Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2023

Marking Scheme

Applied Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

In considering this marking scheme for the written examination, the following points should be noted.

1. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods. The descriptions, methods and definitions in the scheme are not exhaustive and alternative valid answers are acceptable.

2. The detail required in any answer is determined by the context and manner in which the question is asked, and also by the number of marks assigned to the answer in the examination paper. Therefore, in any instance, it may vary from year to year.

3. A solidus (/) indicates different valid attempts.

4. A number of different types of penalties are applied to candidates' work, including:

- mathematical error ("blunder") –3
- mathematical/numerical slip –1
- misreading (if not serious or leading to oversimplification) –1

5. A misreading or slip or omission which oversimplifies the question may be regarded as equivalent to a mathematical error and is marked accordingly.

6. In cases where a question item is marked using a marking scale, the scale is provided in **bold**. For a 20 mark item marked using a marking scale:

- 17 marks are awarded where candidate work shows one systemic error.
- 14 marks are awarded where candidate work shows two systemic errors.
- 8 marks are awarded where a valid attempt is presented which cannot be awarded higher marks.

For a 15 mark item marked using a marking scale:

- 12 marks are awarded where candidate work shows one systemic error.
- 9 marks are awarded where candidate work shows two systemic errors.
- 6 marks are awarded where a valid attempt is presented which cannot be awarded higher marks.

For a 10 mark item marked using a marking scale:

- 7 marks are awarded where candidate work shows one systemic error.
- 4 marks are awarded where candidate work shows two systemic errors or where a valid attempt is presented which cannot be awarded higher marks.

7. A zero should only be recorded when the candidate has attempted the question item but does not merit marks. If a candidate does not attempt a question item examiners should record NR.

8. Examiners are expected to annotate parts of the responses as directed at the marking conference. (See below.)

Symbol	Name	Use
	Cross	Incorrect element
	Tick	Correct element
	Slip	Deduct one mark
	Box 2	Partially correct element – award 2 marks
	^	Missing element
	Horizontal wavy line	To be noticed
	Vertical wavy line	Additional page

9. Bonus marks at the rate of 5% of the marks obtained will be given to a candidate who answers the written examination paper entirely through Irish and who obtains 75% or less of the total mark available (i.e. 300 marks or less). In calculating the bonus to be applied decimals are always rounded down, not up ¬ e.g., 4.5 becomes 4; 4.9 becomes 4, etc. See below for when a candidate is awarded more than 300 marks in the written examination paper.

Marcanna Breise as ucht freagairt trí Ghaeilge

Léiríonn an tábla thíos an méid marcanna breise ba chóir a bhronnadh ar iarrthóirí a ghnóthaíonn níos mó ná 75% d'iomlán na marcanna.

N.B. Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don scrúdú. Ba chóir freisin an marc bónais sin **a shlánú síos**.

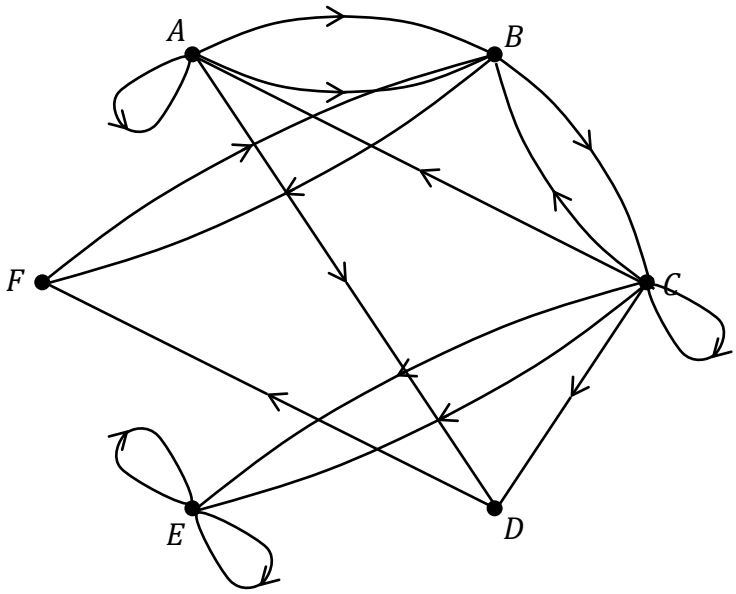
Tábla 400 @ 5%
Bain úsáid as an tábla seo i gcás na n-ábhar a bhfuil 400 marc san iomlán ag gabháil leo agus inarb é 5% gnáthráta an bhónais.

Bain úsáid as an ngnáthráta i gcás 300 marc agus faoina bhun sin. Os cionn an mharc sin, féach an tábla thíos.

