JUNIOR CERTIFICATE 2008

MARKING SCHEME

MATHEMATICS

HIGHER LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
    *   Blunders - mathematical errors/omissions   (-3)
    *   Slips- numerical errors   (-1)
    *   Misreadings (provided task is not oversimplified)   (-1).

    Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
    *   any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
    *   if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
    *   a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5·50 may be written as €5,50.

QUESTION 1

Part (a)	10 marks	Att 3
Part (b)	25 (15,10) marks	Att 8(5,3)
Part (c)	15 (10,5)marks	Att 5(3,2)

Part (a)	10 marks	Att 3

(a) ✍ Given that the speed of sound in air is 330 metres per second, express this speed in km/h.

(a)	10 marks	Att 3

330 metres per second = 330×60 metres per minute (19800)

= 330×60 ×60 metres per hour

= 1188000 metres per hour

=1188000 ÷1000 km/h

= 1188 km/h

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Error in decimal point
B3   Conversion error - apply to both distance and time
B4   Error in using S/D/T formula
B5   Mathematical error

Slips (-1)
S1   Numerical errors to a max of 3 -include incorrect or early rounding off

Attempts (3 marks)
A1   Correct expression for S/D/T and stops
A2   Mentions 1minute = 60 seconds

Worthless (0)
W1   Incorrect answer and no work shown

(b)	**(i)**	✍	Wendy estimates the value of 527 + 889 + 436 by rounding each number to the nearest hundred. Find the estimated value.

(i)

$$527 + 889 + 436 \rightarrow 500 + 900 + 400 = 1800$$

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Error in rounding off - once if consistent
B3   Mathematical error
B4   Adds then rounds off correctly - also incurs B2

Slips (-1)
S1   Fails to find total of correctly estimate values

Attempts (5 marks)
A1   Correctly estimate one number and stops
A2   Adds figures on paper and stops

Worthless (0)
W1   Incorrect answer and no work shown

b(ii)	10 marks	Att 3

(ii)	Three students rent a house for a period of 8 months.

The refuse charges are €16·80 per month.
The electricity bill amounts to €84 every two months.
The television and broadband charges are €324 for the period of the rental.

✎ How much should each of the three students pay monthly for these charges?

b(ii)	10 marks	Att 3

Method I | | | | Method II
Refuse = €16·80 ×8 = €134·4 or €16·80
Electricity =€84 ×4 = €336·00 €42·00
TV and broadband = €324·00 €40·50
Total = €794·40 Total €99·30

Total per student per month
=€794·40 ÷24 = €33·1 €99·30÷3 = €33·10

* Candidates may offer other correct methods

Blunders (-3)

B1 Correct answer but no work shown (✎)
B2 Mathematical error
B3 Each incorrect or missing entry in methods above to a maximum of 2

Slips (-1)
S1 Numerical errors to a max of 3

Attempts (3 marks)
A1 Adds amounts given in question and stops - work shown

Worthless (0)
W1 Incorrect answer and no work shown

Part (c)   **15 (10,5) marks**   **Att (3,2)**

c(i)   **10 marks**   **Att 3**

(i)   Walter borrows €5000 for three years at 4% per annum compound interest.
He repays €1800 at the end of each of the first two years.

    ✍   How much must he repay at the end of the third year to clear his loan?

c(i)   **10marks**   **Att 3**

P Year 1	€5000	
I Year 1	+ €200	
Owes end year 1	€5200	(€5000 × 1· 04)
Repays	- €1800	
P Years 2	€3400	
I Year 2	+ €136	
Owes end year 2	€3536	(€3400×1·04)
Repays	- €1800	
P Year 3	€1736	
I Year 3	€69·44	
Final Payment	€1805·44	(€1736 ×1·04)

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Decimal Error
B3   Mathematical error
B4   Mishandles repayment
B5   Omits a year

Slips (-1)
S1   Numerical to a maximum of 3

Attempts (3 marks)
A1   No compounding of interest
A2   Correctly calculates amount with no work on repayments or leaves all work on repayments to end of year 3

Worthless (0)
W1   Incorrect answer and no work shown

| c(ii) | 5 marks | Att 2 |

| c(ii) | 5 marks | Att 2 |

Let the equal instalment be x

P Year 1 = €5000
P   Year 2 = €5000 × 1·04 - x = €5200 -x
P Year 3 = (€5200 -x) × 1·04 = x Final payment
$(€5200 – x) \times 1·04 = x$

$€5408 – 1·04x = x$

$€5408 = 2·04x$

$x = \dfrac{€5408}{2·04} = €2650·980 = €2650·98$

Blunders (-3)

B1   Correct answer but no work shown (✎)
B2   Decimal Error
B3   Mathematical error
B4   Instalments error

Slips (-1)
S1   Numerical to a maximum of 3

Misreadings (-1)
M1   Carries down answer from c(i) and continues correctly

Worthless (0)
W1   Incorrect answer and no work shown
W2   No compounding of interest
W3 $x = \dfrac{€5000(1·04)^2}{2} = €2704$ or $€5000(1·04)^2$

QUESTION 2

Part (a)	10 (5,5) marks	Att 4(2,2)
Part (b)	20 (10,5,5) marks	Att 7 (3,2,2)
Part (c)	20 (10,10) marks	Att 6 (3,3)
Part (a)	10 (5,5) marks	Att 4(2,2)

(a)	A is the set of prime numbers less than 13.

