Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2023

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate Examination 2023

Mathematics

Higher Level

Paper 1

Marking scheme

300 marks

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10 mark scales	0, 10	0, 5, 10	0, 4, 7, 10	0, 3, 5, 8, 10
15 mark scales			0, 6, 12, 15	0, 4, 8, 12, 15
20 mark scales				0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Section A

Question 1	**(30 marks)**
(a)	**15D**
(b)	**10D**
(c)	**5D**
Question 2	**(30 marks)**
(a)	**15D**
(b)	**5C**
(c)(i)(ii)	**10C**
Question 3	**(30 marks)**
(a)	**10D**
(b)(i)	**10D**
(b)(ii)	**10C**
Question 4	**(30 marks)**
(a)	**5C**
(b)	**15D**
(c)(i)(ii)(iii)	**10D**
Question 5	**(30 marks)**
(a)	**15C**
(b)	**5D**
(c)(i)(ii)	**10D**
Question 6	**(30 marks)**
(a)(i)	**10C**
(a)(ii)	**15D**
(b)	**5D**

Section B

Question 7	**(50 marks)**
(a)	**5B**
(b)	**10C**
(c)	**10C**
(d)	**10D**
(e)	**5C**
(f)	**5B**
(g)	**5D**
Question 8	**(50 marks)**
(a)	**5B**
(b)(i)(ii)	**10D**
(c)	**5C**
(d)(i)(ii)	**15D**
(e)	**10C**
(f)	**5B**
Question 9	**(50 marks)**
(a)(i)(ii)(iii)	**15D**
(b)(i)(ii)(iii)	**20D**
(c)(i)	**10D**
(c)(ii)	**5D**
Question 10	**(50 marks)**
(a)	**5B**
(b)	**10C**
(c)	**5C**
(d)	**10D**
e(i)	**10C**
e(ii)	**10D**

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error / Misreading	
〰	Horizontal wavy	Error	
P	P		The work presented in the body of the script merits *Partial Credit*
L	L		The work presented in the body of the script merits *Low Partial Credit*
M	M		The work presented in the body of the script merits *Mid Partial Credit*
H	H		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit – 1*
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
⦚	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and 〰 and 〰 appear in the body of the work, then L should be placed in the right margin.

In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	**Model Solution –30 Marks**	**Marking Notes**
(a)	**Method 1**: $5+3m=11$ $\quad$ $5+3m=-11$ $3m=6$ $\quad$ $3m=-16$ $m=2$ $\quad$ $m=-\frac{16}{3}$ **OR** **Method 2**: $(5+3m)^2=11^2$ $25+30m+9m^2=121$ $9m^2+30m-96=0$ $3m^2+10m-32=0$ $(3m+16)(m-2)=0$ $m=-\frac{16}{3},\ m=2$	**Scale 15D (0, 4, 8, 12, 15)** **Method 1** *Low Partial Credit:* • 1 linear equation. • One correct value of m found without work. • Attempts at trial and improvement. *Mid Partial Credit:* • One value of m found with work. *High Partial Credit* • One value of m correctly found and work of merit in finding the second value. **Method 2** Note: If quadratic does not have an m term award Mid Partial Credit at most *Low Partial Credit:* • Indication of squaring *Mid Partial Credit:* • relevant quadratic in m expanded (Line 2 of the solution) • Quadratic is missing the m term, otherwise correct *High Partial Credit* • quadratic factorised • quadratic formula fully substituted

Q1	Model Solution –30 Marks	Marking Notes
(b)	$j + k = hk$ $k - hk = -j$ $k(1 - h) = -j$ $k = -\frac{j}{1-h}$ or $k = \frac{j}{h-1}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit in eliminating fractions *Mid Partial Credit* • Terms with k transposed to one side of the equation *High Partial Credit* • $k(1 - h) = -j$ or equivalent

Q1	Model Solution –30 Marks	Marking Notes

(c)

Method 1

$$\begin{array}{r} x + p \\ x^2 - px + 1 \overline{\smash{)}\, x^3 + 0x^2 - 2x - 3r} \\ \underline{x^3 - px^2 + x } \\ px^2 - 3x - 3r \\ \underline{px^2 - p^2x + p} \\ (p^2 - 3)x - 3r - p \end{array}$$

$$p^2 - 3 = 0$$
$$p^2 = 3$$
$$p = -\sqrt{3}\ [p < 0]$$

$$-3r - p = 0$$
$$-3r - \left(-\sqrt{3}\right) = 0$$
$$\sqrt{3} = 3r$$
$$r = \frac{\sqrt{3}}{3}$$

OR

Method 2

$$(x^2 - px + 1)(x + k) = x^3 - 2x - 3r$$

$$x^3 - px^2 + x + kx^2 - pkx + k = x^3 - 2x - 3r$$

$$x^3 + (k - p)x^2 + (1 - pk)x + k = x^3 - 2x - 3r$$

$$k - p = 0$$
$$k = p$$

$$1 - pk = -2$$
$$1 - p^2 = -2$$
$$p^2 = 3$$
$$p = -\sqrt{3}\ \ [p < 0]$$

$$k = -3r$$
$$r = -\frac{k}{3}$$
$$= -\frac{-\sqrt{3}}{3}$$
$$= \frac{\sqrt{3}}{3}$$

OR

Method 3

	x	$-3r$
x^2	x^3	$-3rx^2$
$-px$	$-px^2$	$+3rpx$
$+1$	x	$-3r$

$$-p - 3r = 0$$
$$p = -3r$$
$$r = -\frac{p}{3}$$

$$1 + 3rp = -2$$
$$rp = -1$$
$$-\frac{p}{3}(p) = -1$$
$$p^2 = 3$$
$$p = -\sqrt{3}\ \ [p < 0]$$
$$r = \frac{\sqrt{3}}{3}$$

Scale 5D (0, 2, 3, 4, 5)

Note: Full credit -1 if $\boldsymbol{p} = \sqrt{\mathbf{3}}$ but otherwise correct

Method 1

4 steps:

1. Sets up long division

2. First cycle in long division correct

3. Value of $\boldsymbol{p}$ found

4. Value of $\boldsymbol{r}$ found

Low Partial Credit:

Work of merit, for example, some correct division, or sets up long division.

Mid Partial Credit:

- 2 steps correct

High Partial Credit

- 3 steps correct

Method 2

4 steps:

1. Equation set up

2. Expansion of the product (Allow with 3 or more terms correct)

3. Value of p found

4. Value of r found

Low Partial Credit:

- Work of merit, for example, mentions linear factor

Mid Partial Credit:

- 2 steps correct

High Partial Credit

- 3 steps correct.

Method 3

1. Grid set up

2. Grid completed (Allow with 3 or more terms correct)

3. Value of p found

4. Value of r found

Low Partial Credit:

- Work of merit, for example, mentions linear factor.

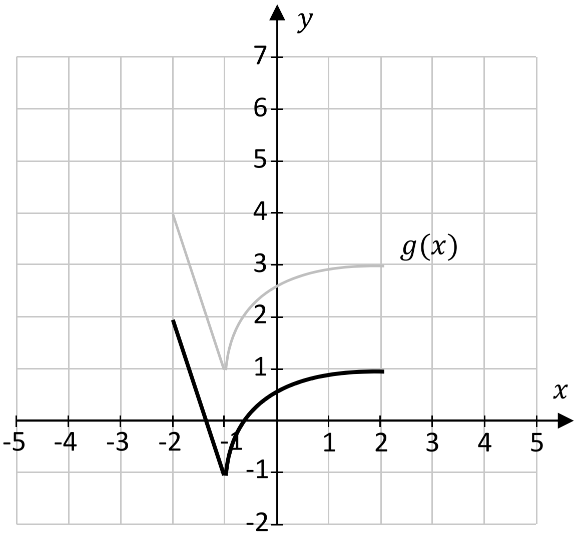
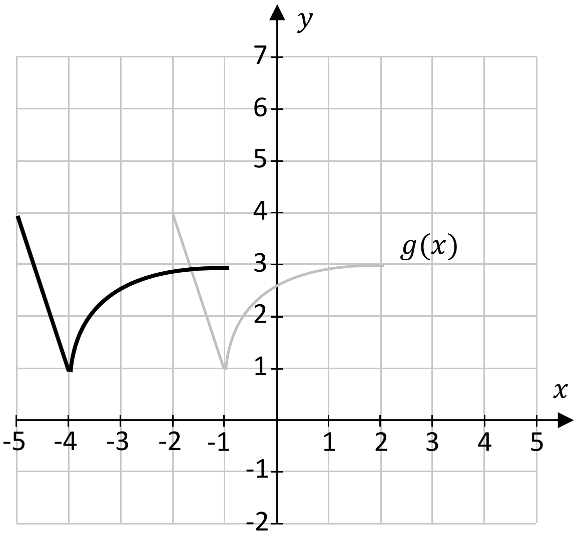
Mid Partial Credit:

- 2 steps correct.

High Partial Credit

- 3 steps correct.

Q2	**Model Solution – 30 Marks**	**Marking Notes**
(a)	$f'(x) = 2x + b$ $f'(3) = 2(3) + b = 0$ $b = -6$ $f(3) = (3)^2 - 6(3) + c = -1$ $9 - 18 + c = -1$ $c = 8$ **OR** $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$ $-\frac{b}{2} = 3 \quad \text{so} \quad b = -6$ $-\frac{b^2}{4} + c = -1 \quad \text{so} \quad c = 8$ **OR** $f(x) = (x-3)^2 - 1$ $\quad = x^2 - 6x + 8$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • Work of merit, for example, $f(3)$ or some correct differentiation • Work of merit at completing the square • $(x-h)^2 + k$ *Mid Partial Credit:* • b correct • Uses $f(3)$ to find a correct equation in b and c • $\left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$ • Work of merit in finding both b and c • $(x-3)^2 + k$, where $k \neq -1$ • $(x-h)^2 - 1$, where $h \neq 3$ *High Partial Credit* • Finds b and work of merit in finding c • $(x-3)^2 - 1$
(b)	$\lim_{n\to\infty}\left(\frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n\right)$ $= \lim_{n\to\infty}\left(\frac{n}{n+1}\right) + \lim_{n\to\infty}\left(\frac{n+1000}{n}\right) + \lim_{n\to\infty}\left(\left(\frac{1}{3}\right)^n\right)$ $\left[= \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right) + \lim_{n\to\infty}\left(\frac{1+\frac{1000}{n}}{1}\right) + \lim_{n\to\infty}\left(\left(\frac{1}{3}\right)^n\right)\right]$ $= \frac{1}{1+0} + \frac{1+0}{1} + 0$ $= 2$	**Scale 5C (0, 2, 3, 5)** Note: Full credit for correct answer without work. *Low Partial Credit:* • Work of merit, for example, indicates sum of limits, divides by highest power of n in one of first two terms • Substitutes ∞ for n • Finds two or more terms of the sequence, $T_n = \frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n$ *High Partial Credit:* • One limit correctly evaluated and work of merit in any one of the other two limits

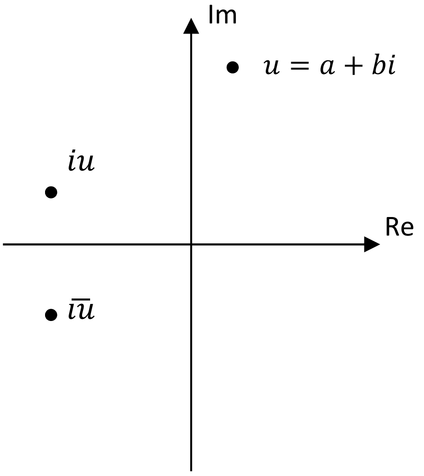
Q2	Model Solution – 30 Marks	Marking Notes
(c)	 	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit in one part, for example, one point correctly transformed • In part (i) any vertical translation of $g(x)$ • In part (ii) any horizontal translation of $g(x)$ • In part (i) finds $g(x+2)$ or $g(x-2)$ • In part (ii) finds $g(x)-3$ or $g(x)+3$ *High Partial Credit:* • One part correct • Work of merit in both parts

