Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate 2014

Marking Scheme

Mathematics
(Project Maths – Phase 2)

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Mathematics
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Paper 1

Higher Level

Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Question 1 **20 Marks**

(a) Place the following numbers in order, starting with the smallest:

$$\frac{3}{2} \qquad 1\cdot4 \qquad \sqrt{2}$$

$1\cdot4,\ \sqrt{2},\ \frac{3}{2}.$ $\quad \left(\sqrt{2}=1\cdot414...,\quad \frac{3}{2}=1\cdot5.\right)$

(b) Which one of the following is **not** a rational number? Explain your answer.

$$3\frac{1}{7} \qquad 3\cdot142 \qquad \frac{22}{7} \qquad \pi$$

Answer: π Reason: It cannot be written as a fraction.

(c) **(i)** Find the values of $\frac{4n^2+1}{13}$, where $n \in \{17, 19, 21\}$.

n	$\frac{4n^2+1}{13}$
17	$\frac{4\times(17)^2+1}{13} = \frac{1157}{13} = 89$
19	$\frac{4\times(19)^2+1}{13} = \frac{1445}{13}$ or $111^2/_{13}$
21	$\frac{4\times(21)^2+1}{13} = \frac{1765}{13}$ or $135^{10}/_{13}$

(ii) State which **one** of your answers is a natural number, and explain your choice.

Answer: 89. Reason: It is a positive whole number.

Question 2 **15 Marks**

(a) John thinks that he has a method for finding **all** prime numbers.
He says that if he uses the formulas in the table below, he will generate the prime numbers.
He also says that these formulas will generate **only** the prime numbers.

(i) Complete the table.

p	$6p + 1$	$6p + 5$
0	**1**	**5**
1	7	11
2	13	17
3	19	23
4	25	29
5	31	35

(ii) Give two reasons why his method is not fully correct.

*There are a number of different reasons – **any** two will suffice.*
*Reasons related to "**all** prime numbers":*

The formulas do not generate 2, which is prime.

The formulas do not generate 3, which is prime.

*Reasons related to "**only** prime numbers":*

The formulas generate 1, which is not prime.

The formulas generate 25, which is not prime.

The formulas generate 35, which is not prime.

(b) The Swiss mathematician and physicist, Euler, first noticed (in 1772) that the expression $n^2 - n + 41$ gives a prime number for all positive integer values of n less than 41.

Explain why it does not give a prime number for $n = 41$.

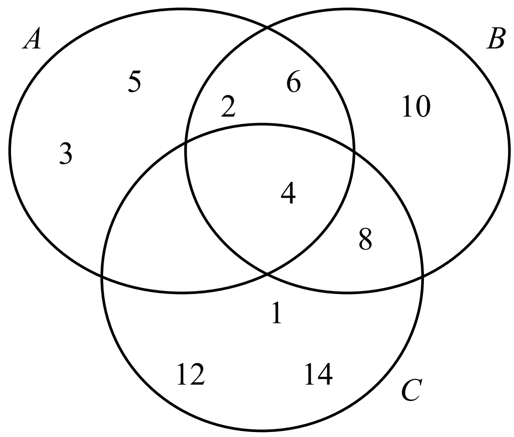
$41^2 - 41 + 41 = 41^2$, which has 41 as a factor.

Question 3 **25 Marks**

(a) The sets A, B, and C are as follows:

$A = \{2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$, and $C = \{1, 4, 8, 12, 14\}$.

(i) Complete the Venn diagram.



(ii) List the elements of each of the following sets:

$A \cap B =$	$\{2, 4, 6\}$
$B \setminus (A \cap C) =$	$\{2, 6, 8, 10\}$
$(B \setminus A) \cup (B \setminus C) =$	$\{2, 6, 8, 10\}$

(iii) Write down a null set, in terms of A, B, and C.

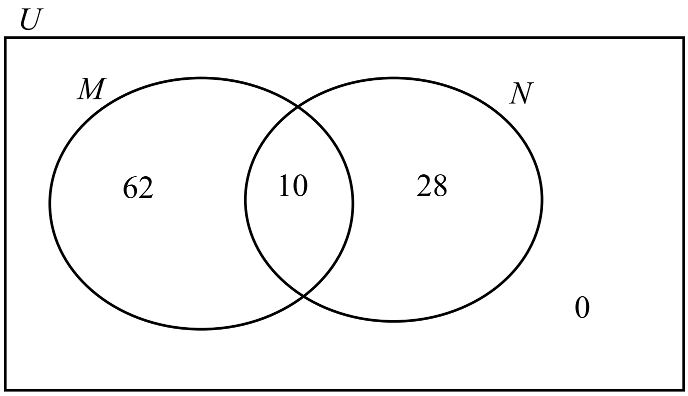
$(A \cap C) \setminus B$ *or equivalent.*

(b) In a table quiz, 100 questions were asked. Team M answered 72 questions correctly. Team N answered 38 questions correctly.

(i) Find, with the aid of the Venn diagram, the minimum number of questions answered correctly by both teams.

$72 + 38 = 110.$
$110 - 100 = 10.$
Minimum $= 10$

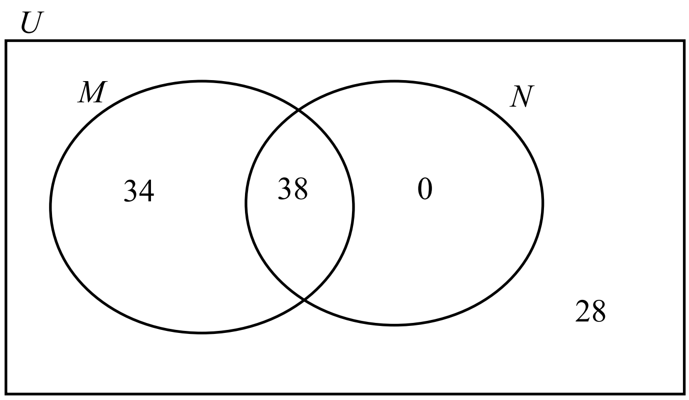
To make $\#(M \cap N)$ *as small as possible, make* $\#(M \cup N)' = 0$.



(ii) Find, with the aid of the Venn diagram, the maximum number of questions answered correctly by both teams.

Maximum $= 38$

To make $M \cap N$ *as big as possible, make the smaller set a subset of the larger set.*



Question 4 **25 Marks**

(a) Factorise fully $9a^2 - 6ab + 12ac - 8bc$.

$$\begin{aligned} 9a^2 - 6ab + 12ac - 8bc &= 3a(3a-2b) + 4c(3a-2b) \\ &= (3a-2b)(3a+4c). \end{aligned}$$

(b) Factorise $9x^2 - 16y^2$.

$$9x^2 - 16y^2 = (3x-4y)(3x+4y).$$

(c) Use factors to simplify the following: $\dfrac{2x^2+4x}{2x^2+x-6}$.

$$\begin{aligned} \frac{2x^2+4x}{2x^2+x-6} &= \frac{2x(x+2)}{(x+2)(2x-3)} \\ &= \frac{2x}{2x-3}. \end{aligned}$$

Question 5 **10 Marks**

Solve the following inequality and show the solution on the number line.

$$-17 \le 1 - 3x < 13, \quad x \in \mathbb{Z}$$

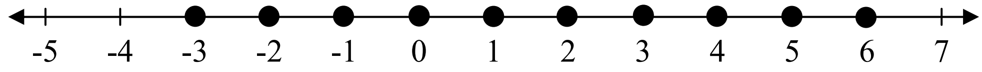
One method:

$$\begin{array}{lccccc} & -17 & \le & 1-3x & < & 13 \\ -1: & -18 & \le & -3x & < & 12 \\ \div(-3): & 6 & \ge & x & > & -4 \\ \text{i.e.} & -4 & < & x & \le & 6. \end{array}$$

Or:

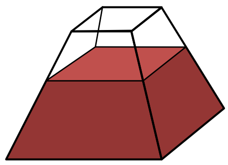
$$\begin{array}{lccccccc} & -17 & \le & 1-3x & \text{and} & 1-3x & < & 13 \\ & 3x & \le & 18 & \text{and} & -3x & < & 12 \\ & x & \le & 6 & \text{and} & x & > & -4 \end{array}$$

i.e. $-4 < x \le 6$.

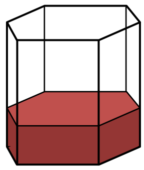


Question 6 **10 Marks**

Below are three containers, labelled **1**, **2**, and **3**.
Water is poured into each container at a constant rate, until it is full.



1



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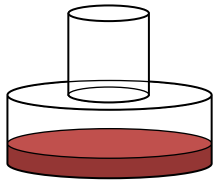
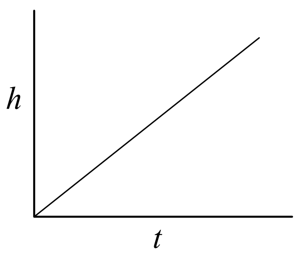


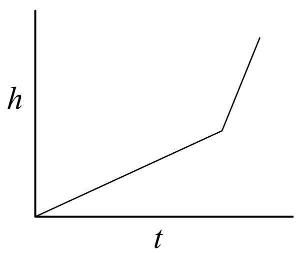
3

The three graphs, **A**, **B**, and **C**, show the height of the water, *h*, in the containers after time *t*.

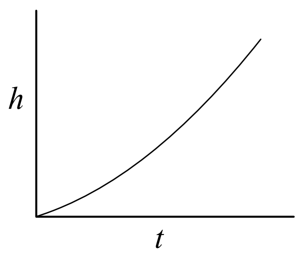
A



B



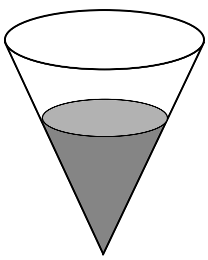
C

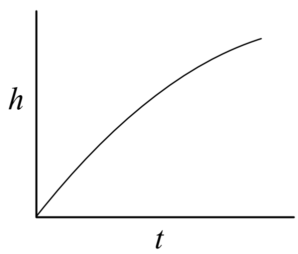


(a) Write **A**, **B**, and **C** in the table below to match each container to its corresponding graph.

Container	**1**	**2**	**3**
Graph	C	A	B

(b) Another container is shown below. Water is also poured into this container at a constant rate until it is full. Sketch the graph you would expect to get when plotting height (*h*) against time (*t*) for this container.





Question 7 **25 Marks**

Last year Elena had a gross income of €36 960.
She had to pay Universal Social Charge (USC) and income tax on her gross income.
The rates and bands of USC are as follows.

Income band	Rate of USC
Up to €10 036	2%
Between €10 036 and €16 016	4%
Above €16 016	7%

(i) Find the amount of USC that was deducted from Elena's gross income last year.

USC @ 2%: $0\cdot02\times10036 = €200\cdot72$.

USC @ 4%: $16016-10036=€5980$, and $0\cdot04\times5980=€239\cdot20$.

USC @ 7%: $36960-16016=€20944$, and $0\cdot07\times20944=€1466\cdot08$.

Total USC $=€1906$.

(ii) The standard rate of income tax was 20% and the higher rate was 41%.
The standard rate cut-off point was €32 800.
Elena paid a total of €4965·60 income tax last year.

Find Elena's tax credits for the year.

Tax @ 20%: $0\cdot20\times32800=€6560\cdot00$.

Tax @ 41%: $36960-32800=€4160$, and $0\cdot41\times4160=€1705\cdot60$.

Gross Tax: $€8265\cdot60$.

