MARKING SCHEME 2001

JUNIOR CERTIFICATE EXAMINATION

MATHEMATICS

HIGHER LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work, as follows:
   - Blunders - mathematical errors / sign errors / omissions (−3)
   - Slips - numerical errors (−1)
   - Misreadings (provided task is not oversimplified) (−1)

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3, .. . S1, S2, S3, .. M1, M2, .. . etc.

2. When awarding attempt marks, e g. Att(3), it is essential to note that
   - any correct relevant step in a part of a question merits, *at least*, the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is not awarded.

3. Worthless work must be awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2, . etc

4. The *same* error in the *same* section of a question is penalised *once* only

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for the attempt mark only

7. The phrase "and stops" means that no more work is shown by the candidate.

QUESTION 1

Each Part	10 marks	Att 3

(i)	10 marks	Att 3

A car was bought for IR£18 750 At the end of the first year the value of the car had fallen by 20%.
Find its value at the end of the first year.

	1 %	=	£187·50	or	$\frac{1}{5}$	=	£3750
$\Rightarrow$	80 %	=	£15 000	$\Rightarrow$	$\frac{4}{5}$	=	£15 000

* Accept correct answer and no work

Blunders (– 3)

B1 Error with decimal point. e.g. 1% = £1875.

B2 Mathematical error, e g 20 % taken as some fraction, but $\neq \frac{1}{5}$.

B3 Fails to get 80% - stops at 1% → £187 50.

B4 Gets 20 % or $\frac{1}{5}$ of 18 750 and stops → £3750

B5 Takes 18750 as 120% and gets 100% → £15 625.
But **note** $\frac{18750}{120}$ = 156 25 and stops also incurs B3 → 4 marks.

B6 Takes 18750 as 80% and gets 100% → £23 437 50.
But **note** $\frac{18750}{80}$ = 234 375 and stops also incurs B3 → 4 marks.

Slips (– 1)

S1 Numerical errors to a max of 3.

Misreadings (– 1)

M1 Reads as IR£17 850 or similar.

Attempts (3 marks)

A1 20 % = $\frac{1}{5}$ or 80 % = $\frac{4}{5}$ and stops.

A2 Cost price = 100 % and stops.

Worthless (0)

W1 Incorrect answer with no work.

(ii) **10 marks** **Att 3**

A train travelled 155 km at an average speed of 62 km/hr.
How long did the journey take?

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \Rightarrow \quad \text{Time} = \frac{155}{62} \quad \Rightarrow \quad 2.5 \text{ hours (2 hrs 30 min.)}$$

* No penalty if units omitted

Blunders (– 3)

B1 Error with decimal point

B2 Incorrect (relevant) formula e.g time = distance x speed or $\text{Time} = \frac{\text{Speed}}{\text{Distance}}$

B3 No division e g stops at $\frac{155}{62}$, but **note** stops at $\frac{62}{155}$ incurs 2 blunders ➔ 4 marks.

B4 Each error in conversion – if done, but **note** 2.5 hours = 2 hrs 50 min ➔ **no** penalty.

Slips (– 1)

S1 Numerical errors to a max of 3.

Attempts

A1 Some correct conversion relevant to values given

A2 $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$ and no work.

A3 Correct answer without work

Worthless (0)

W1 Incorrect answer with no work.

(iii) **10 marks** **Att 3**

The length of a rectangle is three times its width

The area of the rectangle is 48 cm^2.

Calculate the length of the rectangle

Let x = width and length = $3x$

Area A = $x \; 3x$ = $3x^2$ = 48

$\Rightarrow$ x^2 = 16 $\Rightarrow$ x = 4

$\Rightarrow$ length = 12

* Accept correct answer and no work

* Ignore second solution i.e. $x = -4$.

Blunders (– 3)

B1 Each transposition error.

B2 Error in getting square root or no root, but may also incur S1 below

Slips (– 1)

S1 Fails to get length i.e stops at $x = 4$.

Attempts

A1 Uses perimeter to give length = 18.

A2 Some correct algebraic statement of $l = 3w$

A3 No quadratic (oversimplification) e.g $3x = 48$ to get $x = 16$

A4 $\frac{48}{3} = 16$ and no other work.

A5 Area of rectangle = length x width

Worthless (0)

W1 Incorrect answer and no work

(iv) **10 marks** **Att 3**

Evaluate $\sqrt{\frac{1\,6}{(0\ 2)^2}} - 4.$

$$\sqrt{\frac{1\,6}{0\ 04}} - 4 = \sqrt{\frac{160}{4}} - 4 = \sqrt{40 - 4}$$

$$= \sqrt{36} = 6$$

Blunders (– 3)

B1 Error with decimal point e.g $(0{\cdot}2)^2 = 0\ 4$. but **note** gives answer of 0 ➔ 7 marks.

B2 Mathematical error in reading the tables e g wrong page

B3 Evaluates the root of each individual item under the $\sqrt{}$ sign ➔ 4 32.

B4 Error in calculating reciprocal or ignores reciprocal

B5 Error in getting square root or no root

B6 Error in squaring or failure to square

B7 Incorrect use of common denominator e.g $\frac{1.6}{0\ 04} - 4 = \frac{-2\ 4}{0\ 04}$

B8 Ignores a minus in square root. e.g $\sqrt{-36}$ taken as $\sqrt{36}$

B9 Failure to perform subtraction.

Slips (–1)

S1 Numerical errors to a max of 3.

S2 Slip in reading tables. e g. reads adjacent row or column.

Attempts

A1 Evaluates $\frac{1\ 6}{0\ 2}$ as 8 or similar and stops.

A2 Evaluates $(0\ 2)^2$ as 0 04 or similar and stops.

Worthless (0)

W1 Mishandles the square e g evaluates $(0\ 2)^2$ as 0 4 and stops

(v) **10 marks** **Att 3**

If $y = \frac{x - zt}{3}$, express t in terms of x, y and z

$3y = x - zt \Rightarrow zt = x - 3y$

$\Rightarrow t = \frac{x - 3y}{z}$ **or** $t = \frac{3y - x}{-z}$

Blunders (– 3)

B1 Each different transposition error or error with sign

B2 Mishandles cross multiplication e.g. $y - 3 = x - zt$ and continues.

B3 Error in Distributive Law e g. $3(x - zt) = 3x - zt$

B4 Stops at $zt = x - 3y$, but **note** if stops at $3y = x - zt$ also incurs B1 ➔ 4 marks

B5 Stops at $-t = \frac{x - 3y}{-z}$ or $-t = \frac{3y - x}{z}$.

Attempts

A1 Some correct relevant work, e g. one correct transposition or cross multiplication, but **note** B4

A2 Correct answer without work.

Worthless (0)

W1 Incorrect answer and no work.

W2 Interchanges t and y to get $t = \frac{x - zy}{3}$

(vi) **10 marks** **Att 3**

$A \cup B = \{1, 3, 6, 7, 9, 12\}$ and $A \Delta B = \{3, 6, 9, 12\}$

Write down the elements of $A \cap B$.

$A \Delta B$	=	$A \cup B$	\	$A \cap B$
$\{3, 6, 9, 12\}$	=	$\{1, 3, 6, 7, 9, 12\}$	\	$A \cap B$
$\Rightarrow \quad A \cap B$	=	$\{1, 7\}$		

* Accept correct answer and no work.

Blunders (– 3)

B1 Each element incorrect or omitted with or without work, e g. $A \cap B = \{3, 7\}$ ➔ 4 mks

B2 $A \cap B$ correct on Venn diagram, but not identified or identified incorrectly.

Attempts

A1 One operation correct and stops, e g $A \setminus B = \{3, 9\}$

A2 Symmetric difference defined or illustrated

A3 Any Venn diagram showing intersecting sets.

A4 A set with any of the elements 1, 3, 6, 7, 9, 12 merits Attempt <u>at least</u>

Worthless (0)

W1 Incorrect operation, e g. $A \setminus B = \{3, 7\}$ or similar.

(vii) **10 marks** **Att 3**

If $x \bullet y = 2x - 3y$, find the value of a for which $a \bullet (4 \bullet 1) = 0$

	$4 \bullet 1$	=	$2(4) - 3(1)$	=	$8 - 3$	=	5
	$a \bullet 5$	=	$2(a) - 3(5)$	=	0		
$\Rightarrow$	$2a$	=	15	$\Rightarrow$	a	=	7 ½

* Accept $a = 7\frac{1}{2}$ verified correctly

Blunders (– 3)
B1 Each different transposition error or no transposition
B2 Each different mathematical error.
B3 Interchanges x and y once or twice, e.g gets $1 \bullet 4$, but note M1 below.
B4 Assumes associative law, e.g solves $(a \bullet 4) \bullet 1 = 0$ ➔ $a = 6.75$

Slips (– 1)
S1 Numerical errors to a max of 3

Misreadings (– 1)
M1 Reads as $2y - 3x$ or similar

Attempts
A1 Correct answer without work or verification.
A2 Some correct substitution of a
A3 Correct substitution of 4 and/or 1 and stops e g. 2(4)
A4 Gets $4 \bullet 1 = 5$ and stops

Worthless (0)
W1 Treats $\bullet$ as multiplication

(viii) **10 marks** **Att 3**

Solve the equation $3x^2 + 10x - 8 = 0$

$3x^2 + 10x - 8 = (3x - 2)(x + 4) = 0$

$\Rightarrow 3x = 2$ and $x = -4$

$\Rightarrow x = \frac{2}{3}$ and $x = -4$

or

Guide (key) No. $= -24$

$\Rightarrow 3x^2 + 10x - 8 = 3x^2 + 12x - 2x - 8 = 3x(x + 4) - 2(x + 4)$

$\Rightarrow (3x - 2)(x + 4) = 0$

$\Rightarrow x = \frac{2}{3}$ and $x = -4$

* Accept $x = \frac{2}{3}$ and $x = -4$ without work **provided** both are verified correctly.

* Accept other correct methods, e.g. using formula, big 'X', etc. and mark slips and blunders

Blunders (– 3)

B1 Incorrect factors each time and continues,
but **note** $(3x + 2)(x - 4)$ or $(3x + 4)(x + 2)$, incurs 1 Blunder

B2 Correct factors but roots not stated or incorrect roots.