Bunmharc	Marc Bónais
301 - 306	14
307 - 313	13
314 - 320	12
321 - 326	11
327 - 333	10
334 - 340	9
341 – 346	8
347 – 353	7

Bunmharc	Marc Bónais
354 - 360	6
361 - 366	5
367 - 373	4
374 - 380	3
381 - 386	2
387 - 393	1
394 - 400	0

1(a) (i)



10 [**0/4/7**]

1(a) (ii)

e.g. $B \to C \to D \to F \to B$ / $B \to C \to B$ / $B \to C \to A \to B$ etc. 5

1(a) (iii)

in a directed graph the edges have direction /

in an undirected graph the edges do not have an arrow 5

1(b) (i)

$\int ds = \int 2te^{-t}dt$ 5

Let $u = 2t$ and let $dv = e^{-t}dt$

$du = 2dt$ and $v = -e^{-t}$ 5

$\int udv = uv - \int vdu$, so $\int 2te^{-t}dt = -2te^{-t} + 2\int e^{-t}\,dt$

$= -2te^{-t} - 2e^{-t} = -2e^{-t}(t+1)$ 5

$s = -2e^{-t}(t+1) + c$, so $0 = -2 + c$, i.e. $c = 2$ 5

$s = -2e^{-t}(t+1) + 2$ 5

1(b) (ii)

$s(3) = -2e^{-3}(3+1) + 2 = 1.60$ to 2 decimal places 5

2(a)

X: $A(3400)$, ~~$B(2500)$~~, $C(1250)$

C: $B(2150)$, ~~$F(5150)$~~

B: $F(4300)$, $G(6350)$

A: ~~$D(6050)$~~, $E(4200)$

E: $D(5950)$

F: $H(5900)$, $I(6500)$

D: ~~$G(6600)$~~, $J(8950)$

H: ~~$G(7250)$~~, ~~$I(7700)$~~

G: $K(8850)$

I: ~~$K(9850)$~~, ~~$L(11250)$~~

K: ~~$J(10750)$~~, $L(9800)$

J: $M(9900)$

L: ~~$Y(12100)$~~, $N(11750)$

M: $Y(11450)$

Path $= X \to A \to E \to D \to J \to M \to Y$ Cost $= €11\,450$

20 [**0/8/14/17**]

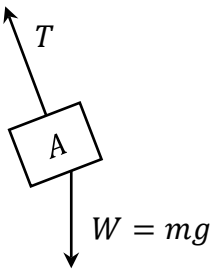
2(b)

		before impact (m s^{-1})	after impact (m s^{-1})	
P	m	$4\cos\alpha\,\vec{i} + 4\sin\alpha\,\vec{j}$	$v_1\vec{i} + 4\sin\alpha\,\vec{j}$	
		$2.4\vec{i} + 3.2\vec{j}$	$v_1\vec{i} + 3.2\vec{j}$	5
Q	$2m$	$0\vec{i} + 3.2\vec{j}$	$v_2\vec{i} + 3.2\vec{j}$	5
PCM		$m(2.4) + 2m(0) = m(v_1) + 2m(v_2)$		5
		$v_1 + 2v_2 = 2.4$		
NEL		$v_1 - v_2 = -2.4e$		5

$v_1 = 0.8(1 - 2e)$ $\quad$ $v_2 = 0.8(1 + e)$

$v_P = 0.8(1 - 2e)\vec{i} + 3.2\vec{j}$ $\quad$ $v_Q = 0.8(1 + e)\vec{i} + 3.2\vec{j}$ 5, 5

3 (i)



5

3 (ii)

$T \sin \alpha = mr\omega^2$

$r = 3.5 + 4.3 \sin \alpha$

$T \cos \alpha = mg$ 5, 5

dividing: $\tan \alpha = \frac{(3.5+4.3 \sin \alpha)\omega^2}{g}$, i.e. $\omega = \sqrt{\frac{g \tan \alpha}{3.5+4.3 \sin \alpha}}$ 5

3 (iii)

$\sqrt{\frac{\text{m s}^{-2}}{\text{m}}} = \sqrt{\text{s}^{-2}} = \text{s}^{-1}$ which are the units for ω 5

3 (iv)

when $\alpha = 25°$, $\omega = \sqrt{\frac{g \tan 25°}{3.5+4.3 \sin 25°}} = 0.927$ (rad) s^{-1} 5

$T' = \frac{2\pi}{\omega} = 6.78$ s 5

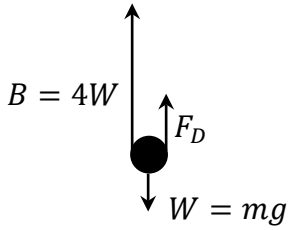
$\frac{60}{T'} = 8.85 = 9$ rotations in one minute, to the nearest whole number (or 8 complete revolutions) 5

3 (v)