Q3	Model Solution – 30 Marks	Marking Notes
(a)	Assume that $\sqrt{2}$ is rational. $\sqrt{2} = \frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$ and $\text{HCF}(a\,, b) = 1$ $2 = \dfrac{a^2}{b^2}$ $2b^2 = a^2$ $\Rightarrow a^2$ is even If a^2 is even, then a is even. $\therefore\ a = 2k$, where $k \in \mathbb{Z}$ $2b^2 = (2k)^2$ $2b^2 = 4k^2$ $b^2 = 2k^2$ $\therefore\ b^2$ is even If b^2 is even, then b is even. If both a and b are even, then they have 2 as a common factor. This contradicts the assumption that $\text{HCF}(a, b) = 1$.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example $\sqrt{2} = \frac{a}{b}$ • Work of merit in showing that a is even *Mid Partial Credit:* • Shows that a is even *High Partial Credit* • Shows both a and b are even
(b)	**Method 1** $\log_3 t + \dfrac{\log_3 t}{\log_3 9} + \dfrac{\log_3 t}{\log_3 27} + \dfrac{\log_3 t}{\log_3 81} = 10$ $\log_3 t + \dfrac{\log_3 t}{2} + \dfrac{\log_3 t}{3} + \dfrac{\log_3 t}{4} = 10$ $12\log_3 t + 6\log_3 t + 4\log_3 t + 3\log_3 t = 120$ $25\log_3 t = 120$ $\log_3 t = \dfrac{120}{25}$ $t = 3^{\frac{120}{25}} = 3^{\frac{24}{5}}$ **OR** **Method 2** $\frac{1}{\log_{\text{t}} 3} + \frac{1}{\log_{\text{t}} 9} + \frac{1}{\log_{\text{t}} 27} + \frac{1}{\log_{\text{t}} 81} = 10$ $\dfrac{1}{\log_{\text{t}} 3} + \dfrac{1}{2\log_{\text{t}} 3} + \dfrac{1}{3\log_{\text{t}} 3} + \dfrac{1}{4\log_{\text{t}} 3} = 10$ $\frac{25}{12\log_{\text{t}} 3} = 10$ $\log_{\text{t}} 3 = \frac{25}{120}$ $t^{\frac{25}{120}} = 3$ $t = 3^{\frac{120}{25}}$	**Scale 10D (0, 3, 5, 8, 10)** **3 steps:** **1.**Changing all to the same base **2.** Simplifies to an equation in ***t*** with one log **3.** Finds ***t*** *Low Partial Credit:* • Work of merit, for example, changes the base of one log (from the given equation) • Writes either 9, 27 or 81 in the form 3^k *Mid Partial Credit:* • One correct step *High Partial Credit* • 2 correct steps

Q3	Model Solution – 30 Marks	Marking Notes
(c) **(i)** **(ii)**	**(i)** *Any valid explanation, for example*: the power you need to raise 6 to, to get m. **(ii)** $\log_6 m > 1$	**Scale 10C (0, 4, 7, 10)** Note: Accept $\boldsymbol{6^x = m}$ as a valid explanation for (i) *Low Partial Credit:* • Work of merit in (i) or (ii), for example, some reference to indices • $\log_6 m > 0$ or $\log_6 m$ is positive *High Partial Credit* • (i) or (ii) correct

Q4	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $(1+i)^2+(3-2i)(1+i)+p=0$ $1+2i+i^2+3+i-2(i)^2+p=0$ $5+3i+p=0$ $p=-5-3i$ **Method 2** Let the second root = z_2 Sum of roots: $1+i+z_2=-3+2i$ $z_2=-4+i$ Product of roots: $(1+i)(-4+i)=p$ $p=-5-3i$ **Method 3** $z=\frac{-(3-2i)\pm\sqrt{(3-2i)^2-4p}}{2}$ $2z=-(3-2i)\pm\sqrt{(3-2i)^2-4p}$ $2z+3-2i=\pm\sqrt{(3-2i)^2-4p}$ $[2z+3-2i]^2=(3-2i)^2-4p$ $z=1+i$ satisfies this equation $[2(1+i)+3-2i]^2=(3-2i)^2-4p$ $5^2=(3-2i)^2-4p$ $4p=-20-12i$ $p=\frac{-20-12i}{4}$ $=-5-3i$ **Method 4** $z+(4-i)$ $z-1-i/\overline{z^2+(3-2i)z+p}$ $\underline{z^2-(1+i)z}$ $(4-i)z+p$ $\underline{(4-i)z-5-3i}$ $p+5+3i$ $p+5+3i=0$ $p=-5-3i$	**Scale 5C (0, 2, 3, 5)** Note: Any attempt involving the conjugate of $\mathbf{1+i}$, award Low Partial Credit at most. **Method 1** *Low Partial Credit:* • Work of merit, for example, some correct substitution or some correct multiplication *High Partial Credit*: • Fully correct substitution and multiplication **Method 2** *Low Partial Credit:* • $z^2-(\text{sum})z+\text{product}$ • States p is the product of the roots • Sum of the roots = $-3+2i$ *High Partial Credit:* • Finds 2nd root • States sum of the roots = $3-2i$, but finishes correctly **Method 3** *Low Partial Credit:* • Some correct substitution in the quadratic formula *High Partial Credit:* • Formula fully substituted and $1+i$ substituted for z • Formula fully substituted and set equal to $1+i$ **Method 4** *Low Partial Credit:* Sets up long division but divisor must be of the form $z-a+bi$, where $a=1$ and $b=-1$ (Accept $b=1$ here) *High Partial Credit:* • First cycle of long division done correctly.

Q4	**Model Solution – 30 Marks**	**Marking Notes**
(b)	Reference Angle: $\alpha = \tan^{-1}\frac{\sqrt{3}}{1} = 60° \left(\frac{\pi}{3}\text{ rads}\right)$ Argument: $\theta = 180^0 - 60^0 = 120^0 \left(\frac{2\pi}{3}\text{ rads}\right)$ Modulus: $r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2}$ $= \sqrt{4}$ $= 2$ General Polar Form: $2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)$ $w^2 = 2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)$ $w = \left[2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)\right]^{\frac{1}{2}}$ De Moivre: $w = 2^{\frac{1}{2}}\left[\left(\cos\frac{1}{2}\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\frac{1}{2}\left(\frac{2\pi}{3} + 2n\pi\right)\right)\right]$ $= 2^{\frac{1}{2}}\left[\cos\left(\frac{\pi}{3} + n\pi\right) + i\sin\left(\frac{\pi}{3} + n\pi\right)\right]$ $n = 0$: $w = \sqrt{2}\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$ $= \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ $= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$ $n = 1$: $w = \sqrt{2}\left(\cos\left(\frac{\pi}{3} + \pi\right) + i\sin\left(\frac{\pi}{3} + \pi\right)\right)$ $= \sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ $= -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$	**Scale 15D (0, 4, 8, 12, 15)** Note: polar form must be used to achieve any credit. Note: Accept correct polar form without work (i.e., finding $\boldsymbol{r}$ and $\boldsymbol{\theta}$) Note: if $(\boldsymbol{w}^2)^2$ is found, award *Mid Partial Credit* at most. Note: general polar form is not required to find the roots. Note: Accept solution in decimal form. **4 steps:** **1**. Finds θ **2**. Finds r **3.** One root evaluated from De Moivre's expression **4.** 2nd root found *Low Partial Credit:* • Work of merit, for example, plots $-1 + \sqrt{3}i$ • Work of merit towards finding r or θ • $w = \left(-1 + \sqrt{3}i\right)^{\frac{1}{2}}$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Roots found correctly, but one or both in polar form

Q4	**Model Solution – 30 Marks**	**Marking Notes**
(c) **(i)** **(ii)** **(iii)**	**(i)** $iu = i(a+bi)$ $= ai + bi^2$ $= -b + ai$ $\overline{iu} = -b - ai$ **(ii)**  **(iii)** 90° counterclockwise rotation about the origin followed by axial symmetry in Re axis (x axis) Axial symmetry in the Im axis (y axis) followed by a 90° counterclockwise rotation about the origin. Axial symmetry in a line through the origin with slope -1 or similar	**Scale 10D (0, 3, 5, 8, 10)** Note: 4 elements required: iu, $\overline{iu}$, plot, transformation Note: Accept conjugate plot from either candidate's work in (i) or by reflection of their $\overline{iu}$ in the real axis *Low Partial Credit:* - Work of merit in one element, for example, $i(a+bi)$ *Mid Partial Credit* - 1 element correct **and** work of merit in a 2nd element *High Partial Credit* - 2 elements correct **and** work of merit in a 3rd element *Full Credit -1* - Diagram not labelled, otherwise correct

Q5	**Model Solution – 30 Marks**	**Marking Notes**
(a)	$f(x) = (5x^2+7)^{-1}$ $f'(x) = -1(5x^2+7)^{-2}(10x)$ $= -10x(5x^2+7)^{-2}$ $f'(x) = \dfrac{-10x}{(5x^2+7)^2}$ **OR** $u(x) = 1$ so $u'(x) = 0$ $v(x) = 5x^2+7$ so $v'(x) = 10x$ $f'(x) = \dfrac{(5x^2+7)(0)-1(10x)}{(5x^2+7)^2}$ $= \dfrac{-10x}{(5x^2+7)^2}$	**Scale 15C (0, 6, 12, 15)** Accept $-10x(5x^2+7)^{-2}$ for full credit. *Low Partial Credit:* • Some correct differentiation *High Partial Credit:* • Correct substitution into quotient rule • One error in substitution into quotient rule, but finishes correctly *Full Credit –1* • $f'(x) = -1(5x^2+7)^{-2}(10x)$
(b)	$u = \tan\frac{x}{2}$ $\quad v = \ln x$ $\frac{du}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$ $\quad \frac{dv}{dx} = \frac{1}{x}$ $g'(x) = \left(\tan\frac{x}{2}\right)\left(\frac{1}{x}\right) + (\ln x)\left(\frac{1}{2}\sec^2\frac{x}{2}\right)$ At $x = \frac{\pi}{2}$: $g'(x) = \left(\tan\frac{\left(\frac{\pi}{2}\right)}{2}\right)\left(\frac{1}{\left(\frac{\pi}{2}\right)}\right) + (\ln\frac{\pi}{2})\left(\frac{1}{2}\sec^2\frac{\left(\frac{\pi}{2}\right)}{2}\right)$ $= 1\left(\frac{2}{\pi}\right) + \ln\frac{\pi}{2}\left(\frac{1}{2}(2)\right)$ $= \frac{2}{\pi} + \ln\frac{\pi}{2}$	**Scale 5D (0, 2, 3, 4, 5)** No differentiation no credit 4 steps **1.** Finds $\frac{du}{dx}$ **2.** Finds $\frac{dv}{dx}$ **3.** Applies the product rule correctly **4.** Evaluates at $x = \frac{\pi}{2}$ *Low Partial Credit:* • Any correct differentiation *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Answer not in the correct form

Q5	Model Solution – 30 Marks	Marking Notes
(c) **(i)** **(ii)**	**(i)** $g\big(f(3)\big) = g(🌍) = w$ **(ii) Injective**: no element of *C* is used more than once **or** no two elements in B go to the same element, **or any other valid reason.** **Not surjective**: one element of *C* is not used, **or** Range ≠ Codomain, **or** $\#B < \#C$, or **any other valid reason.**	**Scale 10D (0, 3, 5, 8, 10)** Three parts to check: (i) and injective and not surjective. In part (i) accept $g\big(f(3)\big)$, where g and f are the functions from part (b). *Low Partial Credit:* • Shows some understanding of injective **or** surjective functions. • Work of merit in any part, for example, $f(3)$ correct, merit in explanation of injective or surjective (e.g., in injective, $\#B \leq \#C$) *Mid Partial Credit* • One of the 3 parts correct *High Partial Credit* • Two parts correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)(i)	$x + 4 = x^2 - 2$ $x^2 - x - 6 = 0$ $(x - 3)(x + 2) = 0$ $x = 3, \qquad x = -2$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit, for example, equation correctly established *High Partial Credit:* • Factors correct • Quadratic formula fully substituted • One correct answer verified

Q6	Model Solution – 30 Marks	Marking Notes
(a)(ii)	**Method 1** $f(x)-g(x)=x+4-(x^2-2)$ $=-x^2+x+6$ $\text{Area}=\int_{-1}^{2}(-x^2+x+6)\,dx$ $=\left[-\frac{x^3}{3}+\frac{x^2}{2}+6x\right]_{-1}^{2}$ $=\left(-\frac{(2)^3}{3}+\frac{2^2}{2}+6(2)\right)-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+6(-1)\right)$ $=\frac{33}{2}$ [units^2] **OR** **Method 2** $\text{Area}_1=\int_{-1}^{2}(x+4)\,dx$ $=\left[\frac{x^2}{2}+4x\right]_{-1}^{2}$ $=\left[\left(\frac{(2)^2}{2}+(8)\right)-\left(\frac{(-1)^2}{2}+(-4)\right)\right]$ $=\frac{27}{2}$ $\text{Area}_2=\int_{-1}^{2}(x^2-2)\,dx$ $=\left[\frac{x^3}{3}-2x\right]_{-1}^{2}$ $=\left[\left(\frac{(2)^3}{3}-2(2)\right)-\left(\frac{(-1)^3}{3}-2(-1)\right)\right]$ $=\|-3\|$ $\text{Area}=\frac{27}{2}+3=\frac{33}{2}$ [units^2] **OR** **Method 3** x intercept of $g(x)$: $x^2-2=0 \rightarrow x=\sqrt{2}$ $A_1=\left\|\int_{-1}^{\sqrt{2}}(x^2-2)dx\right\|=\frac{4\sqrt{2}+5}{3}$ $A_2=\int_{\sqrt{2}}^{2}(x^2-2)dx=\frac{4\sqrt{2}-4}{3}$ $A_3=\int_{-1}^{2}(x+4)dx=\frac{27}{2}$ Total Area $=\frac{4\sqrt{2}+5}{3}+\frac{27}{2}-\left(\frac{4\sqrt{2}-4}{3}\right)=\frac{33}{2}$ [units^2]	**Scale 15D (0, 4, 8, 12, 15)** Consider as 4 steps: 1. Integrate f 2. Integrate g 3. Combine 4. Evaluate with Limits They may do these in a different order, e.g., combine f and g and then integrate this function (= 3 steps) *Low Partial Credit:* • Integration indicated • Some correct integration • Sets up integration of original function(s) • Some work of merit towards finding an approximate area using the Trapezoidal Rule • Work of merit towards finding x intercept of $g(x)$ • $f(-1)$ or $f(2)$ found *Mid Partial Credit:* • 2 steps correct • 1 relevant area calculated *High Partial Credit:* • 2 relevant areas calculated correctly • Steps 1 and 2 correct and one relevant area calculated • 3 steps correct *Full Credit -1* • Area $=-16.5$