B3 Errors in use of quadratic formula.

B4 Mathematical errors e.g $10^2 = 20$

B5 Mathematical errors in reading tables

B6 Stops at $3x(x + 4) - 2(x + 4)$, but **note** also incurs B2 above. ➔ 4 marks.

B7 Leaves answer as $\begin{matrix} 3x & -2 \\ x & +4 \end{matrix}$ but **note** also incurs B2 above. ➔ 4 marks

Slips (– 1)

S1 Correct factors and one root correct **but** other root incorrect

Attempts

A1 Indicates correct guide (key) number and stops

A2 Any correct factors of x^2 and / or 8

A3 Writes down correct quadratic formula and stops.

Worthless (0)

W1 Treats as linear, e g. $6x + 10x - 8 = 0$, etc.

(ix) **10 marks** **Att 3**

Express $\frac{1\ 26\times10^{n}}{2\ 8\times10^{12}}$ in the form $a \times 10^{n}$ where $1 \le a < 10$ and $n \in \mathbf{Z}$

$$\frac{1\ 26\times10^{1}}{2\ 8\times10^{12}} = \frac{1\ 26}{2\ 8}\times10^{1-12} = 0\ 45\times10^{-11} = 4\ 5\times10^{-4}$$

Blunders (– 3)

B1 Not in scientific notation, e.g $0\ 45\times10^{-11}$ or 0 00045

B2 Each decimal error, but **note** answer 0 45 also incurs B1 above ➔ 4 marks

B3 Each error in handling indices, but penalise once only when converting to whole numbers

B4 Stops at $\frac{1\ 26}{2\ 8}\times10^{-11}$ but also incur B1 ➔ 4 marks

B5 Multiplies instead of divides, but handles indices correctly ➔ $3{\cdot}528\times10^{-3}$.
But **note** $3\ 528\times10^{21}$ or $3\ 528\times10^{3}$ also incurs B3 ➔ 4 marks.

Attempts

A1 Indicates some knowledge of indices, e.g. gets 10^{23}, but **note** B4 above.

A2 Any correct cancelling and stops

A3 Converts either expression to whole number and stops

A4 Stops at $\frac{1\ 26}{2\ 8\times10^{3}}$.

Worthless (0)

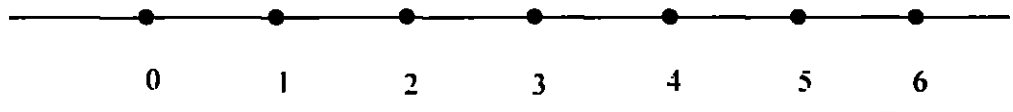
W1 Attempts to add or subtract but shows no knowledge of indices

(x) **10 marks** **Att 3**

Graph on the number line the solution set of $3x - 5 \leq x + 7$, $x \in \mathbf{N}$.

$3x - 5 \leq x + 7 \Rightarrow 2x \leq 12 \Rightarrow x \leq 6$

$\Rightarrow$ {0, 1, 2, 3, 4, 5, 6} or {1, 2, 3, 4, 5, 6}



* Accept correct indication on number line and no work

Blunders (– 3)

B1 Each different transposition error
B2 Mishandles inequality, but **note** S1 below.
B3 Indication on number line inconsistent with candidate's inequality.
B4 No indication on number line, e.g stops at $x \leq 6$
B5 $x \in \mathbf{R}$

Slips (– 1)

S1 Uses $x < 6$, e.g {0, 1, 2, 3, 4, 5} or {1, 2, 3, 4, 5}
S2 $x \in \mathbf{Z}$
S3 Gets $x \leq 6$, but plots points 6, 7, 8 ..

Attempts

A1 Solves equation $3x - 5 = x + 7$ to get $x = 6$
A2 Tests for one value of x and stops
A3 Any correct transposition and stops
A4 Tries to solve two inequalities, e.g $3x - 5 \leq x$ and $-5 \leq x + 7$
A5 Scaled number line.

Worthless (0)

W1 Number line drawn with no points indicated or no scales.

QUESTION 2

Part (a)	**25 marks**	**Att 8**
Part (b)	**25 marks**	**Att 8**

Part (a) **25 (10, 15) marks** **Att 8 (3, 5)**

(a) A window is in the shape of a rectangle and a semicircle as in the diagram

The rectangular part of the window is 50 cm high and

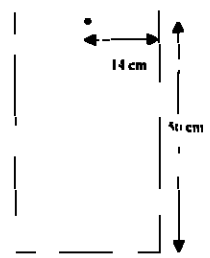
The radius of the semicircle is 14 cm

Calculate:

(i) the perimeter of the window

(ii) the area of the window

Take $\pi = \frac{22}{7}$.



(a) (i) **10 marks** **Att 3**

Length of semicircle = πr = $\frac{22}{7} \times 14$ = 44

Perimeter of rectangular section = 50 + 28 + 50 = 128

$\Rightarrow$ Perimeter = 172

* Accept answer using reasonable approximation for π, e.g. 22/7, 3, 3·1, 3·14 etc.

Blunders (– 3)

B1 Uses $2\pi r$ for semicircle.
B2 Includes diameter once or twice
B3 Uses incorrect value for radius e.g r = 28 or 7.
B4 Mathematical error.
B5 Misplaced decimal. e.g when using a value for π
B6 Perimeter of semicircle only, but also incurs S1 below. → 6 marks.
B7 Perimeter of semicircle in terms of π, but also incurs S1 below. → 6 marks

Slips (–1)

S1 Fails to add two results.

Attempts (3 marks)

A1 Perimeter of rectangle only
A2 Correct answer without work.
A3 Some correct substitution into a relevant length formula
A4 Perimeter of semicircle = πr and stops

Worthless (0)

W1 Perimeter = $2\pi r$ and stops.

(a) (ii) **15 marks** **Att 5**

Area of semicircle	=	$\frac{1}{2}\pi r^2$ =	$\frac{1}{2} \times \frac{22}{7} \times (14)^2$	=	308
Area of rectangular	=	50 x 28		=	1400
Area of window	=	308 + 1400		=	1708

* Accept answer using reasonable approximation for π. e.g. 3. 3 1. 3·14 etc
* Do not penalise the same error if already penalised in (i) above

Blunders (– 3)
B1 Uses πr^2 for semicircle.
B2 Mathematical error e g $14^2 = 28$
B3 Incorrect substitution into correct formula.
B4 Misplaced decimal, e.g when using a value for π.
B5 Uses incorrect value for radius e.g r = 28 or 7
But **note** do not penalise again if penalised in *a* (i) above.
B6 Area of semicircle only, but also incurs S1 below. ➔ 11 marks

Slips (–1)
S1 Fails to add two results

Attempts (5 marks)
A1 Correct answer without work.
A2 Some correct substitution into correct formula for area of circle / semicircle / rectangle
A3 Area of semicircle = $\frac{1}{2}\pi r^2$ and stops.
A4 Area of rectangle only

Worthless (0)
W1 Area = πr^2 and stops

Part (b) **25 (10,15) marks** **Att 8 (3, 5)**

(b) A solid cylinder, made of lead, has radius 4 cm and height 10 cm

(i) Find the volume of the cylinder in terms of π.

The cylinder is melted down and all of the lead is used to make three identical right circular cones.

The height of each cone is 8 cm

(ii) Find the length of the radius of each cone

Give your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{N}$.

(b) (i) **10 marks** **Att 3**

Volume of cylinder = $\pi r^2 h$ = $\pi (4)^2 \times 10$ = 160π.

* Accept answer using reasonable approximation for π, e g 22/7, 3, 3 1, 3 14 etc

* Accept correct answer and no work.

Blunders (– 3)

B1 Uses formula for volume of cone.

B2 Mathematical error e.g. $4^2 = 8$

B3 Each incorrect (inconsistent) substitution into correct formula.

B4 Uses incorrect value for radius e.g r = 2 or 8

B5 Misplaced decimal, e g when using a value for π

B6 Stops at $\pi (4)^2 \times 10$, but **note** also incurs B2 above ➔ 4 marks

but stops at $\pi .16 \times 10$ only 1 Blunder ➔ 7 marks

Attempts (3 marks)

A1 Gets curved surface area of cylinder

A2 Some correct substitution into a relevant volume formula

A3 Cylinder drawn showing correct radius and/or height

Worthless (0)

W1 Volume of cylinder = $\pi r^2 h$ and stops.

W2 Substitution into formula for sphere or circle

(b) (ii) **15 marks** **Att 5**

Volume of each cone $= \frac{160\pi}{3}$

$\Rightarrow \frac{160\pi}{3} = \frac{1}{3}\pi r^2 \times 8 \Rightarrow r^2 = 20$

$\Rightarrow r = \sqrt{20} = 2\sqrt{5}$

* Accept candidate's results from *b*(i) above

Blunders (– 3)

B1 Each incorrect (inconsistent) substitution into correct formula

B2 Assumes volume of cylinder equals volume of 1 cone

B3 Incorrect height for cone

B4 Gets $\sqrt{20}$ as final answer

B5 Error in square root or no square root, e.g stops at $r^2 = 20$.
but **note** also incurs B4. ➔ 9 marks

B6 Stops at $\frac{160\pi}{3} = \frac{1}{3}\pi r^2 \times 8$.
but **note** also incurs B4 and B5 ➔ 6 marks.

B7 Transposition error

Attempts (5 marks)

A1 Some correct substitution into a relevant volume formula.

A2 Divides answer from (b) (i) above by 3 and stops

QUESTION 3

Part (a)	**15 marks**	**Att 6**
Part (b)	**15 marks**	**Att 5**
Part (c)	**20 marks**	**Att 6**

Part (a) **15 (5,5,5) marks** **Att (2,2,2)**

(a) Factorise fully each of the following:

(i) $20xy - 4x^2$

(ii) $5x^2 - 9x - 2$

(iii) $x^3 - 27y^3$

(a) (i) **5 marks** **Att 2**

$20xy - 4x^2 = 4x(5y - x)$

Blunders (– 3)

B1 Not fully factorised, e g gets $2x(10y - 2x)$ or similar

B2 Error in sign.