$s = ut + \frac{1}{2}at^2$ so $4.9t^2 - 4t - 6 = 0$ 5

i.e. $t = 1.59$ s to 2 decimal places, $t > 0$ 5

4 (i)



5

4 (ii)

$F = ma = W - 4W - F_D = -3mg - mv^2$ so $a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} = -3g - v^2$ 5

4 (iii)

$\int \frac{vdv}{29.4+v^2} = -\int ds$ 5

$\int ds = -s + c$ 5

Let $u = 29.4 + v^2$, so $du = 2vdv \therefore \int \frac{vdv}{29.4+v^2} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(29.4 + v^2)$ 5

$v = 15$ when $s = 0$ so $c = \frac{1}{2}\ln 254.4$ 5

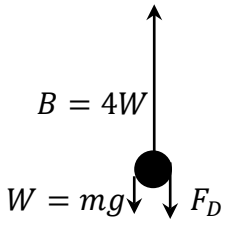
$\ln\frac{29.4+v^2}{254.4} = -2s$ so $\frac{29.4+v^2}{254.4} = e^{-2s}$, i.e. $v = \sqrt{254.4e^{-2s} - 29.4}$ m s^{-1} 5

4 (iv)

$v = 0$ so $e^{-2D} = \frac{29.4}{254.4}$

$D = 1.08$ m to 2 decimal places 5

4 (v)

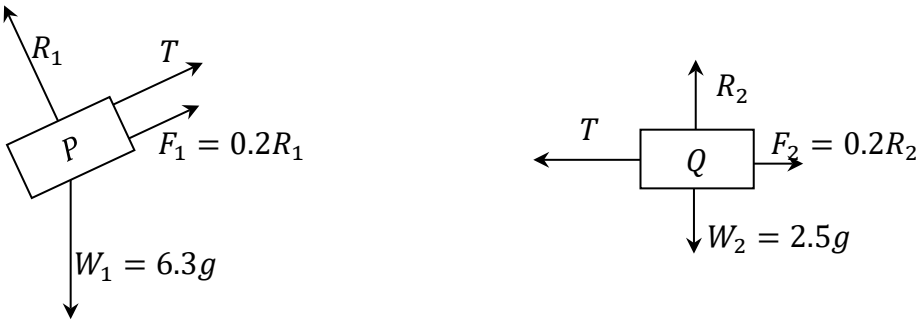


5

4 (vi)

$\frac{dv}{dt}\left[= v\frac{dv}{ds}\right] = 3g - v^2 = 29.4 - v^2$ 5

5(a) (i)



5, 5

5(a) (ii)

$R_1 = 6.3g \cos 25°$ and $R_2 = 2.5g$

$6.3g \sin 25° - T - 1.26g \cos 25° = 6.3a$ 5

$T - 0.5g = 2.5a$ 5

$6.3g \sin 25° - 1.26g \cos 25° - 0.5g = 6.3a + 2.5a = 8.8a$

$8.8a = 10.00$ so $a = 1.14$ m s^{-2} to 2 decimal places 5

5(b)

$s_M = s_T$ and $T_M = T_T + 140$ 5

$s_M = s_1 + s_2 + s_3$ and $T_M = t_1 + t_2 + t_3$

$s_T = s_4 + s_5 + s_6$ and $T_T = t_4 + t_5 + t_6$

$a_1 = \frac{22.5}{40} = 0.5625$

$s_M = [450] + [10800] + [s_3]$

$T_M = 40 + 480 + t_3$

$v_1 = u_2 = 1.5 \times 20 = 30$ so $s_T = [300] + [10800] + [s_6]$

$T_T = 20 + 360 + t_6$

5, 5

$40 + 480 + t_3 = 20 + 360 + t_6 + 140$, i.e. $t_3 = t_6 = t$

$a_3 = -\frac{22.5}{t}$ so $0^2 = 22.5^2 - \frac{45s_3}{t}$, i.e. $s_3 = 11.25t$

$a_6 = -\frac{30}{t}$ so $0^2 = 30^2 - \frac{60s_6}{t}$, i.e. $s_6 = 15t$

$\therefore 450 + 10800 + 11.25t = 300 + 10800 + 15t$, i.e. $t = 40$ s 5

$T_T = 20 + 360 + 40 = 7$ minutes

Áine leaves her house at 08: 23 5

6 (i)

$U_2 = 2U_1 + 3U_0 = 2(2) + 3(1) = 4 + 3 = 7$ [pups]

$U_3 = 2U_2 + 3U_1 = 2(7) + 3(2) = 14 + 6 = 20$ [pups] 5

6 (ii)