(a)   A is the set of prime numbers less than 13.
          (i)   List the elements of the set A.
          B $=$ {1, 3, 5, 7, 9, 11}.
          (ii)   Write down the elements of the set B \ A.

(i)	5 marks	Att 2

(i)   A $=$ {2,3,5,7,11}

Slips (-1)
S1   Each incorrect missing or extra number to a maximum of 3 -must have one element correct
Attempt (2 marks)
A1   One correct Prime number
Worthless (0)
W1   No Prime number listed

(ii)	5 marks	Att 2

(ii)   B \ A. $=$ {1,9}

*   Accept Candidates A from part (i)
*   Accept marked/identified correctly on correct Venn diagram
Blunders (-3)
B1   Correct Venn diagram and stops
B2   Incorrect set operation

Misreading (-1)
M1   A\B
Slips (-1)
S1   Each incorrect missing or extra number to a maximum of 3 -must have at least one element correct
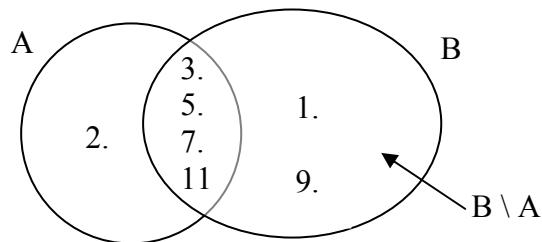*Attempts (2 marks)*
A1   Marks correct region on Venn diagram but does show elements

Part (b)   **20 (10,5,5) marks**   **Att 7(3,2,2)**

(b)	Two brands of blackcurrant squash drinks contain concentrated juice and sugar. In brand A, the ratio of concentrated juice to sugar is 19:1. In brand B, the ratio of concentrated juice to sugar is 9:1.

(i) ✍ What is the volume of concentrated juice in 500 ml of brand A?

(ii) ✍ What is the volume of sugar in 300 ml of brand B?

500 ml of brand A is mixed with 300 ml of brand B.

(iii) ✍ What is the ratio of the concentrated juice to the sugar in the mixture?

b(i)   **10 marks**   **Att 3**

(i)   Volume concentrated juice in A

$$19 + 1 = 20$$

$$\frac{19}{20} \times 500 = 475\,ml$$

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Incorrect numerator
B3   Incorrect denominator
B4   Mathematical error

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (3 marks)
A1   Finds 19 + 1 =20 and stops
A2   Answer ≥ 500 with work

Worthless (0)
W1   Incorrect answer and no work shown

b(ii)   **5 marks**   **Att 2**

(ii)	Volume sugar in B
	$9 + 1 = 10$
	$\dfrac{1}{10} \times 300 = 30 \; ml$

Blunders (-3)

B1   Correct answer but no work shown (✍)

B2   Incorrect numerator

B3   Incorrect denominator

B4   Mathematical error

Slips (-1)

S1   Numerical errors to a max of 3

Attempts (2 marks)

A1   Finds $9 + 1 = 10$ and stops

A2   Answer ≥ 300 with work

Worthless (0)

W1   Incorrect answer and no work shown

b(iii)   **5 marks**   **Att 2**

	Vol. concentrate	Vol. sugar
A	475	25 (500 - 475)
B	270 (300-30)	30
Mixture	745	55
Ratio	745: 55 or 149:11	

*   No back marking for (i) and (ii)

Blunders (-3)

B1   Correct answer but no work shown (✍)

B2   Ratio error

Slips (-1)

S1   Numerical errors to a max of 3

Attempts (2 marks)

A1   Finds one other volume and stops

Worthless (0)

W1   Incorrect answer and no work shown

W2   28:2 or 14:1

W3   475:30 or similar

(c)   In 2006, the average costs of running a car for the year were as follows:
            road tax €485, petrol €1440, servicing €650 and insurance €425.

    (i)   ✍   What was the total cost of running the car in 2006?

    In 2007, the petrol costs went up by 5%, the cost of servicing went up by 15%
    and the cost of insurance went down by 10%.

    (ii)   ✍   Given that the total running costs increased by 4·6% in 2007, calculate
          the percentage (%) increase in the road tax for 2007, giving your
          answer correct to one decimal place.