B3 Error in distributive law, e.g. gets $4x(5y - 4x^2)$ or similar.

Attempts (2 marks)

A1 Correct factors of either term.

(a) (ii) **5 marks** **Att 2**

$5x^2 - 9x - 2 = (5x + 1)(x - 2)$

* Accept other correct methods, e.g. using formula, big 'X', guide number etc and mark slips and blunders.

Blunders (– 3)

B1 Incorrect factors.

B2 Leaves answer as

$$5x \quad +1$$

$$x \quad -2$$

B3 Stops at $5x(x - 2) + 1(x - 2)$

B4 Errors in use of quadratic formula.

B5 Uses formula to get $x = -\frac{1}{5}$ and $x = 2$ but fails to form factors.

Attempts (2 marks)

A1 Any correct factors of x^2 and / or 2

A2 Some correct substitution into correct quadratic formula

A3 Indicates correct guide (key) number and stops

A4 Writes down correct quadratic formula and stops.

(a) (iii) **5 marks** **Att 2**

$$x^3 - 27y^3 = (x - 3y)(x^2 + 3xy + 9y^2)$$

* Apply slips and blunders if candidate divides $x^3 - 27y^3$ by $x - 3y$.

Blunders (– 3)

B1 Each error in sign

B2 Treats as $x^3 - (27y)^3$ to get $(x - 27y)(x^2 + 27xy + 729y^2)$

B3 Each incorrect term, if it is not a slip or if it is not the same error being repeated.

Attempts (2 marks)

A1 Some correct relevant work, e g $(3y)^3$

A2 Treats as difference of two squares, i.e. $(x - \sqrt{27}y)(x + \sqrt{27}y)$

A3 Correct formula for sum / difference of two cubes and stops

A4 Indicates some knowledge of the sum / difference of two cubes.

A5 Expands $(x - 3y)^3$.

A6 Gets $(x - 3y)$ and stops

Worthless (0)

W1 $(x^3 - 27y^3)(x^3 + 27y^3)$

Part (b) **15 marks** **Att 5**

(b) Find the value of $x^2 - 2xy + 3$ when $x = \frac{1}{2}$ and $y = \frac{2}{3}$

(b) **15 marks** **Att 5**

$$x^2 - 2xy + 3 = \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + 3 = \frac{1}{4} - \frac{2}{3} + \frac{3}{1}$$

$$= \frac{31}{12} = 2\frac{7}{12}$$

Blunders (– 3)

B1 Each incorrect substitution

B2 Errors in sign

B3 Error in squaring / multiplication. e.g $\left(\frac{1}{2}\right)^2 = 1$

B4 Each error in distributive law. e.g. $2\left(\frac{1}{2}\right) = 2\frac{1}{2} = \frac{5}{2}$

B5 Incorrect common denominator or mishandles common denominator.

B6 Stops at $\frac{1}{4} - \frac{2}{3} + 3$, but **note** also incurs B5 ➔ 9 marks.

Slips (–1)

S1 Mixes up x and y values

S2 Numerical errors to a max of 3

S3 Rounding off error

Attempts (5 marks)

A1 Some correct substitution and stops

Part (c) **20 (10, 10) marks** **Att (3, 3)**

(c) Solve, correct to two decimal places, the equation

$$\frac{4}{x+5} - \frac{1}{x+1} = -1, \quad x \neq -5, x \neq -1$$

(c) Quadratic **10 marks** **Att 3**

$$\frac{4(x+1) - 1(x+5)}{(x+5)(x+1)} = \frac{-1(x+5)(x+1)}{(x+5)(x+1)}$$

$$\Rightarrow \quad 4x + 4 - x - 5 = -(x^2 + 6x + 5) = -x^2 - 6x - 5$$

$$\Rightarrow \quad x^2 + 9x + 4 = 0$$

Blunders (– 3)

B1 Errors in sign and multiplication.
B2 Incorrect common denominator or mishandles common denominator
B3 Each error in distributive law
B4 No final transposition or error in transposition
B5 Failure to add like terms. i e stops at $4x + 4 - x - 5 = -(x^2 + 6x + 5)$
But **note** also incurs B4 → 4 marks.

Attempts (3 marks)

A1 Correct common denominator and stops

(c) Solving Quadratic **10 marks** **Att 3**

Solving $x^2 + 9x + 4 = 0$ using formula

$\Rightarrow \quad x = -0.47$ and $x = -8.53$

* Accept candidate's quadratic from above

Blunders (– 3)

B1 Each error in formula, e g. + b $\pm \sqrt{}$ etc
B2 Each different incorrect substitution into formula.
But **note** b = 4 ; c = 9 → 1 Blunder
B3 Mathematical error in sign, e g. – 4(1)(4) = 16
B4 Mathematical error in squaring, e.g $9^2 = 18$ or similar
B5 Mathematical error in tables (wrong page).
B6 Ignores a minus in square root, e.g. $\sqrt{-65}$ taken as $\sqrt{65}$
B7 One solution where there should be two, but **note** A3 below
B8 Misplaced decimal
B9 Gets $\frac{-9 \pm \sqrt{65}}{2}$ and stops, incurs **2 blunders** and S1 below → 3 marks

Slips (–1)

S1 Failure to round off or rounds off incorrectly, once or twice.
S2 Numerical to max of 3.
S3 Ignores minus sign at final step → S(– 1) each time.

Attempts (3 marks)

A1 Incorrect relevant formula with some correct substitution.
A2 Correct formula and stops.
A3 No quadratic due previous errors, merits attempt at most

Worthless (0)

W1 Some attempt at factorising.

QUESTION 4

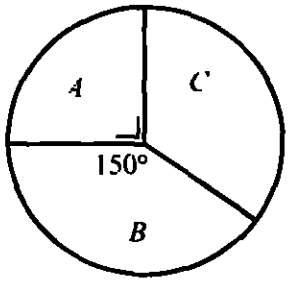
Part (a)	**15 marks**	**Att 5**
Part (b)	**35 marks**	**Att 12**

Part (a) **15 (10, 5) marks** **Att (3, 2)**

An survey was taken to find out which of the political parties, A, B or C, people would vote for in an election
The results of the poll are shown in this pie chart
420 people would vote for party B

(i) How many people took part in the survey?

(ii) How many people would vote for party C?



(a) (i) **10 marks** **Att 3**

$$150° \rightarrow 420 \text{ people} \Rightarrow 1° \rightarrow \frac{420}{150} \text{ people}$$

$$\Rightarrow 360° \rightarrow 1008 \text{ people}$$

* Accept other correct methods

Blunders (– 3)
B1 Errors in cross multiplication.
B2 Full circle not 360° or no use of 360°
B3 Some incorrect ratio or ratio inverted.
B4 Errors with decimal point
B5 No final multiplication, but **note** may also incur B2 above.

Slips (– 1)
S1 Each slip to max 3

Attempts (3 marks)
A1 Writes 420 = 150° and stops.
A2 Some correct ratio
A3 Some use of 360°

(a) (ii) **5 marks** **Att 2**

Angle for (= 120° → No. of people = $\frac{1008}{3}$ = 336

* Accept candidate's total from part **(i)** above.
* Accept correct answer and no work consistent with part **(i)** above
* Do not penalise the same error if already penalised in part **(i)** above.

Blunders (– 3)
B1 Errors in cross multiplication
B2 Full circle not 360° or no use of 360°
B3 Some incorrect ratio or ratio inverted
B4 Errors with decimal point.

Slips (– 1)
S1 Each slip to max 3.

Attempts (2 marks)
A1 Calculates / states angle 120° and stops
A2 Some use of 360°

Part (b) **35 (10, 10, 5, 5, 5) marks** **Att 12 (3, 3, 2, 2, 2)**

The following frequency distribution table shows the scores obtained by 50 people in a quiz

Score	0 -10	10 - 20	20 - 30	30 - 60	60 - 100
No of people	5	12	15	14	4

(note 0 -10 means 0 or more but less than 10, etc)

Copy and complete the following cumulative frequency table:

Score	< 10	< 20	< 30	< 60	< 100
No. of people	5				

(i) On graph paper, draw the ogive (cumulative frequency curve), putting the number of people on the vertical axis.

Use your graph to estimate:

(ii) the median score in the quiz

(iii) the number of people who scored between 25 and 50

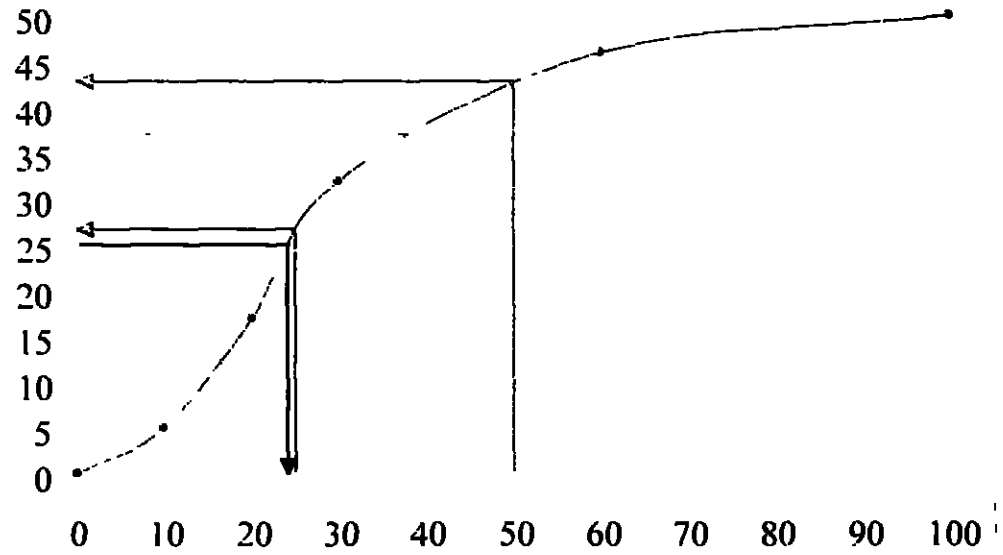
(iv) the interquartile range.

