Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2004

MATHEMATICS — HIGHER LEVEL

PAPER 2 (300 marks)

MONDAY, 14 JUNE — MORNING, 9:30 to 12:00

Attempt **FIVE** questions from Section **A** and **ONE** question from Section **B.**
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

SECTION A
Answer FIVE questions from this section.

1. **(a)** A circle has centre (−1, 5) and passes through the point (1, 2).
Find the equation of the circle.

(b) The point $a(5, 2)$ is on the circle K: $x^2 + y^2 + px - 2y + 5 = 0$.

(i) Find the value of p.

(ii) The line $L: x - y - 1 = 0$ intersects the circle K.
Find the co-ordinates of the points of intersection.

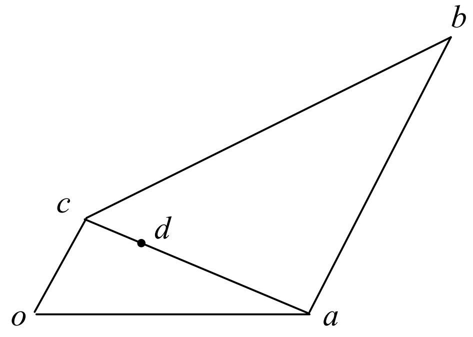
(c) The y-axis is a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

(i) Prove that $f^2 = c$.

(ii) Find the equations of the circles that pass through the points (–3, 6) and (–6, 3) and have the y-axis as a tangent.

2. **(a)** $\vec{r} = 12\vec{i} - 35\vec{j}$. Find the unit vector in the direction of $\vec{r}$.

(b) $oabc$ is a quadrilateral, where o is the origin.
$\vec{ad} = 3\vec{dc}$ and $\vec{ab} = 3\vec{c}$.

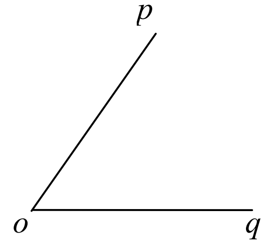


(i) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\vec{db}$ in terms of $\vec{a}$ and $\vec{c}$.

(iii) Show that o, d and b are collinear.

(c) p and q are points and o is the origin.
p, q and o are not collinear and $\left|\vec{p}\right| = \left|\vec{q}\right|$.



(i) Prove that $\vec{pq}$ is perpendicular to $\left(\vec{p} + \vec{q}\right)$.

(ii) Prove that $\vec{po} \cdot \vec{pq} = \frac{1}{2}\left|\vec{pq}\right|^2$.

3. **(a)** $a(-1, 4)$ and $b(9, -1)$ are two points and p is a point in $[ab]$.
Given that $|ap| : |pb| = 2 : 3$, find the co-ordinates of p.

(b) **(i)** Calculate the perpendicular distance from the point $(-1, -5)$ to the line
$$3x - 4y - 2 = 0.$$

(ii) The point $(-1, -5)$ is equidistant from the lines $3x - 4y - 2 = 0$ and $3x - 4y + k = 0$, where $k \neq -2$. Find the value of k.

(c) f is the transformation $(x, y) \to (x', y')$, where $x' = 2x - y$ and $y' = x + y$.
L is the line $y = mx + c$.
K is the line through the origin that is perpendicular to L.

(i) Find the equation of $f(L)$ and the equation of $f(K)$.

(ii) Find the values of m for which $f(K) \perp f(L)$.
Give your answer in surd form.

4. **(a)** A is an acute angle such that $\tan A = \dfrac{8}{15}$.
Without evaluating A, find

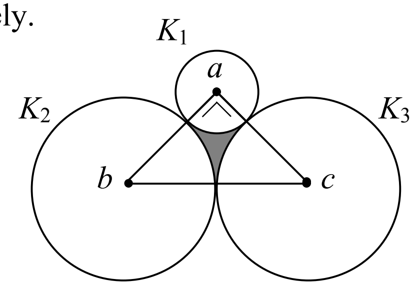
(i) $\cos A$

(ii) $\sin 2A$.

(b) **(i)** Prove that $\cos 2A = \cos^2 A - \sin^2 A$.
Deduce that $\cos 2A = 2\cos^2 A - 1$.