Tax Credits: $8265\cdot60-4965\cdot60=€3300$.

(iii) Find Elena's total deduction (USC and income tax) as a percentage of her gross income.
Give your answer correct to the nearest percent.

Total Deductions: $1906+4965\cdot60=€6871\cdot60$.

Total Deductions as % of Gross Income:

$$\frac{6871\cdot60}{36960}\times100=18\cdot59...=19\%\text{, correct to the nearest percent.}$$

Question 8 **15 Marks**

The table shows the height, in metres, of a ball at various times after being kicked into the air.

(i) Is the pattern of heights in the table linear, quadratic, or exponential? Explain your answer.

Time (seconds)	0	0·5	1	1·5	2	2·5	3
Height (metres)	0·3	3·4	5·7	7·2	7·9	7·8	6·9

First difference: 3·1 2·3 1·5 0·7 –0·1 –0·9

Second difference: –0·8 –0·8 –0·8 –0·8 –0·8

Answer: Quadratic.

Reason: The first differences are not all the same, but the second differences are.

(ii) Estimate the height of the ball after 3·5 seconds.

5·2 metres.

Second difference:		–0·8		–0·8			
First difference:	–0·1		–0·9		–1·7		
Height (m):	7·9		7·8		6·9		5·2
Time (s):	2		2·5		3		3·5

(iii) Estimate the total time the ball spends in the air. Justify your answer.

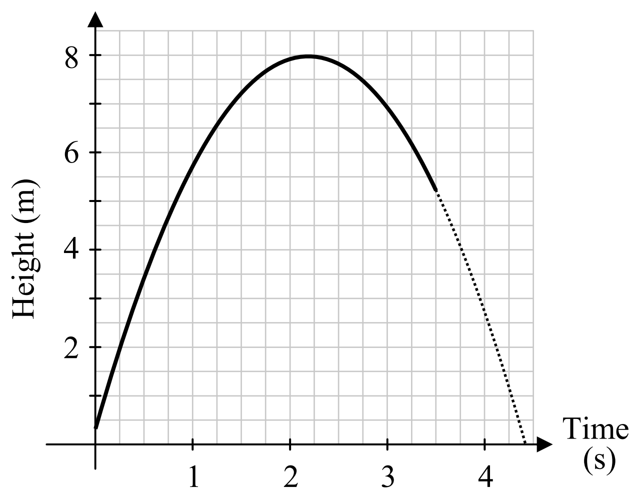
Continuing the method for **(ii)**:

Second difference:		–0·8		–0·8		–0·8		–0·8		
First difference:	–0·1		–0·9		–1·7		–2·5		–3·3	
Height (m):	7·9		7·8		6·9		5·2		2·7	–0·6
Time (s):	2		2·5		3		3·5		4	4·5

Answer: The ball spends roughly 4·4 seconds in the air. Its height is 0 just before 4·5 seconds.

Or, graphically:

From the graph, the ball spends roughly 4·4 seconds in the air



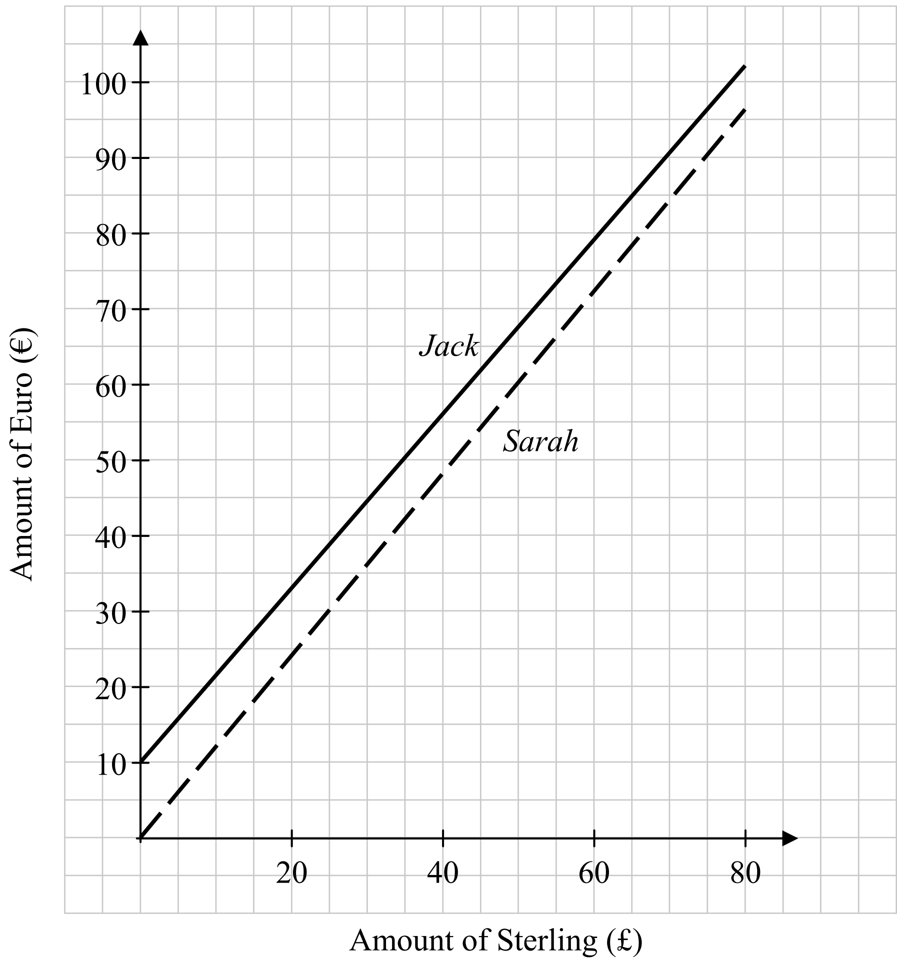
Question 9 **30 Marks**

Jack and Sarah are going on a school tour to England. They investigate how much different amounts of sterling (£) will cost them in euro (€). They each go to a different bank.

Their results are shown in the table below.

Amount of sterling (£)	Cost in euro (€) for Jack	Cost in euro (€) for Sarah
20	33	24
40	56	48
60	79	72
80	102	96

(i) On the grid below, draw graphs to show how much the sterling will cost Jack and Sarah, for up to £80.



(ii) Using the table, or your graph, find the slope (rate of change) of Jack's graph. Explain what this value means. Refer to both euro and sterling in your explanation.

> $\text{Slope} = \dfrac{56-33}{40-20} = \dfrac{23}{20}$, *or* $1\cdot15$.
>
> Explanation: Each extra £1 costs Jack an extra €1·15.
>
> *Or*:
>
> Explanation: Each £1 costs Jack €1·15, after an initial fee of €10.

(iii) Write down a formula to represent what Jack must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

> $e = 1\cdot15s + 10$, where s is the amount, in sterling, and e is the amount, in euro.

(iv) Write down a formula to represent what Sarah must pay, in euro, for any given amount of sterling. State clearly the meaning of any letters you use in your formula.

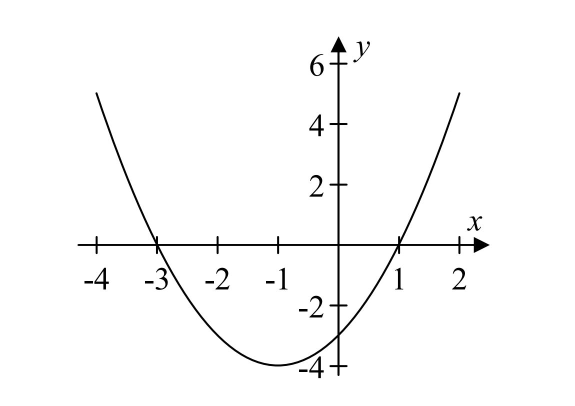
> $\text{Slope} = \dfrac{48-24}{40-20} = \dfrac{6}{5}$, *or* $1\cdot2$. y-intercept $= 0$
>
> $e = 1\cdot2s$, where s is the amount, in sterling, and e is the amount, in euro.

(v) Using your formulas from **(iii)** and **(iv)**, or otherwise, find the amount of sterling Jack and Sarah could buy that would cost them the same amount each in euro.

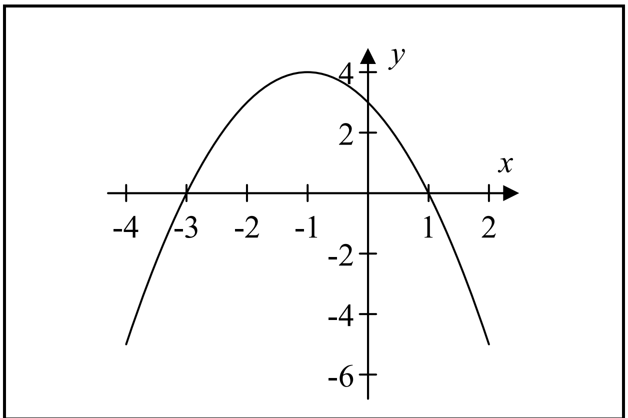
> *Using formulas*:
>
> $e = 1\cdot15s + 10$ and $e = 1\cdot2s$, so $1\cdot15s + 10 = 1\cdot2s$,
>
> i.e. $s = 200$ and $e = 240$.
>
> Amount of sterling: £200.
>
> *From table*:
>
> Each time the amount of sterling goes up by 20, the difference between the costs decreases by €1.
>
> This difference is €9 for £20.
>
> So after 9 increases, i.e. increase of $9 \times 20 = £180$, the costs are the same, i.e. for £200.

Question 10 **15 Marks**

(a) The graphs of the functions $f(x) = x^2 + 2x - 3$ and $g(x) = -x^2 - 2x + 3$ are shown below.
Identify each graph by writing $f(x)$ or $g(x)$ in the space provided below the graph.

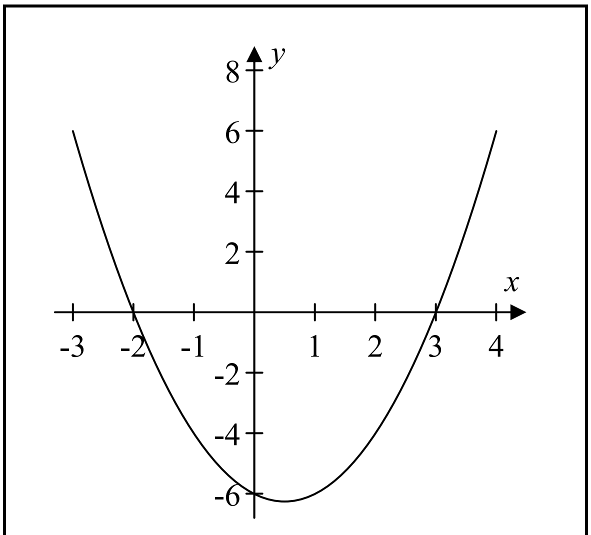


Answer: $f(x)$

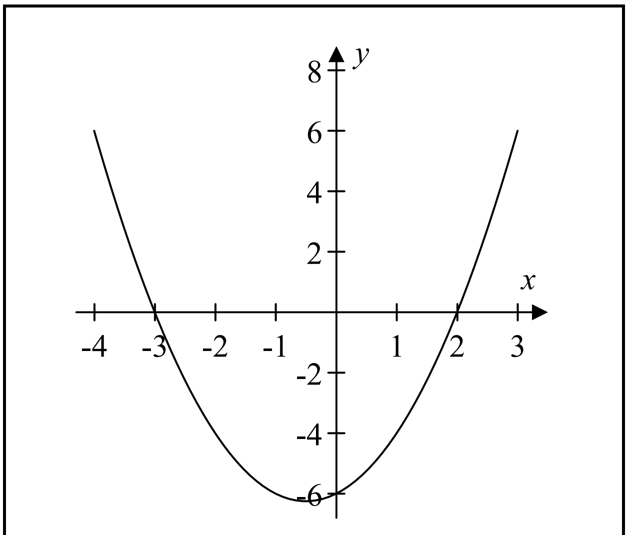


Answer: $g(x)$

(b) The graphs of the functions $y = h(x)$ and $y = k(x)$ are shown below.



