Coimisiún na Scrúduithe Stáit
State Examinations Commission

JUNIOR CERTIFICATE EXAMINATION, 2005

MATHEMATICS – HIGHER LEVEL

PAPER 1 (300 marks)

THURSDAY, 9 JUNE – MORNING, 9:30 to 12:00

Attempt **ALL** questions.

Each question carries 50 marks.

Graph paper may be obtained from the superintendent.

The symbol ✍ indicates that supporting work must be shown to obtain full marks.

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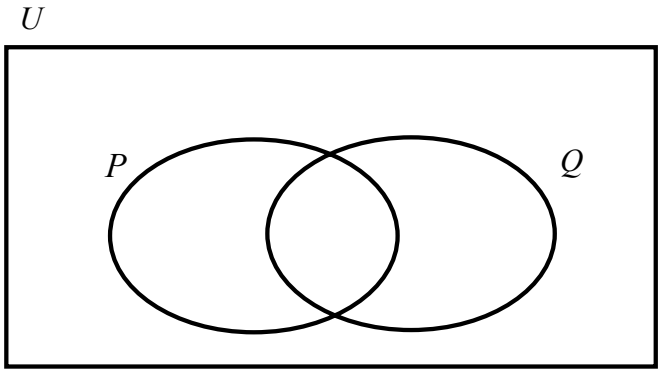
1. **(a)** ✍ U is the universal set. P and Q are two subsets of U. Copy the Venn diagram into your answerbook and shade in the set $(P \cup Q)'$.



(b) **(i)** ✍ Light travels at a speed of approximately $(2 \cdot 9 \times 10^5)$ km / sec.

How many kilometres will light travel in 8 minutes?

Express your answer in the form $a \times 10^n$,

where $n \in \mathbf{N}$ and $1 \leq a < 10$.

(ii) ✍ A tourist paid \$4620 to a travel agent for a holiday in Ireland, where €1 = \$1·32.

The cost to the travel agent of organising the holiday was €2985.

Calculate, in euro, the profit made by the travel agent.

(c) **(i)** ✍ By rounding to the nearest whole number, estimate the value of

$$\left(\frac{5 \cdot 9 + \sqrt[3]{27 \cdot 24}}{3 \cdot 06}\right)^2 .$$

Then, evaluate $\left(\frac{5 \cdot 9 + \sqrt[3]{27 \cdot 24}}{3 \cdot 06}\right)^2$, correct to two decimal places.

(ii) ✍ Simplify $\sqrt{3}(2\sqrt{6} - 4\sqrt{3}) \; - \; \sqrt{10}(3\sqrt{5} - 2\sqrt{10})$,

without the use of a calculator.

Express your answer in the form $a + b\sqrt{2}$, where $a, b \in \mathbf{Z}$.

2. **(a)** **(i)** Write down the reciprocal of $\frac{7}{2}$.

(ii) Find the value of this reciprocal, correct to 2 decimal places.

(b) **(i)** There are 25 000 fish in a fish farm.
The number of fish in the farm increases by 40% each year.

✍ How many fish will be in the farm at the end of 3 years?

(ii) The monthly line rental on Peter's mobile phone amounts to €12·70.
During May, the duration of his calls is 1 hr 41 mins and 50 secs.
Calls are charged at 0·6 cent per second.

✍ Calculate Peter's total bill for May.

(c) **(i)** The standard rate of income tax is 20% and the higher rate is 42%.
Sheila has tax credits of €2700 for the year and a standard rate cut-off point of €22 000.
Sheila has a gross income of €45 000 for the year.

✍ Calculate the total tax payable by Sheila for the year.

(ii) Tony pays tax at the same rates as Sheila.
Tony has tax credits of €2900 for the year and has the same standard rate cut-off point as Sheila.
His total tax payable amounts to €13 680 for the year.

✍ Calculate Tony's gross income for the year.

3. **(a)** ✍ Write $\sqrt[3]{16}$ in the form 2^k, $k \in \mathbf{Q}$.

(b) **(i)** Factorise $3x^2 + 8x - 3$.

(ii) ✍ Factorise $3p - c + 3pc - c^2$.

(iii) ✍ Simplify $(2x-1)^2 - (x-1)^2$.

(c) A box of drinking chocolate powder costs €3·60.

(i) If the box contains x grams of powder, write an expression in x to represent the cost of 1 gram of the powder.

During a promotion, the manufacturer adds in to the box an extra 30 grams of powder. The cost of the box of drinking chocolate remains at €3·60.

(ii) Write an expression in x to represent the cost of 1 gram of the powder during the promotion.

Each gram of powder, in this case, now costs 1 cent less.

(iii) Write an equation in x to represent the above information.

(iv) ✍ Solve this equation to find how many grams of powder are in the box during the promotion.

4. **(a)** ✍ Let f be the function $f: x \to x^2 + x - 7$, $x \in \mathbf{R}$.

Find $f(-3)$.

(b) Helen buys stamps costing 48 cent and 60 cent.

She buys a total of 50 stamps costing €25·68.

(i) Taking x to be the number of 48 cent stamps and y to be the number of 60 cent stamps, write down two equations in x and y to represent this information.

(ii) ✍ Solve the equations to find the number of each type of stamp that Helen has purchased.

(c) **(i)** ✍ Express in its simplest form:

$$\frac{1}{x-1} + \frac{1}{x+1}.$$

(ii) ✍ Hence, or otherwise, solve the equation:

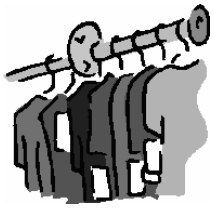
$$\frac{1}{x-1} + \frac{1}{x+1} = 3.$$

Express your answer in the form $a \pm b\sqrt{10}$, where $a, b \in \mathbf{Q}$.

5. **(a)** Seven shirts and two sweaters cost €202 · 50.

A sweater costs the same as four shirts.

✍ Find the cost of one shirt.



(b) In a school of 430 students, 250 students study History, 240 students study Geography.

Let x represent the number of students who study neither History nor Geography.

The number of students who study both History and Geography is 3 times the number who study neither of these subjects.

(i) ✍ Represent this information on a Venn diagram.

(ii) ✍ Write down and simplify an expression in x for the total number of students in the school.

(iii) ✍ Use this expression to find the number of students who study neither History nor Geography.

(c) Let f be the function $f: x \to x^2 + bx + c,\ x \in \mathbf{R}$ and $b, c \in \mathbf{Z}$.

The graph of f cuts the x axis at the points where $x = -3$ and $x = 2$.

(i) ✍ Find the value of b and the value of c.

(ii) ✍ Find the value of x for which $f(x) = f(x + 2)$.

6. **(a)** ✍ Find the solution set of the inequality: $6 - 2x \leq 12$, $x \in \mathbf{R}$.

(b) Let f be the function $f: x \to 5 - 3x - 2x^2$ and g be the function $g: x \to -2x - 1$.

✍ Using the same axes and scales, draw the graph of f

and the graph of g, for $-3 \leq x \leq 2$, $x \in \mathbf{R}$.

(c) Use your graphs from part **(b)** to estimate:

(i) ✍ the maximum value of $f(x)$

(ii) ✍ the values of x for which $f(x) = g(x)$

(iii) ✍ the range of values of x for which $f(x) \geq g(x)$.