(b) Cumulative frequency table **10 marks** **5 x 2 marks**

Score	< 10	< 20	< 30	< 60	< 100
No of people	5	17	32	46	50

* **Note** Each value filled in correctly gets **2 marks.** Errors do **not** carry forward.

(b) (i) Ogive **10 marks** **Att 3**



* Use candidate's values from their cumulative frequency table

Blunders (– 3)
B1 Scale not uniform (1 x -3 each axis).
B2 Each point omitted or plotted incorrectly (if not consistent or slip).
B3 Points not joined or not a smooth curve
B4 Number of people on the horizontal axis

Slips (–1)
S1 Slip in plotting points (to max 3)

Attempts (3 marks)
A1 Axes scaled or partly scaled and stops
A2 Frequency polygon /curve.
A3 Cumulative frequency histogram.
A4 Couples named. e.g (10.5) and stops
A5 Uses the frequency distribution table given.

Worthless (0)
W1 Pie chart, bar chart or histogram

(b) (ii) **5 marks** **Att 2**

Median Score	⇒	24

* Accept answer consistent with candidate's curve (within tolerance of ± 2).

Blunders (– 3)
B1 Value read from wrong starting point of correct axis
B2 Value taken using wrong axis as starting point and continues.

Slips (–1)
S1 Lines drawn correctly but value not indicated or written down
S2 Written value just outside tolerance

Attempts (2 marks)
A1 A line drawn from correct axis to graph and stops

(b) (iii) **5 marks** **Att 2**

Score of 25	⇒	27 people
Score of 50	⇒	43 people
Score between 25 and 50	⇒	16 people

* Accept answer consistent with candidate's curve (within tolerance of ± 2).

Blunders (– 3)
B1 Value(s) read from wrong starting point of correct axis.
B2 Value(s) taken using wrong axis as starting point and continues.
B3 Two lines drawn correctly and stops.

Slips (–1)
S1 Lines drawn correctly but value not indicated or written down
S2 Written value just outside tolerance.
S3 No subtraction.

Attempts (2 marks)
A1 A line drawn from correct axis to graph and stops

(b) (iv) **5 marks** **Att 2**

3rd Quarter	⇒	38
1st Quarter	⇒	17
The interquartile range	⇒	21

* Accept answer consistent with candidate's curve (within tolerance of ± 2).

Blunders (– 3)
B1 Value(s) read from wrong starting point of correct axis
B2 Value(s) taken using wrong axis as starting point and continues.
B3 Two lines drawn correctly and stops.

Slips (–1)
S1 Lines drawn correctly but value not indicated or written down.
S2 Written value just outside tolerance.
S3 No subtraction

Attempts (2 marks)
A1 A line drawn from correct axis to graph and stops

QUESTION 5

Part (a)	**35 marks**	**Att 13**
Part (b)	**15 marks**	**Att 5**

Part (a) **35 (20,5,5,5) marks** **Att 13 (7,2,2,2)**

(a) Graph the function $f: x \to 3 + 3x - 2x^2$ in the domain $-2 \leq x \leq 3$, $x \in \mathbf{R}$

Use your graph to estimate

(i) $f(2.5)$

(ii) the roots of the equation $3 + 3x - 2x^2 = 0$

(iii) the maximum value of $f(x)$.

(a) Graph **20 marks** **Att 7**

x	-2	-1	0	1	2	3
3	3	3	3	3	3	3
$+3x$	-6	-3	0	3	6	9
$-2x^2$	-8	-2	0	-2	-8	-18
$f(x)$	-11	-2	3	4	1	-6

or

$f(-2) = 3 + 3(-2) - 2(-2)^2 = -11$

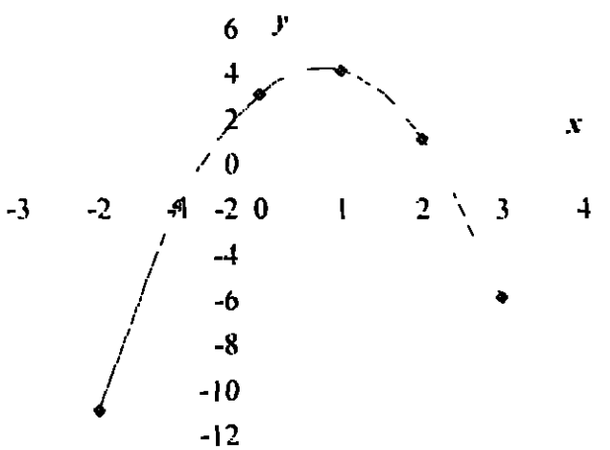
$f(-1) = 3 + 3(-1) - 2(-1)^2 = -2$

$f(0) = 3 + 3(0) - 2(0)^2 = 3$

$f(1) = 3 + 3(1) - 2(1)^2 = 4$

$f(2) = 3 + 3(2) - 2(2)^2 = 1$

$f(3) = 3 + 3(3) - 2(3)^2 = -6$



Values for quadratic graph

Blunders (– 3)

B1 Each incorrect $f(x)$ without work.
B2 x row added in, i.e top row.
B3 $f(x) = 2x^2 - 3x - 3$ and continues correctly
B4 Omits $+3x$ or 3 row (– 3 for each omitted)
B5 Treating the domain as $-2 < x < 3$, can incur 2 Blunders if both omitted.
B6 Each different blunder which yields an incorrect row (full or part).
e g. $-(2x)^2$ for $-2x^2$.
B7 Avoids square for some (not all) values.
B8 Mathematical errors in tots, e.g $-5 + 3 = 2$
B9 $-2x^2$ treated as $(-2x)^2$ but penalise once only.
B10 $2x^2$ written for $-2x^2$ and work completed correctly
B11 3 row treated as $3x$.

Slips (–1)

S1 Numerical slips to a max. of 3.

Attempts (7 marks)

A1 Omits $-2x^2$ or does not treat as x^2 (Treats as linear expression)

A2 Correct or partly correct table / values but no graph drawn

Plotting the quadratic graph

* Accept candidate's values from the table.
* Accept correct graph without work (20 marks)
* Accept max $f(x)$ as 4

Blunders (– 3)

B1 Points not joined to form a reasonable graph
B2 (x, y) plotted as (y, x)
B3 + and – sides confused, e.g $(-2, -11)$ plotted as $(2, -11)$
B4 Scale not reasonably uniform. 1 v (–3) each axis
B5 Each different blunder in plotting points from candidate's table / values.
B6 Each point omitted, if graph does not go reasonably close to where point should be.
B7 Points joined with straight lines

Attempts (7 marks)

A1 No marks awarded so far but scaled axis drawn

(a) (i) **5 marks** **Att 2**

$f(2.5) = -2$

* Accept values in range – 1 8 to – 2 2 inclusive.
* Accept an answer consistent with candidate's graph

Blunders (– 3)
B1 Solves $f(x) = 2\ 5$, either indicated on graph or calculated.
B2 Correct indication on graph, but no value given

Slips (–1)
S1 Written value just outside tolerance.

Attempts (2 marks)
A1 Finds $f(2\ 5)$ algebraically or any correct substitution of 2 5 into $f(x)$

Worthless (0)
W1 Answer inconsistent with candidate's graph

(a) (ii) **5 marks** **Att 2**

$3 + 3x - 2x^2 = 0 \Rightarrow x = 2\ 2$ and $-0\ 7$

* Accept an answer consistent with candidate's graph, within tolerance ± 0 2.

Blunders (– 3)
B1 One value only
B2 Correct indication on graph but no values given.

Slips (–1)
S1 Correct indication on graph, but roots misread.
S2 Written value just outside tolerance

Attempts (2 marks)
A1 Correct solution algebraically
A2 Some indication on graph, but no values given

Worthless (0)
W1 Tries to solve algebraically, but **note** A1 above

(a) (iii) **5 marks** **Att 2**

Maximum value of $f(x)$ = 4 1

* Accept values in range 4 3 to 4 0 inclusive for max of $f(x)$
* Accept an answer consistent with candidate's graph.

Blunders (– 3)
B1 x value of maximum only.

Slips (–1)
S1 Written value just outside tolerance
S2 Gives coordinates of the maximum rather than the x value

Attempts (2 marks)
A1 Point indicated on graph only.

Worthless (0)
W1 Answer inconsistent with candidate's graph

Part (b) **15 (5, 5, 5) marks** **Att 6 (2,2,2)**

(b) $g \quad x \to 9 - x^2$ is a function defined on **R**.

(i) What is $g(-4)$?

(ii) Find the value of x for which $g(x) = 0$.

(iii) Verify that $g(4) > (g \circ g)(2)$

(b) (i) **5 marks** **Att 2**

$g(-4) = 9 - (-4)^2 = 9 - 16 = -7$

* Accept correct answers and no work

Blunders (– 3)
B1 Mathematical errors, e.g $(-4)^2 = 8$
B2 Sign errors, e.g $(-4)^2 = -16$

Slips (–1)
S1 Numerical to max of 3.

Attempts (2 marks)
A1 Some effort at substitution and stops.

(b) (ii) **5 marks** **Att 2**

$g(x) = 0 \quad \Rightarrow \quad 9 - x^2 = 0 \quad \Rightarrow \quad x = \pm 3$

* Accept correct answers and no work

Blunders (– 3)

B1 Transposition errors

B2 Sign errors.

B3 Incorrect factors (if used)

B4 $x = +3$ only.

Attempts (2 marks)

A1 Correct factors and stops.

A2 $9 - x^2 = 0$ and stops

A3 $x^2 = 9$ and stops

A4 Finds $g(0) = 9$

(b) (iii) **5 marks** **Att 2**

$g(4) = 9 - 4^2 = -7$

$g(2) = 9 - 2^2 = 5$

$g(5) = 9 - 5^2 = -16$

$\Rightarrow \quad -7 > -16$

Blunders (– 3)

B1 Errors in squaring.

B2 Sign errors.

Slips (–1)

S1 Work correct but no final statement.

Attempts (2 marks)

A1 Any $g(x)$ correct

A2 Ignores $g \circ g$.

QUESTION 6

Part (a)	**20 marks**	**Att 7**
Part (b)	**15 marks**	**Att 6**
Part (c)	**15 marks**	**Att 5**

Part (a) **20 marks** **Att 7**

(a) Solve the simultaneous equations.