$h(x)$



$k(x)$

Write down the roots of each function.
Hence, or otherwise, write down an equation for each function.

Roots of $h(x)$: $x = -2$ and $x = 3$.

Equation: $h(x) = (x+2)(x-3)$, or $h(x) = x^2 - x - 6$.

[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $h(0) = -6$, which corresponds to the graph.]

Roots of $k(x)$: $x = -3$ and $x = 2$.

Equation: $k(x) = (x+3)(x-2)$, or $k(x) = x^2 + x - 6$.

[Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $k(0) = -6$, which corresponds to the graph.].

Question 11 **20 Marks**

x is a real number.
One new number is formed by increasing x by 1.
A second new number is formed by decreasing x by 2.

(i) Write down each of these new numbers, in terms of x.

Increase x by 1:	$x+1$
Decrease x by 2:	$x-2$

(ii) The product of these two new numbers is 1.
Use this information to write an equation in x.

$(x+1)(x-2)=1$ *or equivalent.*

(iii) Solve this equation to find the two possible values of x.
Give each of your answers correct to 3 decimal places.

$$(x+1)(x-2)=1$$
$$\Rightarrow \quad x^2-x-3=0$$
$$\Rightarrow \quad x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}$$
$$\Rightarrow \quad x=2\cdot 3028... \quad \text{and} \quad x=-1\cdot 3028...$$
$$\Rightarrow \quad x=2\cdot 303 \quad \text{and} \quad x=-1\cdot 303,$$ correct to three decimal places.

Question 12 **25 Marks**

(a) Simplify $(6x-3)(2x-1)$.

$(6x-3)(2x-1) = 12x^2 - 12x + 3$

(b) Simplify $\left(3x^3-2x^2-3x+2\right)\div(x-1)$.

$$
\begin{array}{r}
3x^2+x-2 \\
x-1\overline{)\,3x^3-2x^2-3x+2} \\
\underline{3x^3-3x^2} \\
x^2-3x+2 \\
\underline{x^2-x} \\
-2x+2 \\
\underline{-2x+2} \\
0
\end{array}
$$

Answer = $3x^2 + x - 2$.

Or:

	$3x^2$	x	-2
x	$3x^3$	x^2	$-2x$
-1	$-3x^2$	$-x$	2

Answer = $3x^2 + x - 2$.

(c) **(i)** Solve the simultaneous equations:

$2x - 3y = 18$ ①

$5x + 9y = -10$ ②

① × 3: $6x - 9y = 54$

②: $\underline{5x + 9y = -10}$

$11x = 44$

÷11: $x = 4$

Sub in x = 4 in ①:

$2(4) - 3y = 18$

$8 - 3y = 18$

$-3y = 18 - 8$

$-3y = 10$

×(–1): $3y = -10$

÷3: $y = -10 \div 3 = -10/3$ *or equivalent*

Answer: $x = 4$ and $y = -10/3$.

(ii) Verify your answer to **(c)(i)**.

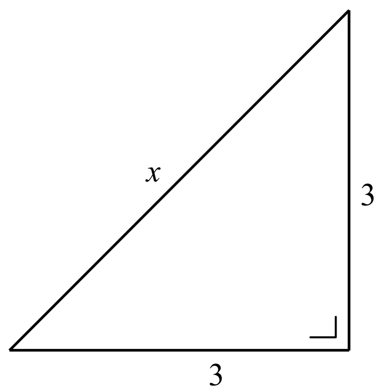
Note: *Only need to check the equation that* ***wasn't*** *used to find the second variable. In this case, we only need use* ②.

$$5(4)+9\left(-\frac{10}{3}\right) \quad = \quad 20-30 \quad = \quad -10.$$

Question 13 **15 Marks**

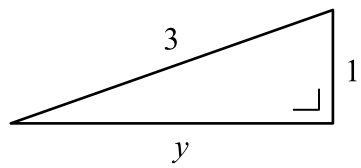
(i) Use the diagram on the right to calculate the value of x.
Give your answer in surd form.

$x=\sqrt{3^2+3^2}$ $=\sqrt{18}$ or $3\sqrt{2}$	*Or*: $\sin 45^\circ=\dfrac{3}{x}$ $\dfrac{1}{\sqrt{2}}=\dfrac{3}{x}$ $x=3\sqrt{2}$



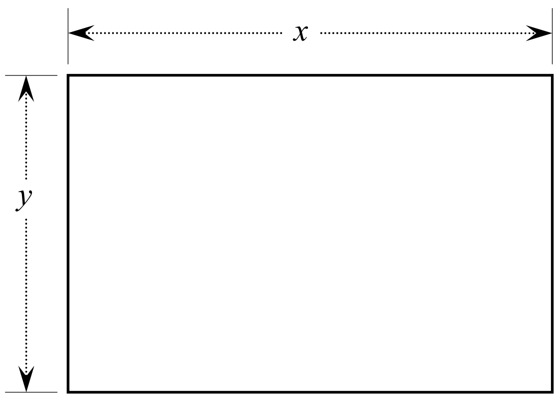
(ii) Use the diagram below to calculate the value of y. Give your answer in surd form.

$y=\sqrt{3^2-1^2}=\sqrt{8}$ or $2\sqrt{2}$.



(iii) A rectangle with sides of length x and y is drawn using the values of x and y from parts **(i)** and **(ii)**, as shown below.

Write the **perimeter** of this rectangle in the form $a\sqrt{2}$, where $a\in\mathbb{N}$.



$$\begin{aligned}\text{Perimeter}&=2x+2y\\&=2\sqrt{18}+2\sqrt{8}\\&=2\left(3\sqrt{2}\right)+2\left(2\sqrt{2}\right)\\&=10\sqrt{2}.\end{aligned}$$

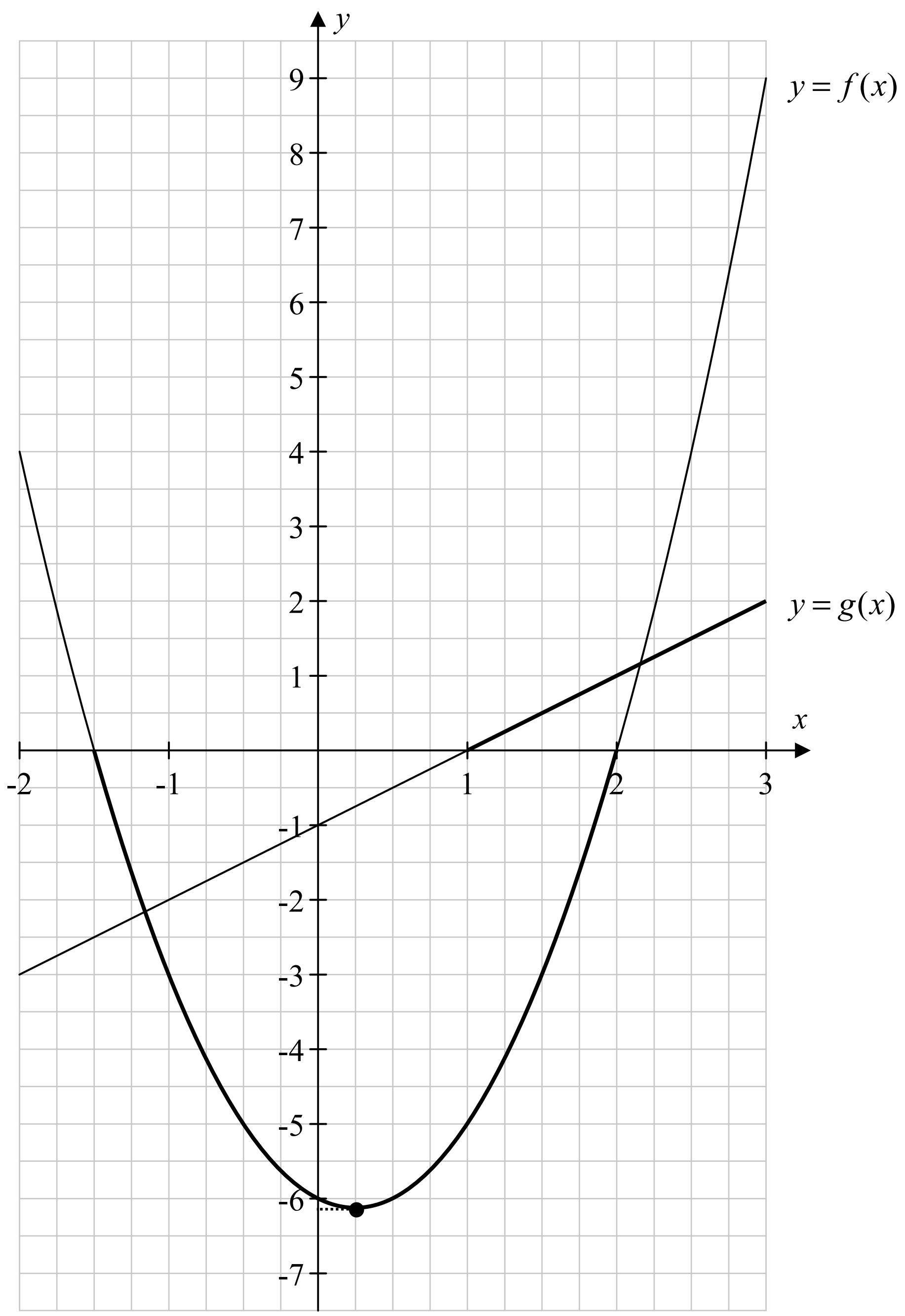
Question 14 **50 Marks**

(i) g is the function $g : x \mapsto x - 1$, where $x \in \mathbb{R}$. Find the value of each of the following.

$g(3) = 3 - 1 = 2.$
$g(-2) = -2 - 1 = -3.$

(ii) f is the function $f : x \mapsto 2x^2 - x - 6$, where $x \in \mathbb{R}$.

Using the same axes and scales, draw the graphs of the functions $y = f(x)$ and $y = g(x)$ in the domain $-2 \leq x \leq 3$.



Graphing g:	*Graphing f*:
Straight line, so only need the two points from **(i)**: $(3,2)$ and $(-2,-3)$.	$f(-2)=4$ $f(-1)=-3$ $f(0)=-6$ $f(1)=-5$ $f(2)=0$ $f(3)=9$
Or: $g(x) = x - 1$	*Or*: $f(x) = 2x^2 - x - 6$

x	x	- 1	y
-2	-2	- 1	-3
-1	-1	- 1	-2
0	0	- 1	-1
1	1	- 1	0
2	2	- 1	1
3	3	- 1	2

x	$2x^2$	- x	- 6	y
-2	8	+2	- 6	4
-1	2	+ 1	- 6	- 3
0	0	0	- 6	- 6
1	2	- 1	- 6	- 5
2	8	- 2	- 6	0
3	18	- 3	- 6	9

Use your graphs from **(ii)** to estimate:

(iii) the minimum value of $f(x)$

$f_{\min}(x) = -6\cdot 1$ *... see graph*

(iv) the range of values of x for which $f(x) < 0$

$-1\cdot 5 < x < 2$ *... see graph*

(v) the range of values of x for which $g(x) \geq 0$.

