, S_N, R_{180^0}$.

Blunders (–3)

B1 Each incorrect element.

B2 Each missing element.

Slips (–1)

S1 Arithmetic error.

Attempts (3 marks)

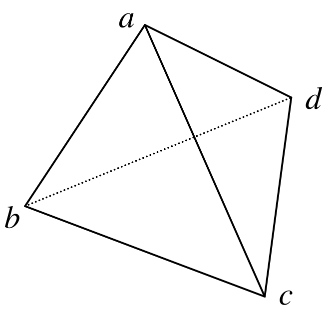
A1 One correct element.

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**
Part (c) (i) **5 marks** **Att 2**

10 (c) A regular tetrahedron has twelve rotational symmetries.
These form a group under composition.
The symmetries can be represented as permutations of the vertices a, b, c and d.

$$X = \left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix} \right\}, \circ \text{ is a subgroup}$$

of this tetrahedral group.



(i) Write down one other subgroup of order 2.

Subgroup of order two **5 marks** **Att 2**

10 (c) (i) $\left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ d & c & b & a \end{pmatrix} \right\}$ or $\left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix} \right\}$.

* If subgroup is not of order 2 then 0 marks.

Blunders (–3)
B1 Incorrect element.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 One correct element.

Part (c) (ii) **5 marks** **Att 2**

10 (c) (ii) Write down a subgroup of order 3.

Subgroup of order three **5 marks** **Att 2**

10 (c) (ii) $\left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ a & d & b & c \end{pmatrix} \right\}$.

or

10 (c) (ii) $\left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ c & b & d & a \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ d & b & a & c \end{pmatrix} \right\}$.

or

10 (c) (ii) $\left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ b & d & c & a \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ d & a & c & b \end{pmatrix} \right\}$.

or

10 (c) (ii) $\left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ b & c & a & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ c & a & b & d \end{pmatrix} \right\}$.

* If subgroup is not of order 3 then 0 marks.

Blunders (–3)
B1 Each incorrect element.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 One correct element.

Part (c) (iii) **5 marks** **Att 2**

10 (c) (iii) Write down the only subgroup of order four.

Subgroup of order four **5 marks** **Att 2**

10 (c) (iii) $\left\{ \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ d & c & b & a \end{pmatrix}, \begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix} \right\}.$

* If subgroup is not of order 4 then 0 marks.

Blunders (–3)
B1 Each incorrect element.

Slips (–1)
S1 Arithmetic error.

Attempts (2 marks)
A1 One correct element.

QUESTION 11

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

11(a) Find the equation of an ellipse with centre (0, 0), eccentricity $\frac{5}{6}$ and one focus at (10, 0).

Value of a	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

11 (a) Focus $= (10, 0) = (ae, 0) \Rightarrow ae = 10. \therefore \frac{5}{6}a = 10 \Rightarrow a = 12.$

$$b^2 = a^2(1-e^2) \Rightarrow b^2 = 144\left(1-\frac{25}{36}\right) = 144\left(\frac{11}{36}\right) \Rightarrow b^2 = 44.$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{144} + \frac{y^2}{44} = 1.$$

Blunders (–3)
B1 Values for *a* and *b* found but final equation not given.

Slips (–1)
S1 Arithmetic error.

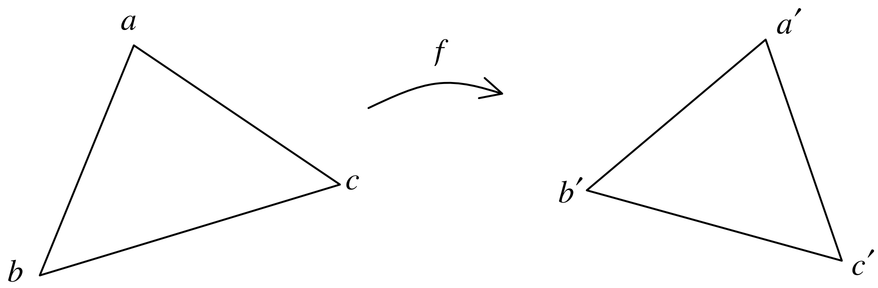
Attempts (2, 2 marks)
A1 ae=10.
A2 $b^2 = a^2(1-e^2)$.
A3 Correct value for b^2 and stops.

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

11 (b) f is a similarity transformation having magnification ratio k.
A triangle abc is mapped onto a triangle $a'b'c'$ under f.
Prove that $|\angle abc| = |\angle a'b'c'|$.