$U_{n+2} = 2U_{n+1} + 3U_n$ i.e. $U_{n+2} - 2U_{n+1} - 3U_n = 0$

$x^2 - 2x - 3 = 0$ i.e. $(x-3)(x+1) = 0$ i.e. $x = 3$ or $x = -1$ 5

$U_n = \alpha 3^n + \beta(-1)^n$ 5

$U_0 = 1$ so $\alpha + \beta = 1$ and $U_1 = 2$ so $3\alpha - \beta = 2$ 5

$\alpha = \frac{3}{4}$ and $\beta = \frac{1}{4}$, i.e. $U_n = \frac{3}{4}3^n + \frac{1}{4}(-1)^n$ 5

6 (iii)

$U_{10} = \frac{3}{4}3^{10} + \frac{1}{4}(-1)^{10} = 44287$ [pups] 5

6 (iv)

$V_{n+2} = 2V_{n+1} + 3V_n - 2(n+2)$ has a particular solution of the form $f(n) = an + b$ 5

$f(n+2) = 2f(n+1) + 3f(n) - 2n - 4$

$an + 2a + b = 2an + 2a + 2b + 3an + 3b - 2n - 4$ i.e. $2an + 2b = n + 2$ for all n

$a = \frac{1}{2}$ and $b = 1$ 5

$V_n = \alpha 3^n + \beta(-1)^n + \frac{n}{2} + 1$

$V_0 = 1$ so $\alpha + \beta + 1 = 1$

$V_1 = 2$ so $3\alpha - \beta + \frac{1}{2} + 1 = 2$

$\alpha = \frac{1}{8}$ and $\beta = -\frac{1}{8}$, i.e. $V_n = \frac{1}{8}3^n - \frac{1}{8}(-1)^n + \frac{n+2}{2}$ 5

6 (v)

$V_{10} = \frac{1}{8}3^{10} - \frac{1}{8}(-1)^{10} + \frac{10+2}{2} = 7387$ [pups] 5

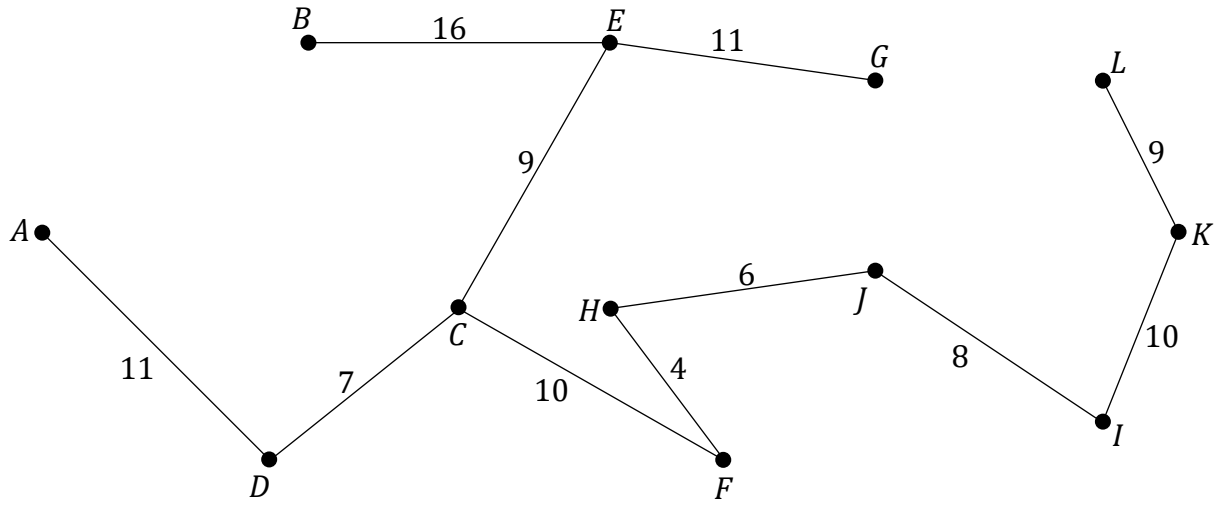
7(a) (i)

Kruskal's algorithm	*Prim's algorithm*
$\|FH\| = 4$	Choose node A, say.
$\|HJ\| = 6$	$\|AD\| = 11$
$\|CD\| = 7$	$\|CD\| = 7$
$\|IJ\| = 8$	$\|CE\| = 9$
$\|CE\| = 9$	$\|CF\| = 10$
$\|KL\| = 9$	$\|FH\| = 4$
$\|CF\| = 10$	$\|HJ\| = 6$
$\|IK\| = 10$	$\|IJ\| = 8$
$\|AD\| = 11$	$\|IK\| = 10$
$\|EG\| = 11$	$\|LK\| = 9$
~~$\|EH\| = 11$~~	$\|EG\| = 11$
~~$\|GJ\| = 12$~~	$\|BE\| = 16$
~~$\|FI\| = 13$~~	
~~$\|HG\| = 14$~~	
$\|BE\| = 16$	

15 [**0/6/9/12**]

Deduct 3 marks if the algorithm used is not correctly named.