$3x - y = 7$

$$\frac{x-1}{3} - \frac{y+4}{2} = 0$$

(a) **20 marks** **Att 7**

$$\frac{x-1}{3} - \frac{y+4}{2} = 0 \quad \Rightarrow \quad \frac{2(x-1)}{6} - \frac{3(y+4)}{6} = 0$$

$$\Rightarrow \quad 2x - 2 - 3y - 12 = 0 \quad \Rightarrow \quad 2x - 3y = 14$$

and $-9x + 3y = -21$

$$\Rightarrow \quad x = 1 \quad \text{and} \quad y = -4$$

* Accept $x = 1$ and $y = -4$, if verified in both equations **or** shown as intersection of lines

Blunders (–3)

B1 Incorrect common denominator
Note: An error in common denominator could still result in candiate getting correct answer, e g. $6(x - 1) - 6(y + 4) = 0$ will give correct values for x and y.
➔ 17 marks

B2 Each different error in transposing or signs

B3 Does not multiply every term of equation, each time, e g. $-9x + 3y = -7$.

B4 Mathematical error, e g. $-3y - 3y = 0$

B5 Calculates the value of x **or** y correctly and stops ➔ 17 marks

Slips (–1)

S1 Finds $x = 1$ but subs in some other value of x to find y, e.g. $x = -1$.

S2 Numerical to a max of 3, **note** $6 \times 0 = 6$

Attempts (7 marks)

A1 $x = 1$ and $y = -4$ without work

A2 Some correct relevant work, e g. $-9x + 3y = -21$ and stops

A3 Writes x in terms of y or vice-versa, e.g $y = 3x - 7$

Worthless (0)

W1 $x = 1$ **or** $y = -4$ without work

Part (b) **15 (5,5,5) marks** **Att 6 (2,2,2)**

(b) Let $p = \text{Log}_{10} 7$ and $q = \text{Log}_{10} 2$

Express each of the following in the form $\text{Log}_{10} n$. $n \in \mathbf{Q}$, $n > 0$:

(i) $p + q$

(ii) $p - q$

Express in term of p and q:

(iii) $\text{Log}_{10} 56$

(b) (i) **5 marks** **Att 2**

$p + q = \text{Log}_{10} 7 + \text{Log}_{10} 2 = \text{Log}_{10} 14$

* No penalty for omission of base 10 in any part of question

Blunders (–3)

B1 $\text{Log}_{10} 9$

Slips (–1)

S1 Stops at $\text{Log}_{10} 7 \times 2$.

Attempts (2 marks)

A1 Any correct substitution of value for p or q

A2 Correct value for $\text{Log}_{10} 7$ or $\text{Log}_{10} 2$ from the tables

A3 Indicates some knowledge of logs / indices

A4 *List relevant rule. e.g* $\log A + \log B = \log A \times B$

A5 $\text{Log}_{10} 7 \times \text{Log}_{10} 2$ and stops

(b) (ii) **5 marks** **Att 2**

$p - q = \text{Log}_{10} 7 - \text{Log}_{10} 2 = \text{Log}_{10} \frac{7}{2}$ or $\text{Log}_{10} 3\ 5$

* Do not penalise a similar error if already penalised in (i) above.

Blunders (–3)

B1 $\text{Log}_{10} 5$.

Slips (–1)

S1 Stops at $\text{Log}_{10} 7 \div 2$, but accept $\text{Log}_{10} \frac{7}{2}$ for 5 marks.

Attempts (2 marks)

A1 Any correct substitution of value for p or q

A2 Correct value for $\text{Log}_{10}\ 7$ or $\text{Log}_{10}\ 2$ from the tables

A3 Indicates some knowledge of logs / indices.

A4 List relevant rule, e.g $\log A - \log B = \log \frac{A}{B}$

A5 $\text{Log}_{10}\ 7 - \text{Log}_{10}\ 2$ and stops.

(b) (iii) **5 marks** **Att 2**

$$\text{Log}_{10}\ 56 = \text{Log}_{10}\ 7\times 8 = \text{Log}_{10}\ 7 + \text{Log}_{10}\ 8$$
$$= \text{Log}_{10}\ 7 + \text{Log}_{10}\ 2^3 = \text{Log}_{10}\ 7 + 3\text{Log}_{10}\ 2$$
$$= p + 3q$$

Blunders (–3)

B1 $\text{Log}_{10}\ 7 + \text{Log}_{10}\ 8$ and stops

B2 $\text{Log}_{10}\ 8 = 4\text{Log}_{10}\ 2$

B3 $\text{Log}_{10}\ 7 + \text{Log}_{10}\ 2$ and stops.

Slips (–1)

S1 Stops at $\text{Log}_{10}\ 7 + 3\text{Log}_{10}\ 2$ without writing $p + 3q$.

Attempts (2 marks)

A1 Writes 56 as a product

A2 Any correct substitution of value for p or q

A3 Correct value for $\text{Log}_{10}\ 56$ from the tables.

A4 Indicates some knowledge of logs / indices.

A5 List relevant rule, e g. $\log A^n = n \log A$

Worthless (0)

W1 $\text{Log}_{10}\ (7+8)$ and stops

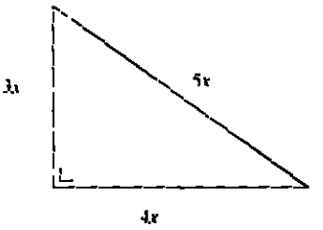
Part (c) **15 (5, 10) marks** **Att 5 (2, 3)**

(c) The sides of a right angled triangle are $3x$, $4x$ and $5x$ in length

The area of the triangle is 121 5 cm^2

Use this information to write an equation in x

Solve the equation and hence, find the lengths of the sides of the triangle



(c) Equation **5 marks** **Att 2**

Area	=	$\frac{1}{2}(4x)(3x)$	=	121 5

Blunders (–3)
B1 Area = (4x)(3x) – omits ½.
B2 Uses 5x

Attempts (2 marks)
A1 Use of Pythagoras theorem
A2 Finds area of triangle of sides 3, 4, 5 = 6.
A3 Some correct substitution into area of triangle formula.
A4 ½ *ah* = 121 5

Worthless (0)
W1 Area of triangle = ½ *ah*

(c) Solve **10 marks** **Att 3**

	$\frac{1}{2}(4x)(3x)$	=	$6x^2$	=	121 5
	x^2	=	$\frac{121\ 5}{6}$	=	20·25
⇒	x	=	$\sqrt{20\ 25}$	=	4 5
⇒	3 x	=	13 5		
⇒	4 x	=	18		
⇒	5 x	=	22 5		

Blunders (-3)
B1 Error in the distributive law.
B2 Errors in transposition
B3 Error in square root or no square root, e g stops at x^2 = 20 25.
but **note** also incurs S1 × 3 ➔ 4 marks
B4 Decimal errors.
B5 Correct factors and stops, but **note** also incurs S1 × 3 ➔ 4 marks

Slips (–1)
S1 *Finds the value of x but fails to get 3x, 4x and 5x → 1 Slip (– 1) each time*

Attempts (3 marks)
A1 No quadratic due previous errors, merits attempt at most.
A2 No use of algebraic equation, e g $\frac{121\ 5}{6}$
A3 Any correct use of distributive law.

MARKING SCHEME 2001

JUNIOR CERTIFICATE EXAMINATION

MATHEMATICS

HIGHER LEVEL

PAPER 2

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work, as follows:
   - Blunders - mathematical errors / sign errors / omissions (–3)
   - Slips - numerical errors (–1)
   - Misreadings (provided task is not oversimplified) (–1)

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3, S1, S2, S3, .. . M1, M2, . etc.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that
   - any correct relevant step in a part of a question merits, *at least*, the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is not awarded

3. Worthless work must be awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2, ... etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for the attempt mark only.

7. The phrase "and stops" means that no more work is shown by the candidate.

QUESTION 1

Each Part **10 marks** **Att 3**

(i) **10 marks** **Att 3**

A sum of money is divided in the ratio 5 : 6 The smaller amount is IR£25.
What is the total amount of money?

$\frac{5}{11}$ = £25 (step 1) $\Rightarrow$ $\frac{1}{11}$ = £5 (step 2) $\Rightarrow$ $\frac{11}{11}$ = £55 (step 3)
OR any comparable method

- Accept correct answer without work for full marks

B1 Each step omitted or incorrect B2 Leaves answer as $\frac{25}{5} \times 6 = 30$

B3 Evaluates $\frac{25}{6} \times 5$ and continues correctly (45 83)

B4 $\frac{5}{11}$ as 5 & continues

S1 Each numerical slip (max 3)

A1 Some recognition of ratio and stops e g. $\frac{1}{5}$, 11 etc

W1 Incorrect answer without work
W2 Evaluates $\frac{25}{6} \times 5$ & stops

(ii) **10 marks** **Att 3**

A sum of money, invested at 6% per annum interest, amounted to IR£1,590
after one year
What sum of money was invested?

106% = 1590 (step 1) 1% = 15 (step 2) 100% = 1500 (step 3)
OR comparable method

B1 Each step omitted or incorrect
B2 $\frac{1590}{106} \times 106$ & continues
B3 Error in decimal point

S1 Each numerical slip to max of 3
A1 Any mention of 106% or 1 06 A2 6% = $\frac{6}{100}$
A3 Gets 6% of IR£1,590 with or without further work.