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c(i)   **10 marks**   Att 3

(i)
    2006
    Road tax   = €485
    Petrol   = €1440
    Servicing =   €650
    Insurance = €425
    Total   = €3000

Blunders (-3)

B1   Correct answer but no work shown (✍)

Slips (-1)
S1   Numerical errors to a max of 3

Misreadings (-1)
M1 One cost omitted
M2 Adjusts cost with work shown

Attempts (3 marks)
A1   Some correct relevant work

Worthless (0)
W1 Incorrect answer and no work shown

2006 costs		**2007 costs**	**Changes 07**
Road tax	= €485	?	?
Petrol	= €1440	€1440 + €72 = €1512	+ €72
Servicing	= €650	€650 + €97·5 = €747·5	+€97·5
Insurance	= €425	€425 - €42·5 = €382·5	- €42·5
Total	= €3000	€3000 +€138 = €3138	+ €138

€3138- €2640= €496   €138- €127

Method I   Method II

€496 - €485= €11 = €138- €127

$$\frac{496}{485}\times100 = 102{\cdot}268 \qquad\qquad \frac{11}{485}\times100 = 2{\cdot}268 = 2{\cdot}3\%$$

$$102{\cdot}268 - 100 = 2{\cdot}268$$

$$\%\ \ Increase = 2{\cdot}268 = 2{\cdot}3\%$$

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Percentage error - once if consistent
B3   Each incorrect or missing figure for 2007 from the above to a max of 2 blunders
B4   Percentage of incorrect figure in final part

Slips (-1)

S1   Numerical errors to a max of 3
S2   Fails to round off or rounds off incorrectly
S3   Fails to subtract in method I above

Attempts (3 marks)

A1   Any one increase or decrease correct

Worthless (0)

W1   Incorrect answer and no work shown

QUESTION 3

Part (a)	15 marks	Att 5
Part (b)	20(10,10) marks	Att 6(3,3)
Part (c)	15 (5,10) marks	Att 5(2,3)
Part (a)	**15 marks**	**Att 5**

(a) When 23 is added to 4 times a certain number, the answer is 11.

✎ Find this number.

(a)	**15 marks**	**Att 5**

(a)

$$4x + 3 = 11$$

$$4x = 11 - 23 \qquad 11 - 23 = -12$$

$$\text{or}$$

$$4x = -12 \qquad -12 \div 4 = -3$$

$$x = -3$$

Blunders (-3)

B1   Correct answer but no work shown (✎)
B2   Each missing or incorrect entry from equation e.g. $x + 4 \times 23 = 11$
B3   Transposing error
B4   Mathematical error

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (5 marks)
A1   Some correct relevant work

(b)

An examination paper consists of 40 questions.

5 marks are given for each correct answer.

3 marks are deducted for each incorrect answer.

Kenny answered all 40 questions, getting x correct and getting y incorrect.

His total score for the examination was 56 marks.

(i)   Write two equations to represent the above information.

(ii)   ✍   Solve these equations to find how many questions Kenny answered correctly.

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b(i)   **10 marks**   **Att 3**

(i)
$$x + y = 40 \qquad equation\ 1$$
$$5x - 3y = 56 \qquad equation\ 2$$

*   One correct equation merits 7 marks

Blunders (-3)

B1   One correct equation only and stops

B2   Incorrect term, once per equation

B3   Mathematical error

Attempts (3 marks)

A1   Mentions $5x$ or $3y$

| b(ii) | 10 marks | Att 3 |

(ii)

$x + y = 40$ $\quad \times 3$

$5x - 3y = 56$

$3x + 3y = 120$

$5x - 3y = 56$

$8x = \dfrac{176}{8} = 22$ $\quad Correct - merits\ full\ marks,\ not\ asked\ for\ number\ incorrect$

*   Verifies correct values in both equations accept for 10 marks

Blunders (-3)

B1   Correct answer, but no work shown (✍)
B2   Error in balancing equations e.g. fails to multiply across completely
B3   Mathematical error
B4   Calculates the value of y correctly and stops
B5   Substitutes for incorrect variable at final step, if applicable
B6   Unrealistic value for x and no conclusion

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (3 marks)
A1   Some correct relevant work
A2   Writes x in terms of y or vice versa and stops
A3   Arithmetic methods leading to the correct answer

Worthless (0)
W1   Incorrect answer and no work shown

Part (c)   **15(5,10) marks**   **Att 5(2,3)**

(c)   **(i)**   ✍   Express in its simplest form:

$$\frac{1}{2x-3} \;-\; \frac{1}{x+3}.$$

(ii)   ✍   Hence, or otherwise, solve the equation:

$$\frac{1}{2x-3} \;-\; \frac{1}{x+3} \;=\; 2,$$

giving your answers correct to two decimal places.

(i)   **5 marks**   **Att 2**

(i)

$$\frac{1}{2x-3} - \frac{1}{x+3}$$

$$= \frac{1(x+3) - 1(2x-3)}{(2x-3)(x+3)}$$

$$\frac{x+3-2x+3}{(2x-3)(x+3)}$$

$$= \frac{6-x}{(2x-3)(x+3)}$$

*   Accept common denominator as $(2x-3)(x+3)$ - penalise incorrect multiplication in part c(ii)

Blunders (-3)

B1   Correct answer, but no work shown (✍)
B2   Incorrect common denominator or mishandles common denominator
B3   Error in distributive law
B4   Mathematical /sign errors
B5
Slips (-1)
S1   Numerical slips to a max of 3

Attempts (2 marks)
A1   Correct common denominator and stops
A2   Leaves out denominator

Worthless (0)
W1   Incorrect answer and no work shown
W2   Subtracts to get $\dfrac{0}{x}$ or similar

