Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2016

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE – AFTERNOON, 2:00 to 4:30

Six questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be $9{\cdot}8$ m s^{-2}.

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

1. **(a)** A car has an initial speed of u m s^{-1}. It moves in a straight line with constant acceleration f for 4 seconds. It travels 40 m while accelerating.
The car then moves with uniform speed and travels 45 m in 3 seconds.
It is then brought to rest by a constant retardation $2f$.

(i) Draw a speed-time graph for the motion.

(ii) Find the value of u.

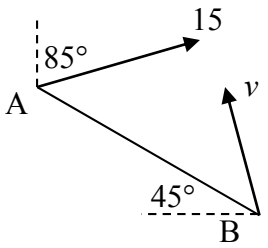
(iii) Find the total distance travelled.

(b) A particle is projected vertically upwards with a velocity of u m s^{-1}.
After an interval of $2t$ seconds a second particle is projected vertically upwards from the same point and with the same initial velocity.

They meet at a height of h m.

Show that $h = \dfrac{u^2 - g^2t^2}{2g}$.

2. **(a)** At 12 noon, ship A is north west of ship B as shown.



Ship A is moving north 85° east at a uniform speed of 15 km h^{-1}.

Ship B is moving in a straight line with uniform speed v km h^{-1}.

Ship B intercepts ship A.

(i) Find the least possible value of v.

(ii) If $v = 13$ km h^{-1}, find the two possible directions that ship B can travel in order to intercept ship A.

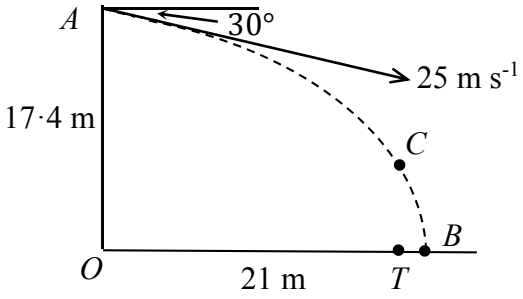
(b) A man can swim at $\dfrac{5}{6}$ m s^{-1} in still water. He swims across a river 125 m wide.

The river flows at a constant speed of $\dfrac{25}{18}$ m s^{-1} parallel to the straight banks.
How long will it take him if he swims so as to reach the opposite bank

(i) as quickly as possible

(ii) as little downstream as possible?

3. **(a)** A ball is thrown from a point A at a target T, which is on horizontal ground. The point A is 17·4 m vertically above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The distance OT is 21 m.
The ball misses the target and hits the ground at the point B, as shown in the diagram.

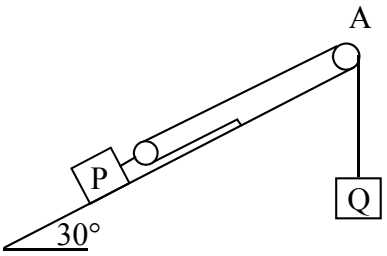


Find
(i) the time taken for the ball to travel from A to B
(ii) the distance TB.

The point C is on the path of the ball vertically above T.
(iii) Find the speed of the ball at C.

(b) A plane is inclined at an angle of 60° to the horizontal.
A particle is projected up the plane with initial speed u m s^{-1} at an angle θ to the inclined plane. The plane of projection is vertical and contains the line of greatest slope.
The maximum range of the particle is $\frac{ku^2}{g}$.
Find the value of k correct to one decimal place.

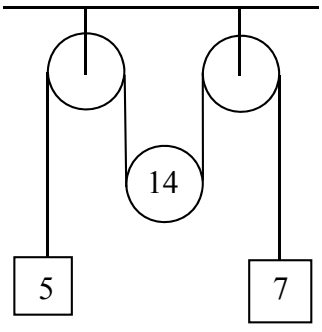
4. **(a)** The block P has a light pulley fixed to it. The two blocks P and Q, of mass 40 kg and 30 kg respectively, are connected by a taut light inextensible string passing over a light smooth fixed pulley, A, as shown in the diagram.



P is on a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between P and the inclined plane is $\frac{1}{4}$.
Q is hanging freely. The system is released from rest.