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$\left[b\left(\frac{1}{b}e^{bx}\right)\right]_0^b = e$ $\left(e^{b(b)} - e^{b(0)}\right) = e$ $e^{b^2} - 1 = e$ $e^{b^2} = e + 1$ $b^2 = \ln(e+1)$ $b = \sqrt{\ln(e+1)} = 1 \cdot 15 \ldots.$	**Scale 5D (0, 2, 3, 4, 5)** Note: if there is an error in integration, max of Mid Partial Credit is awarded and only if both limits are substituted correctly *Low Partial Credit:* • Some correct integration, for example, ke^{bx} $(k \neq 1)$ appears without the integral sign • $+C$ *Mid Partial Credit:* • Fully correct integration *High Partial Credit:* • $e^{b^2} - 1 = e$

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$v(0) = \frac{2}{3}(0)^3 - 6(0)^2 + 13(0) + 109$ $v(0) = 109$ km/hr	**Scale 5B (0, 2, 5)** Accept correct answer without work. *Partial Credit:* • Some correct substitution • Identifies $t = 0$ • Substitutes $t = 5$ *Full Credit -1* • No unit or incorrect unit
(b)	$v'(t) = 2t^2 - 12t + 13$ $v'(5) = 2(5)^2 - 12(5) + 13$ $= 3$ [km/hr/min]	**Scale 10C (0, 4, 7, 10)** Accept correct answer without unit *Low Partial Credit:* • Some correct differentiation *High Partial Credit* • Substitution into correct derivative • At most one error in differentiation and finishes correctly

Q7	**Model Solution – 50 Marks**	**Marking Notes**
(c)	Maximum speed when $v'(t) = 0$ $2t^2 - 12t + 13 = 0$ $t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(13)}}{2(2)}$ $t = 4 \cdot 58$ or $t = 1 \cdot 42$ Maximum at $t = 1 \cdot 42$ [as coefficient of $t^3 > 0$ and domain of interest is $[0, 4]$, with local min at $t = 4 \cdot 58$] **OR** $[v''(t) = 4t - 12$ $v''(1.42) < 0$ $\Rightarrow$ maximum at $t = 1.42]$	**Scale 10C (0, 4, 7, 10)** Note: Accept candidate's derivative from part (b) Note: If candidate's derivative is linear, award *Low Partial Credit* at most 3 steps: **1.** $\boldsymbol{v'(t) = 0}$ **2.** Substitutes into formula **3**. Evaluates for $\boldsymbol{t}$ *Low Partial Credit:* • Work of merit in finding $v'(t)$ or brings $v'(t)$ from (b) • States $v'(t) = 0$ or similar • $v''(t)$ appears *High Partial Credit:* • 2 steps correct *Full Credit –1* • $t = 4.58$ written down and not explicitly excluded • Rounded incorrectly or no rounding, otherwise correct

Q7	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{1}{5-0}\left[\int_0^5 \left(\frac{2}{3}t^3 - 6t^2 + 13t + 109\right) dt\right]$ $= \frac{1}{5}\left[\frac{t^4}{6} - 2t^3 + \frac{13t^2}{2} + 109t\right]_0^5$ $= \frac{1}{5}\left[\left(\frac{(5)^4}{6} - 2(5)^3 + \frac{13(5)^2}{2} + 109(5)\right) - (0)\right]$ $= 112 \cdot 333$ …. km/hr $= 112 \cdot 33$ km/hr [2 d.p.]	**Scale 10D (0, 3, 5, 8, 10)** Note: Indication of integration is required to be awarded any credit **4 steps:** Note: If $\frac{1}{5}$ is omitted, treat step 1 as not fully correct, but all other steps can be accepted as correct Note: If speed is treated as $\boldsymbol{v'(t)}$ in (a) a correct solution must include the line, $\frac{1}{5}\left[\int_0^5 \boldsymbol{v'(t)dt}\right]$ **1.** $\frac{1}{5}\left[\int_0^5 v(t)dt\right]$ **2.** Integrates correctly **3.** Subs in limits **4.** Evaluates correctly *Low Partial Credit:* • Work of merit, for example, integration indicated *Mid Partial Credit:* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1* • Rounded incorrectly or no rounding, otherwise correct • Incorrect unit or no unit, otherwise correct

Q7	**Model Solution – 50 Marks**	**Marking Notes**
(e)	Answer: B Justification: $v'(1) > 0$: Function is increasing so the slope is positive. $v''(1) < 0$: Rate of increase is slowing. **OR** The slope is decreasing.	**Scale 5C (0, 2, 3, 5)** Note: Justification needs to explicitly deal with both $\boldsymbol{v}'$ and $\boldsymbol{v}''$, but can be a single combined sentence. Note: Substitution into $\boldsymbol{v}'(\boldsymbol{t})$ or $\boldsymbol{v}''(\boldsymbol{t})$ is not considered work of merit. Note: Accept "the function is concave down", or similar as a justification using $\boldsymbol{v}''(\mathbf{1}) < \mathbf{0}$ 3 elements required: **1.** Answer B **2**. Justification for $\boldsymbol{v}'(\mathbf{1}) > \mathbf{0}$ **3.** Justification for $\boldsymbol{v}''(\mathbf{1}) < \mathbf{0}$ *Low Partial Credit:* • Answer correct • Work of merit in either justification *High Partial Credit:* • 2 elements correct • Answer given as B or D, justified correctly using $v'(1)$ • Answer given as B justified correctly using $v''(1)$ • Answer given as A justified correctly using $v''(1)$ • No answer given, but 2 justifications are correct

(f)	Time $=$ Distance/Speed $= \frac{10}{100} = 6$ [minutes]	**Scale 5B (0, 2, 5)** Note: Accept correct answer without units *Partial Credit:* - Work of merit in finding time *Full Credit –1* - $1/10$ (i.e. incorrect unit)
(g)	<u>120 km/hr for 2 minutes:</u> $\text{Distance} = 120 \times \frac{2}{60} = 4\text{km}$ $10 - 4 = 6$ km remaining to get to B <u>Average Speed:</u> $$\frac{10}{\text{total time}} = 100$$ $$\text{total time} = \frac{1}{10}\text{hrs}$$ $$= 6 \text{ minutes}$$ $\Rightarrow$ 4 minutes remaining to get to B <u>Average speed for last 6 km:</u> $\text{Avg Speed} = 6 \div \left(\frac{1}{15}\right) = 90 \text{ km/hr}$ $\frac{120+v}{2} = 90$, where v is the speed at B $v = 60\ km/hr$ Decelerates from 120 to 60 over 4 minutes. So, deceleration $=$ 15 km/hr per minute **OR** Average speed for last 6 km: $\frac{120+v}{2}$, where v is the speed at B $$\text{Distance} = \left(\frac{120+v}{2}\right) \times \text{time}$$ $$6 = \left(\frac{(120+v)}{2}\right) \times \frac{4}{60}$$ $$v = 60$$ Decelerates from 120 to 60 over 4 minutes. Deceleration $=$ 15 km/hr per minute **OR** Average speed for the last 6km: $$\frac{1}{4}\int_0^4 (120 - at)dt = 90$$ $$\frac{1}{4}\left[120t - \frac{1}{2}at^2\right]_0^4 = 90$$ $$\frac{1}{4}\left[120(4) - \frac{1}{2}a(4)^2\right] = 90$$ $$120 - 2a = 90$$ $$a = 15\text{kmh}^{-1} \text{ per minute}$$	**Scale 5D (0, 2, 3, 4, 5)** Accept -15km/hr/min *Low Partial Credit:* - Some correct substitution in relation to distance for first 2 minutes or time / distance for remaining part - Indicates integration *Mid Partial Credit:* - Identifies 4 km, 6 km or 4 minutes. *High Partial Credit* - Calculates 60 km/hr after the extra 4 minutes or 90km/hr after extra 2 minutes. *Full Credit -1* - Answer in km/hr/hr

Q8	**Model Solution – 50 Marks**	**Marking Notes**
(a)	$F = 3000(1 + 0.024)^5$ $= €3377.70$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula; finds 2.4% as a decimal
(b) **(i),** **(ii)**	**(i)** It is the amount that should be invested today to amount to €1000 in 1 years' time at the particular interest rate. **(ii)** $4000 = P(1 + 0.024)^6$ $\frac{4000}{1.024^6} = P$ $P = €3469 \cdot 45$	**Scale 10D (0, 3, 5, 8, 10)** Note: In (i) Accept $\boldsymbol{P} = \frac{\mathbf{1000}}{(\mathbf{1}+\boldsymbol{i})}$ *Low Partial Credit:* • Work of merit in (i) or (ii), for example, formula in (i), correct substitution into relevant formula in (ii) • 2.4% written as a decimal *Mid Partial Credit:* • (i) or (ii) correct • Work of merit in both parts *High Partial Credit* • One part correct and work of merit in the other part
(c)	$1 \cdot 024 = (1 + i)^4$ $(1.024)^{\frac{1}{4}} = 1 + i$ $(1.024)^{\frac{1}{4}} - 1 = i$ $0 \cdot 005947.. = i$ Rate $= 0 \cdot 59\%$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some correct substitution into relevant formula • 2.4% written as a decimal *High Partial Credit:* • $(1.024)^{\frac{1}{4}} = 1 + i$ • Evaluates correctly $i = (1.024)^4 - 1$ • Uses 3 or 12 instead of 4, but otherwise correct *Full Credit -1* • Answer given as a decimal

Q8	**Model Solution – 50 Marks**	**Marking Notes**
(d) **(i)** **(ii)**	**(i)** $A(1\cdot 0011)^{36}+A(1\cdot 0011)^{35}+\cdots$ $\ldots+A(1\cdot 0011)^{2}+A(1\cdot 0011)$ **OR** $=A[(1\cdot 0011)^{36}+(1\cdot 0011)^{35}+\cdots$ $\ldots+(1\cdot 0011)^{2}+(1\cdot 0011)]$ **(ii)** $A[1{\cdot}0011+(1\cdot 0011)^{2}+\cdots$ $\ldots+(1\cdot 0011)^{35}+(1\cdot 0011)^{36}]$ $a=1\cdot 0011,\ \ r=1\cdot 0011,\quad n=36$ $A\left[\dfrac{1\cdot 0011\,(1-1\cdot 0011^{36})}{1-1\cdot 0011}\right]=12000$ $A=\dfrac{12000}{\dfrac{1\cdot 0011(1-1\cdot 0011^{36})}{1-1\cdot 0011}}$ $A=€326\cdot 60$ [2 D.P.]	**Scale 15D (0, 4, 8, 12, 15)** Consider as requiring 3 steps: **1.** Finds geometric series **2.** Substitutes into geometric formula **3.** Finds A *Low Partial Credit:* - Work of merit in either part, for example, in **(i)**Writes 0.11% as a decimal; in **(ii)**, sets answer in **(i)** equal to 12000 *Mid Partial Credit:* - 1 step correct - Substantial work of merit in both parts *High Partial Credit:* - 2 steps correct *Full Credit –1:* - Correct solution, but excludes second and/or second last term - Investments made at the end of each month, otherwise correct
(e)	$E(x)=11(0\cdot 52)+(x-5)(0\cdot 15)$ $+x(0\cdot 33)=13\cdot 85$ $0{\cdot}15x+0{\cdot}33x=13{\cdot}85-5{\cdot}72+0{\cdot}75$ $0\cdot 48x=8\cdot 88$ $x=€18\cdot 50$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Work of merit, for example, some correct term in $E(x)$ *High Partial Credit* - Fully correct equation
(f)	Cost Price $=$ 82% of Selling Price Profit $=$ 18% of Selling Price Mark-up $=\frac{0{\cdot}18}{0{\cdot}82}\times 100$ $=0\cdot 2195=22\%$ [nearest percent] **OR** Let $x=$ selling price and $y=$ cost price $\dfrac{x-y}{x}=0.18\ \rightarrow y=0.82x$ Mark up: $\frac{x-y}{y}=\frac{x-0.82x}{0.82x}=\frac{9}{41}$ Mark up: $\frac{9}{41}\times 100=21.95$ $=22\%$ [nearest percent]	**Scale 5B (0, 2, 5)** *Partial Credit:* - Work of merit, for example, states CP = 82% of SP - Mentions 82% - Finds 18% of a number