$x \geq 1$ *... see graph*

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect), scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale	0, 5	0, 2, 5	0, 2, 3, 5	
10-mark scale			0, 4, 7, 10	0, 4, 6, 8, 10
25-mark scale				0, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may be awarded. Thus, for example, in Scale 10C, 9 marks may be awarded.

Unless otherwise specified, accept correct answer with or without work.

Accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Question 1 (20)

(a)	5B
(b)	5B
(c)	10C

Question 2 (15)

(a)(i)	5C
(a)(ii)	5B
(b)	5B

Question 3 (25)

(a)(i)	5C
(a)(ii)&(iii)	10D
(b)	10C

Question 4 (25)

(a)	10D
(b)	5B
(c)	10C

Question 5 (10)

	10D

Question 6 (10)

	10C

Question 7 (25)

(i)	10D
(ii)	10D
(iii)	5C

Question 8 (15)

(i)	5C
(ii)	5B
(iii)	5B

Question 9 (30)

(i)	10C
(ii)	5C
(iii)&(iv)	10C
(v)	5B

Question 10 (15)

(a)	5A
(b)	10C

Question 11 (20)

(i)	5B
(ii)	5B
(iii)	10C

Question 12 (25)

(a)	5B
(b)	5C
(c)(i)	10D
(c)(ii)	5C

Question 13 (15)

(i)	5C
(ii)	5C
(iii)	5C

Question 14 (50)

(i)	10C
(ii)	25D
(iii)	5B
(iv)&(v)	10C

Detailed marking notes

Question 1 (20)

(a)	Scale 5B (0, 2, 5) *Partial credit*:	 $\sqrt{2}$ or 3/2 as a decimal; *or* Reverse order; *or* Any 2 numbers in increasing order, except one given.
(b)	Scale 5B (0, 2, 5) *Partial credit*: *Full credit*:	 π; *or* Shows some understanding of rational / irrational numbers. Accept: "It goes on forever and doesn't repeat."
(c)	Scale 10C (0, 4, 7, 10) *Low partial credit*: *High partial credit*: *Full credit*:	 Some correct substitution. 3 correct answers; *or* 1 correct answer in **(i)**, and **(ii)** correct; *or* 3 substantially correct answers in **(i)**, and work of merit in **(ii)**. "Positive" is not needed in answer. Accept "17" as the natural number in **(ii)** (indentifying the output by the input).

Question 2 (15)

(a)(i)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 1 correct value. 5 correct values.
(a)(ii)	Scale 5B (0, 2, 5) *Partial credit*:	 1 correct reason only; *or* Shows understanding of prime numbers.
(b)	Scale 5B (0, 2, 5) *Partial credit*:	 Some work of merit e.g. correct substitution.

Question 3 (25)

(a)(i)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 2 elements correctly placed. 4 parts of the Venn diagram filled in correctly.
(a) (ii)&(iii)	Scale 10D (0, 4, 6, 8, 10) *Low partial credit*: *Middle partial credit*: *High partial credit*:	 1 element correct in 1 part of **(a)(ii)**. 2 parts of **(a)(ii)** correct; *or* **(a)(iii)** correct. 3 parts of **(a)(ii)** correct; *or* **(a)(iii)** and 1 part of **(a)(ii)** correct.

(b)	Scale 10C (0, 4, 7, 10) *Low partial credit*: *High partial credit*: *Full credit*:	 Some work of merit in one part. 1 fully correct; *or* Answers swapped for **(b)(i)** and **(b)(ii)**. Accept answers on grid with no work on Venn diagram. Accept answer shown only on diagram.

Question 4 (25)

(a)	Scale 10D (0, 4, 6, 8, 10) *Low partial credit*: *Middle partial credit*: *High partial credit*:	 1 common factor; *or* Grouping indicated. 1 correct factorisation, using HCF; *or* 2 factorisations, using HCF, with sign errors. Almost correct answer, e.g. 2 correct factorisations, using HCF; *or* Last line of correct solution with sign errors.
(b)	Scale 5B (0, 2, 5) *Partial credit*:	 Some work of merit.
(c)	Scale 10C (0, 4, 7, 10) *Low partial credit:* *High partial credit*:	 Some work of merit. Correct factorisation of nominator or denominator.

Question 5 (10)

	Scale 10D (0, 4, 6, 8, 10) *Low partial credit*: *Middle partial credit*: *High partial credit*:	 Some work of merit, including subbing in value for x. $x > -4$ *or* $x \leq 6$. Accept with/without inequality sign. Solution to inequality or number line correct.

Question 6 (10)

	Scale 10C (0, 4, 7, 10) *Low partial credit*: *High partial credit*:	 Any correct entry in **(a)**. **(a)** or **(b)** correct.

Question 7 (25)

(i)	Scale 10D (0, 4, 6, 8, 10) *Low partial credit*: *Middle partial credit*: *High partial credit*: *Full credit –1*:	 Exhibits basic knowledge of percentages. Finds relevant percentage of a relevant number (i.e. gets €200·72 or €239·20 or €1466·08). Almost correct answer, e.g. All 3 percentages correct; *or* All 3 percentages substantially correct, and totalled. Units omitted.

(ii)	Scale 10D (0, 4, 6, 8, 10)	
	Low partial credit:	Exhibits basic knowledge of percentages.
	Middle partial credit:	Finds relevant percentage of a relevant number (i.e. gets €6560 or €1705·60).
	High partial credit:	Finds gross tax (€8265·60).
	Full credit –1:	Units omitted.
(iii)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Finds total deductions.
	High partial credit:	$\frac{6871\cdot 60}{36960}$; *or* Finds percentage using USC or tax, i.e. fails to add.

Question 8 (15)

(i)	Scale 5C (0, 2, 3, 5)	
	Low partial credit:	Some work of merit, e.g. One first difference, or "Quadratic" without a valid reason.
	High partial credit:	One second difference. Accept 0·8 for 2nd difference.
	Full credit:	Accept explanation such as: "It is quadratic because the height goes up and then down", without any further work, for full credit.
(ii)	Scale 5B (0, 2, 5)	
	Partial credit:	Some work of merit, e.g. Some use of first or second difference, or of a graph; *or* Value given between 3·8 and 6·1 inclusive, no work shown.
	Full credit –1:	Units omitted.
(iii)	Scale 5B (0, 2, 5)	
	Partial credit:	Some work of merit, including value of t in the range $4 \le t \le 4\cdot 5$, without work.
	Full credit:	Accept answer consistent with work shown.
	Full credit − 1:	Units omitted (once per question).

Question 9 (30)

(i)	Scale 10C (0, 4, 7, 10) *Low partial credit*: *High partial credit*: *Full credit*:	 1 point correctly plotted; *or* Co-ordinates reversed on graph and missing point(s). 7 points correctly plotted (incorrectly joined / not joined); *or* End points plotted for each line; *or* 1 line correctly plotted; *or* Co-ordinates reversed. Treat any other type of graph (e.g. bar chart) as points plotted but not joined. Accept graph for $20 \le x \le 80$ for full credit.
(ii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Some work of merit, e.g. a correct slope formula. Slope or explanation correct.
(iii)&(iv)	Scale 10C (0, 4, 7, 10) *Low partial credit*: *High partial credit*:	 Some work of merit. 1 equation fully correct.
(v)	Scale 5B (0, 2, 5) *Partial credit*:	 Some work of merit.

Question 10 (15)

(a)	Scale 5A (0, 5)	 Hit or miss. Must be fully correct.
(b)	Scale 10C (0, 4, 7, 10) *Low partial credit*: *High partial credit*:	 1 correct root; *or* Effort to create quadratic function. Roots of $h(x)$ and $k(x)$; *or* 1 correct function.

Question 11 (20)

(i)	Scale 5B (0, 2, 5) *Partial credit*:	 1 correct response.
(ii)	Scale 5B (0, 2, 5) *Partial credit*:	 Something =1; *or* Exhibits understanding that product means multiply.
(iii)	Scale 10C (0, 4, 7, 10) *Low partial credit*: *High partial credit*:	 Some work of merit. Accept writing down relevant formula / identifying a or b or c. Accept further work presented in **(ii)** for low partial credit here. If **(ii)** results in a linear equation, it must be solved correctly to be awarded *Low partial credit*. Almost correct answer, e.g. one correct solution; *or* 2 solutions presented, with work of substantial merit.

Question 12 (25)

(a)	Scale 5B (0, 2, 5) *Partial credit*:	 Some relevant work.
(b)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Sets up division. Substantial work with 1 or more critical omissions.
(c)(i)	Scale 10D (0, 4, 6, 8, 10) *Low partial credit*: *Middle partial credit*: *High partial credit*:	 Some relevant work. Finds 1 value. Substantial work with 1 or more critical omissions.
(c)(ii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*: *Full credit:*	 Substitution into either equation, accept interchanging x and y. Values do not equate and no conclusion stated. Verifies using the other equation.

Question 13 (15)

(i)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Some work of merit. 3^2+3^2; *or* $\frac{3}{x}=\frac{1}{\sqrt{2}}$.
(ii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Some work of merit. 3^2-1^2.
(iii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Identifies x or y; *or* Exhibits knowledge of perimeter. Answer correct, but not in required form; *or* Half of perimeter in correct form.

Question 14 (50)

(i)	Scale 10C (0, 4, 7, 10)	
	Low partial credit:	Some correct substitution; *or* Solves $g(x) = 3$ or $g(x) = -2$ correctly.
	High partial credit:	$g(3)$ or $g(-2)$ correct; *or* Solves $g(x) = 3$ and $g(x) = -2$ correctly. (Accept $x = 4$ and $x = -1$ with or without work for *High partial credit*.)
(ii)	Scale 25D (0, 10, 15, 20, 25)	
	Low partial credit:	Some work of merit.
	Middle partial credit:	Substantial work in preparing and/or plotting $f(x)$, with some omissions; *or* $g(x)$ plotted correctly. If $f(x)$ drawn as linear graph, it must be calculated and drawn completely correctly for *Middle partial credit*.
	High partial credit:	Almost correct, e.g. Substantial work in plotting $f(x)$, with some omissions, and $g(x)$ plotted correctly; *or* $f(x)$ plotted correctly; *or* Scale error, but graphs consistent with scale used.
	Full credit −1:	Correct separate graphs.
(iii)	Scale 5B (0, 2, 5)	
	Partial credit:	Clear indication on graph.
	Full credit −1:	Minimum point.
(iv) &(v)	Scale 10C (0, 4, 7, 10)	
	Low partial credit:	One correct value i.e. −1·5 or 2 or 1; *or* Some work of merit e.g. indication on graph.
	High partial credit:	**(iv)** or **(v)** correct; *or* All three values correct (−1·5, 2, and 1), but inequalities not used, or used incorrectly.

2014. S235

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2014

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Question 1 **20 Marks**

Pauline flips a fair coin 3 times, and records the outcomes.
She writes H for each head and T for each tail.

(i) Complete the table below to show all of the possible outcomes.
Two outcomes have already been filled in for you.

$H\ H\ H$	$T\ H\ H$
$H\ H\ T$	$T\ H\ T$
$H\ T\ H$	$T\ T\ H$
$H\ T\ T$	$T\ T\ T$

(ii) Find the probability of getting two heads and one tail.

$$\Pr(2\ H, 1\ T) = \frac{3}{8}$$

(iii) Jamie says: "You have the same probability of getting three heads as you do of getting two heads and one tail."
Do you agree with Jamie? Give a reason for your answer.