(ii)

$$\frac{6-x}{(2x-3)(x+3)} = 2$$

$$6 - x = 2(2x-3)(x+3)$$

$$6 - x = 2(2x^2 + 3x - 9)$$

$$6 - x = 4x^2 + 6x - 18$$

$$4x^2 + 7x - 24 = 0$$

$$a = 4, \; b = 7, \; c = -24$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-7 \pm \sqrt{7^2 - 4(4)(-24)}}{2(4)}$$

$$= \frac{-7 \pm \sqrt{49 + 384}}{8}$$

$$= \frac{-7 \pm \sqrt{433}}{8}$$

$$= \frac{-7 \pm 20{\cdot}8086}{8}$$

$$x = \frac{-7 + 20{\cdot}8086}{8} \quad \text{or} \quad x = \frac{-7 - 20{\cdot}8086}{8}$$

$$x = 1{\cdot}72608 \qquad \text{or} \quad x = -3{\cdot}47608$$

$$x = 1{\cdot}73 \qquad\qquad \text{or} \quad x = -3{\cdot}48$$

*   If part (i) mishandled but correct common denominator in (ii) award attempt (2marks) for (i) - assuming 0 at part (i)

*   Error in quadratic formula and/or mathematical errors in its use apply a maximum of 2 blunders

Blunders (-3)

B1   Correct answer, but no work shown (✍)
B2   Error when simplifying algebra once only -see A2 below
B3   Transposing error
B4   Error in quadratic formula
B5   Error when applying quadratic formula
B6   Only finds one solution

Slips (-1)
S1   Numerical errors to a max of 3
S2   Fails to round off or rounds off incorrectly - once only

Attempts (3 marks)
A1   Correct formula or identifies *a*, *b* or *c* correctly and stops
A2   Has simplified equation to linear and solves correctly for single value of *x* -max att 3
A3   Carries down answer from (i)
A4   Multiplies out common denominator correctly and stops

Worthless (0)
W1   Incorrect answer and no work shown

QUESTION 4

Part (a)	10 marks	Att 3
Part (b)	20 (10,10) marks	Att 6(3,3)
Part (c)	20 (5,5,5,5) marks	Att 8(2,2,2,2)

Part (a)	10 marks	Att 3

(a) ✍ Given that $f(x) = kx + 8$ and that $f(9) = 44$, find the value of k.

(a)	10 marks	Att 3

(a)

$$f(x) = kx + 8$$
$$f(9) = k(9) + 8 = 44$$
$$9k + 8 = 44$$
$$9k = 44 - 8$$
$$9k = 36$$
$$k = 4$$

* Accept verification for full marks

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Incorrect substitution - e.g. interchanges x and y
B3   Transposing error solving equation
B4   Mathematical error

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (3 marks)
A1   Some correct substitution e.g. finds (0,8)

Worthless (0)
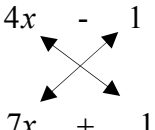W1   Incorrect answer and no work shown

Part (b)   **20(10, 10) marks**   **Att 6(3,3)**

b(i)   **10 marks**   **Att 3**

(b)   **(i)**   Factorise $28x^2 - 3x - 1$.

(i)

Method 1	Method II	Method III
$(4x-1)(7x+1)$	$28x^2 - 7x + 4x - 1$	
	$7x(4x-1)+1(4x-1)$	
	$(4x-1)(7x+1)$	

Method III:

$$\begin{matrix} 4x & - & 1 \\ \times & & \\ 7x & + & 1 \end{matrix}$$
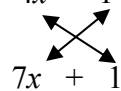
$$(4x-1)(7x+1)$$

* Accept without work - no (✍)

B1   Factors of $28x^2$ Note: $4x$ and $7x$ required
B2   Incorrect factors of -1
B3   Incorrect middle term
B4   Stops at $7x(4x-1)+1(4x-1)$ or

$$\begin{matrix} 4x & - & 1 \\ \times & & \\ 7x & + & 1 \end{matrix}$$

B5   Incorrect use of formula
B6   Incorrect formula

Attempts (3 marks)
A1   Sets up factors
A2   Finds guide number, -28, in Method II and stops
A3   Uses formula and stops at roots

Note   *(4x+1) (7x-1)* one blunder all other attempts apply B1 B2 and/or B3

(ii)   ✎   Solve $\dfrac{-47x-30}{7} = x^2$.

b(ii)

$$\frac{-47x-30}{7} = x^2$$

$$-47x-30 = 7x^2$$

$$7x^2 + 47x + 30 = 0$$

$$(7x+5)(x+6) = 0 \qquad \text{or} \qquad \text{Formula}$$

$$(7x+5) = 0 \quad (x+6) = 0$$

$$7x = -5 \qquad x = -6$$

$$x = -\frac{5}{7}$$

*   Both solutions, by trial and error or similar, verified merits 10 marks
*   Error in quadratic formula and/or mathematical errors in its use apply a maximum of 2 blunders

Blunders (-3)

B1   Correct answer but no work shown (✎)
B2   Transposing error -once only if consistent
B3   Error in factors -maximum loss 3 marks
B4   Error in roots from factors or no roots
B5   Incorrect formula
B6   Incorrect use of formula

Misreadings (-1)

M1   Uses $\dfrac{47x-30}{7} = x^2$ and solves correctly

Attempts (3 marks)

A1   Generates and solves a linear equation attempt mark at most
A2   One correct solution by trial and error
A3   States quadratic formula correctly and stops

Worthless (0)

W1   Incorrect solutions without work

(c)   In a certain week, x people shared equally in a club lotto prize of €2000.