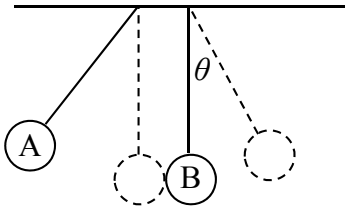
Find
(i) the acceleration of P and the acceleration of Q
(ii) the speed of P when it has moved 30 cm.

(b) A light inextensible string passes over a small smooth fixed pulley, under a small smooth moveable pulley, of mass 14 kg, and then over a second small smooth fixed pulley. A 5 kg mass is attached to one end of the string and a 7 kg mass is attached to the other end.



The system is released from rest.
(i) Find the tension in the string.
(ii) If instead of the system starting from rest, the moveable pulley is given an initial upward velocity of 0·8 m s^{-1}, find the time taken until the moveable pulley reverses direction.

5. **(a)** Two small smooth spheres A, of mass 2 kg, and B, of mass 3 kg, are suspended by light strings from a ceiling as show in the diagram. The distance from the ceiling to the centre of each sphere is 2 m.



Sphere A is drawn back 60° and released from rest. A collides with B and rebounds. B swings through an angle θ.

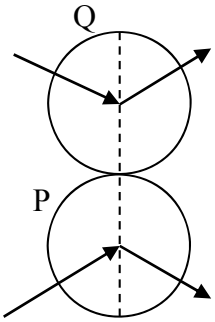
The coefficient of restitution between the spheres is $\frac{3}{4}$.

(i) Show that A strikes B with a speed of $\sqrt{2g}$ m s^{-1}.

(ii) Find the speed of each sphere after the collision.

(iii) Find the value of θ.

(b) Two identical smooth spheres P and Q collide.



The velocity of P **after** impact is $3\vec{i} - \vec{j}$ and the velocity of Q **after** impact is $2\vec{i} + \vec{j}$, where $\vec{j}$ is along the line of the centres of the spheres at impact.

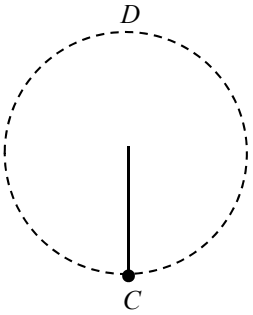
The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find

(i) the velocities, in terms of $\vec{i}$ and $\vec{j}$, of the two spheres before impact

(ii) to the nearest degree, the angle through which the direction of motion of P is deflected by the collision.

6. **(a)** A small particle hanging on the end of a light inextensible string 2 m long is projected horizontally from the point *C*.



(i) Calculate the least speed of projection needed to ensure that the particle reaches the point *D* which is vertically above *C*.

(ii) If the speed of projection is 7 m s^{-1} find the angle that the string makes with the vertical when it goes slack.

(b) A particle P of mass 2 kg is hanging from one end of a light elastic string, of natural length 1 m and elastic constant 98 N m^{-1}. The other end of the string is attached to a fixed point *A*.

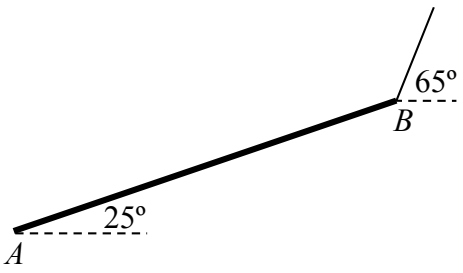
The particle is now pulled down to a point *Q* which is 0·4 m vertically below the equilibrium position and released from rest.

(i) Prove that, while the string is taut, P moves with simple harmonic motion.

(ii) Find the speed of P when the string first becomes slack (no longer taut).

(iii) Find the time taken, from release, for P to reach the highest point in its motion.

7. **(a)** A uniform beam AB of length 30 m and mass 200 kg is held in limiting equilibrium by a light inextensible cable attached to B as shown in the diagram.



End A of the beam rests on a smooth horizontal surface.