<table>
<tr><th>Q9</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i) (ii)
(iii)</td><td>

(i) $2^0,\ 2^1,\ 2^2,\ 2^3,\ 2^4$

OR

$\mathbf{1},\ \mathbf{2},\ \mathbf{4},\ \mathbf{8},\ \mathbf{16}$

(ii) 8

(iii) 2^{10} and 3^{12} have no common factors
2^{10} will have 11 factors
3^{12} will have 13 factors

So $2^{10} \times 3^{12}$ will have $(11)(13) = 143$ factors

</td><td>

Scale 15D (0, 4, 8, 12, 15)

Accept for full credit correct answers without work

In (i) accept any 5 factors including negative factors

In (ii) accept 16 for full credit

In (iii) accept 286 factors for full credit

Low Partial Credit:

- Work of merit in one part, for example, one correct factor in (i) or lists 2 or more factors in (ii)
- Some valid relevant statement in (ii) or (iii)

Mid Partial Credit

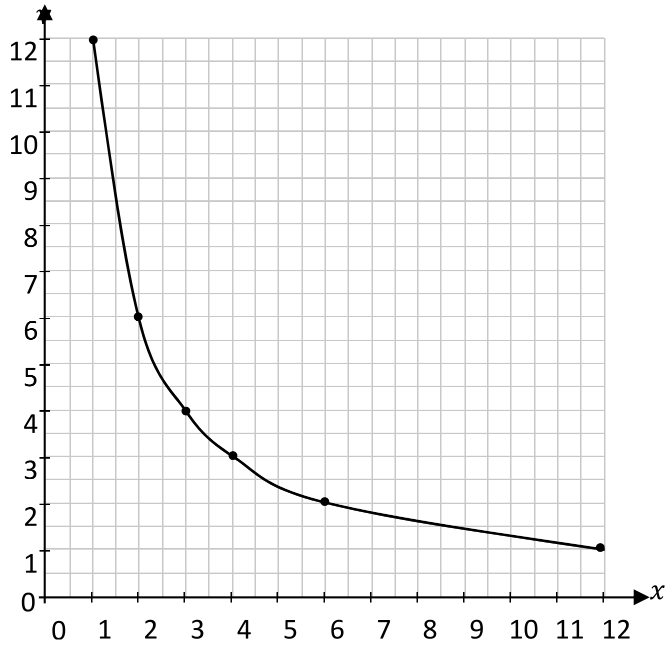
- One part correct

High Partial Credit

- Two parts correct
- Either (i) or (ii) correct **and** work of merit in (iii)

</td></tr>
<tr><td>(b)
(i)
(ii)
(iii)</td><td>

x	1	2	3	**4**	6	**12**
y	**12**	**6**	4	**3**	**2**	1



</td><td>

Scale 20D (0, 5, 10, 15, 20)

Solution consists of 12 parts:

- 6 values in table
- 5 points plotted
- Points joined appropriately (not with line segments)

Low Partial Credit:

- 3 parts correct

Mid Partial Credit

- 7 parts correct

High Partial Credit

- 9 parts correct

Full Credit -1

- 11 parts correct
- Correct graph with no table entries

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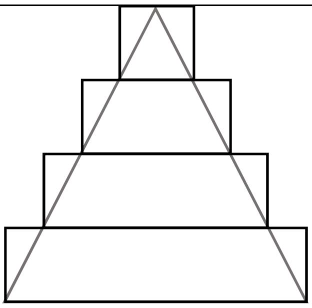
</table>

(c)(i)	**Method 1** Derivative: $y=\frac{12}{x} \quad \frac{dy}{dx}=-\frac{12}{x^2}$ $\frac{dy}{dx}(x=p)=-\frac{12}{p^2}$ Equation at point $\left(p,\frac{12}{p}\right)$: $y-\frac{12}{p}=-\frac{12}{p^2}(x-p)$ $y=-\frac{12}{p^2}x+\frac{24}{p}$ **OR** **Method 2** Derivative: $y=\frac{12}{x} \quad \frac{dy}{dx}=-\frac{12}{x^2}$ $\frac{dy}{dx}(x=p)=-\frac{12}{p^2}$ Equation at point $\left(p,\frac{12}{p}\right)$: Line is of the form $y=-\frac{12}{p^2}x+c$ $\frac{12}{p}=-\frac{12}{p^2}(p)+c$ $\left[\frac{12}{p}=-\frac{12}{p}+c\right]$ $c=\frac{24}{p}$ **OR** **Method 3** From the given equation the slope is $-\frac{12}{p^2}$ $\frac{dy}{dx}=-\frac{12}{x^2}$ $\frac{dy}{dx}(x=p)=-\frac{12}{p^2}$ Substitute $\left(p,\frac{12}{p}\right)$ in the given equation $\frac{12}{p}=-\frac{12}{p^2}(p)+\frac{24}{p}$ $\frac{12}{p}=-\frac{12}{p}+\frac{24}{p}$ $\frac{12}{p}=\frac{12}{p}$	**Scale 10D (0, 3, 5, 8, 10)** **Method 1 & Method 2** 4 steps: **1.** Finds $\frac{dy}{dx}$ **2.** Finds slope at $x=p$ **3.** Subs slope and point $\left(p,\frac{12}{p}\right)$ into equation of line formula **4.** Finds equation in required form *Low Partial Credit:* • Work of merit, for example, some correct differentiation, some correct substitution into equation of line formula *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct **Method 3** 4 steps: **1.** From the given equation writes down the slope of the tangent **2.** Finds $\frac{dy}{dx}$ **3.** Substitutes $x=p$ in $\frac{dy}{dx}$ **4.** Verifies that the point $\left(p,\frac{12}{p}\right)$ is on the given equation *Low Partial Credit:* • Work of merit, for example, some correct differentiation, some correct substitution into the given equation *Mid Partial Credit* • 2 steps correct. *High Partial Credit* • 3 steps correct.

(c)(ii)	y-intercept: $y = -\frac{12}{p^2}(0) + \frac{24}{p}$ $y = \frac{24}{p}$ $\left(0, \frac{24}{p}\right)$ height $= \frac{24}{p}$ x-intercept: $0 = -\frac{12}{p^2}x + \frac{24}{p}$ $\frac{12}{p^2}x = \frac{24}{p}$ $\frac{12}{p}x = 24$ $x = 2p$ $(2p, \ 0)$ base $= 2p$ Area $= \frac{1}{2}(2p)\left(\frac{24}{p}\right)$ $= 24 \text{ units}^2$ **OR** x intercept $(2p, 0)$ $\int_0^{2p}\left(-\frac{12}{p^2}x + \frac{24}{p}\right)dx$ $= \left(-\frac{12x^2}{2p^2} + \frac{24x}{p}\right)_0^{2p}$ $= -24 + 48$ $= 24$ [units^2]	**Scale 5D (0, 2, 3, 4, 5)** **Method 1** **4 steps:** **1.** Finds the x-intercept **2.** Finds the y-intercept **3.** Substitution into area formula **4.** Finds k Note: Accept where candidates substitute a value for *p* into the equation of the tangent, find both intercepts of the subsequent equation, and then find *k* *Low Partial Credit:* • Work of merit, for example, lets $x = 0$ or $y = 0$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct **Method 2** **1.** Finds x intercept **2.** Integrates the function **3.** Substitutes limits **4.** Finds k *Low Partial Credit:* • Work of merit, for example, lets $y = 0$ • Indicates integration *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q10	**Model Solution – 50 Marks**	**Marking Notes**
(a)		**Scale 5B (0, 2, 5)** *Partial Credit:* - Work of merit, for example, each rectangle of height 2 units
(b)	**Method 1** $h = \frac{8}{3}$ Using similar triangles $\frac{w_1}{2} = \frac{8}{3} \div 8$ (w_1 = length of middle rectangle) $w_1 = \frac{2}{3}$ $w_2 = \frac{4}{3}$ (w_2 = length of top rectangle) $\text{Area} = 2\left(\frac{8}{3}\right) + \frac{4}{3}\left(\frac{8}{3}\right) + \frac{2}{3}\left(\frac{8}{3}\right)$ $= \frac{32}{3}\text{units}^2$ **Method 2** 6 small triangles of length $\frac{2}{6} = \frac{1}{3}$ $\text{Area}_{\text{small }\Delta'\text{s}} = 6 \times \frac{1}{2} \times \frac{1}{3} \times \frac{8}{3} = \frac{8}{3}$ $\text{Area}_{\text{big }\Delta} = \frac{1}{2} \times 2 \times 8 = 8$ $\text{Area}_{\text{rectangles}} = \frac{8}{3} + 8 = \frac{32}{3}\ \text{units}^2$ **Method 3** Sum of lengths of horizontal sides of small triangles (i.e., excess of rectangles over large triangle) $= 2$ Height of each small triangle $= \frac{8}{3}$ $\sum$ areas of small Δ's $= \frac{1}{2} \times 2 \times \frac{8}{3} = \frac{8}{3}$ Area of large triangle $= \frac{1}{2} \times 2 \times 8 = 8$ $\therefore$ Area of rectangles $= \frac{8}{3} + 8 = \frac{32}{3}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Work of merit in finding dimensions **or** area of one rectangle or one small triangle, - Finds $h = \frac{8}{3}$ - Finds the area of the large triangle *High Partial Credit:* - w_1 and w_2 found - Areas of 2 rectangles found - Sum of the areas of the small triangles found with work shown

Q10	Model Solution – 50 Marks	Marking Notes
(c)	**Method 1** $T_4=\frac{8}{4}\left[\frac{2}{4}+\frac{4}{4}+\frac{6}{4}+\frac{8}{4}\right]$ Similarly, $T_n=\frac{8}{n}\left[\frac{2}{n}+\frac{4}{n}+\cdots+\frac{2n}{n}\right]$ $=\frac{8}{n^2}[2+4+6+\cdots+2n]$ $2+4+\cdots+2n$ is an A.P. with $a=2$ and $d=2$ and n terms $S_n=\frac{n}{2}[2(2)+(n-1)(2)]$ $=n(n+1)$ $T_n=\frac{8}{n^2}[n(n+1)]$ $=\frac{8}{n}(n+1)$ or equivalent **Method 2** Total area = area of given triangle plus sum of areas of small triangles. $h=\frac{8}{n}$ $2n$ small triangles Base of each small $\Delta=\frac{2}{2n}=\frac{1}{n}$ $\text{Area of small } \Delta's = 2n\times\frac{1}{2}\times\frac{1}{n}\times\frac{8}{n}$ $=\frac{8}{n}$ Area $=\left(\frac{8}{n}\right)+\left(\frac{1}{2}\times 2\times 8\right)$ $=\frac{8}{n}+8$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, work towards establishing pattern by writing $T_{k,}\ k\neq 3$ • $\frac{8}{n}$ mentioned as height of rectangle • Identifies $2n$ (or n) small triangles • Base length of each small triangle found *High Partial Credit:* • $T_n=\frac{8}{n}\left[\frac{2}{n}+\frac{4}{n}+\cdots+\frac{2n}{n}\right]$ or equivalent not in closed form • Finds sum of the areas of the small triangles

Q10	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{8(n-1)}{n} > 0\cdot 95(8)$ $\frac{n-1}{n} > 0\cdot 95$ $n-1 > 0\cdot 95n$ $0\cdot 05n > 1$ $n > 20$ $n = 21$ **OR** $\frac{8n^2(n-1)}{n} > 0.95(8)n^2$ $8n^2 - 7.6n^2 - 8n > 0$ $0.4n^2 - 8n > 0$ $n^2 - 20n > 0$ $n(n-20) > 0$ $n > 20$ $n = 21$	**Scale 10D (0, 3, 5, 8, 10)** Note: where candidates multiply both sides by n^2, they must find $n = 20$ to be awarded *High Partial Credit.* *Low Partial Credit:* - Work of merit in establishing inequality, for example, finds the area of triangle *Mid Partial Credit* - Forms the correct inequality *High Partial Credit* - $n - 1 > 0\cdot 95n$
(e) (i)	$\int_0^h \frac{x^2\,c^2}{h^2}dx = \frac{c^2}{h^2}\int_0^h x^2dx$ $= \frac{c^2}{h^2}\left[\frac{x^3}{3}\right]\begin{matrix}h\\0\end{matrix}$ $= \frac{c^2}{h^2}\left[\frac{h^3}{3} - 0\right]$ $= \frac{c^2h}{3}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Integral set up correctly *High Partial Credit* - Integration is correct - Mishandles $\frac{c^2}{h^2}$, but otherwise correct.