      (i)   Write down an expression in x for the amount that each person received.

    The following week, $x + 1$ people shared equally in the prize of €2000.

      (ii)   Write down an expression in x for the amount that each person received that week.

    In the second week, each winner received €100 less.

      (iii)   Write down an equation in x to represent the above information.

      (iv)   ✍   Solve this equation to find the value of x

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c(i)   **5 marks**   **Att 2**

Week 1

(i) $\dfrac{€2000}{x}$

Blunders (-3)

B1   Amount per person $= \dfrac{x}{€2000}$

Attempts (2 marks)

A1   Gives answer as €2000.x

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c(ii)   **5 marks**   **Att 2**

Week 2

(ii) $\dfrac{€2000}{x+1}$

Blunders (-3)

B1   Inverts e.g. $\dfrac{x+1}{€2000}$ but do not penalise if already blundered in (i)

B2   Uses $x - 1$

Parts c(i) and c(ii) as a Table
Accept for 5 marks + 5 marks

	Week 1	Week 2
Prize fund	2000	2000
No Winners	x	$x+1$
Value Prize	$\dfrac{€2000}{x}$	$\dfrac{€2000}{x+1}$

c(iii)   **5 marks**   **Att 2**

(iii)

$$\frac{2000}{x} - \frac{2000}{x+1} = 100 \quad \text{or equivalent}$$

*   Accept candidate's answer from (i) and (ii) above
*   If no work, or no work of merit, at parts (i) and/or (ii) but states above, award Att 2 and
       Att 2 from parts (i) and/or (ii) here

Blunders (-3)

B1   Error in setting up equation has $\dfrac{2000}{x} + \dfrac{2000}{x+1} = 100$ or $\dfrac{2000}{x+1} - \dfrac{2000}{x} = 100$

Slips (-1)

S1   Writes correct expression with required terms but no equal sign

Attempts (2 marks)

A1   Must construct an equation or expression using at least two of:
       100, answer (i), answer (ii)

c(iv)   **5 marks**   **Att 2**

(iv)

$$\frac{2000}{x} - \frac{2000}{x+1} = 100$$

$$2000(x+1) - 2000x = 100(x)(x+1)$$
$$2000x + 2000 - 2000x = 100x^2 + 100x$$
$$100x^2 + 100x - 2000 = 0$$
$$x^2 + x - 20 = 0$$
$$(x+5)(x-4) = 0$$
$$x = -5 \quad \text{or} \quad x = 4$$

Blunders (-3)

B1   Correct answer, but no work shown ($\mathscr{L}$)
B2   Mathematical error e.g. balancing/forming equation
B3   Error in distributive law
B4   Correct factors and stops
B5   Incorrect factors
B6   Errors using quadratic formula

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (2 marks)
A1   Not a quadratic equation, due to previous errors, merits attempt marks at most

Worthless (0)
W1   Incorrect answer and no work shown

QUESTION 5

Part (a)	10 marks	Att 3
Part (b)	20 (10,10) marks	Att 6(3,3)
Part (c)	20 (15,5) marks	Att 7(5,2)

Part (a)	10 marks	Att 3

(i) (a) ✍   Given that $3d = b(c + a)$, write c in terms of a, b and d.

(a)	10 marks	Att 3

(a)

$$3d = b(c + a) \qquad \text{or} \qquad 3d = b(c + a)$$

$$3d = bc + ab \qquad\qquad \frac{3d}{b} = c + a$$

$$3d - ab = bc \qquad\qquad \frac{3d}{b} - a = c$$

$$\frac{3d - ab}{b} = c$$

or   Equivalent

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Transposing error
B3   Distribution error

Misreadings (-1)
M1   Expresses a in terms of b, c and d correctly with work shown

Attempts (3 marks)
A1   Assigns numbers to a, b and d and finds a correct value for c
A2   Expresses d in terms of b, c and a
A3   Correctly expresses b in terms of d, c and a

Worthless (0)
W1   Incorrect answer and no work shown

(i)   ✍   When $x = \dfrac{1}{2}$, find the value of $\dfrac{3}{x+2} - \dfrac{1}{2x+4}$.

(i)

$$x = \frac{1}{2} = 0{\cdot}5$$

Method   I   **Method   II**   **Method   III**

Method I:
$$\frac{3}{x+2} - \frac{1}{2x+4}$$
$$= \frac{3}{\frac{1}{2}+2} - \frac{1}{2\left(\frac{1}{2}\right)+4}$$
$$= \frac{3}{2\frac{1}{2}} - \frac{1}{1+4}$$
$$= \frac{6}{5} - \frac{1}{5} = \frac{5}{5}$$
$$= 1$$

or

Method II:
$$\frac{3}{0{\cdot}5+2} - \frac{1}{2(0{\cdot}5)+4}$$
$$\frac{3}{2{\cdot}5} - \frac{1}{5}$$
$$1{\cdot}2 - 0{\cdot}2$$
$$= 1$$