Answer: No

Reason: $\Pr(3\ H) = \frac{1}{8}$ but $\Pr(2\ H, 1\ T) = \frac{3}{8}$

Or:

Reason: There is only 1 way to get three heads. There are 3 ways to get two heads and one tail.

(iv) Max says: "You have the same probability of getting $H\ H\ H$ as you do of getting $H\ T\ H$."
Do you agree with Max? Give a reason for your answer.

Answer: Yes

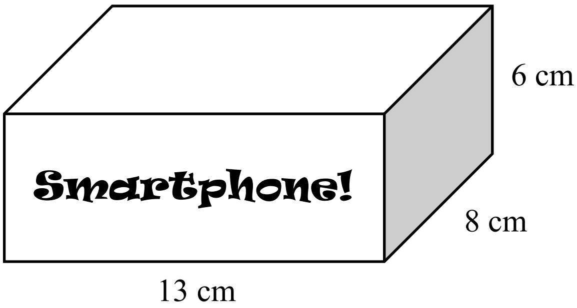
Reason: $\Pr(H\ H\ H) = \frac{1}{8}$ and $\Pr(H\ T\ H) = \frac{1}{8}$

Or:

Reason: There is only one way to get each event.

Question 2 **35 Marks**

The box for an individual mobile phone is 13 cm long, 8 cm wide, and 6 cm high, as shown.

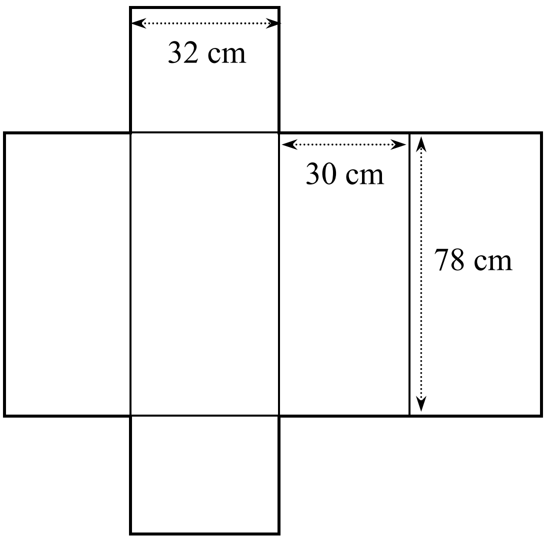


(i) Find the volume of an individual mobile phone box.

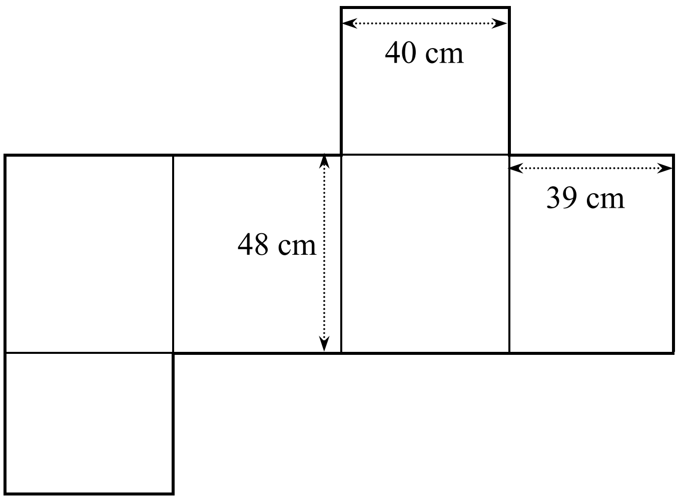
Volume $=13\times8\times6=624$ cm^3

These individual mobile phone boxes will be shipped in a large rectangular box.
Below are diagrams of the nets of two large boxes that could be used, Box **A** and Box **B**.

Box **A**



Box **B**



(ii) Show that Box **A** and Box **B** have the same volume.

Box **A**: Volume $=32\times30\times78=74\ 880$ cm^3	Box **B**: Volume $=48\times40\times39=74\ 880$ cm^3

(iii) What is the largest number of individual mobile phone boxes that will fit in each large box?

> Box **A**: $32 \div 8 = 4$; $30 \div 6 = 5$; $78 \div 13 = 6$; so Box **A** can be filled completely.
>
> Box **B**: $48 \div 6 = 8$; $40 \div 8 = 5$; $39 \div 13 = 3$; so Box **B** can be filled completely.
>
> Total: $4 \times 5 \times 6 = 120$ individual phone boxes.

(iv) Find the surface area of each large box.

Box **A**:	Box **B**:
Surface Area $= 2(32\times30 + 32\times78 + 30\times78)$ $= 11\,592 \text{ cm}^2$.	Surface Area $= 2(48\times40 + 48\times39 + 40\times39)$ $= 10\,704 \text{ cm}^2$.

(v) The large boxes are made from cardboard. The cardboard costs €0·67 per m^2. The cardboard just covers the net of a box. Find the cost of the box that uses the least amount of cardboard.

> *Use Box **B**. The cost is given per m^2, so convert surface area to m^2 (or cost to per cm^2).*
>
> $1 \text{ cm} = 0\cdot01 \text{ m}$, so $1 \text{ cm}^2 = 0\cdot01^2 \text{ m}^2 = 0\cdot0001 \text{ m}^2$.
>
> Surface area $= 10\ 704 \text{ cm}^2 = 10\ 704 \times 0\cdot0001 \text{ m}^2 = 1\cdot0704 \text{ m}^2$.
>
> Cost of box $= €1\cdot0704 \times 0\cdot67$
> $= €0\cdot717168$
> $= €0\cdot72$, to the nearest cent.

(vi) An average of 140 large boxes is produced each month. Find the saving, per annum, if you choose to make the box that uses the least amount of cardboard.

> Cost of Box A $= €(11592 \times 0\cdot0001 \times 0\cdot67)$
> $= €0\cdot776664$
> $= €0\cdot78$, to the nearest cent.
>
> Saving per annum $= €(0\cdot78 - 0\cdot72) \times 140 \times 12$
> $= €(0\cdot06) \times 1680$
> $= €100\cdot80$.
>
> *Or*:
>
> Difference in area $= (11592 - 10704) \text{ cm}^2 = 888 \text{ cm}^2 = 0\cdot0888 \text{ m}^2$.
>
> Saving per annum $= €0\cdot67 \times 0\cdot0888 \times 140 \times 12 = €99\cdot95$.

Question 3 **40 Marks**

All of the students in a class took *IQ Test 1* on the same day. A week later they all took *IQ Test 2*. Their scores on the two IQ tests are shown in the tables below.

IQ Test 1				
86	104	89	105	96
96	103	94	104	119
115	79	97	111	108

IQ Test 2				
83	120	105	111	114
99	111	108	106	97
97	102	94	108	117

(i) Draw a back-to-back stem-and-leaf plot below to display the students' scores.

IQ Test 1						*IQ Test 2*				
				9	7					
			9	6	8	3				
	7	6	6	4	9	4	7	7	9	
8	5	4	4	3	10	2	5	6	8	8
		9	5	1	11	1	1	4	7	
					12	0				

Key: $9|7 =$ a score of 97

(ii) Find the range of scores for each IQ test.

Range of *IQ Test 1* $= 119 - 79 = 40$.	Range of *IQ Test 2* $= 120 - 83 = 37$.

(iii) Find the median score for each IQ test.

15 data points in each set, so median is the $\frac{15+1}{2} = 8th$ *data point.*	
Median of *IQ Test 1* $= 103$.	Median of *IQ Test 2* $= 106$.

(iv) Find the mean score for each IQ test.

Mean of *IQ Test 1* $= \frac{1506}{15} = 100 \cdot 4$.	Mean of *IQ Test 2* $= \frac{1572}{15} = 104 \cdot 8$.

(v) Compare the scores on the two IQ tests. Refer to **at least one** measure of central tendency and **at least one** measure of variability (spread) in your answer.

> In general, the scores in *IQ Test 2* are slightly higher than in *IQ Test 1*, as both the mean and median are higher for *IQ Test 2*.
>
> The scores are slightly more spread out in *IQ Test 1* than in *IQ Test 2*, as the range is bigger for *IQ Test 1*; *or* The spread of scores is very similar, as the two ranges are almost the same.

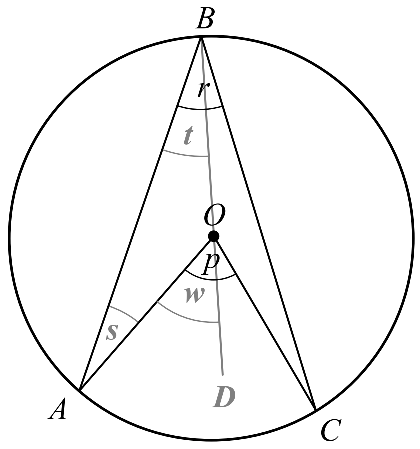
(vi) Marshall says that every student in the class must have done better on *IQ Test 2* than on *IQ Test 1*. Is Marshall correct? Explain your answer.

	Answer:	No.
	Explanation:	The person who got 119 on *IQ Test 1* could have got less, e.g. 94, on *IQ Test 2*.
Or:		
	Explanation:	The maximum score on *IQ Test 1* is greater than the minimum score on *IQ Test 2*.

Question 4 **30 Marks**

(a) Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

Diagram:



Given: A circle with centre O. Points A, B, and C on the circle. Angles p and r, as shown.

To Prove: $p = 2r$.

Construction: Join B to O, and extend to D. Mark the angles s, t, and w.

Proof:

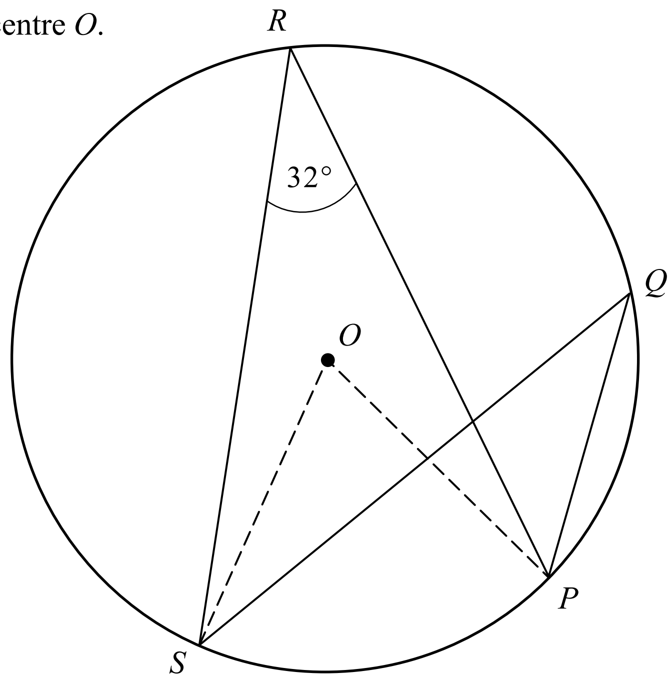
$\lvert OA\rvert = \lvert OB\rvert$	radii of circle	*Step 1*
$\therefore s = t$	isosceles triangle	*Step 2*
$w = s + t$	exterior angle	*Step 3*
$\therefore w = 2t$		*Step 4*

Similarly, $(p - w) = 2(r - t)$.

So $p = (p - w) + w$
$= 2(r - t) + 2t$
$= 2r$ *Step 5*

(b) P, Q, R, and S are points on a circle with centre O.