(iii) **10 marks** **Att 3**

Three tennis balls, each of radius 3.5 cm, fit exactly into a cylindrical tube
Find, in terms of π, the volume of the tube

Height of tube = $7 \times 3 = 21$ cm Radius of tube = 3.5 cm (step 1)

$\Rightarrow V = \pi \times (3.5)^2 \times 21$ (step 2)

$\Rightarrow V = 12.25 \times 21\pi = 257.25\,\pi$ cm^3 (step 3)

- No penalty for substitution for π or cm^3 omitted

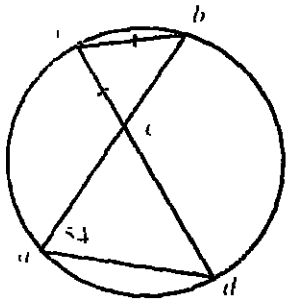
B1 Each step omitted or incorrect B2 Incorrect height and/or radius
B3 Substitution & no simplifying i.e. step omitted B4 Error in decimal point
B5 Error in indices B6 Uses $2\pi rh$ & continues (147π)

A1 Diameter = 7 & stops A2 $\pi r^2 h$ & stops
A3 Substitutes for r or h & stops A4 Correct diagram & stops
A5 $\frac{4}{3}\pi r^3$ with or without work

W1 Uses any other formula except $\pi r^2 h$ or $2\pi rh$ or $\frac{4}{3}\pi r$

(iv) **10 marks** **Att 3**

In the circle, the chords [ab] and [cd]
Intersect at the point e
$|\angle bad| = 54°$ and $|eb| = |ce|$
Find $|\angle cbe|$



$|\angle dcb| = |\angle bad| = 54°$ (step 1) $180° - 54° = 126°$ (step 2)

$\Rightarrow |\angle cbe| = \frac{126}{2} = 63°$ (step 3)

B1 Each step omitted or incorrect B2 $|\angle ade| = 54°$ gives $|\angle cbe| = 72°$ or $54°$

B3 $|ce|.|ae| = |be|.|ed| \Rightarrow \frac{|ce|}{|ed|} = \frac{|be|}{|ae|} \Rightarrow$ Ans $= 54°$

A1 Assumes Δ cbe is equiangular $\Rightarrow |\angle cbe| = 60°$

A2 $|ce|.|ed| = |eb|.|ae|$ & stops A3 $|ce|.|eb| = |ae|.|ed|$ & continues

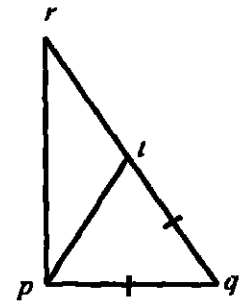
A4 Statement or use of any relevant theorem

W1 Assumes cb // ad $\Rightarrow |\angle cbe| = 54°$ (no work)

(v) **10 marks** **Att 3**

The triangle pqr has a right angle at p
The point t is on [qr] such that $|qt| = |qp|$
$|qr| = 17$ and $|pr| = 15$

Find $|pq|$ and hence find $|tr|$



$|pq|^2 = 17^2 - 15^2 \Rightarrow |pq|^2 = 289 - 225 = 64$ (step 1)
$\Rightarrow |pq| = 8$ (step 2) $\Rightarrow |tr| = 17 - 8 = 9$ (step 3)

B1 Each step omitted or incorrect
B2 Blunder in use of Pythagoras
B3 Blunder in indices (once only)
B4 Reads wrong page of tables
B5 Error in decimal point

S1 Slips in reading tables

A1 Any mention of theorem of Pythagoras & stops

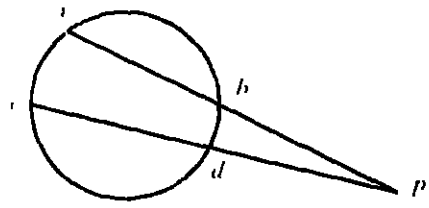
W1 Equal angles from isosceles triangle
W2 $|rt| = 8.5$
Note: Candidate may use trigonometric method

(vi) **10 marks** **Att 3**

Two chords, [ab] and [cd], of a circle intersect externally at p.

$|ab| = 7$, $|bp| = 9$ and $|cp| = 18$

Find $|cd|$



$|ap||bp| = |cp||dp| \Rightarrow 16 \times 9 = 18 \times |dp|$ (step 1)
$\Rightarrow |dp| = \frac{144}{18} = 8$ (step 2) $\Rightarrow |cd| = 18 - 8 = 10$ (step 3)

B1 Each step omitted or incorrect
B2 Blunder in use of theorem e.g. $7 \times 9 = 9$

A1 States or proves theorem correctly & stops

(vii) **10 marks** **Att 3**

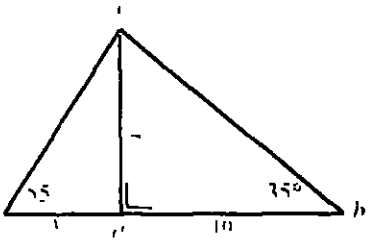
In the diagram cd ⊥ ab

$|\angle cbd| = 35°$ and $|\angle cad| = 55°$

$|cd| = 7$. $|db| = 10$ and $|ad| = x$

Show that the triangles cad and cdb are equiangular

Hence, find x



Each triangle has a right angle, and $|\angle acd| = 35° = |\angle cbd|$ step 1

$\Rightarrow$ (triangles are equiangular) $\Rightarrow \frac{x}{7} = \frac{7}{10}$ (step 2) $\Rightarrow 10x = 49 \Rightarrow x = 4.9$ (step 3)

or may use trigonometry $\tan 35° = \frac{x}{7}$ (2) $\Rightarrow x = 7\tan 35° \Rightarrow x = 7(0.7) = 4.9$ (3)

B1 Each step omitted or incorrect
B2 Trigonometric blunder (once only) B3 Reads wrong page of tables

S1 Slips in reading tables

A1 Any use of Pythagoras' Theorem
A2 States ∠acb is right angled & stops

(viii) **10 marks** **Att 3**

Find the area of the triangle with vertices (-3, 2), (-1, -2) and (3, 0)

Method 1		Method 2
(-3,2) (-1,-2) (3,0)	(step 1)	Slope bc = $\frac{1}{2}$. $\Rightarrow$ slope ab = - 2
$\Rightarrow$ (0,0) (2,-4) (6,-2)	(step1)	$\lvert bc\rvert = \sqrt{20}$ $\lvert ab\rvert = \sqrt{20}$
$\frac{1}{2}\lvert(2)(-2)-(-4)(6)\rvert$	(step 2)	$\Delta = \frac{1}{2}\sqrt{20}\sqrt{20} = 10$
= 10	(step 3)	

B1 Each step omitted or incorrect. B2 Omits the half in the formula
B3 Not translating to the origin i.e. omitting step 1
B4 Incorrect relevant formula

S1 Numerical error

A1 Writes any correct relevant formula (no substitution) & stops
A2 Mentions correct translation & stops

W1 Any use of irrelevant formula
Note If candidate uses long area formula mark by slips and blunders

(ix) **10 marks** **Att 3**

(7, 3) is the mid-point of the line segment joining (3, a) and (b, 8) Find a and b

$(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$	(step 1)	(3, a) → (7, 3) i e right 4
$(\frac{3+b}{2}, \frac{a+8}{2})$	(step 2)	(b, 8) → (7, 3) i e down 5
$\frac{3+b}{2} = 7 \Rightarrow b = 11$		$7 \rightarrow 7+4 = 11 = b$
$\frac{a+8}{2} = 3 \Rightarrow a = -2$	(step 3)	$3 \rightarrow 3-5 = -2 = a$

- Accept correct answer without work for full marks

B1 Incorrect mid-point formula, e.g minus for plus in both (but one is a slip)
B2 Incorrect translation
B3 Incorrect cross multiplication

S1 Numerical slips
S2 One sign incorrect in mid-point formula

(x) **10 marks** **Att 3**

SinA = 0 54 and $0° \leq A \leq 90°$ Use the tables to find the value of cos2A

A = 32°41'	(step 1)	$Cos2A = 1-2\sin^2 A$
2A = 65°22'	(step 2)	$= 1 - 2(0\ 54)^2$
Cos2A = 0 4168	(step 3)	= 1 – 0.5832 = 0 4168

B1 1° = 100'
B2 Gets cosA i e omits step 2 in method 1
B3 Decimal blunder e.g reads $\sin^{-1}.054$ (3°6') & continues
B4 Reads wrong page of tables

S1 Slips in reading tables

A1 Draws a right angled triangle with opposite = 54, hypotenuse = 100 & stops

QUESTION 2

Part (a)	**30 marks**	**Att 3,3,3**
Part (b)	**20 marks**	**Att 2,2,3**

Part (a) **30 (10,10,10) marks** **Att 3,3,3**

(a) A person earns a gross income of IR£494 and has tax-free allowances of IR£144

Tax is paid at 44% of taxable income

(i) Calculate the person's income after tax is paid

The tax free-allowances are increased by IR£20 and the rate of tax is reduced to 42%

(ii) Calculate the increase in the person's income after tax is paid

A second person with the same tax-free allowances and tax rate as in (ii) pays IR£105 in tax

(iii) Calculate the second person's gross income

(a) (i) **10 marks** **Att 3**

Taxable Income is 494 – 144 = 350 (step 1)

Tax is $\frac{44}{100} \times 350 = 154$ (step 2) Income after tax is 494 – 154 = 340 (step 3)

OR

Taxable income = 350 (step 1) 56% x 350 = 196 (step 2) 196 + 144 = 340 (step 3)

B1 Each step omitted or incorrect
B2 Error in decimal point
B3 Gets 44% of gross income (£217 36) and finishes with £276 64
B4 Adds tax-free allowances to gross income and finishes correctly (£213 28)
B5 Gets 1% of £350 (3 50) and subtracts from £494 correctly (£490.50)
B6 Double blunder: gets 44% of £494 correctly & stops

S1 Each numerical slip to a max. of 3

A1 Gross income – tax-free allowances & stops
A2 States 44% = $\frac{44}{100}$ or 0.44 & stops
A3 Gets 1% of £494 correctly & stops

(a) (ii) **10 marks** **Att 3**

Taxable income = 494 – 164 = 330 (step 1)

Tax is $\frac{42}{100} \times 330$ = £138.60 and income after tax is 494 – 138.60 = 355.40 (step 2)

Increase in income is 355.40 – 340.00 = £15.40 (step 3) or 154 – 138.60 = 15.40

(Uses 44% tax rate and T F A of £144 getting £382.64 Accept for full marks)

B1 Each step omitted or incorrect
B2 Error in decimal point

S1 Each numerical slip to a max of 3

A1 Gross Income – Tax Free Allowances & stops
A2 Adds £20 to £144 to get £164 & stops
A3 States 42% = $\frac{42}{100}$ or 0.42 & stops
A4 Gets 2% of £20 = 40p

(a) (iii) **10 marks** **Att 3**

Tax is $\frac{42}{100} \times$ Taxable income = £105 (step 1)