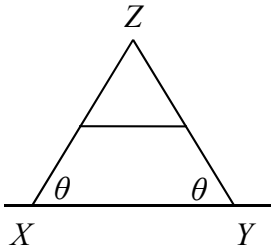
The angle between the beam and the surface is 25º and the cable makes an angle of 65º with the horizontal.

Find

(i) the tension in the cable

(ii) the magnitude of the reaction at A.

(b) Two uniform rods, XZ and YZ, of length 2 m and weight W, are freely jointed at Z, and rest in equilibrium in a vertical plane with the ends X and Y on a smooth horizontal plane. Each rod is inclined at an angle θ to the horizontal.



A string connects the mid points of the rods.

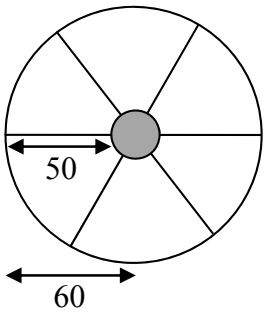
(i) Show that the tension in the string is $\dfrac{W}{\tan\theta}$.

A weight $2W$ is placed 25 cm from X on XZ.

(ii) Show that the tension of the string is increased by 25%.

8. **(a)** Prove that the moment of inertia of a uniform rod, of mass m and length 2ℓ, about an axis through its centre, perpendicular to its plane, is $\frac{1}{3}m\ell^2$.

(b) A wheel, of radius 60 cm, is formed of a thin uniform rim (hoop), six uniform spokes and an axle in the shape of a disc.
The mass of the rim is 4 kg.
Each spoke has a mass of 0·05 kg and length 50 cm.
The mass of the axle is 1 kg and it has a radius of 10 cm.



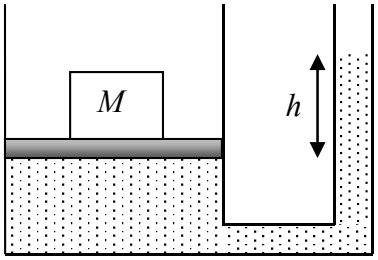
The wheel is rolling on a horizontal road at a speed of 5 m s^{-1}.

(i) Find the moment of inertia of the wheel about an axis through the centre of the axle, perpendicular to its plane.

(ii) Calculate the kinetic energy of the wheel.

(iii) If the wheel comes to an incline of $\sin^{-1}\frac{1}{5}$ how far will it travel up the incline before it stops?

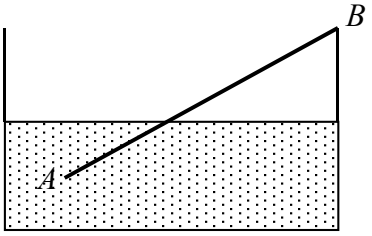
9. **(a)** A load of mass M acts on a light circular piston of diameter d.
The piston sits on a reservoir of oil.
The density of the oil is ρ.
The reservoir is connected to a round tube.
The oil rises in the open tube to a height h.



Find h in terms of M, ρ and d.

(b) A thin uniform rod AB is in equilibrium in an inclined position in a container of water.

End B is supported by the edge of the container as shown in the diagram.



The relative density of the rod is s.

Find in terms of s the fraction of the length of the rod that is immersed in the water.

[Density of water = 1000 kg m^{-3}]

10. **(a)** At time t seconds the acceleration a m s^{-2} of a particle, P, is given by

$$a = 8t + 4.$$

At $t = 0$, P passes through a fixed point with velocity -24 m s^{-1}.

(i) Show that P changes its direction of motion only once in the subsequent motion.

(ii) Find the distance travelled by P between $t = 0$ and $t = 3$.

(b) A particle moves along a straight line in such a way that its acceleration is always directed towards a fixed point O on the line, and is proportional to its displacement from that point.
The displacement of the particle from O at time t is x.
The equation of motion is

$$v\frac{dv}{dx} = -\omega^2 x$$

where v is the velocity of the particle at time t and ω is a constant.

The particle starts from rest at a point P, a distance A from O.

Derive an expression for
(i) v in terms of A, ω and x
(ii) x in terms of A, ω and t.

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