Allow 3 marks for the name of a correct algorithm if no other work is presented.



7(a) (ii)

$11 + 7 + 2(9 + 11 + 16) + 10 + 4 + 6 + 8 + 10 + 9 = 137$ minutes 5

7(b) (i)

$\frac{dN}{dt} = k(2000 - N)$ so $\int \frac{dN}{2000-N} = \int k dt$ 5

$\ln \frac{1}{2000-N} = kt + c$ 5, 5

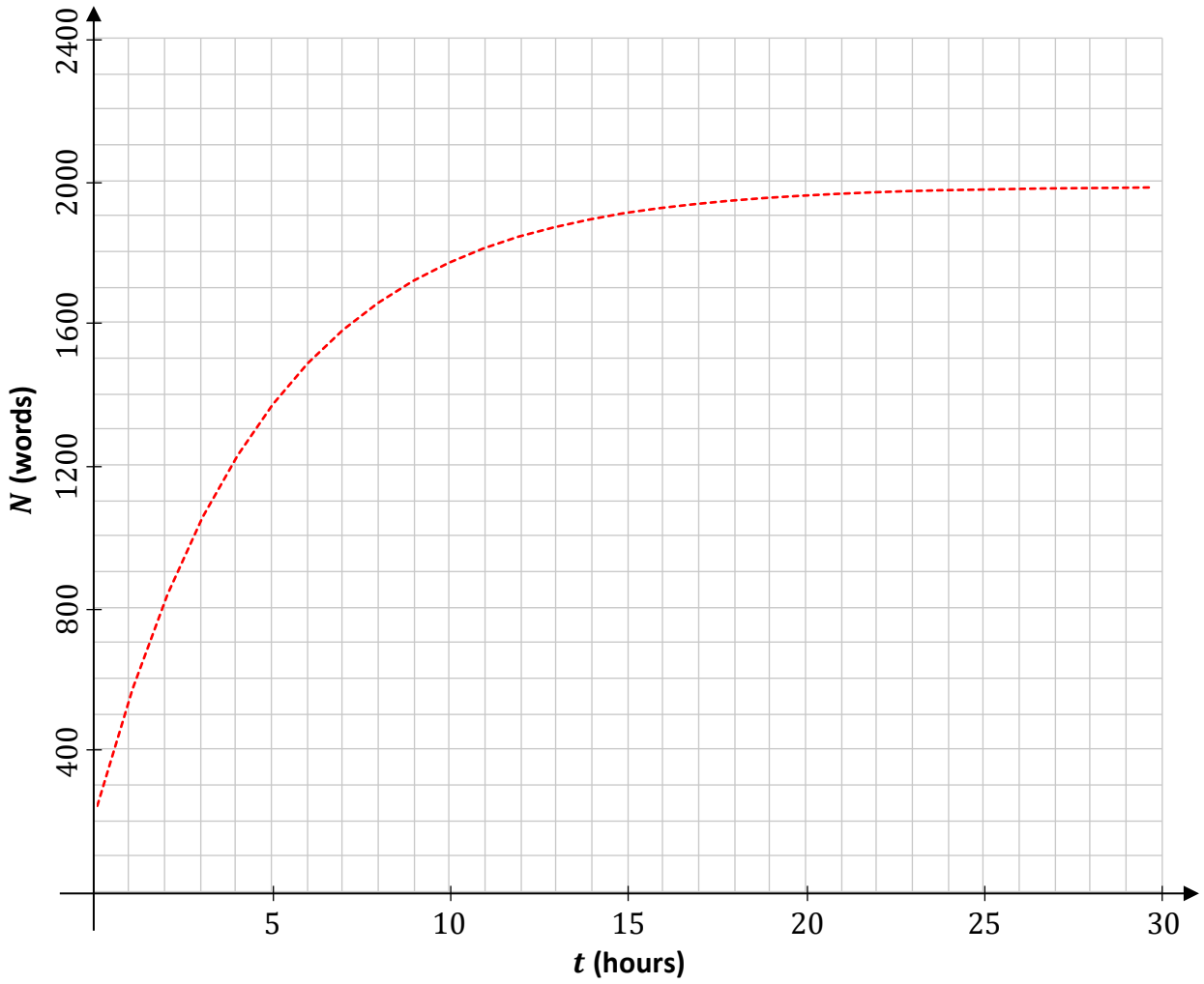
$N = 250$ when $t = 0$ so $c = \ln \frac{1}{1750}$

$\frac{1750}{2000-N} = e^{kt}$ so $N = 2000 - 1750e^{-kt}$ 5

7(b) (ii)

$N = 1500$ when $t = 6$ so $1500 = 2000 - 1750e^{-6k}$, i.e. $k = \frac{\ln 3.5}{6} \cong 0.209$ hour^{-1} 5