Q10	**Model Solution – 50 Marks**	**Marking Notes**
(e)(ii)	$\frac{dx}{dt} = 3 \qquad \frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ $S(x) = \frac{x^2\,c^2}{h^2}$ $\frac{dS}{dx} = \frac{c^2}{h^2}(2x)$ $\frac{dS}{dt} = \frac{c^2}{h^2}(2x)(3)$ $= \frac{6c^2x}{h^2}$ When $x = \frac{h}{2}$ $\frac{dS}{dt} = \frac{6c^2\left(\frac{h}{2}\right)}{h^2} = \frac{3c^2}{h}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • States a relevant derivative, for example, $\frac{dS}{dx}$ or $\frac{dS}{dt}$ • $x = \frac{h}{2}$ • Some correct differentiation *Mid Partial Credit* • Any **two** of the following: ○ $\frac{dx}{dt} = 3$ ○ $x = \frac{h}{2}$ ○ $\frac{dS}{dx} = \frac{c^2}{h^2}(2x)$ ○ $\frac{ds}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ or similar *High Partial Credit* • $\frac{ds}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$, and any two others from the *MPC* list above

Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate Examination 2023

Mathematics

Higher Level

Paper 2

Marking scheme

300 marks

Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales		0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10 mark scales		0, 5, 10	0, 4, 7, 10	0, 3, 5, 8, 10
15 mark scales				0, 4, 8, 12, 15
20 mark scales				0, 6, 12, 17, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Section A

Question 1	**(30 marks)**
(a)	**10C**
(b)	**10D**
(c)	**10D**

Question 2	**(30 marks)**
(a)	**10C**
(b)	**10C**
(c)	**10D**

Question 3	**(30 marks)**
(a)	**15D**
(b)(i)	**5B**
(b)(ii)	**10D**

Question 4	**(30 marks)**
(a)(i)	**10C**
(a)(ii)	**15D**
(b)	**5D**

Question 5	**(30 marks)**
(a)(i)(ii)(iii)	**20D**
(b)	**10C**

Question 6	**(30 marks)**
(a)	**5B**
(b)(i)	**15D**
(b)(ii)	**10D**

Section B

Question 7	**(50 marks)**
(a)	**10C**
(b)	**10D**
(c)(d)	**10D**
(e)(i)	**5B**
(e)(ii)	**10C**
(f)	**5D**

Question 8	**(50 marks)**
(a)	**10D**
(b)(i)	**10C**
(b)(ii)	**5C**
(c)	**10D**
(d)(i)	**10D**
(d)(ii)	**5C**

Question 9	**(50 marks)**
(a)(i)	**5B**
(a)(ii)	**10C**
(b)(i)	**10C**
(b)(ii)	**10D**
(c)(i)	**5B**
(c)(ii)	**10C**

Question 10	**(50 marks)**
(a)(i)	**10C**
(a)(ii)	**15D**
(a)(iii)	**5D**
(b)(i)	**5B**
(b)(ii)	**5C**
(b)(iii)	**10C**

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error / Misreading	
〰	Horizontal wavy	Error	
P	P		The work presented in the body of the script merits *Partial Credit*
L	L		The work presented in the body of the script merits *Low Partial Credit*
M	M		The work presented in the body of the script merits *Mid Partial Credit*
H	H		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit – 1*
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
⌇	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and 〰 and 〰 appear in the body of the work, then L should be placed in the right margin.

In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	**Model Solution –30 Marks**	**Marking Notes**
(a)	$P(€6, €9, €6) = \left[\frac{5}{12} \times \frac{3}{12} \times \frac{5}{12}\right]$ $= \frac{75}{1728} = \frac{25}{576} = 0 \cdot 04340\ldots.$ $= 0 \cdot 0434$ [4 d.p.]	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Any correct relevant probability stated *High Partial Credit:* • $P(€6) = \frac{5}{12}$ and $P(€9) = \frac{3}{12}$ and some multiplication indicated • $\frac{5}{12} \times \frac{3}{12} \times \frac{5}{12}$ • $\frac{5}{12} \times \frac{3}{11} \times \frac{5}{10}$ and continues *Full Credit -1* • Incorrect rounding or no rounding
(b)	Success = getting a 9 P(success)= $\frac{1}{4}$ Failure = not getting a 9 P(failure)= $\frac{3}{4}$ 2 successes in first 7 spins and then success $= \binom{7}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^5 \times \frac{1}{4}$ $= 0 \cdot 07786\ldots$ $= 0 \cdot 0779$ [4 d.p.]	**Scale 10D (0, 3, 5, 8, 10)** Consider the solution as being the product of four terms: $\binom{7}{2}$, $\left(\frac{1}{4}\right)^2$, $\left(\frac{3}{4}\right)^5$ and $\frac{1}{4}$ *Low Partial Credit:* • P(success)= $\frac{1}{4}$ • P(failure)= $\frac{3}{4}$ • $\binom{7}{2}$ • $\frac{1}{4}$ for the last day • $\binom{8}{3}$ *Mid Partial Credit:* • Product of two correct terms evaluated • Product of three correct terms *High Partial Credit:* • $\binom{7}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^5 \times \frac{1}{4}$ *Full Credit -1* • Incorrect rounding or no rounding

<table>
<tr><th>Q1</th><th>Model Solution –30 Marks</th><th>Marking Notes</th></tr>
<tr>
<td>(c)</td>
<td>
Will get less than €16 unless $9, 9$, so:
$1 - P(9,9) = 1 - \frac{1}{4} \times \frac{1}{4} = \frac{135}{144}$ or $\frac{15}{16}$
$= 0 \cdot 9375$
OR
<table>
<tr><td>$P(0,0)$</td><td>$\frac{4}{12} \times \frac{4}{12}$</td><td rowspan="3">$P(0 \text{ first})$</td><td rowspan="3">$\frac{4}{12}$</td></tr>
<tr><td>$P(0,6)$</td><td>$\frac{4}{12} \times \frac{5}{12}$</td></tr>
<tr><td>$P(0,9)$</td><td>$\frac{4}{12} \times \frac{3}{12}$</td></tr>
<tr><td>$P(6,0)$</td><td>$\frac{5}{12} \times \frac{4}{12}$</td><td rowspan="3">$P(6 \text{ first})$</td><td rowspan="3">$\frac{5}{12}$</td></tr>
<tr><td>$P(6,6)$</td><td>$\frac{5}{12} \times \frac{5}{12}$</td></tr>
<tr><td>$P(6,9)$</td><td>$\frac{5}{12} \times \frac{3}{12}$</td></tr>
<tr><td>$P(9,0)$</td><td>$\frac{3}{12} \times \frac{4}{12}$</td><td>$P(9,0)$</td><td>$\frac{1}{12}$</td></tr>
<tr><td>$P(9,6)$</td><td>$\frac{3}{12} \times \frac{5}{12}$</td><td>$P(9,6)$</td><td>$\frac{5}{48}$</td></tr>
<tr><td>TOTAL</td><td>$= \frac{15}{16}$</td><td></td><td>$= \frac{15}{16}$</td></tr>
</table>
$= 0 \cdot 9375$
</td>
<td>
Scale 10D (0, 3, 5, 8, 10)
Note:
Low Partial Credit:

Relevant probability
Relevant work on establishing the condition, for example, indicates $P(9,9)$, or lists three that are in line with the condition (e.g.,$(0,0)$, $(0,6)$, $(0,9)$)

Mid Partial Credit:

$\frac{1}{4} \times \frac{1}{4}$ indicated
Probability calculated correctly for three pairs on the table

High Partial Credit:

$\frac{1}{16}$ or equivalent
Probability calculated correctly for six pairs on the table

Full Credit -1

No rounding or incorrect rounding

</td>
</tr>
</table>

Q2	**Model Solution – 30 Marks**	**Marking Notes**
(a)	$\cos(A-B)=\cos A\cos B+\sin A\sin B$ Replace A with $90-A$: $\cos(90-A-B)=\cos(90-A)\cos B+\sin(90-A)\sin B$ ....as $\sin A=\cos(90-A)$ $\cos(90-(A+B))=\sin A\cos B+\cos A\sin B$ $\sin(A+B)=\sin A\cos B+\cos A\sin B$ **OR** $\sin A=\cos(90-A)$ $\sin(A+B)=\cos\big(90-(A+B)\big)=\cos\big((90-A)-B\big)$ $=\cos(90-A)\cos B+\sin(90-A)\sin B$ ... by $\cos(A-B)$ formula $=\sin A\cos B+\cos A\sin B$	
	Scale 10C (0, 4, 7, 10) *Low Partial Credit:* • Work of merit, for example, $\cos(A-B)$ formula • Verified with one or more values for A and B • $\sin A=\cos(90-A)$ or $\cos A=\sin(90-A)$ *High Partial Credit:* • $\cos(90-A-B)=\cos(90-A)\cos B+\sin(90-A)\sin B$	
(b)	$\sin(30+45)=\sin 30\cos 45+\cos 30\sin 45$ $\sin 75=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)+\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$ $\sin 75=\frac{1}{2\sqrt{2}}+\frac{\sqrt{3}}{2\sqrt{2}}=\frac{1+\sqrt{3}}{2\sqrt{2}}=\frac{\sqrt{2}+\sqrt{6}}{4}$	**Scale 10C (0, 4, 7, 10)** Note: Accept $\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)+\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$ for full credit. *Low Partial Credit:* • Work of merit, for example $30+45$, or some correct substitution into relevant formula *High Partial Credit:* • $\sin 30\cos 45+\cos 30\sin 45$ or equivalent

(c)	**Method 1** $\sin t[1-\ 2\cos t]=0$ $\sin t=0$ and $1-2\cos t=0$ $\sin t=0$ when $t=0^\circ, 180^\circ$ and 360° $1-2\cos t=0$ $\cos t=\dfrac{1}{2}$ $\cos t=\frac{1}{2}$ when $t=60^\circ$ and $t=300^\circ$ $t=0^\circ, 60^\circ, 180^\circ, 300^\circ$ and 360° **OR** $sin2t-sint=0$ $2cos\dfrac{3t}{2}sin\dfrac{t}{2}=0$ $cos\dfrac{3t}{2}=0$ $t=60^\circ, 180^\circ\ and\ 300^\circ$ $sin\dfrac{t}{2}=0$ $t=0^\circ\ and\ 360^\circ$ **OR** **Method 2** Trial & Improvement: $t=0$ $\sin 0=\sin(2(0))$ $0=0$ $t=60$ $\sin 60=\sin(2(60))$ $\dfrac{\sqrt{3}}{2}=\dfrac{\sqrt{3}}{2}$ $t=180$ $\sin 180=\sin(2(180))$ $0=0$ $t=300$ $\sin \mathbf{300}=\sin(2(300))$ $\dfrac{\sqrt{3}}{2}=\dfrac{\sqrt{3}}{2}$ $t=360$ $\sin 360=\sin(2(360))$ $0=0$	**Scale 10D (0, 3, 5, 8, 10)** **Method 1** Consider as involving 4 steps: **1.** Replaces $sin\ 2t$ with $2\ sin\ t\ cos\ t$ **2.** $sin\ t[1-\ 2\ cos\ t]=0$ stated or implied **3.** Solves $sin\ t=0$ **4.** Solves $1-2\ cos\ t=0$ **OR** **1.** $sin2t-sint=0$ **2.** $2cos\frac{3t}{2}sin\frac{t}{2}=0$ **3.** Solves $cos\frac{3t}{2}=0$ **4.** Solves $sin\frac{t}{2}=0$ *Low Partial Credit:* - Work of merit, for example, effort to expand $sin\ 2t$ or some correct transposition *Mid Partial Credit:* - 2 steps correct *High Partial Credit:* - 3 steps correct *Full Credit –1:* - Apply a * for each solution omitted from Step 3 or Step 4 **Method 2** *Low Partial Credit:* - One correct solution *Mid Partial Credit:* - 3 correct values verified *High Partial Credit:* - 4 correct values verified - All values correct but no work shown

Q3	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $(\underset{\downarrow}{4},6), (-\underset{\downarrow}{3},-1), (\underset{\downarrow}{0},11).$ -11 $(4,-5), (-3,-12), \ (0,0)$ AREA $= \frac{1}{2}\lvert 4(-12)-(-3)(-5)\rvert$ $= \frac{1}{2}\lvert -63\rvert$ $= 31 \cdot 5$ **OR**	**Scale 15D (0, 4, 8, 12, 15)** **Method1** *Low Partial Credit:* - Work of merit in translating one point to $(0,0)$ *Mid Partial Credit:* - Three points correctly translated - Two of the given points subbed in to the area formula and evaluated *High Partial Credit* - Correct substitution into Area formula - One error in translating points and finishes correctly
	Method 2 Uses any one of the following formulae: 1. Area $= \frac{1}{2} ab\sin c$ 2. Area $= \frac{1}{2} \times \text{base} \times \text{perpendicular height}$	**Method 2** *Low Partial Credit:* - Work of merit, for example, finds one relevant piece of data eg. length of one side *Mid Partial Credit:* - All information relevant to one formula calculated, for example, the lengths of 2 sides and the included angle; or the length of one side and the perpendicular height *High Partial Credit* - Correct substitution into Area formula

Q3	**Model Solution – 30 Marks**	**Marking Notes**
(b) **(i)**	Mid-point $= \left(\frac{-1+5}{2}, \frac{k+l}{2}\right)$ $= \left(2, \frac{k+l}{2}\right)$ **OR** -1 to 5 is 6 steps, then $x = -1 + 3 = 2$ k to l is $(l-k)$ steps, then $y = k + \frac{l-k}{2} = \frac{k+l}{2}$ Mid-point $= \left(2, \frac{k+l}{2}\right)$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula

Q3	Model Solution – 30 Marks	Marking Notes
(b)(ii)	Slope $AB = \frac{l-k}{5-(-1)} = \frac{l-k}{6}$ Perpendicular slope $= -\frac{6}{l-k}$ Slope of $3x + 2y - 14 = 0$ is $-\frac{3}{2}$ $-\frac{6}{l-k} = -\frac{3}{2}$ so $\boldsymbol{l - k = 4}$ … Eqn 1 or Slope $AB = \frac{2}{3}$, then $(-1, k)$and $(5, l) \in$ $y = mx + c$, also gives $\boldsymbol{l - k = 4}$ … Eqn 1 $\left(2, \frac{k+l}{2}\right) \to 3(2) + 2\left(\frac{k+l}{2}\right) - 14 = 0$ $\boldsymbol{k + l = 8}$ … Eqn 2 $\begin{cases} l + k = 8 \\ l - k = 4 \end{cases}$ $2l = 12$ ….. $l = 6,\ \ k = 2$ **OR** Slope $AB = \frac{l-k}{6}$ and perpendicular $= -\frac{6}{l-k}$ Eqn of perp bisector: $y - \frac{k+l}{2} = -\frac{6}{l-k}(x - 2)$ $2y - k - l + \frac{12}{l-k}x - \frac{24}{l-k} = 0$ or $\frac{12}{l-k}x + 2y - k - l - \frac{24}{l-k} = 0$ Equating coefficients: x: $\frac{12}{l-k} = 3$ so $4 = l - k$ … **Eqn 1** Const.: $-k - l - \frac{24}{l-k} = -14$ $-k - l - \frac{24}{4} = -14$ So $k + l = 8$ … **Eqn 2** Solve for $l = 6, k = 2$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example, relevant use of midpoint of $[A, B]$, or finds a relevant slope (of AB or of bisector) *Mid Partial Credit:* • Equation 1 or 2 correct found • Finds equation of perpendicular bisector in terms of l and k *High Partial Credit:* • Equation 1 and 2 correct found

Q4	Model Solution – 30 Marks	Marking Notes
(a) **(i)**	Centre $= (h, -3)$ Radius $= \sqrt{12}\ or\ 2\sqrt{3}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit towards finding x-ordinate **or** y-ordinate of the centre *High Partial Credit* • Centre or radius correct
(ii)	$\frac{\|h-4(-3)+7\|}{\sqrt{(1)^2+(-4)^2}} = 5$ $\|h+19\| = 5\sqrt{17}$ $h+19 = 5\sqrt{17}$ or $h+19 = -5\sqrt{17}$ $h = 5\sqrt{17} - 19$ or $h = -5\sqrt{17} - 19$ **OR** $(h+19)^2 = 425$ $h^2 + 38h - 64 = 0$ $h = 5\sqrt{17} - 19$ or $h = -5\sqrt{17} - 19$	**Scale 15D (0, 4, 8, 12, 15)** **3 steps:** **1.** $\frac{\mathbf{\|h-4(-3)+7\|}}{\sqrt{\mathbf{(1)^2+(-4)^2}}}$ **2.** $\frac{\|h-4(-3)+7\|}{\sqrt{(1)^2+(-4)^2}} = 5$ **3.** Find values of h *Low Partial Credit:* • Work of merit, for example, some substitution into relevant formula, or draws diagram with relevant figures (5, centre marked, and line) *Mid Partial Credit:* • 1 step correct *High Partial Credit:* • 2 steps correct

Q4	Model Solution – 30 Marks	Marking Notes
(b)	Centre $= (h, k)$ $k = \dfrac{3-5}{2} = -1$ $(x-h)^2 + (y+1)^2 = \left(\sqrt{20}\right)^2$ $x^2 + y^2 - 2hx + h^2 + 2y + 1 = 20$ $(8,1)$ is on the circle… $(8)^2 + (1)^2 - 2h(8) + h^2 + 2(1) + 1$ $= 20$ $h^2 - 16h + 48 = 0$ $(h-4)(h-12) = 0$ $h = 4$, ~~$h = 12$~~ s: $(x-4)^2 + (y+1)^2 = 20$ **OR** $(a, 3)$: $a^2 + 2ga + 6f + c = -9$ $(a, -5)$: $a^2 + 2ga - 10f + c = -25$ So: $f = 1$ $(8, 1)$: $16g + 2f + c = -65$ So: $16g + c = -67$ … **Eqn A** $\sqrt{g^2 + f^2 - c} = \sqrt{20}$ So: $g^2 - c = 19$ … **Eqn B** From **A** and **B**: $g^2 + 16g + 48 = 0$ so $g = -4$ So $c = -3$ And eqn is $x^2 + y^2 - 8x + 2y - 3 = 0$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Some correct substitution of relevant point *Mid Partial Credit* • Finds $k = -1$ • Finds $f = 1$ • 4 independent equations in g, f, c and a *High Partial Credit:* • $k = -1$ and quadratic equation in h • Finds $f = 1$ and quadratic equation in g *Full Credit –1:* • Finds the relevant constants, but equation of circle not stated • Finds equation of circles for both values of *h* but does not select the correct one

<table>
<tr><th>Q5</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)
(ii)
(iii)</td><td>(i)
Mean $\frac{0+3+2+2+4+5+1}{7} = \frac{17}{7} = 2\cdot42$ ...
= 2.4 [1 d.p.]

Standard deviation $= 1\cdot59.. = 1.6$ [1 d.p.]

(ii) $r = -0\cdot76204$ ...
$r = -0\cdot762$ [3 d.p.]

(iii) <i>Any valid explanation, for example:</i>
If the number of red cubes increases
then there will be less green cubes</td><td>Scale 20D (0, 6, 12, 17, 20)
Note: Accept correct answer without supporting work

Consider solution as requiring 4 items:
1. Mean
2. Standard deviation
3. r
4. Explanation
<i>Low Partial Credit:</i>
• Work of merit, for example, finds the total number of red cubes
<i>Mid Partial Credit:</i>
• 1 item correct and work of merit in any other item
<i>High Partial Credit:</i>
• 3 items correct

<i>Full Credit -1</i>
• One or more answers not to required number of decimal places</td></tr>
<tr><td>(b)</td><td><table><tr><td>3 faces:</td><td>8</td></tr><tr><td>2 faces:</td><td>24</td></tr><tr><td>1 face:</td><td>22</td></tr><tr><td>no faces:</td><td>6</td></tr></table></td><td>Scale 10C (0, 4, 7, 10)
Accept correct answer without work
<i>Low Partial Credit:</i>
• Work of merit, for example, relevant work on the diagram
• One correct value
• Reference to total of 60 cubes
<i>High Partial Credit</i>
• Two correct values</td></tr>
</table>

Q6	**Model Solution – 30 Marks**	**Marking Notes**
(a)	Answer: FALSE Justification: *Describes or draws any situation where two angles are equal in size without being vertically opposite. Eg. equilateral triangle, isosceles triangle, opposite angles in a parallelogram etc.*	**Scale 5B (0, 2, 5)** *Partial Credit:* • Correct answer with no justification • States True and justifies (ignores “and only if
(b) **(i)**	**1.** $\lvert\angle EHD\rvert = \lvert\angle DBC\rvert = \theta$ … alternate angles $\lvert\angle EFD\rvert = \lvert\angle EHD\rvert$ … both $= \theta$ **2.** $\lvert\angle FED\rvert = \lvert\angle HED\rvert$ …rectangle & straight angle $\lvert\angle FDE\rvert = \lvert\angle HDE\rvert$ … angles in tri. sum to 180** **3.** $\lvert ED\rvert = \lvert ED\rvert$ … common side **Conclusion:** So $FED \equiv HED$ … by ASA $\lvert FE\rvert = \lvert EH\rvert$ **OR** **1.** $\lvert\angle EHD\rvert = \lvert\angle DBC\rvert = \theta$ … alternate angles $\lvert\angle EFD\rvert = \lvert\angle EHD\rvert$ … both $= \theta$ $\lvert FD\rvert = \lvert DH\rvert$ … isosceles triangle **2.** $\lvert\angle FED\rvert = \lvert\angle HED\rvert = 90°$ …rectangle & straight angle **3.** $\lvert ED\rvert = \lvert ED\rvert$ … common side **Conclusion:** So $FED \equiv HED$ … by RHS $\lvert FE\rvert = \lvert EH\rvert$ *Or similar*	**Scale 15D (0, 4, 8, 12, 15)** Consider the solution as having 4 elements: 3 steps and a conclusion: **Steps 1, 2 & 3:** 3 correct statements for congruency (with justifications) **and** Conclusion (with reason) Note: To prove by SAS, candidates will have to establish that $\lvert\angle FDE\rvert = \lvert\angle HDE$ *Low Partial Credit:* • Work of merit, for example, $\lvert\angle EHD\rvert = \lvert\angle DBC\rvert$ indicated *Mid Partial Credit:* • 2 correct steps (no justifications) *High Partial Credit:* • 3 correct steps (including one side) with at least one justification • All 3 steps correct and conclusion stated (no justifications) *Full Credit -1* • $\lvert FE\rvert = \lvert EH\rvert$ not stated

Q6	Model Solution – 30 Marks	Marking Notes
(b) **(ii)**	$\lvert FE\rvert = \frac{1}{2}\lvert AB\rvert = 10$ $\lvert FG\rvert = 40$ $\tan\theta = \frac{\lvert AG\rvert}{\lvert FG\rvert} = \frac{90}{40}$ $\theta = 66 \cdot 037..^\circ$ $\theta = 66^\circ$ [nearest degree] **OR** $\lvert FE\rvert = \frac{1}{2}\lvert AB\rvert = 10$ Triangles FED and BCD are similar So $\lvert ED\rvert$ = 90/4 = 22.5 $\tan\theta = \frac{\lvert ED\rvert}{\lvert FE\rvert} = \frac{22.5}{10}$ $\theta = 66 \cdot 037..^\circ$ $\theta = 66^\circ$ [nearest degree] **OR** $\lvert FE\rvert = \frac{1}{2}\lvert AB\rvert = 10$ Triangles FED and BCD are similar So $\lvert DC\rvert$ = 90 x $\frac{3}{4} = 67.5$ $\tan\theta = \frac{\lvert CD\rvert}{\lvert BC\rvert} = \frac{67.5}{30}$ $\theta = 66 \cdot 037..^\circ$ $\theta = 66^\circ$ [nearest degree]	**Scale 10D (0, 3, 5 , 8, 10)** **4 steps:** **1.** $\lvert FE\rvert = \frac{1}{2}\lvert AB\rvert = 10$ **2.** Finds 2nd side in a relevant triangle **3.** Trignometric equation set up **4.** Finds θ *Low Partial Credit:* • Work of merit in finding any side of a relevant triangle *Mid Partial Credit* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit -1* • Incorrect rounding or no rounding

Q7	**Model Solution – 50 Marks**	**Marking Notes**
(a)	$\|AC\|^2 + 9^2 = 70^2$ $\|AC\|^2 = 70^2 - 9^2$ $\|AC\|^2 = 4819$ $\|AC\| = \sqrt{4819}$ Gradient $= \frac{9}{\sqrt{4819}} \times 100 = 12.96..$ $=13\%$ [nearest percent]	**Scale 10C (0, 4, 7, 10)** **3 steps:** **1.** Sets up Pythagoras **2.** Finds $\|\boldsymbol{AC}\|$ **3.** Finds the gradient *Low Partial Credit:* • Work of merit, for example, $70^2 = 4900$, gradient = $\frac{9}{70}$ *High Partial Credit* • 2 steps correct *Full Credit -1* • Incorrect rounding or no rounding
(b)	$\|< POR\| = 5°$ $\frac{\|RO\|}{\sin 87} = \frac{20}{\sin 5}$ $\|RO\| = \frac{20 \sin 87}{\sin 5}$ $\|RO\| = 229 \cdot 16$ m $\tan 17 = \frac{\|HO\|}{229 \cdot 16}$ $\|HO\| = 229 \cdot 16 \tan 17$ $\|HO\| = 70$ [m]	**Scale 10D (0, 3, 5, 8, 10)** **4 steps:** **1.** Substitution into 'sine rule' **2.** Finds $\|\boldsymbol{RO}\|$ **3.** Sets up trigonometric equation to solve $\|\boldsymbol{HO}\|$ **4.** Finds $\|\boldsymbol{HO}\|$ *Low Partial Credit:* • Work of merit, for example, finds $\|< POR\|$, some correct substitution into 'sine rule' *Mid Partial Credit:* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1* • Calculator in incorrect mode • No rounding or incorrect rounding