(ii) Hence, or otherwise, find the value of θ for which
$$2\cos\theta - 7\cos\left(\frac{\theta}{2}\right) = 0, \text{ where } 0^0 \leq \theta \leq 360^0.$$
Give your answer correct to the nearest degree.

(c) a, b and c are the centres of circles K_1, K_2 and K_3 respectively.
The three circles touch externally and $ab \perp ac$.
K_2 and K_3 each have radius $2\sqrt{2}$ cm.



(i) Find, in surd form, the length of the radius of K_1.

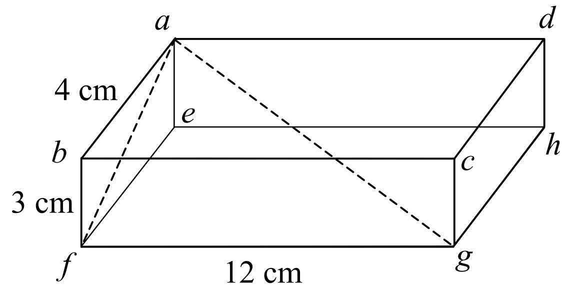
(ii) Find the area of the shaded region in terms of π.

5. **(a)** Prove that $\cos^2 A + \sin^2 A = 1$, where $0^\circ \leq A \leq 90^\circ$.

(b) **(i)** Show that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2$ simplifies to a constant.

(ii) Express $1 - (\cos x - \sin x)^2$ in the form $a \sin bx$, where $a, b \in \mathbf{Z}$.

(c) The diagram shows a rectangular box.
Rectangle *abcd* is the top of the box and rectangle *efgh* is the base of the box.



$|ab| = 4$ cm, $|bf| = 3$ cm and $|fg| = 12$ cm.

(i) Find $|af|$.

(ii) Find $|ag|$.

(iii) Find the measure of the acute angle between $[ag]$ and $[df]$.
Give your answer correct to the nearest degree.

6. **(a)** A committee of five is to be selected from six students and three teachers.

(i) How many different committees of five are possible?

(ii) How many of these possible committees have three students and two teachers?

(b) **(i)** Solve the difference equation $3u_{n+2} - 2u_{n+1} - u_n = 0$, where $n \geq 0$, given that $u_0 = 3$ and $u_1 = 7$.

(ii) Evaluate $\lim_{n \to \infty} u_n$.

(c) Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black.
Four cards are selected at random from the eight cards.

Find the probability that the four cards selected are:

(i) all of different colours

(ii) two odd-numbered cards and two even-numbered cards

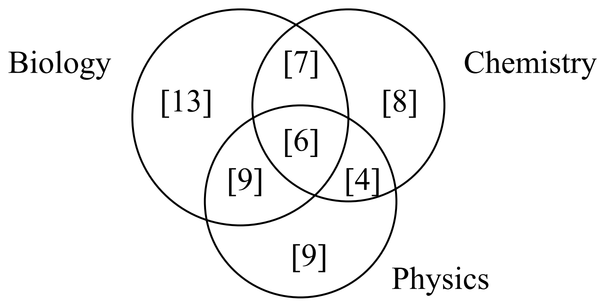
(iii) all of different colours, two odd-numbered and two even-numbered.

7. **(a)** At the Olympic Games, eight lanes are marked on the running track. Each runner is allocated to a different lane. Find the number of ways in which the runners in a heat can be allocated to these lanes when there are

(i) eight runners in the heat

(ii) five runners in the heat and any five lanes may be used.

(b) In a class of 56 students, each studies at least one of the subjects Biology, Chemistry, Physics. The Venn diagram shows the numbers of students studying the various combinations of subjects.



(i) A student is picked at random from the whole class. Find the probability that the student does not study Biology.

(ii) A student is picked at random from those who study at least two of the subjects. Find the probability that the student does not study Biology.

(iii) Two students are picked at random from the whole class. Find the probability that they both study Physics.

(iv) Two students are picked at random from those who study Chemistry. Find the probability that exactly one of them studies Biology.

(c) The mean of the real numbers p, q and r is $\bar{x}$ and the standard deviation is σ.