7(b) (iii)



5

8 (i)

At the point of collision, the balls have the same height. 5

$\therefore 38 \sin 41° (3) - \frac{1}{2} g(3)^2 = u \sin 64° (2) - \frac{1}{2} g(2)^2$ 5

i.e. $u = 27.98$ m s^{-1} to 2 decimal places 5

8 (ii)

$D = 38 \cos 41° (3) + 27.98 \cos 64° (2) = 86.04 + 24.53 = 110.57$ m 5

8 (iii)

$\overrightarrow{v_P}(t) = 38 \cos 41° \vec{\imath} + (38 \sin 41° - 9.8 \times 3)\vec{\jmath} = 28.68\vec{\imath} - 4.47\vec{\jmath}$ m s^{-1} 5

$\overrightarrow{v_Q}(t) = -27.98 \cos 64° \vec{\imath} + (27.98 \sin 64° - 9.8 \times 2)\vec{\jmath} = -12.27\vec{\imath} + 5.55\vec{\jmath}$ m s^{-1} 5

8 (iv)

$\overrightarrow{v_P} \cdot \overrightarrow{v_Q} = (28.68\vec{\imath} - 4.47\vec{\jmath}) \cdot (-12.27\vec{\imath} + 5.55\vec{\jmath}) = (28.68)(-12.27) + (-4.47)(5.55)$ 5

i.e. $\overrightarrow{v_P} \cdot \overrightarrow{v_Q} = -376.71$ [units not required] 5

8 (v)

$\overrightarrow{v_P} \cdot \overrightarrow{v_Q} = |\overrightarrow{v_P}|\left|\overrightarrow{v_Q}\right| \cos\theta$ 5

$|\overrightarrow{v_P}| = 29.03$ and $\left|\overrightarrow{v_Q}\right| = 13.47$ so $\cos\theta = -0.96$, i.e. $\theta = 164.44°$, i.e. angle $= 15.56°$ 5

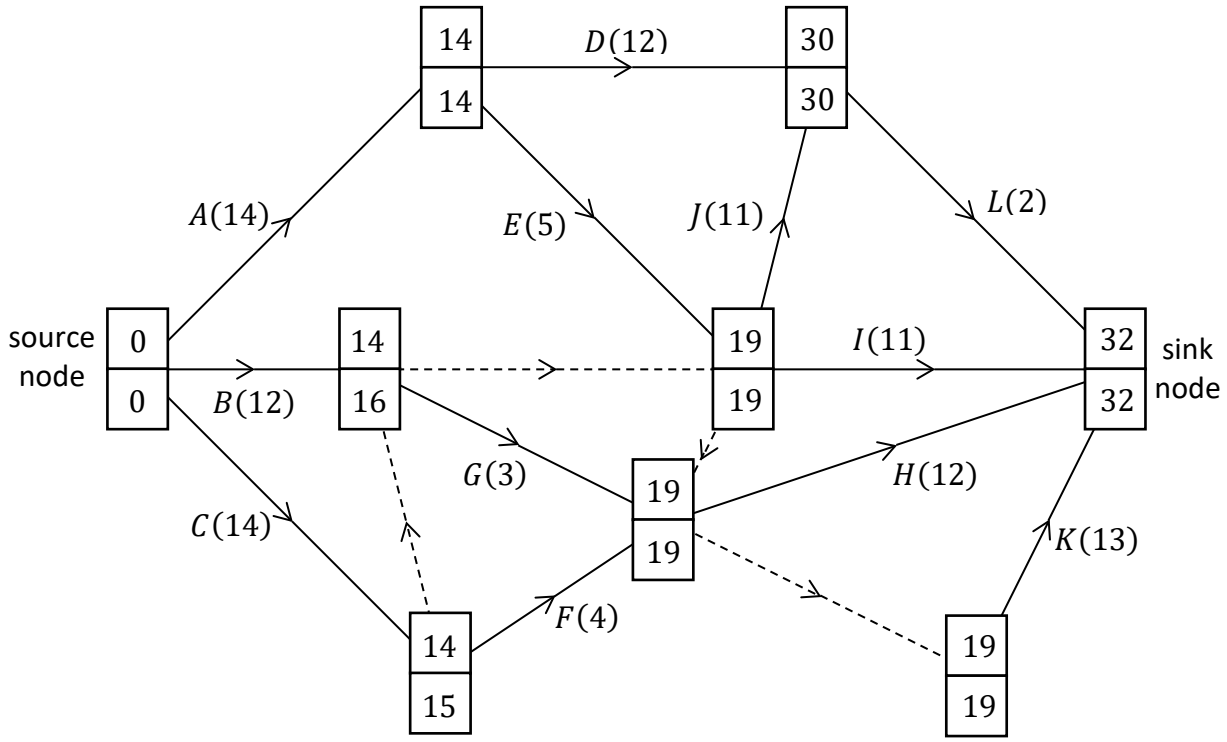
9 (i)