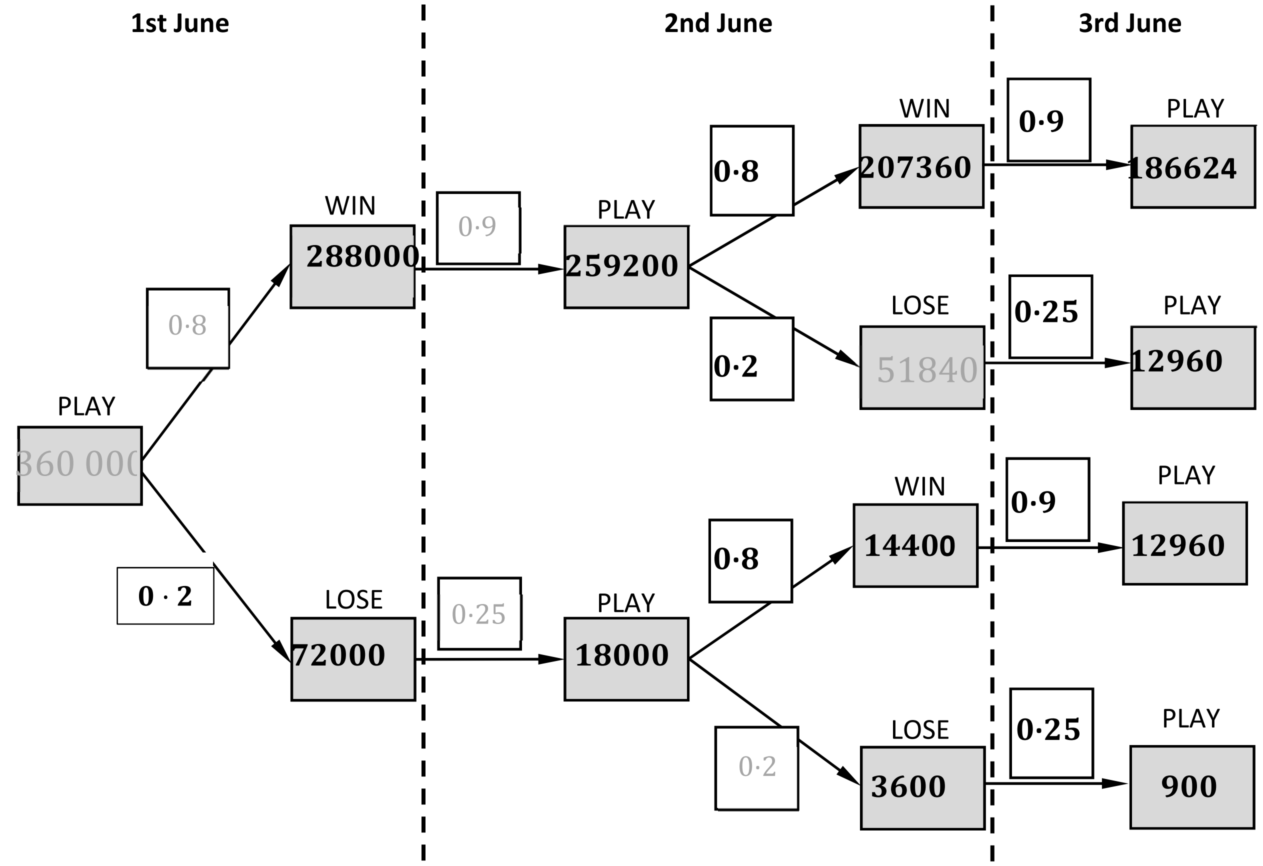
Q7	Model Solution – 50 Marks	Marking Notes
(c) **(d)**	**(c)** Range $= 2 \pm 0 \cdot 4$ $a = 1 \cdot 6 \quad b = 2 \cdot 4$ **OR** $V'(t) = -0 \cdot 4\left[(-\sin\frac{\pi}{2}t)\ \left(\frac{\pi}{2}\right)\right] = 0$ $\sin\frac{\pi}{2}t = 0$ $\frac{\pi}{2}t = 0$ or $\frac{\pi}{2}t = \pi$ $t = 0$ or $t = 2$ $V(0) = 1.6 \qquad V(2) = 2.4$ $a = 1 \cdot 6 \qquad b = 2 \cdot 4$ **(d)** If $\mathrm{V}'(t) > 0$ then the volume of air is increasing so she is breathing in. If $\mathrm{V}'(t) < 0$ then the volume of air is decreasing so she is breathing out.	**Scale 10D (0, 3, 5, 8, 10)** Note: in (d), accept correct explanation for breathing in or breathing out. *Low Partial Credit:* • Work of merit in finding either a or b • Work of merit in part **(d)**, for example, indicates that V' is the rate of change of volume of air *Mid Partial Credit:* • **(c)** or **(d)** correct • Work of merit in both **(c)** and **(d)** *High Partial Credit* • One part correct and work of merit in other part
(e) **(i)**	$V(0 \cdot 5) = 2 - 0 \cdot 4\cos\frac{\pi}{2}(0 \cdot 5)$ $= 1 \cdot 7171 \ldots$ $= 1 \cdot 717$ litres [3 d.p.]	**Scale 5B (0, 2, 5)** *Partial Credit:* • Some relevant substitution *Full Credit -1* • Calculator in incorrect mode • No rounding or incorrect rounding • No units or incorrect units
(e) **(ii)**	$V'(t) = -0 \cdot 4\left[(-\sin\frac{\pi}{2}t)\ \left(\frac{\pi}{2}\right)\right]$ $V'(0 \cdot 5) = 0 \cdot 4\left(\frac{\pi}{2}\right)\left[\sin\frac{\pi}{2}(0 \cdot 5)\right]$ $= 0 \cdot 4442..$ $= 0 \cdot 444$ litres/sec [3 d.p.]	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit in finding $V'(t)$ *High Partial Credit* • $V'(t)$ correct *Full Credit -1* • Calculator in incorrect mode • No rounding or incorrect rounding • No units or incorrect units

Q7	**Model Solution – 50 Marks**	**Marking Notes**
(f)	Form: $a + b\cos(ct)$ Maximum $= 3 \cdot 6$ Minimum $= 1 \cdot 3$ Range $= [3 \cdot 6,\ 1 \cdot 3]$ Mid-line $= \frac{1}{2}(3 \cdot 6 + 1 \cdot 3) = 2 \cdot 45 = a$ $b = \dfrac{1}{2}(3 \cdot 6 - 1 \cdot 3) = 1 \cdot 15$ Period $=$ 2 seconds $\frac{2\pi}{c} = 2$ $c = \pi$ $E(t) = 2 \cdot 45 - 1 \cdot 15\cos\pi\, t$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Work of merit in finding any of a, b or c • Any work on graph towards finding $a, b,$ or c *Mid Partial Credit* • One of a, b or c correct *High Partial Credit* • Two of a, b or c correct *Full Credit -1* • $E(t) = 2 \cdot 45 + 1 \cdot 15\cos\pi\, t$

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$z = \frac{x-\mu}{\sigma} = \frac{3 \cdot 5 - 3 \cdot 87}{0 \cdot 36} = -1 \cdot 03$ $P(x < 3 \cdot 5)$ $= P(z < -1 \cdot 03)$ $= 1 - P(z < 1 \cdot 03)$ $= 1 - 0 \cdot 8485$ $= 0 \cdot 1515$	**Scale 10D (0, 3, 5, 8, 10)** **1.** Find $\boldsymbol{z}$ -score **2.** Find $\mathbf{0 \cdot 8485}$ **3.** Find solution Note: Accept $\boldsymbol{z = 1.03}$ as correct $\boldsymbol{z}$ -score in Step 1, but must be handled correctly for Step 3 *Low Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula, relevant diagram drawn, indicates μ or σ *Mid Partial Credit* • Correct z-score or $\frac{3 \cdot 5 - 3 \cdot 87}{0 \cdot 36}$ *High Partial Credit* • Finds z-score **and** further work, for example, finds 0.8485 or indicates $1 - P(z < 1 \cdot 03)$
(b) **(i)**	$\bar{x} \pm 1 \cdot 96 \frac{\sigma}{\sqrt{n}}$ $3 \cdot 74 - 1 \cdot 96 \left(\frac{0 \cdot 36}{\sqrt{64}}\right) = 3 \cdot 6518$ $3 \cdot 74 + 1 \cdot 96 \left(\frac{0 \cdot 36}{\sqrt{64}}\right) = 3 \cdot 8282$ C.I. : $3 \cdot 6518 \leq \mu \leq 3 \cdot 8282$	**Scale 10C (0, 4, 7, 10)** Note: If $\sqrt{\mathbf{64}}$ is omitted, award Low Partial Credit at most *Low Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula *High Partial Credit* • Confidence interval fully substituted • One side of interval only caculated

Q8	Model Solution – 50 Marks	Marking Notes
(ii)	$H_0: \mu = 3 \cdot 87$ $H_1: \mu \neq 3 \cdot 87$ We reject null hypothesis. Galway players do take a different average number of attempts. Confidence Interval: $3 \cdot 6518 \leq \mu \leq 3 \cdot 8282$ $3 \cdot 87$ is NOT within the confidence interval **OR** Test statistic: $z = \frac{3\cdot74-3\cdot87}{\frac{0\cdot36}{\sqrt{64}}} = -2 \cdot 89$ $-2 \cdot 89 < -1 \cdot 96$ so test statistic is in the critical zone of rejection **OR** Test statistic: $z = \frac{3\cdot74-3\cdot87}{\frac{0\cdot36}{\sqrt{64}}} = -2 \cdot 89$ $P(z \leq -2 \cdot 89) = 0 \cdot 9981$ p-value: $2 \times P(z < -2 \cdot 89) = 2(0.0019) = 0 \cdot 0038$ $0 \cdot 0038 < 0 \cdot 05$	**Scale 5C (0, 2, 3, 5)** Note: If $\boldsymbol{H_0}$ and $\boldsymbol{H_1}$ are reversed, treat as one error Note: treat solution as requiring the four parts laid out in answer grid *Low Partial Credit:* - Work of merit, for example, some correct calculation *High Partial Credit:* - Two parts correct

Q8	Model Solution – 50 Marks	Marking Notes
(c)	$\hat{p} + 1 \cdot 96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0 \cdot 435$ $0{\cdot}35 + 1{\cdot}96\sqrt{\frac{0{\cdot}35(1-0{\cdot}35)}{n}} = 0{\cdot}435$ $\sqrt{\frac{0 \cdot 2275}{n}} = \frac{0 \cdot 435 - 0 \cdot 35}{1 \cdot 96}$ $\frac{0 \cdot 2275}{n} = \left(\frac{0 \cdot 085}{1 \cdot 96}\right)^2$ $\frac{0 \cdot 2275}{\left(\frac{0 \cdot 085}{1 \cdot 96}\right)^2} = n$ $n = 120.963.$ $n = 121$	**Scale 10D (0, 3, 5, 8, 10)** **4 steps:** **1.** $\mathbf{1{\cdot}96}\sqrt{\frac{\mathbf{0{\cdot}35(1-0{\cdot}35)}}{\boldsymbol{n}}}$ **2.** Finds 0.085 or equivalent **3.** Sets up equation **4.** Solves Note: If margin of error is used ie.$\boldsymbol{E} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{n}}}$ award Mid Partial Credit at most *Low Partial Credit:* - Some correct substitution into relevant formula *Mid Partial Credit* - 2 correct steps *High Partial Credit* - 3 correct steps *Full Credit -1* - Incorrect rounding

Q8	Model Solution – 50 Marks	Marking Notes
(d) **(i)**		
	(d)(i) Scale 10D (0, 3, 5, 8, 10) 19 entries to check – note: candidates work will need to be followed from left to right as a single error may lead to subsequent answers differing from those shown in solution. *Low Partial Credit:* - At least one correct entry *Mid Partial Credit* - Eight correct entries *High Partial Credit* - 14 correct entries *Full Credit -1* - One incorrect entry	

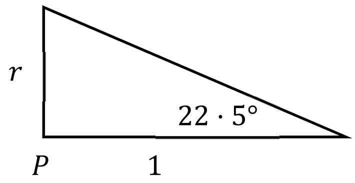
(d) **(ii)**	**Method 1** Total players= $186624+12960+12960+900$ $=213444$ Lost on 1st and/or 2nd $=12960+12960+900$ $=26820$ Probability $=\frac{26820}{213444}=\frac{745}{5929}$ **OR** **Method 2** $P(win, win)=\frac{186624}{213444}$ $1-P(win, win)=1-\frac{186624}{213444}$ $=\frac{26820}{213444}=\frac{745}{5929}$ **Method 3** $P(\text{win, play, win, play})=0.8\times 0.9\times 0.8\times 0.9=\frac{324}{625}$ $P(\text{win, play, lose, play})=0.8\times 0.9\times 0.2\times 0.25=\frac{9}{250}$ $P(\text{lose, play, win, play})=0.2\times 0.25\times 0.8\times 0.9=\frac{9}{250}$ $P(\text{lose, play, lose, play})=0.2\times 0.25\times 0.2\times 0.25=\frac{1}{400}$ $P(lost\ on\ 1st\ and/or\ 2nd, given\ that\ they\ played\ 3\ days)=$ $\frac{\frac{9}{250}+\frac{9}{250}+\frac{1}{400}}{\frac{324}{625}+\frac{9}{250}+\frac{9}{250}+\frac{1}{400}}=\frac{745}{5929}$	**Scale 5C (0, 2, 3, 5)** **Method 1** **3 steps:** **1.** Finds the total number of players who played on all three days **2.** Finds the number of players who lost **3.** Finds probability *Low Partial Credit:* • Work of merit in finding total players or number of players who lost *High Partial Credit:* • 2 steps correct **Method 2** *Low Partial Credit:* • Work of merit, for example, finds the total number of players who played on all three days *High Partial Credit:* • $1-P(win, win)=1-\frac{186624}{213444}$ **Method 3** *Low Partial Credit* • Work of merit, for example, $0.2\times 0.25\times 0.8\times 0.9$ *High Partial Credit* • $\frac{9}{250}+\frac{9}{250}+\frac{1}{400}$ or equivalent • $\frac{324}{625}+\frac{9}{250}+\frac{9}{250}+\frac{1}{400}$ or equivalent