Method III:
$$\frac{5x+10}{(x+2)(2x+4)}$$
$$\frac{5}{(2x+4)} \text{ or Equivalant}$$
$$\frac{5}{\left(2\left(\frac{1}{2}\right)+4\right)}$$
$$\frac{5}{5}$$
$$= 1$$

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Fraction error - Method I
B3   Decimal error - Method II
B4   Algebraic error - Method III
B5   Substitution error
B6   Mathematical error
B7   Unfinished

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (3 marks)
A1   Finds Common Denominator and stops
A2   Equation $\dfrac{3}{x+2} - \dfrac{1}{2x+4} = \dfrac{1}{2}$ plus some correct step
A3   Drops common denominator in algebraic method (may apply in other methods)

Worthless (0)
W1   Incorrect answer and no work shown

b(ii)	**10 marks**	**Att 3**

(ii) ✎ Divide $6x^3 - 13x^2 + 27x - 14$ by $3x - 2$.

b(ii)	**10 marks**	**Att 3**

(ii)

$$
\begin{array}{r}
2x^2 - 3x + 7 \\
3x - 2 \overline{)\, 6x^3 - 13x^2 + 27x - 14} \\
\underline{6x^3 - 4x^2} \\
-9x^2 + 27x \\
\underline{-9x^2 + 6x} \\
21x - 14 \\
\underline{21x - 14}
\end{array}
$$

*   Candidates may offer other correct methods

Blunders (-3)

B1   Correct answer but no work shown (✎)
B2   Mathematical error once if consistent
B3   Excess terms in quotient - (with work)

Slips(-1)
S1   Numerical errors to a max of 3

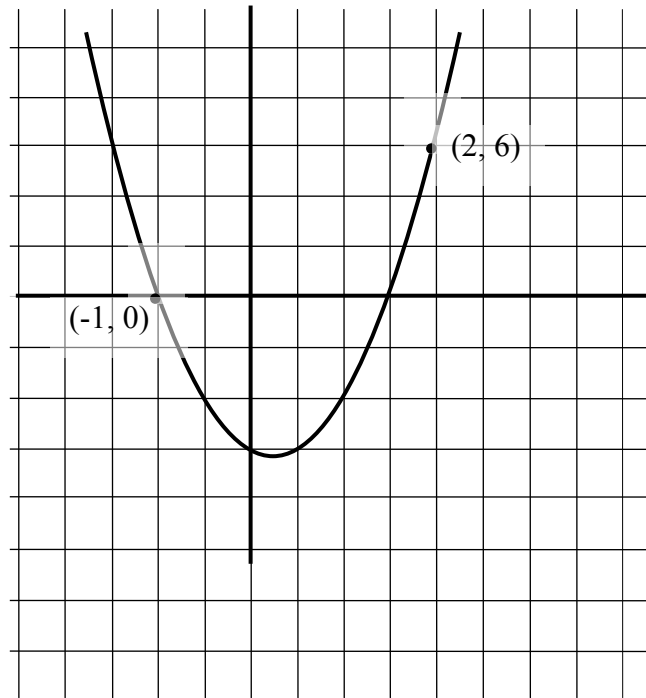*Attempts (3 marks)*
A1   Sets up division
A2   Any correct term of answer with work
A3   Multiplies instead of dividing, with at least one correct term

Worthless (0)
W1   Incorrect answer and no work shown

(c)   Let f be the function $f : x \rightarrow 4x^2 + bx + c$, $x \in \mathbf{R}$ and $b, c \in \mathbf{Z}$.
The points $(2, 6)$ and $(-1, 0)$ lie on the graph of f, as shown in the diagram.

(i)   ✐   Find the value of b and the value of c.

(ii)   ✐   Solve $f(x) = -6$.

Forming Equations

$f(x) = 4x^2 + bx + c$

$f(2) = 6$ $f(-1) = 0$

$4(2)^2 + b(2) + c = 6$ $4(-1)^2 + b(-1) + c = 0$

$16 + 2b + c = 6$ $4 - b + c = 0$

$2b + c + 10 = 0$ $-b + c = -4$

....

Solving

$2b + c = -10 \quad \times -1$

$-b + c = -4$

$-2b - c = 10$

$\underline{-b + c = -4}$

$-3b = 6$

$b = -2$

$b = -2$

$2b + c = -10$

$2(-2) + c = -10$

$-4 + c = -10$

$c = -6$

* Verifies correct values in both equations accept

Blunders (-3)

B1   Correct answer but no work shown (✍)
B2   Only one correctly simplified equation and stops - will also incur B6
B3   Incorrect or missing term, once per equation
B4   Mathematical error
B5   Incorrect or incomplete substitution apply to both equations
B6   Fails to solve correctly for *b* and *c*

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (5 marks)
A1   Some correct relevant work
A2   Arbitrary value for one unknown used to find second unknown (unless other work of merit)

Worthless (0)
W1   Incorrect answer and no work shown

c(ii)	**5 marks**	**Att 2**

(ii)	✍ Solve $f(x) = -6$	

c(ii)

$$4x^2 - 2x - 6 = -6$$

$$4x^2 - 2x = 0$$

$$2x(2x - 1) = 0$$

$$2x - 1 = 0 \quad \text{or} \quad 2x = 0$$

$$x = \frac{1}{2} \qquad x = 0$$

* Accept candidates equation from part (i)
* Verifies correct values in equation accept