Activity	Depends directly on …	Activity	Depends directly on …
A	–	G	B, C
B	–	H	E, F, G
C	–	I	B, C, E
D	A	J	B, C, E
E	A	K	E, F, G
F	C	L	D, J

10

–1 for each incorrect part A to L

9 (ii)

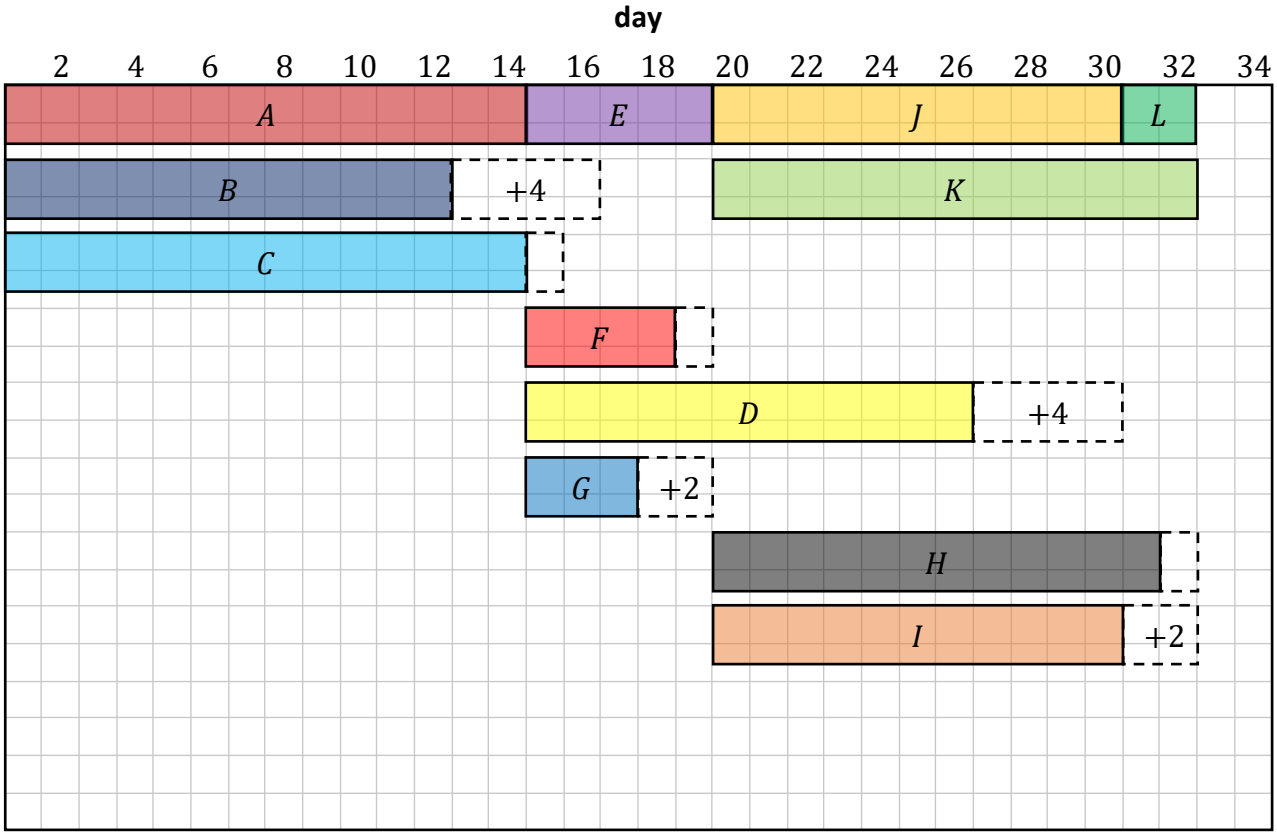


20 [**0/8/14/17**]

9 (iii)

A, E, J, L and A, E, K 5

9 (iv)



10 [**0/4/7**]

9 (v)

E, F, D, G 5

10(a) (i)

$U_1 = 1.2(175) - 30 = 180$ [grasshoppers]

$U_2 = 1.2(180) - 30 = 186$ [grasshoppers] 5

10(a) (ii)

$U_{n+1} = 1.2U_n - 30$ 5

10(a) (iii)

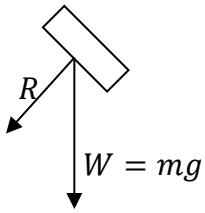
$U_n = a^n U_0 + b\left(\frac{1-a^n}{1-a}\right)$ or $U_n = Ca^n + D$ 5