Taxable Income = $\frac{105 \times 100}{42}$ = £250 (step 2) Income =250 + 164 = £414 (step 3)

B1 Each step omitted or incorrect
B2 Error in decimal point
B3 Double error $\frac{105}{42}$ = £2.50 & stops

S1 Numerical slips

A1 58% is rest of taxable income & stops
A2 Answer is £164 + taxable income & stops
A3 Mention of $\frac{1}{42}$ or $\frac{42}{100}$ or 42 or $\frac{58}{100}$ or 58

Part (b) **20 (5,5,10) marks** **Att 2,2,3**

(b) $a = u + v$ and $b = u - v$

(i) Express $a^2 - b^2$ in terms of u and v

(ii) Hence, or otherwise, evaluate uv when $a = 29$ and $b = 21$

(b) (i) **10 (5,5) marks** **Att 2,2**

A $a^2 - b^2 = (u+v)^2 - (u-v)^2$ 5 marks att 2

OR **B** $a^2 - b^2 = (a+b)(a-b) = [(u+v)+(u-v)][(u+v)-(u-v)]$

AND then continues to get $a^2 - b^2 = 4uv$ 5 marks att 2

A

B1 $(u+v)^2$ or $(u-v)^2$ & stops

B2 $(u+v+u-v)(u+v-u-v)$

MR $(u+v)^2 + (u-v)^2$

B

B1 Incorrect factors

B2 Says a^2 and/or $b^2 = u^2 + v^2$

B3 Writes $-b^2$ as $-u^2 - 2uv + v^2$

(b) (ii) **10 marks** **Att 3**

$a^2 - b^2 = 4uv$ (from (b)(i))	OR	$a^2 - b^2 = 4uv$ (from (b)(i))
$29^2 - 21^2 = 4uv$	(step 1)	$(a+b)(a-b) = 4uv$
$841 - 441 = 4uv$	(step 2)	$(50)(8) = 4uv$
$4uv = 400$ $uv = 100$	(step 3)	$4uv = 400$ $uv = 100$

Special Case

$u + v = 29$ and $u - v = 21$ (5 marks) [if not awarded in (b)(i)]

Solves for $u = 25$ (step1) Solves for $v = 4$ (step 2) $uv = (25)(4) = 100$ (step 3)

B1 Each step omitted or incorrect

B2 Error in transposing

B3 Errors in indices (once only)

S1 Numerical slips to a max of 3

A1 Squares 29 and/or 21 & stops

A2 States $u + v = 29$ or $u - v = 21$ & stops

A3 States $a^2 - b^2 = (a-b)(a+b)$ & stops

QUESTION 3

Part (a)	**25 marks**	**Att 8**
Part (b)	**25 marks**	**Att 5,3**

Part (a) **25 marks** **Att 8**

(a) Prove that any point on the bisector of an angle is equidistant from the arms of the angle.

(a) **25 marks** **Att 8**

Given Line P bisector of $\angle aob$ and $x \in P$

Prove x is equidistant from oa and ob

Const Draw $xy \perp oa$ and $xz \perp ob$ (step 1)

Proof In triangles xoy and xoz

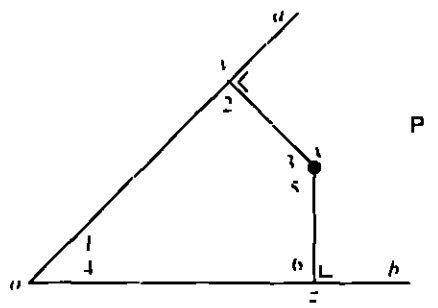
$|ox| = |ox|$ common (step 2)

$|\angle 1| = |\angle 4|$ given (step 3)

$|\angle 2| = |\angle 6|$ const (step 4)

$\Rightarrow \Delta xoy \equiv \Delta xoz$ or A.S A. (step 5)

$\Rightarrow |xy| = |xz|$ (step 6)



B1 Each step omitted or incorrect
B2 Steps not in logical order (once only)

A1 Diagram showing angle and bisector only
A2 Proves "Perpendicular bisector of line segments" theorem correctly

W1 Proves any other theorem

Note 1 $|\angle 3| = |\angle 5|$ may be paired with $|\angle 1| = |\angle 4|$
Note 2: Correct diagram is 10 marks
Note 3. Some steps may be indicated on diagram
Note 4 Memorised proof and no diagram, give att 8 marks if proof can be reconciled with a diagram

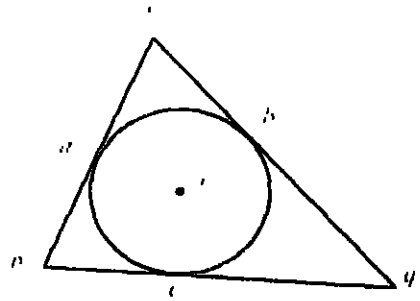
Part (b) **25 (15,10) marks** **Att 5,3**

(b) The circle, centre o, is inscribed in the triangle pqr

The circle touches the sides of the triangle at the points a,b and c

(i) use the angles poa and poc to prove that $|pa| = |pc|$

(ii) Hence show that $|pq| - |pr| = |qb| - |rb|$



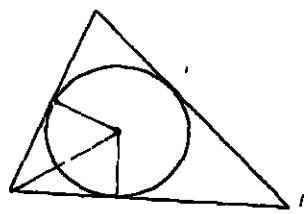
(b) (i) **15 marks** **Att 5**

Method 1

$|oa| = |oc|$ or $|op| = |op|$ (step 1) $|\angle apo| = |\angle cpo|$ (step 2)

$|\angle pao| = |\angle pco|$ (step 3) indication of congruency (step 4)

$\Rightarrow |pa| = |pc|$ (step 5)



Method 2

$|op|^2 = |pa|^2 + |ao|^2$ (step 1) $|op|^2 = |pc|^2 + |co|^2$ (step 2)

$\Rightarrow |pa|^2 + |ao|^2 = |pc|^2 + |co|^2$ (step 3)

$\Rightarrow |pa|^2 = |pc|^2$ (step 4) $\Rightarrow |pa| = |pc|$ (step 5)

B1 Each step omitted or incorrect.

A1 Diagram only showing o joined to a and/or p and/or c

(b) (ii) **10 marks** **Att 3**

$|pq| - |pr| = (|pc| + |cq|) - (|pa| + |ar|)$ (step 1)

$= |pc| + |cq| - |pa| - |ar| \Rightarrow |cq| - |ar|$ (step 2)

$= |qb| - |rb|$ (step 3)

B1 Each step omitted or incorrect

A1 Gives $|cq| = |qb|$ or $|ar| = |rb|$

A2 $|oq|^2 = |oc|^2 + |cq|^2$ or similar

QUESTION 4

Part (a)	**25 marks**	**Att 8**
Part (b)	**25 marks**	**Att 5,3**

Part (a) **25 marks** **Att 8**

(a) Prove that a line is a tangent to a circle at a point t on the circle if it is perpendicular to the diameter through t

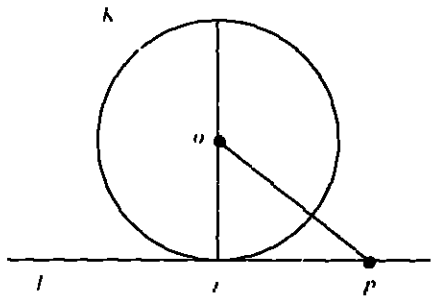
(a) **25 marks** **Att 8**

Given. Circle K with centre o, ot is a diameter of K
And the line T is perpendicular to ot through t
Diagram (step 1)

Prove T is a tangent to K

Const · Let p be any point on T Join op

Proof ∠otp is the largest angle in Δotp (step 2)
⇒ {[op] is its longest side} ⇒ |op| > |ot| (step 3)
⇒ |op| > the radius of K (step 4)
⇒ p is not a point of K (step 5)
But this true for any point on T except t
So T meets K at only one point
and is a tangent (step 6)



B1 Each step omitted or incorrect
B2 Steps not in logical order, (once only)

A1 Attempt at relevant diagram & stops

Part (b) **25 (15,10) marks** **Att 5,3**

(b) pt is a tangent to a circle of centre c

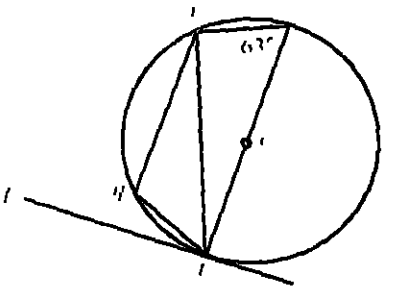
[ts] is a diameter of the circle

r is a point on the circle such that $|\angle tsr| = 63°$

(i) Find $|\angle ptr|$.

q is a point on the circle such that qr//ts.

(ii) Find $|\angle trq|$



(b) (i) **15 marks** **Att 5**

$|\angle srt|$ is a right angle (step 1)

$\Rightarrow |\angle str| = 90° - 63° = 27°$ (step 2) $\Rightarrow |ptr| = 90° - 27° = 63°$ (step 3)

OR

Angle between tangent and chord equals angle in alternate segment

$\Rightarrow |\angle ptr| = 63°$

- Accept correct answer without work for full marks

B1 Each step omitted or incorrect

A1 180° - 63° & stops

W1 Any work with "alternate" or "corresponding" angles (for (b)(i) only)

Note Some of the steps may appear in the diagram.

(b) (ii) **10 marks** **Att 3**

$|\angle trq| = |\angle rts|$ (alternate angles)

$\Rightarrow |\angle trq| = 27°$ (90° - 63°)

- Accept correct answer without work for full marks

A1 Any mention of alternate angles & stops.

QUESTION 5

50 (5,5,10,15,15) Marks **Att 2,2,3,5,5**

The equation of the line L is $3x - 2y + 6 = 0$

Find

(i) The slope of L

(ii) The point (h, -3) is on the line L

Find the value of h

(iii) The line K passes through (h, -3) and is perpendicular to L

Find the equation of K

(iv) K contains the point (-1, -5)

Find the image of this point under S_L, the axial symmetry in L.