$|\angle PRS| = 32°$, as shown.



(i) Find $|\angle SOP|$.

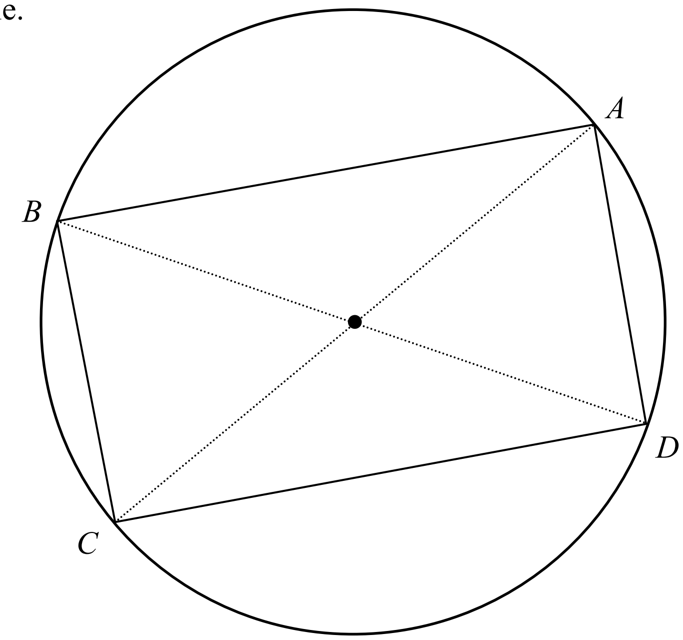
$$|\angle SOP| = 2 \times 32° = 64°.$$

(ii) Find $|\angle SQP|$.

$$|\angle SQP| = |\angle SRP| = 32°.$$

(c) A, B, C, and D are points on a circle, as shown below.
$[AC]$ and $[BD]$ are diameters of the circle.

Prove that $ABCD$ is a rectangle.



We just need to prove that the four angles are 90°.

$|\angle BAD| = |\angle BCD| = 90°$, as $[BD]$ is a diameter.

Similarly, $|\angle CBA| = |\angle CDA| = 90°$.

So $ABCD$ is a rectangle.

Question 5 **45 Marks**

Students in a class are investigating spending in their local area. They each carry out a different survey, and display the results.

(a) John is investigating whether people pay for their weekly shopping with Credit Card, Debit Card, Cash, or Cheque. When people tell him which one of these they usually use, he writes it in a table. His results are shown below.

Credit Card	Debit Card	Debit Card	Cash	Debit Card
Credit Card	Cash	Cash	Credit Card	Debit Card
Debit Card	Debit Card	Cheque	Cash	Cash
Cash	Cash	Debit Card	Cash	Credit Card

(i) What type of data has John collected? Put a tick (✓) in the correct box below.

Numerical Continuous	Numerical Discrete	Categorical Nominal	Categorical Ordinal
☐	☐	☑	☐

(ii) Fill in the frequency table below.

Method of Payment	Credit Card	Debit Card	Cash	Cheque
Frequency	4	7	8	1

(iii) What is the mode of John's data? Mode = Cash

(iv) John says that he cannot find the mean of his data. Explain why this is the case.

He cannot add up his values and divide by 20.

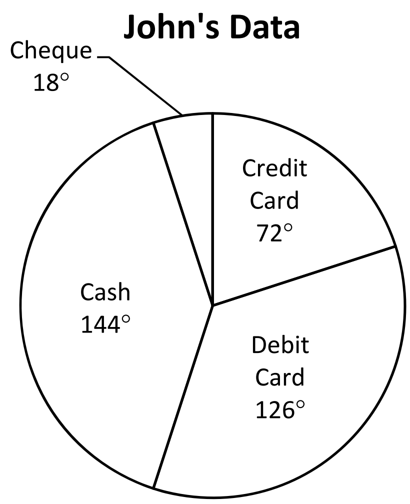
(v) Display John's data in a pie chart. Show all of your calculations clearly.

Credit Card: $\frac{4}{20}\times 360° = 72°$

Debit Card: $\frac{7}{20}\times 360° = 126°$

Cash: $\frac{8}{20}\times 360° = 144°$

Cheque: $\frac{1}{20}\times 360° = 18°$.

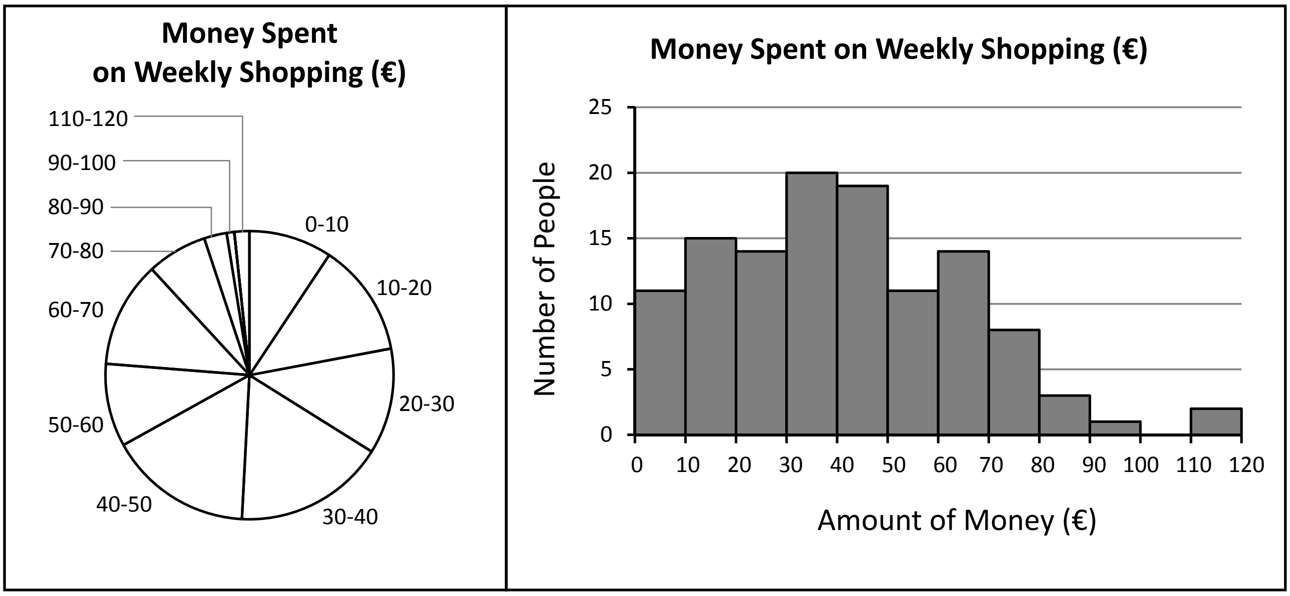


(b) Margaret wants to examine if people prefer to do their weekly shopping in *Tesco*, *Dunnes Stores*, *SuperValu*, or *Lidl*. She stands outside her local *Lidl* shop for one day, and asks everyone as they leave the shop where they prefer to do their weekly shopping.

Give one reason why Margaret's data may be biased.

Margaret's data may be biased because her sample is probably not representative. She will probably have a lot more people answering "Lidl" than she should.

(c) Mary is interested in the amount of money people spend on their weekly shopping. She surveys people as they leave the local supermarket on a Saturday morning, and displays her results in the two graphs below.



(i) Mary wants to show that about half of her sample spent less than €40 on their weekly shopping. Which graph do you think she should use? Give a reason for your answer.

Answer:	Pie chart.
Reason:	It's easy to see where half the pie chart is (180°).

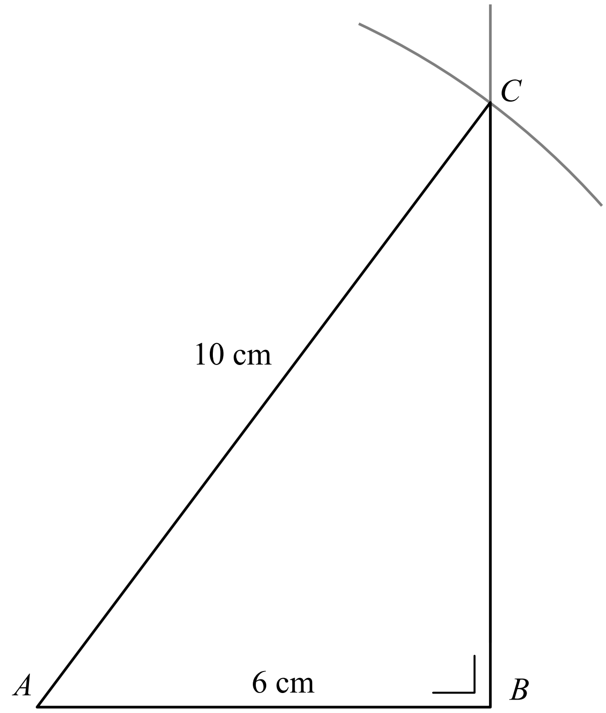
(ii) Mary wants to show that there were more people in the 30–40 group than in any other. Which graph do you think she should use? Give a reason for your answer.

Answer:	Bar chart / Histogram.
Reason:	It's easy to see which bar is highest.

Question 6 **25 Marks**

(i) Construct a right-angled triangle ABC, where:

$$|AB| = 6 \text{ cm}$$
$$|\angle ABC| = 90°$$
$$|AC| = 10 \text{ cm.}$$



Note: *It is also possible to work out the length of the third side,* $[BC]$, *using the Theorem of Pythagoras, and then construct* $[BC]$ *and* $[AC]$.

(ii) On your diagram, measure the angle $\angle CAB$. Give your answer correct to the nearest degree.

$|\angle CAB| =$ $\boxed{53°}$

(iii) Let X be the whole number you wrote as your answer to **(ii)**.

Use a calculator to find $\cos X$. Give your answer correct to 3 decimal places.

$\cos(53°) = 0\cdot 6018... = 0\cdot 602$, correct to three decimal places.

(iv) Jacinta says that $\cos(\angle CAB)$ is exactly 0·6, because $\cos(\angle CAB) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$.

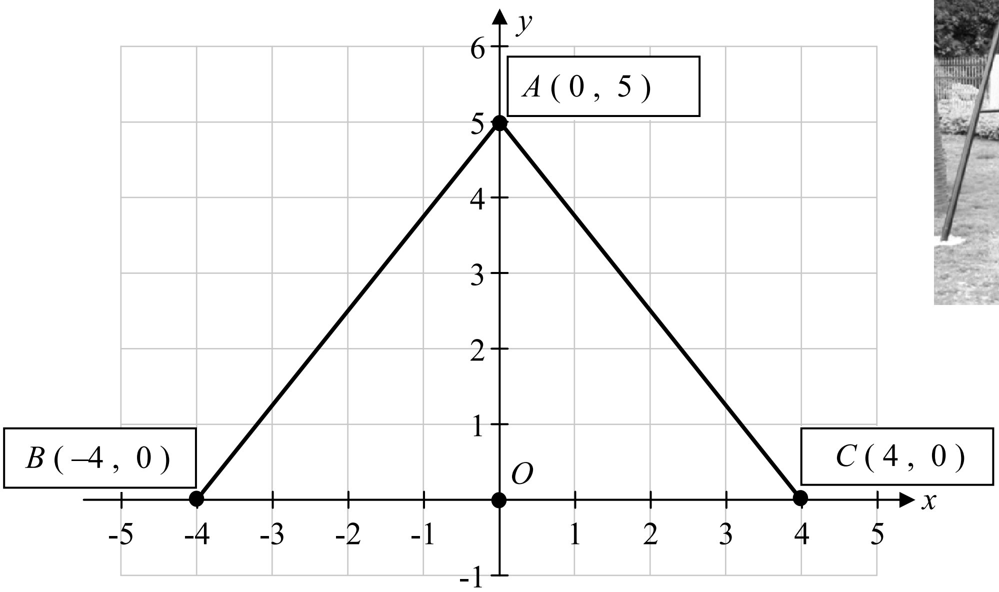
Explain why your answer in **(iii)** is **not** the same as Jacinta's.