$U_n = 1.2^n(25) + 150$ 5

10(a) (iv)

$U_{12} = 1.2^{12}(25) + 150 = 373$ [grasshoppers] 5

10(b) (i)



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10(b) (ii)

$E_A = \frac{1}{2}mu^2 = \frac{mkgr}{2}$

$E_\theta = \frac{1}{2}mv^2 + mg(r + r\cos\theta)$

$E_A = E_\theta$ so $kgr = v^2 + 2g(r + r\cos\theta)$ 5

$\frac{mv^2}{r} = R + mg\cos\theta$ 5

$= mg\cos\theta$ when the car loses contact with the track, i.e. when $R = 0$

$v^2 = gr\cos\theta$ so $kgr = gr\cos\theta + 2g(r + r\cos\theta) = 2gr + 3gr\cos\theta$

$\therefore \cos\theta = \frac{k-2}{3}$ as required 5

10(b) (iii)

$\cos\theta = 1$ so $k = 5$ 5

Higher Level Applied Mathematics Mathematical Modelling Project – Report Structure and Mark Allocations

Section	**Indicative Content**	**Marks**
Introduction and Research	• Background research on brief • Identify specific problem(s) to be modelled • Research specific problem(s) • Identify relevant variables • Present relevant data • Provide citations and references	**20**
The Modelling Process	• Explain and justify model and assumptions • Compute solutions • Present solutions using appropriate mathematical and graphical representations • Analysis of solution(s) – sensitivity to changes in assumptions; comparison with other solutions or real-world data • Iterative process	**50**
Interpretation of Results	• Interpretation of solution(s) in real-world context • Conclusions and reflections	**15**
Communication and Innovation	This is not a distinct section of the report. • Innovative and creative approaches • Overall coherence	**15**

To be noted by examiner:

- Before commencing marking read the entire reporting booklet to familiarise yourself with the content presented for marking.
- Be careful not to penalise skilful brevity, nor to reward unwarranted length.
- Mark descriptors should be interpreted in the context of the challenges and demands of the specific problem which the candidate has chosen.

Higher Level Applied Mathematics Mathematical Modelling Project – Marking Scheme

1 *Introduction & Research* **(20 marks)**	Very thorough 16 – 20	Thorough 11 – 15	Basic 6 – 10	Very basic 0 – 5
	Problem and variables identified, research presented and cited, data presented where relevant.	Work of a good standard, but some issues with, for example, identification of variables or citation of research.	Basic statement of problem with some evidence of research undertaken.	Statement of problem with no evidence of research.
2a *The Modelling Process – Explain & Justify* **(15 marks)**	Very thorough 12 – 15	Thorough 8 – 11	Basic 4 – 7	Very basic 0 – 3
	Model fully explained, including further iterations, assumptions identified and justified.	Model well explained, assumptions clearly described.	Model and assumptions poorly explained.	Model outlined with no explanation of assumption.
2b *The Modelling Process – Compute & Iterate* **(20 marks)**	Very thorough 16 – 20	Thorough 11 – 15	Basic 6 – 10	Very basic 0 – 5
	Computation of mathematical solution(s) fully explained. Full iterative process.	Partial explanation of mathematical solution(s). Iteration(s) presented.	Solution(s) computed without explanation. Some evidence of iterative process	Solution(s) computed without explanation.

2c *The Modelling Process – Present & Analyse* **(15 marks)**	Very thorough 12 – 15	Thorough 8 – 11	Basic 4 – 7	Very basic 0 – 3
	Solution(s) presented using appropriate mathematical/graphical representations. Solution(s) analysed with reference to model's assumptions or other solutions or real-world data.	Solution(s) presented and analysed, but with some issues with, for example, mathematical/graphical representations.	Solution(s) presented. Some evidence of analysis.	Solution(s) presented.
3 *Interpretation of Results* **(15 marks)**	Very thorough 12 – 15	Thorough 8 – 11	Basic 4 – 7	Very basic 0 – 3
	Excellent interpretation of results in a real-world context. Conclusion(s) drawn and project reflected on.	Good interpretation of results in a real-world context. Conclusion(s) drawn *or* project reflected on.	Some interpretation of results presented. Limited conclusion(s) drawn *or* limited reflection on project.	Results interpreted poorly *or* conclusion(s) drawn poorly *or* project reflected on poorly.
4 *Communication & Innovation* **(15 marks)**	Very thorough 12 – 15	Thorough 8 – 11	Basic 4 – 7	Very basic 0 – 3
	Project approached and/or presented in a highly innovative and/or creative way. Excellent overall coherence.	Project approached and/or presented with innovation/creativity. Good overall coherence.	Project approached or presented with limited innovation/creativity. Fair overall coherence.	Little or no evidence of innovation/creativity. Poor overall coherence.