(v) L and K cut the y-axis at the points p and q, respectively

Calculate |pq|

(i) **5 Marks** **Att 2**

L is $3x - 2y + 6 = 0$ $\Rightarrow m = \frac{-3}{-2}$ $(= \frac{3}{2})$

May also use $y = mx + c$ or two points

- Accept correct answer without work for full marks

B1 Error in slope formula used

B2 Transposition error

S1 Numerical slips

A1 A correct slope formula with no substitution

(ii) **5 Marks** **Att 2**

L is $3x - 2y + 6 = 0$ point is (h, -3)

$3(h) - 2(-3) + 6 = 0 \Rightarrow 3h + 6 + 6 = 0 \Rightarrow h = -4$

OR substitutes $y = -3$ into L: $3x - 2y + 6 = 0$ getting $x = -4 = h$

- Accept correct answer without work for full marks

B1 Transposition error

B2 Incorrect substitution ($x = -3$)

B3 Error in sign in multiplying or dividing

S1 Numerical slips

A1 Substitutes & stops

(iii) **10 Marks** **Att 3**

$M_1 = \frac{3}{2}$ (part 1) $\Rightarrow M_K = -\frac{2}{3}$ and point is (-4, -3) Point on K (step 1)

$y - y_1 = m(x - x_1)$ not necessary (step 2)

$\Rightarrow y + 3 = -\frac{2}{3}(x + 4)$ (or $2x + 3y + 17 = 0$) (step 3)

OR may use K $2x + 3y + k = 0$ etc

- Accept correct answer without work for full marks

B1 Incorrect perpendicular slope e.g $\frac{2}{3}$ or $-\frac{3}{2}$ or some slope, and continued e g $-\frac{3}{4}$:

B2 Incorrect line formula $y - 3 = -\frac{2}{3}(x - 4)$ i.e both signs wrong slip if only one wrong

S1 Numeric slips in $2x + 3y + k = 0$

A1 $M_1.M_2 = -1$ or variation and stops

Note Writes $y - y_1 = m(x - x_1)$ or $y = mx + c$ & stops merits 4 marks

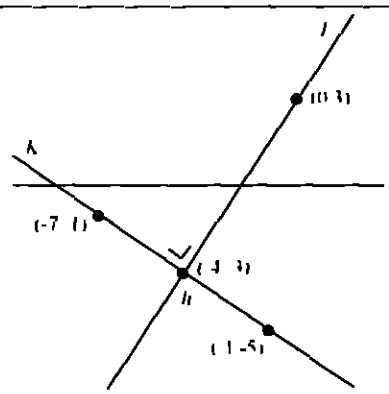
(iv) **15 Marks** **Att 5**

(-1 ,-5) (-4, -3)
-3 +2 -3 +2
(-4, -3) (-7, -1) image of point

OR

$\frac{x_1 - 1}{2} = -4$ and $\frac{y_1 - 5}{2} = -3$

$\Rightarrow x_1 = -7$ and $y_1 = -1$



- Accept candidate's equation of line K from part (iii)

B1 (-4,-3)→(-1,-5) →(2,-7) i.e. 12 marks B2 Incorrect mid-point formula

S1 Numerical slips

A1 Any transformation other than B1
A2 Correct mid-point formula & stops
A3 Tries to solve K∩L & stops

Note: Accept correct answer from accurate diagram for full marks but no diagram no marks.

(v) **15 Marks** **Att 5**

$L \cap y\text{-axis} \Rightarrow x = 0 \Rightarrow 0 - 2y + 6 = 0$	(step 1)
$\Rightarrow y = 3 \Rightarrow p(0,3)$	(step 2)
$K \cap y\text{-axis} \Rightarrow x = 0 \Rightarrow 0 + 3y + 17 = 0$	(step 3)
$\Rightarrow y = -5\frac{2}{3} \Rightarrow q(0, -5\frac{2}{3})$	(step 4)
$\Rightarrow \lvert pq \rvert = 3 + 5\frac{2}{3} = 8\frac{2}{3}$	(step 5)

- Accept candidate's equation of line K from part (iii)

B1 Each step omitted or incorrect

S1 Numerical slips

A1 $x = 0$ & stops or gets $-2y + 6 = 0$ & stops (step 1 or part of it)
A2 Writes down the distance formula & stops

Note A step might be incorrect arising from a slip
e.g candidate in step 4 might have $-5\frac{1}{3}$ deduct 1 mark

Note Some of the steps for this part may have been down in previous parts

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 3,3**
Part (c)	**20 marks**	**Att 2,5**

Part (a) **10 Marks** **Att 3**

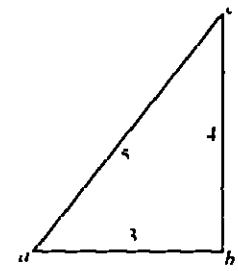
(a) Construct an angle A such that $\text{Cos}A = \frac{3}{5}$

(a) **10 marks** **Att 3**

Draw |ab| = 3units (step 1) $\frac{3}{5} = 0.6$

Draw a perpendicular bc (step 2) $\cos^{-1} 0.6 = 53°$

Draw |ac| = 5 units or |bc| = 4 units and join ac (step 3) Draw angle



To be applied to all parts (a), (b) and (c)

B1 **Each step omitted (max of 3 marks can be lost in any step)**
B2 **Incorrect ratio (sin, cos or tan)**
B3 **Misuse of Pythagoras'Theorem**
B4 **Incorrect ratio in Sine Rule**
B5 **Error in cross multiplication**
B6 **Takes 1° = 100'**
B7 **Incorrect transposition**
B8 **Decimal error**
B9 **Reads wrong page of tables**

S1 **Numerical slips**
S2 **Slips in reading tables**

This Part

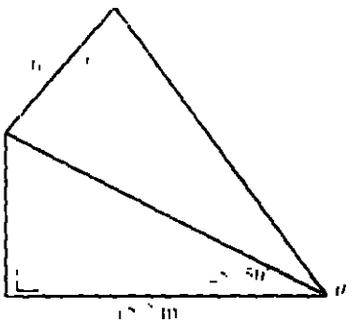
B10 Each step incorrect
B11 Units not consistent

S1 Fails to join a to c
A1 Rough sketch with sides marked.

Part (b) **20 (10,10) marks** **Att 3,3**

(b) A garden pqrs is in the shape of a quadrilateral

$|pq| = 15\ 3$ m, $|\angle pqs| = 25°50'$ and $|\angle qps| = 90°$



(i) Find $|sq|$, correct to the nearest metre

$|sr| = 9$ m and $|\angle qrs| = 69°14'$

(ii) Find $|\angle sqr|$, correct to the nearest degree

(b) (i) **10 marks** **Att 3**

$\cos 25°50' = \dfrac{15\ 3}{|sq|}$ (step 1) $\dfrac{|sq|}{\sin 90°} = \dfrac{15\ 3}{\sin 64°10'}$

$0\ 9 = \dfrac{15\ 3}{|sq|}$ (step 2) $|sq| = \dfrac{15\ 3}{0.9}$

$0\ 9|sq| = 15\ 3 \Rightarrow |sq| = 17$ (step 3) $= 17$

Blunders and slips as listed above (Blunders 1,2,4,5,6,7,8,9)

S1 Failure to round off

A1 Partly fills in sine rule & stops

A2 Look up the sine or cosine of a given relevant angle & stops

(b) (ii) **10 marks** **Att 3**

$\dfrac{9}{\sin\angle sqr} = \dfrac{17}{\sin 69°14'}$ (step 1) $\dfrac{9}{\sin\angle sqr} = \dfrac{17}{0\ 935}$ (step 2)

$17\sin\angle sqr = 8\ 415 \Rightarrow \sin\angle sqr = 0\ 495 \Rightarrow |\angle sqr| = 30°$ (step 3)

Blunders and slips as listed above (Blunders 1,4,5,6,7,8,9)

S1 Failure to round off

A1 Partly fills in sine rule & stops A2 Gets the sine of a given angle & stops

W1 Treats triangle rsq as right-angled

Part (c) **20 (5,15) marks** **Att 2,5**

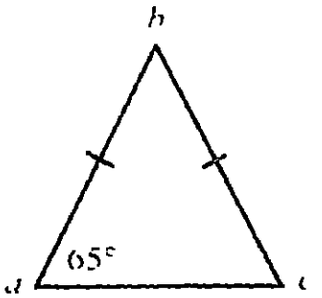
(c) abc is an isosceles triangle with $|ab| = |bc|$

$|\angle bac| = 65°$

(i) Calculate $|\angle abc|$

The area of the triangle is 38 3cm^2

(ii) Find $|ab|$



(c) (i) **5 marks** **Att 2**

$|\angle bac| = |\angle acb| = 65°$ Double to get 130°

$|\angle abc| = 180° - 130° = 50°$

- accept correct answer without work for full marks

Blunders and slips as listed above

B12 Gives $|\angle abc| = 65°$ and then works down to $|\angle acb| = 50°$

A1 States that angles opposite equal sides are equal and stops
A2 States that the three angles of a triangle sum to 180° & stops

(c) (ii) **15 marks** **Att 5**

$\frac{1}{2}|ab||cb|\sin 50° = 38\ 3$ (step 1) $\frac{1}{2}|ab|^2\ 0\ 766 = 38.3$ (step 2)

$$|ab|^2 = \frac{38\ 3 \times 2}{0\ 766}$$ (step 3)

$\Rightarrow |ab|^2 = 100$ (step 4) $\Rightarrow |ab| = 10$ (step 5)

Blunders and slips listed as above (Blunders 1,5,6,7,8,9)

B13 Halves the 50 in sin 50° & cont B14 Uses only one side i e $\frac{1}{2}|ab|\sin 50°$

A1 Any correct relevant formula without substitution e g. $\frac{1}{2}$ a h
A2 Gets sin 50° & stops A3 Multiplies 38 3 by 2

NOTES

A sign error in multiplication or division is a blunder
but
A sign error in addition is a slip

The long form for the area of a triangle in Coordinate Geometry

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

This can also be written in "determinant of a matrix form" (ad-bc)

Premature rounding off, that over-simplifies the work, is a blunder

Previous step(s) may be implied in the statement "Each step omitted or incorrect"