Blunders (-3)

B1 Correct answer, but no work shown (✍)
B2 Error in forming equation
B3 Mathematical error
B4 Incorrect factors or no factors
B5 Incorrect roots from factors
B6 Only finds one root
B7 Error in use of quadratic formula

Slips (-1)
S1 Numerical errors to a max of 3
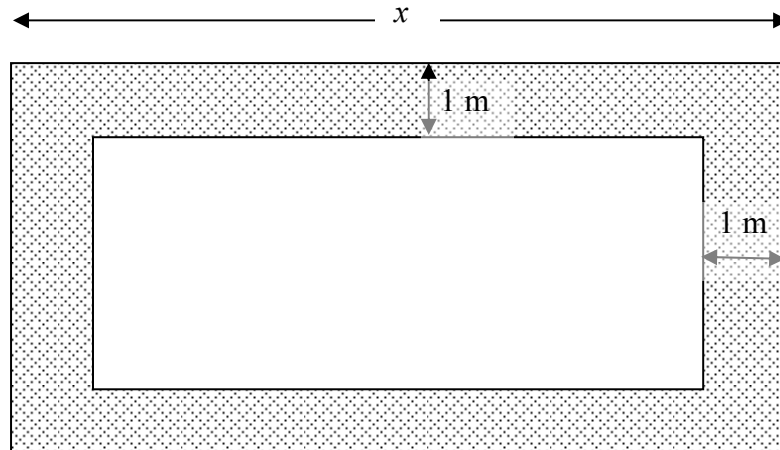
Attempts (2 marks)
A1 Sets up equation
A2 Verifies one solution

QUESTION 6

Part (a)	5 marks	Att 2
Part (b)	10 (5,5) marks	Att 4(2,2)
Part (c)	35 (30,5) marks	Att 12(10,2)

Part (a)	5 marks	Att 2

(a) The diagram shows a rectangular garden of perimeter 24 m.
The length of the garden is *x* m.
Write down an expression in *x* for the width of the garden.

(a)	5 marks	Att 2

(a) $2x + 2w = p$
$2x + 2w = 24$
$2w = 24 - 2x$
$w = 12 - x$

*   Accept $w = \dfrac{24 - 2x}{2}$

*   Accept without work - no (✎)
*   Accept marked on diagram

Blunders (-3)
B1   Incorrect formula for P
B2   Stops at $x+w=12$ or $2x + 2w =24$
B3   Mathematical error

Slips (-1)
S1   Numerical errors to a max of 3

Attempts (2 marks)
A1   Correct formula for P and stops

(b)   Paving of width 1 m is placed around the garden as shown.

         (i)   Write expressions in x for the length and width of the inner section.

         (ii)   ✍   Show that the area, in m^2, of the inner section is $-x^2 + 12x - 20$.

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b(i)   **5 marks**   **Att 2**

Inner section length $= x\text{-}2$

Inner section width $= 12\text{-}x\text{-}2$ or $10\text{-}x$

* Accept without work - no (✍)
* Accept marked on diagram
* Accept candidates' answer from part (a)

Blunders (-3)

B1   Only considers one side of path i.e. *l= x-1* once only if consistent

B2   Only calculates *l* or *w* of inner section

b(ii)   **5 marks**   **Att 2**

Area $= xw$ *(lb)*   or   Factorises

     $= (x\text{-}2)(10\text{-}x)$ $-x^2 + 12x - 20 = (x-2)(-x+10) = lw$

     $= -x^2 + 12x - 20$

*   Accept candidates' work from previous part with correct conclusion

Blunders (-3)

B1   Incorrect area formula

B2   Error in factors

B3   Fails to multiply *(x-2)(10-x)*

Slips (-1)

S1   Incorrect or no conclusion

Attempts(2 marks)

A1   Correct formula for area and stops

(c)   The area of the inner section is represented by the function:
$$f : x \rightarrow - x^2 + 12x - 20.$$

(i)   ✍   Draw the graph of f for $2 \leq x \leq 10$, $x \in \mathbf{R}$.

(ii)   Write down the maximum possible area of the inner section.

$$f(x) = - x^2 + 12x - 20$$

$f(2) = -(2)^2 + 12(2) - 20 = -4 + 24 - 20 = 0$

$f(3) = -(3)^2 + 12(3) - 20 = -9 + 36 - 20 = 7$

$f(4) = -(4)^2 + 12(4) - 20 = -16 + 48 - 20 = 12$

$f(5) = -(5)^2 + 12(5) - 20 = -25 + 60 - 20 = 15$

$f(6) = -(6)^2 + 12(6) - 20 = -36 + 72 - 20 = 16$

$f(7) = -(7)^2 + 12(7) - 20 = -49 + 84 - 20 = 15$

$f(8) = -(8)^2 + 12(10) - 20 = -64 + 96 - 20 = 12$

$f(9) = -(9)^2 + 12(10) - 20 = -81 + 108 - 20 = 7$

$f(10) = -(10)^2 + 12(10) - 20 = -100 + 120 - 20 = 0$

x	2	3	4	5	6	7	8	9	10
$-x^2$	-4	-9	-16	-25	-36	-49	-64	-81	-100
$+12x$	24	36	48	60	72	84	96	108	120
-20	-20	-20	-20	-20	-20	-20	-20	-20	-20
$f(x)$	0	7	12	15	16	15	12	7	0

Points $(x, f(x))$

    (2,0) (3,7) (4,12) (5,15) (6,16) (7,15) (8,12) (9,7) (10,0)

<u>Values for quadratic graph</u>

Blunders (-3)

B1   Each incorrect $f(x)$ without work.