(i) Show that the mean of the four numbers p, q, r and $\bar{x}$ is also $\bar{x}$.

(ii) The standard deviation of p, q, r and $\bar{x}$ is k. Show that $k : \sigma = \sqrt{3} : 2$.

SECTION B
Answer ONE question from this section.

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8. **(a)** Use integration by parts to find $\int x \sin x dx$.

(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series.

(i) Derive the first five terms of the Maclaurin series for e^x.

(ii) Hence write down the first five terms of the Maclaurin series for e^{-x} and deduce the first three non-zero terms of the series for $\frac{e^x + e^{-x}}{2}$.

(iii) Write the general term of the series for $\frac{e^x + e^{-x}}{2}$ and use the Ratio Test to show that the series converges for all x.

(c) A solid cylinder has height h and radius r. The height of the cylinder, added to the circumference of its base, is equal to 3 metres.

(i) Express the volume of the cylinder in terms of r and π.

(ii) Find the maximum possible volume of the cylinder in terms of π.

9. **(a)** z is a random variable with standard normal distribution.
Find the value of z_1 for which $P(z \leq z_1) = 0.9370$.

(b) A child throws a ball at a group of three skittles. The probability that the ball will knock 0, 1, 2 or 3 of the skittles is given in the following probability distribution table:

x	0	1	2	3
$P(x)$	0.1	0.1	0.5	k

(i) Find the value of k.

(ii) Find the mean of the distribution.

(iii) Find the standard deviation of the distribution, correct to two decimal places.

(c) Before local elections, a political party claimed that 30% of the voters supported it. In a random sample of 1500 voters, 400 said they would vote for that party.
Test the party's claim at the 5% level of significance.

10. **(a)** The binary operation $*$ is defined by $a * b = a + b - ab$, where $a, b \in \mathbf{R} \setminus \{1\}$.

(i) Find the identity element.

(ii) Calculate 3^{-1}, the inverse of 3.

(iii) Find x^{-1} in terms of x.

(iv) Show that $(a * b) * c = a * (b * c)$.

(v) Show that $a * b \neq 1$, for all $a, b \in \mathbf{R} \setminus \{1\}$.

(b) Prove that if H and K are subgroups of G, then so also is $H \cap K$.

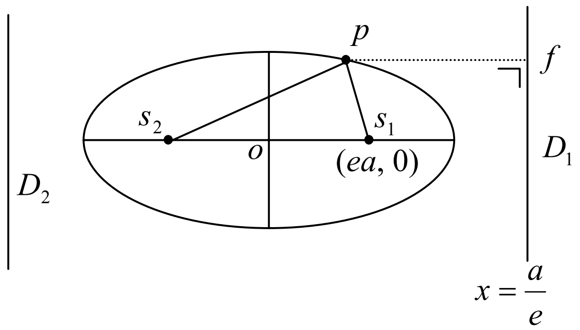
11. **(a)** f is the transformation $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix}$ where $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

o is the point $(0, 0)$, p is the point $(1, 0)$ and q is the point $(0, 1)$.

(i) Find o', p' and q', the images of o, p and q, respectively under f.

(ii) Verify that $|\angle p'o'q'| = 90^0$.

(b) **(i)**

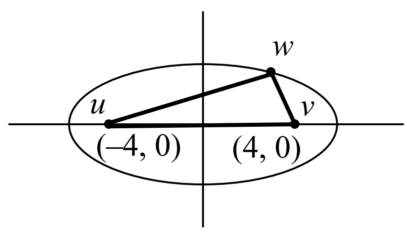


The diagram shows an ellipse with eccentricity e, centred at the origin. One focus is the point s_1 $(ea, 0)$ and the other focus is s_2.

$x = \frac{a}{e}$ is the equation of the directrix D_1. p is any point on the ellipse.

Noting that $|ps_1| = e|pf|$, prove that $|ps_1| + |ps_2| = 2a$.

(ii) u $(-4, 0)$ and v $(4, 0)$ are two points.
w is a point such that the perimeter of triangle uvw has length 18.
The locus of w is an ellipse. Find its equation.



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