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	Square: $l^2 = 140$ $l = \sqrt{140} = 11.83..$ $l = 11.8$ cm [1 d.p.]	**Scale 5B (0, 2, 5)** *Partial Credit:* • l^2 = 140 *Full Credit -1* • Incorrect rounding, or no rounding • No unit or incorrect unit

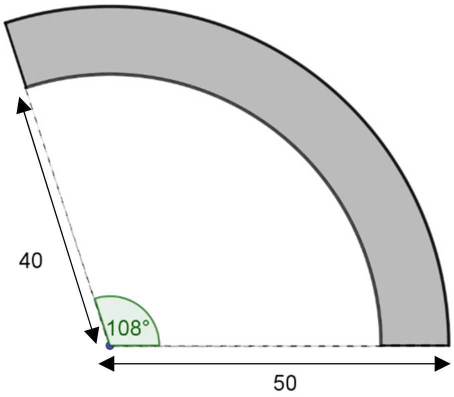
Q9	Model Solution – 50 Marks	Marking Notes
(a) (ii)	Hexagon: $\frac{140}{6}$ area of one triangle $\frac{1}{2}x^2 \sin 60 = \frac{140}{6}$ $x^2 = \frac{140}{3\sin 60}$ $x = \sqrt{\frac{140}{3\sin 60}} = \sqrt{\frac{280}{3\sqrt{3}}}$ $x = 7 \cdot 34 \ldots$ = 7.3 [cm] [1 d.p.] **OR** Hexagon: $\frac{140}{6}$ area of one triangle Let $h =$ perpendicular height of one triangle $h^2 + \left(\frac{1}{2}x\right)^2 = x^2$ $h^2 = \frac{3}{4}x^2$ $h = \frac{\sqrt{3}}{2}x$ $\frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x = \frac{140}{6}$ $x^2 = \frac{280}{3\sqrt{3}}$ $x = 7.34 \ldots.$ = 7.3 [cm] [1 d.p.] **OR** Total Area = area of two identical trapeziums $140 = 2\left(\frac{x+2x}{2}\right)\frac{\sqrt{3}}{2}x$ …….. *h* calculated above $140 = \frac{3\sqrt{3}}{2}x^2$ $x^2 = \frac{280}{3\sqrt{3}}$ $x = 7.34 \ldots.$ = 7.3 [cm] [1 d.p.]	**Scale 10C (0, 4, 7, 10)** Accept correct answer without units *Low Partial Credit:* - Work of merit, for example, finds area of one triangle; finds the area of one trapezium, work towards finding perpendicular height *High Partial Credit:* - Equation in $\boldsymbol{x}$ formed *Full Credit -1* - No rounding or incorrect rounding

Q9	**Model Solution – 50 Marks**	**Marking Notes**
(b) (i)	$4^2 = 6^2 + 8^2 - 2(6)(8)\cos\alpha$ $\cos\alpha = \dfrac{6^2 + 8^2 - 4^2}{2(6)(8)}$ $\cos\alpha = \dfrac{84}{96} = \dfrac{7}{8}$ $\alpha = \cos^{-1}\dfrac{7}{8}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Correct formula with some substitution *High Partial Credit* - $\cos\alpha = \frac{6^2+8^2-4^2}{2(6)(8)}$ or equivalent - $16 = 100 - 96\cos\alpha$ - Uses the cosine rule to find either $\angle ADE$ or $\angle DEA$
(b) (ii)	$\cos\alpha = \frac{8}{\lvert AC\rvert} = \frac{7}{8}$ so $\lvert AC\rvert = \frac{64}{7}$ $\lvert CD\rvert = \sqrt{\left(\frac{64}{7}\right)^2 - 8^2} = \frac{8\sqrt{15}}{7}$ Area $= 2\left[\frac{1}{2}(8)\left(\frac{8\sqrt{15}}{7}\right)\right]$ $= \frac{64\sqrt{15}}{7}$ = 35.410… =35.41 cm^2 [2 d.p.] **OR** $\alpha = 28 \cdot 955°$ $\lvert AC\rvert = \dfrac{64}{7}$ Area $= 2\left[\frac{1}{2}(8)\left(\frac{64}{7}\right)\sin 28\cdot 96\right] = 35\cdot 410..$ $= 35.41\ cm^2$ [2 d.p.], **OR** $\alpha = 28 \cdot 955°$ $\tan 28.955 = \dfrac{\lvert CD\rvert}{8}$ $\lvert CD\rvert = 4.426$ Area $= 2[\frac{1}{2} \times 8 \times 4.426] = 35.410 \ldots$ $= 35.41\ cm^2$ [2 d.p.]	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* - Work of merit in finding a relevant side or angle *Mid Partial Credit:* - Correctly finds relevant sides and/or angles to allow for area to be found, for example, $\lvert CD\rvert$, or α and $\lvert AC\rvert$ *High Partial Credit* - Correctly filled formula for area of triangle ACD *Full Credit -1* - No rounding or incorrect rounding - No unit or incorrect unit

Q9	Model Solution – 50 Marks	Marking Notes
(c) (i)	*Verifies point, for example:* $Q = (\cos 135°, \sin 135°)$ [unit circle] $= \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **OR** Let co-ordinates of Q = (x, y) $\cos 45° = \frac{x}{1} = \frac{1}{\sqrt{2}} \Rightarrow x = -\frac{1}{\sqrt{2}}$ [2nd quadrant] $\sin 45° = \frac{y}{1} \Rightarrow y = \frac{1}{\sqrt{2}}$ **OR** Shows that $\lvert\angle QOP\rvert = 45°$ **and** that distance from Q to $(0, 0)$ is 1, or that Q lies on c **OR** $x^2 + y^2 = 1$ $y = -x$ $\therefore x^2 + x^2 = 1$ $2x^2 = 1$ $x = \pm\frac{1}{\sqrt{2}}$ But *x* in 2nd quadrant. $\therefore x = \frac{-1}{\sqrt{2}}$, and $y = \frac{1}{\sqrt{2}}$,	**Scale 5B (0, 2, 5)** Note: Check drawing for relevant work of merit *Partial Credit:* • Work of merit in finding x or y ordinate

(c) (ii)	**Method 1** $P=(-1,0)$ Tangent at P: $x=-1$ $Q=\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ Slope of tangent at $Q=1$ Tangent at Q: $y-\frac{1}{\sqrt{2}}=1(x--\frac{1}{\sqrt{2}})$ $y=x+\sqrt{2}$ At $x=-1$: $y=-1+\sqrt{2}$ Centre $=(-1,-1+\sqrt{2})$ Radius $=-1+\sqrt{2}$ **OR** **Method 2**  $r=\tan 22\cdot5°$ $\tan 45°=\frac{2\tan 22\cdot5°}{1-\tan^2 22\cdot5°}$ $1=\frac{2r}{1-r^2}$ $r^2+2r-1=0$ $r=\frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2(1)}$ $r=-1+\sqrt{2}$ Centre $=(-1,-1+\sqrt{2})$ Radius $=-1+\sqrt{2}$ **OR** **Method 3** Let $(-1,k)$ be centre of s $(-1,0)$ to $(-1,k)=\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ to $(-1,k)$ $k=\sqrt{\left(-1-(-\frac{1}{\sqrt{2}})\right)^2+\left(k-(\frac{1}{\sqrt{2}})\right)^2}$ $k=\sqrt{\frac{3-2\sqrt{2}}{2}+\frac{2k^2-2\sqrt{2}k+1}{2}}$ $2k^2=4-2\sqrt{2}-2\sqrt{2}k+2k^2$ $2\sqrt{2}k=4-2\sqrt{2}$ $k=\frac{4-2\sqrt{2}}{2\sqrt{2}}=-1+\sqrt{2}$ Centre $=(-1,-1+\sqrt{2})$ Radius $=-1+\sqrt{2}$	**Scale 10C (0, 4, 7, 10)** Consider the solution as having three steps: **1.** Finds that the $\boldsymbol{x}$ co-ordinate of the centre $=\boldsymbol{-1}$ **2.** Indicates that the $\boldsymbol{y}$ co-ordinate of the centre $=\boldsymbol{r}$, the radius of circle $\boldsymbol{s}$ **3.** Finds $\boldsymbol{r}$ *Low Partial Credit:* • Work of merit, for example, any relevant work on the diagram *High Partial Credit* • Two steps correct

Q10	**Model Solution – 50 Marks**	**Marking Notes**
(a)(i)	$\frac{x}{12} = \frac{x+10}{15}$ … or equivalent $15x = 12x + 120$ $3x = 120$ $x = 40$ [cm] **OR** $\frac{40}{12} = \frac{50}{15}$ … or equivalent $600 = 600$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit in establishing equation *High Partial Credit:* • Correct equation set up • $\frac{40}{12} = \frac{50}{15}$ or equivalent
(a)(ii)	Large cone: $R = 15, L = 50$ Small cone: $r = 12,\ l = 40$ Surface area $= \pi RL - \pi rl + \pi r^2$ $= \pi(15)(50) - \pi(12)(40) + \pi(12)^2$ $= 750\pi - 480\pi + 144\pi = 414\pi$ $1300 \cdot 61.. = 1300.6$ cm^2 [1d.p.]	**Scale 15D (0, 4, 8, 12, 15)** Note: Candidate may use $\boldsymbol{\pi(r+R)l}$, where $\boldsymbol{l = 10}$ to find the curved surface area *Low Partial Credit:* • Some correct substitution into πRL or πrl or πr^2 • $\pi RL - \pi rl + \pi r^2$ • $\pi RL - \pi rl$ *Mid Partial Credit* • One of πRL or πrl or πr^2 fully substituted and calculated • Fully correct substitution in to $\pi RL - \pi rl$ • $\pi RL - \pi rl + \pi r^2$ with some substitution *High Partial Credit* • $\pi RL - \pi rl + \pi r^2$ fully substituted • $\pi RL - \pi rl$ calculated *Full Credit -1* • No rounding or incorrect rounding • No unit or incorrect unit

Q10	**Model Solution – 50 Marks**	**Marking Notes**
(a)(iii)	Angle: $\frac{\theta}{360}2\pi(40) = 2\pi(12)$ $\theta = 108°$ 	**Scale 5D (0, 2, 3, 4, 5)** Three measurements required: angle subtended at the centre and 2 relevant lengths of line segments *Low Partial Credit:* • Net of a cone drawn • Work of merit in calculating angle. • Correct structure but no measurements or incorrect measurements given *Mid Partial Credit* • Correct structure with one correct measurement given • Angle correctly calculated *High Partial Credit* • Correct structure with one incorrect measurement

(b)(i)	$9 \times 8 \times 7 \times 6 = 3024$ **OR** $9C_4 \times 4! = 3024$	**Scale 5B (0, 2, 5)** Note: Accept correct answer without work *Partial Credit:* • Work of merit, for example, lists some correct code *Full Credit-1* • $9 \times 9 \times 9 \times 9$ evaluated • $10 \times 9 \times 8 \times 7$ evaluated
(b)(ii)	$4(1 \times 8 \times 7 \times 6) = 1344$ **OR** No 2: $8 \times 7 \times 6 \times 5 = 1680$ $3024 - 1680 = 1344$ **OR** Of the 3024 codes: $\frac{1}{9}$ of them begin with a '2', $\frac{1}{9}$ of them have a '2' in the 2nd position… Therefore, number of codes that contain the digit 2 $= \frac{1}{9} \times 3024 \times 4$	**Scale 5C (0, 2, 3, 5)** Note: Accept correct answer without work *Low Partial Credit:* • Work of merit, for example, lists some correct codes, brings answer from b(i) down to b(ii) *High Partial Credit:* • $1 \times 8 \times 7 \times 6$ or $\frac{1}{9} \times 3024$ • 1680
(b) (iii)	$1 + 2 + 3 = 6$ $1 + 2 + 4 = 7$ $1 + 2 + 5 = 8$ $1 + 2 + 6 = 9$ $1 + 3 + 4 = 8$ $1 + 3 + 5 = 9$ $2 + 3 + 4 = 9$ 7 possible combinations with 6 possible arrangements of each So 7×6 *or* $7 \times 3! = 42$	**Scale10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit in listing some codes *High Partial Credit:* • All 7 combinations listed • One correct combination and mentions 3! or 6 possible arrangements