They are not the same because $|\angle CAB| = \cos^{-1}\left(\dfrac{6}{10}\right) = 53\cdot 1301...°$.

So if X is a whole number then $\cos X$ can never be exactly 0·6.

Question 7 **45 Marks**

The diagram below shows part of the frame of a swing on a co-ordinate grid.
Each unit on the grid represents one metre.
The line segments $[AB]$ and $[AC]$ represent metal bars.



(i) Write the co-ordinates of the points *A*, *B*, and *C* in the spaces provided in the diagram.

(ii) Find the total length of metal bar needed to make this part of the swing.
Give your answer in metres, correct to one decimal place.

$|AB| = \sqrt{4^2 + 5^2} = \sqrt{41}.$

Similarly, $|AC| = \sqrt{41}.$

Total length of metal bar $= 2\sqrt{41} = 12 \cdot 80... = 12 \cdot 8$ m, correct to one decimal place.

(iii) Find the slope of AB and the slope of AC.

AB:	*AC*:
Slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{5}{4}$ or $1 \cdot 25$.	Slope $= \dfrac{5-0}{0-4} = -\dfrac{5}{4}$ or $-1 \cdot 25$.

(iv) Is *AB* perpendicular to *AC*? Give a reason for your answer.

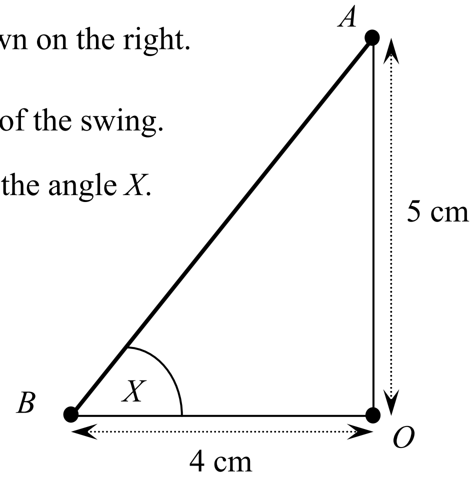
Answer: No

Reason: Product of slopes $= \dfrac{5}{4} \times -\dfrac{5}{4} = -\dfrac{25}{16} \neq -1.$

Or: Reason: When you invert one slope and change the sign, you don't get the other slope.

(v) Madison draws the scale diagram of the triangle OAB shown on the right. She marks in the angle X.
Recall that $[AB]$ is a metal bar, which is part of the frame of the swing.

Write down the value of $\tan X$, and hence find the size of the angle X. Give the size of the angle X correct to two decimal places.



$\tan X = \dfrac{5}{4}$
$\lvert\angle X\rvert = \tan^{-1}\left(\dfrac{5}{4}\right) = 51\cdot340... = 51\cdot34°$, correct to two decimal places.

In order to increase the height of the swing, it is decided to increase X by 20%.
The distance $\lvert AB\rvert$ will be kept the same.

(vi) Find the new height of the swing. Give your answer in metres, correct to one decimal place.

Recall from **(ii)** that $\lvert AB\rvert = \sqrt{41}$ m.

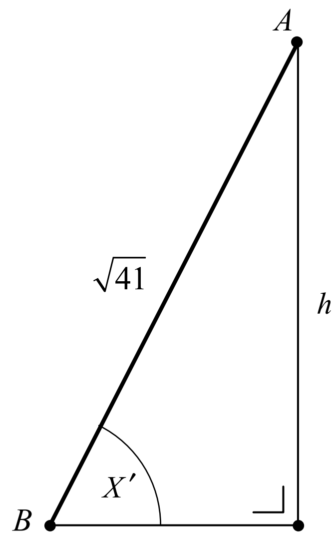
Increase $\lvert\angle X\rvert$ by 20% to get $\lvert\angle X'\rvert$:

$$\lvert\angle X'\rvert = 51\cdot34 \times 1\cdot2 = 61\cdot608°$$

From the diagram, $\sin X' = \sin 61\cdot608 = \dfrac{h}{\sqrt{41}}$.

$$\Rightarrow \quad h = \sqrt{41}\sin 61\cdot608 = 5\cdot632... = 5\cdot6 \text{ m},$$

correct to one decimal place.



Question 8 **20 Marks**

The equation of the line l is $x-3y-6=0$.

(i) Find the slope of the line l.

Method 1:	*Method 2*:
$-3y = -x+6$ *Step 1* $3y = x-6$ $y = \frac{1}{3}x-2$ *Step 2* $\Rightarrow$ Slope $= \frac{1}{3}$ *Step 3*	Slope $= -\frac{a}{b}$ $= -\frac{1}{-3}$ $= \frac{1}{3}$

(ii) Show that the point $(1,-2)$ is **not** on the line l.

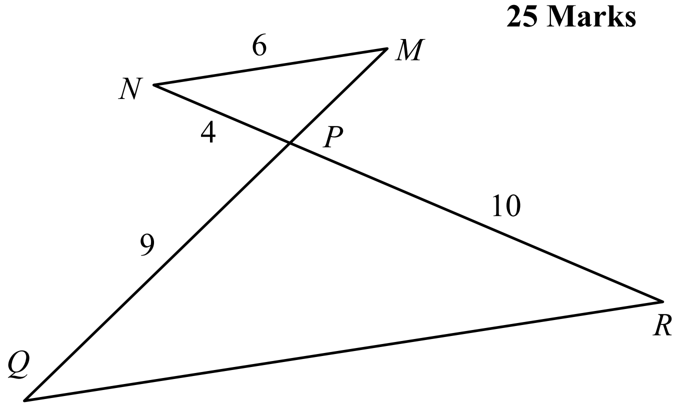
Sub in $(1,-2)$ to l: LHS $=1-3(-2)-6=1\neq 0=$ RHS. Point not on l.

(iii) The line k passes through $(1,-2)$ and is parallel to the line l.
Find the equation of the line k.

Slope of $k=\frac{1}{3}$.	Point on $k=(1,-2)$.
Equation of k: $y-(-2) = \frac{1}{3}(x-1)$ $\Rightarrow$ $y = \frac{x}{3}-\frac{7}{3}$ *or* $x-3y-7=0$	*Or*: Equation of k: $x-3y+c=0$ $\Rightarrow$ $1-3(-2)+c=0$ $\Rightarrow$ $c=-7$ $\Rightarrow$ $x-3y-7=0$

Question 9 **25 Marks**

In the diagram below, $|\angle MNP| = |\angle PRQ|$.



(i) Prove that $\Delta\, MNP$ and $\Delta\, QRP$ are similar.

Proof:	$\lvert\angle MNP\rvert$	$=$	$\lvert\angle PRQ\rvert$	(given)
	$\lvert\angle NPM\rvert$	$=$	$\lvert\angle QPR\rvert$	(vertically opposite)
	$\lvert\angle NMP\rvert$	$=$	$\lvert\angle PQR\rvert$	(third angle)
	$\Rightarrow$ Triangles are similar.			

(ii) Is NM parallel to QR? Give a reason for your answer.

Answer: Yes

Reason: $|\angle MNP| = |\angle PRQ|$ *or* $|\angle NMP| = |\angle PQR|$ *or* alternate angles are equal.

Given $|MN| = 6$, $|NP| = 4$, $|QP| = 9$, and $|PR| = 10$, find:

(iii) $|QR|$

By similar triangles $\triangle MNP$ and $\triangle QRP$:

$$\frac{|QR|}{6} = \frac{10}{4}$$

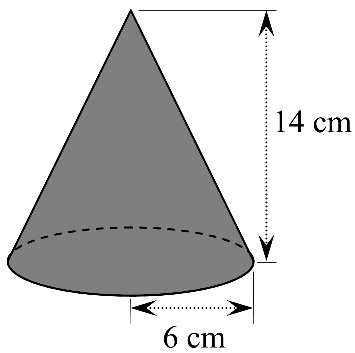
$$\Rightarrow |QR| = 6\times\frac{10}{4} = 15.$$

(iv) $|QM|$.

By similar triangles $\triangle MNP$ and $\triangle QRP$:	
$\frac{\lvert PM\rvert}{9}=\frac{6}{15}$ or $\frac{4}{10}$ $\Rightarrow \lvert PM\rvert=\frac{18}{5}$ or $3\cdot6$ $\Rightarrow \lvert QM\rvert=9+3\cdot6=\frac{63}{5}$ or $12\cdot6$.	*Or*: $\frac{\lvert PM\rvert}{4} = \frac{9}{10}$ $\Rightarrow \lvert PM\rvert = 4\times\frac{9}{10} = \frac{18}{5}$ or $3\cdot6$ $\Rightarrow \lvert QM\rvert = 9+3\cdot6 = \frac{63}{5}$ or $12\cdot6$.

Question 10 **15 Marks**

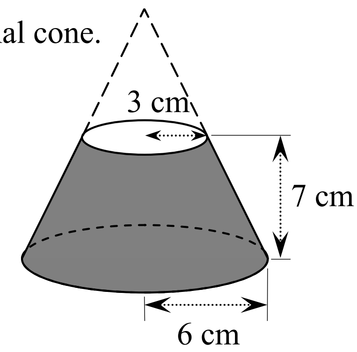
A solid cone has a radius of 6 cm and a height of 14 cm, as shown.



(i) Find the volume of the cone. Give your answer in terms of π.

Volume $= \frac{1}{3}\pi(6)^2\times14 = 168\pi$ cm^3

The shape shown below is a *frustum*. This is made by taking the cone above, cutting it horizontally at a height of 7 cm, and removing the upper portion. The radius of the circular top of the frustum is 3 cm, as shown in the diagram.



(ii) Find the ratio of the volume of the frustum to the volume of the original cone.

Volume of upper (removed) portion: $\frac{1}{3}\pi(3)^2\times7 = 21\pi$ cm^3 Volume of frustum: $168\pi-21\pi = 147\pi$ cm^3
Or: Volume of frustum $=\frac{1}{3}\pi h\left[R^2+Rr+r^2\right]$ $=\frac{1}{3}\pi\times7\left(6^2+6\times3+3^2\right)=147\pi$ cm^3 Required ratio: $\frac{147\pi}{168\pi}=\frac{7}{8}$ *or* $7{:}8$ *or* $0\cdot875$

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect), scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale	0, 5	0, 2, 5	0, 2, 3, 5	
10-mark scale			0, 3, 7, 10	0, 2, 5, 7, 10
15-mark scale				0, 4, 7, 11, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may be awarded. Thus, for example, in Scale 10C, 9 marks may be awarded.

Accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Question 1 (20)

(i)	5B
(ii)	5A
(iii)	5B
(iv)	5B

Question 2 (35)

(i)	5B
(ii)	5C
(iii)	5C
(iv)	10C
(v)	5B
(vi)	5B

Question 3 (40)

(i)	10D
(ii)	5C
(iii)	5B
(iv)	10C
(v)	5C
(vi)	5B

Question 4 (30)

(a)	15D
(b)	5C
(c)	10C

Question 5 (45)

(a)(i)	5A
(a)(ii)	5B
(a)(iii)	5A
(a)(iv)	5A
(a)(v)	10C
(b)	5A
(c)(i)	5B
(c)(ii)	5B

Question 6 (25)

(i)	10C
(ii)	5B
(iii)	5B
(iv)	5A

Question 7 (45)

(i)	5C
(ii)	5C
(iii)	10C
(iv)	5B
(v)	10C
(vi)	10D

Question 8 (20)

(i)	5C
(ii)	5B
(iii)	10C

Question 9 (25)

(i)	5C
(ii)	5B
(iii)	5C
(iv)	10C

Question 10 (15)

(i)	5C
(ii)	10C

Detailed marking notes

Question 1 (20)

(i)	Scale 5B (0, 2, 5) *Partial credit*:	 3 further correct outcomes.
(ii)	Scale 5A (0, 5)	
(iii)	Scale 5B (0, 2, 5) *Partial credit*:	 Correct answer, no reason or incorrect reason given; *or* Indication of one way of getting first outcome and three ways of getting second outcome; *or* 1/8; *or* 3/8.
(iv)	Scale 5B (0, 2, 5) *Partial credit*:	 Correct answer, no reason or incorrect reason given; *or* Indication of one way of getting each outcome; *or* 1/8.

Question 2 (35)

(i)	Scale 5B (0, 2, 5) *Partial credit*:	 Volume = $l \times w \times h$; *or* Volume = $13 \times 8 \times 6$.
(ii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Volume = $l \times w \times h$. Correct volume of Box **A** or Box **B**.
(iii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*: *Full credit*:	 Division of all dimensions of either box by dimensions of the phone. 74 880 / 624. Accept 74 880 / 624 = 120 for *Full credit*.
(iv)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*:	 Surface area = $2[l \times w + l \times h + w \times h]$; *or* 2 sides multiplied. Correct surface area of Box **A** or Box **B**.
(v)	Scale 5B (0, 2, 5) *Partial credit*: *Full credit –1*:	 Any work of merit, e.g. 10 704 × 0·67 = €7 171·68. Cost for Box **A** correctly found.
(vi)	Scale 5B (0, 2, 5) *Partial credit*:	 Any work of merit.

Question 3 (40)

(i)	Scale 10D (0, 2, 5, 7, 10) *Low partial credit*: *Middle partial credit*: *High partial credit*: *Full credit –1*: *Full credit*:	 1 score placed correctly on **either** side. One side fully completed. 10 or more scores placed correctly on **each** side. Fully correct graph, but key omitted or incorrect. An unordered graph may be accepted for *Full credit*.
(ii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*: *Full credit –1*:	 Use of 119 or 79 or 120 or 83. Range of one test correct; *or* One of 119, 79, 120, 83 incorrect; *or* 79 – 119 **and** 83 –120. Ranges of tests swapped.
(iii)	Scale 5B (0, 2, 5) *Partial credit*:	 One median correct; *or* Indication of the 8th or middle score
(iv)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*:	 Sum of scores for either test correct; *or* Indication of division by 15; *or* Indication of sum of 15 correct scores for either test. Mean for one test correct; *or* 1506 / 15 and 1572 / 15.
(v)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Mention of mean or median or mode or average or range. Comparison using measure of central tendency **or** measure of spread only.
(vi)	Scale 5B (0, 2, 5) *Partial credit*:	 Correct answer, no reason or incorrect reason given.

Question 4 (30)

(a)	Scale 15D (0, 4, 7, 11, 15) *Low partial credit*: *Middle partial credit*: *High partial credit*: *Full credit*:	 Diagram. Diagram, Given, To Prove and Construction only; *or* More than one step missing in proof. One step missing in proof; *or* Fully correct but with no reason given; *or* gets as far as $w = 2t$ or equivalent. Given, Construction, and To Prove may be indicated on diagram. Some steps in Proof may be indicated on diagram.

(b)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Indication that the angle at the centre is twice the angle at the circumference; *or* Indication that $\lvert\angle SRP\rvert = \lvert\angle SQP\rvert$. **(i)** or **(ii)** correct.
(c)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*:	 Work of some merit, e.g. States that the angles in a rectangle are right angles; *or* Indication that the angle in a semi-circle is a right angle; *or* Indication that AC and BD bisect each other; *or* Indication of isosceles or congruent triangles; *or* Identification of vertically opposite angles. Accept indication that opposite sides or opposite angles are equal (one pair). $\lvert\angle BAD\rvert = \lvert\angle BCD\rvert = \lvert\angle ABC\rvert = \lvert\angle ADC\rvert = 90^\circ$ with no reason. This may be indicated on the diagram.

Question 5 (45)

(a)(i)	Scale 5A (0, 5)	
(a)(ii)	Scale 5B (0, 2, 5) *Partial credit*: *Full credit –1*:	 One correct frequency. Tally or relative frequency given.
(a)(iii)	Scale 5A (0, 5)	
(a)(iv)	Scale 5A (0, 5) *Full credit*:	 Accept "data is not numerical" or "data is categorical" or similar.
(a)(v)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*: *Full credit*:	 One angle or fraction correct; *or* Correct labelled pie chart, no work and no angle shown. Angles correctly calculated and two angles correct in pie chart with labelling; *or* Angles correctly calculated and correct pie chart drawn but no labelling or incorrect labelling; *or* Angles or fractions correct, no work shown and correct pie chart drawn. Allow a tolerance of $\pm 2^\circ$ in drawing.

(b)	Scale 5A (0, 5)	
(c)(i)	Scale 5B (0, 2, 5) *Partial credit*: *Full credit*:	 Correct answer, no reason or incorrect reason given. Reason must refer to the diagram.
(c)(ii)	Scale 5B (0, 2, 5) *Partial credit*: *Full credit*:	 Correct answer, no reason or incorrect reason given. Reason must refer to the diagram.

Question 6 (25)

(i)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*: *Full credit*:	 One side correctly drawn; *or* Use of Pythagoras theorem; *or* Sketch drawn with given measurements shown; *or* 1 side or angle correctly drawn. Triangle correctly drawn with no construction lines or no work for $\lvert BC \rvert$; *or* Triangle correctly drawn but unlabelled or incorrectly labelled. Allow a tolerance of ± 2 mm or $\pm 2°$.
(ii)	Scale 5B (0, 2, 5) *Partial credit*: *Full credit*:	 Wrong angle correctly measured; *or* Triangle incorrect and unlabelled but an angle correctly measured. Allow a tolerance of $\pm 2°$.
(iii)	Scale 5B (0, 2, 5) *Partial credit*:	 Calculator in incorrect mode.
(iv)	Scale 5A (0, 5) *Full credit*:	 Accept mention of whole number rounding/approximation of candidate's work.

Question 7 (45)

(i)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 1 point correct; *or* All 3 reversed, i.e. (y, x). 2 points correct.
(ii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Some correct substitution into distance formula or Pythagoras theorem; *or* Correct formula from tables; *or* $\|AB\|$ or $\|AC\|$ between 6·4 and 6·6 inclusive, without work. $\|AB\|$ or $\|AC\|$ found, with work ($\sqrt{41}$ or 6·4); *or* Total length between 12·8 and 13·2, without work.
(iii)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*:	 Some correct substitution into slope formula; *or* Indication that slope = $\frac{\text{rise}}{\text{run}}$; *or* Correct formula from tables. Slope of AB or AC correctly found; *or* Correct substitution into slope formula in both cases.
(iv)	Scale 5B (0, 2, 5) *Partial credit*:	 Indication that the product of slopes of perpendicular lines = −1, or of the negative reciprocal of one slope, or that "perpendicular" means lines make 90° angle; *or* Correct answer, no reason or incorrect reason given. *Note*: Reason by measurement alone is not acceptable for *Full credit*.
(v)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*: *Full credit –1*:	 Some use of Pythagoras Theorem; *or* Correct trigonometric ratio. $X = \tan^{-1}\frac{5}{4}$; *or* $\tan X = 4/5$, and $X = 38{\cdot}66°$; *or* Tan X not found and X correct using sin or cos. Tan X correct but X correct using sin or cos.

(vi)	Scale 10D (0, 2, 5, 7, 10) *Low partial credit*: *Middle partial credit*: *High partial credit*:	 Mentions 120%; *or* $X' = 61\cdot608$; *or* 20% = 10·268; *or* Correct diagram; *or* Correct trigonometric ratio; *or* $\lvert AB\rvert = \sqrt{41}$. $X' = 61\cdot608°$ and correct diagram; *or* $\tan 61\cdot608° = \frac{h}{4}$. $\sin 61\cdot608° = \frac{h}{\sqrt{41}}$; *or* $h = 4\tan 61\cdot608° = 7\cdot4$.

Question 8 (20)

(i)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 Some work of merit, e.g. Correct manipulation of line equation; *or* Correct substitution into slope formula; *or* Correct slope formula ($y = mx + c$, Slope = $-a/b$, or similar). First 2 steps correct in Method 1 (as presented in Model Solutions above); *or* 1 error in Method 2.
(ii)	Scale 5B (0, 2, 5) *Partial credit*: *Full credit*:	 Some correct substitution into line equation. Conclusion needed for *Full credit*.
(iii)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*: *Full credit*:	 Correct slope; *or* Some correct substitution into line formula; *or* Correct formula from tables. Correct slope with some correct substitution into line formula; *or* Correct slope but both x and y reversed in substitution. It is not necessary to write the equation in the form $ax + by + c = 0$ for *Full credit*.

Question 9 (25)

(i)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 One of the first two pairs of equal angles correctly identified; *or* Wrong angle in first two pairs of angles. First 2 pairs of equal angles correctly identified and no conclusion; *or* Diagram with 3 pairs of equal angles marked.
(ii)	Scale 5B (0, 2, 5) *Partial credit*:	 Correct answer, no reason or incorrect reason given; *or* "Alternate angles are equal" stated.
(iii)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*:	 One correct relevant ratio; *or* Corresponding sides identified; *or* Indication that corresponding sides are proportional. $\frac{\lvert QR \rvert}{6} = \frac{10}{4}$, or equivalent
(iv)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*:	 One correct relevant ratio; *or* Corresponding sides identified; *or* Indication that corresponding sides are proportional; *or* $\lvert QM \rvert = \lvert QP \rvert + \lvert PM \rvert$, or similar. $\lvert PM \rvert$ correctly found, no addition.

Question 10 (15)

(i)	Scale 5C (0, 2, 3, 5) *Low partial credit*: *High partial credit*: *Full credit –1*:	 Incorrect relevant formula used; *or* Some correct substitution into volume of cone formula; *or* Correct formula from tables. $\frac{1}{3} \times \pi \times 36 \times 14$ Answer correct, but not in terms of π.
(ii)	Scale 10C (0, 3, 7, 10) *Low partial credit*: *High partial credit*: *Full credit –1*:	 Volume of small cone : Volume of large cone = 1 : 8; *or* Incorrect relevant formula used; *or* Some correct substitution into volume of cone or frustum formula; *or* Correct formula from tables. Volume of frustum = 147π ; *or* Volume of small cone : Volume of frustum = 1 : 7. Ratio reversed; *or* Answer not in simplest form.

Bonus marks for answering through Irish

Bonus marks are applied separately to each paper, as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded **down**.
For instance, 198 marks $\times$ 5% $= 9 \cdot 9 \Rightarrow$ bonus $= 9$ marks.

If the mark achieved is above 225, the following table applies:

Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)	Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0