Cos$\angle abc$	**5 marks**	**Att 2**
Cos$\angle a'b'c'$	**5 marks**	**Att 2**
$\lvert p'q'\rvert = k\lvert pq\rvert$	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

11 (b)



$$\cos\angle abc = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab|.|bc|}.$$

$$\cos\angle a'b'c' = \frac{|a'b'|^2 + |b'c'|^2 - |a'c'|^2}{2|a'b'|.|b'c'|}.$$

But $|a'c'| = k|ac|$, $|a'b'| = k|ab|$ and $|b'c'| = k|bc|$ as f is a similarity transformation.

$$\therefore \cos\angle a'b'c' = \frac{k^2|ab|^2 + k^2|bc|^2 - k^2|ac|^2}{2k^2|ab|.|bc|} = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab|.|bc|} = \cos\angle abc.$$

$\cos\angle abc = \cos\angle a'b'c' \Rightarrow |\angle abc| = |\angle a'b'c'|$, as $0^\circ \le |\angle abc| \le 180^\circ$.

Blunders (–3)
B1 Error in cosine formula.
B2 Error in definition of similarity transformation.
B3 Fails to square k.
B4 Reason why $|\angle abc| = |\angle a'b'c'|$ not given.

Slips (–1)
S1 Arithmetic error.

Attempts (2, 2, 2, 2 marks)
A1 Use of cosine rule.
A2 Cos$\angle a'b'c'$ expressed in terms of sides of triangle abc.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

Part (c) (i) **5 marks** **Att 2**

11 (c) (i) g is the transformation $(x, y) \to (x', y')$ where $x' = ax$ and $y' = by$ and $a > b > 0$.

(i) C is the circle $x^2 + y^2 = 1$. Show that $g(C)$ is an ellipse.

Show that $g(C)$ is an ellipse **5 marks** **Att 2**

11 (c) (i) $C: x^2 + y^2 = 1$. $x' = ax$ and $y' = by \Rightarrow x = \frac{x'}{a}$ and $y = \frac{y'}{b}$.

$\therefore\ g(C) = \frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1$. $\therefore g(C)$ is an ellipse.

Blunders (–3)
B1 Error in substitution.

Slips (–1)
S1 Arithmetic error.

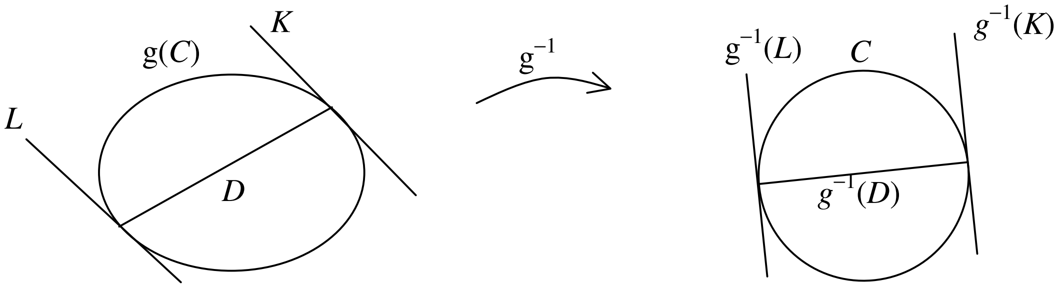
Attempts (2 marks)
A1 x in terms of x' or y in terms of y'.

Part (c) (ii)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

11 (c) (ii) L and K are tangents at the end points of a diameter of the ellipse $g(C)$.
Prove that L and K are parallel.

g^{-1} mapping of $g(C)$, D, L and K	**5 marks**	**Att 2**
Showing $g^{-1}(L)$ or $g^{-1}(K) \perp g^{-1}(D)$	**5 marks**	**Att 2**
Prove L and K are parallel	**5 marks**	**Att 2**

11 (c) (ii)



By g^{-1}, L, K and D map onto $g^{-1}(L)$, $g^{-1}(K)$ and $g^{-1}(D)$ respectively.
But $g^{-1}(L)$ is perpendicular to $g^{-1}(D)$ and $g^{-1}(K)$ is perpendicular to $g^{-1}(D)$, as tangent to circle is perpendicular to diameter at point of contact.
$\therefore$ $g^{-1}(L)$ is parallel to $g^{-1}(K)$.
$\therefore$ L is parallel to K, as parallelism is invariant.

Blunders (–3)
B1 Error in mapping or mapping circle to ellipse..
B2 Reason why $g^{-1}(L)$ or $g^{-1}(K) \perp g^{-1}(D)$ not given.
B3 Reason why L is parallel to K not given.

Slips (–1)
S1 Arithmetic error.

Attempts (2, 2, 2 marks)
A1 One correct mapping.
A2 States $g^{-1}(L)$ or $g^{-1}(K) \perp g^{-1}(D)$ without reason given.
A3 $g^{-1}(L)$ parallel to $g^{-1}(K)$.

BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is less than 226, the bonus is 5% of the mark obtained, rounding *down*. (e.g. 198 marks $\times$ 5% = 9.9 $\Rightarrow$ bonus = 9 marks.)

If the mark awarded is 226 or above, the following table applies:

Marks obtained	Bonus
226 – 231	11
232 – 238	10
239 – 245	9
246 – 251	8
252 – 258	7
259 – 265	6
266 – 271	5
272 – 278	4
279 – 285	3
286 – 291	2
292 – 298	1
299 – 300	0