B2 x row added in, i.e. top row, or adds in extra row.

B3   Treating the domain as $2 < x < 10$, can incur 2 Blunders if both omitted.

B4   Each different blunder which yields an incorrect row (full or part),

B5   Avoids square for some (not all) values. See Attempts below

B6   Mathematical errors in tots, apply once only.

B7 $x^2 + 12x - 20$ with work shown - only award attempt at most in c(ii)

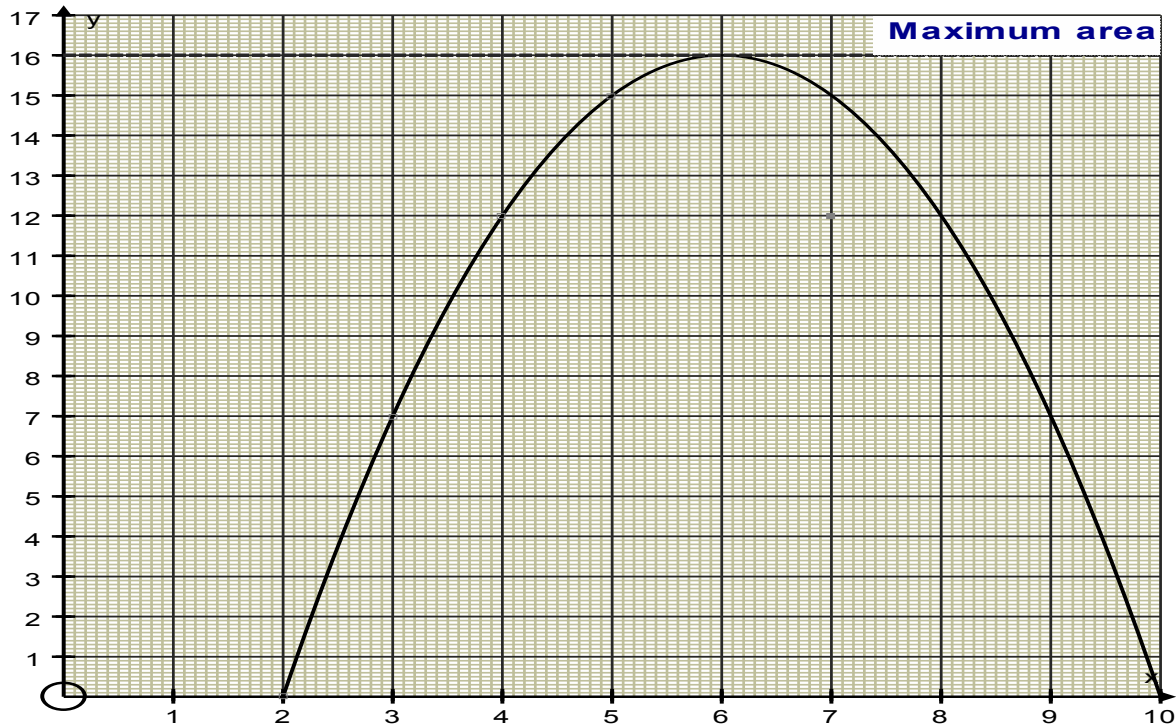B8 $x^2 - 12x + 20$ with work shown - only award attempt at most in c(ii)

Slips (-1)

S1   Numerical errors to a max. of 3.

Attempts (10 marks)

A1   Omits $-x^2$ or treat as, linear expression

A2   Correct or partly correct table / values, but no graph drawn.

Graph of $f : x \rightarrow -x^2 + 12x - 20$

Plotting the quadratic graph

*   Accept candidate's values from his/her table.

Blunders (-3)

B1   Points not joined to form a reasonable graph

B2 (x, y) plotted as (y, x), but apply once only, or reverses axes.

B3   Scale not reasonably uniform can apply to each axis

B4   Each different blunder in plotting points from candidate's table / values.

B5   Each point omitted if graph does not go reasonably close to where point should be

B6   Points joined with straight lines.

Attempts (10 marks)

A1   Scaled axes drawn.

(ii)	**5 marks**	**Att 2**
(ii)	Write down the maximum possible area of the inner section.	

(ii)   Maximum area = 16

*   Accept without work no (✍)

*Accept answer consistent with candidates curve tolerance $\pm 0\cdot3$ see B7 and B8 page 30

Blunders (-3)

B1   Gives answer as 6, x value

B2   16 x 6

Slips (-1)

S1   Fails to identify maximum from coordinates

Attempts (2 marks)

A1   Marks on diagram only

Worthless (0)

W1   96 without work

W2   (16,6)