MARKING SCHEME

JUNIOR CERTIFICATE EXAMINATION 2005

MATHEMATICS –HIGHER LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

Question 1

Part (a)	**10 (5,5) marks**	**Att (2,2)**
Part (b)	**20 (5,15) marks**	**Att (2,5)**
Part (c)	**20 (10,10)marks**	**Att(3,3)**

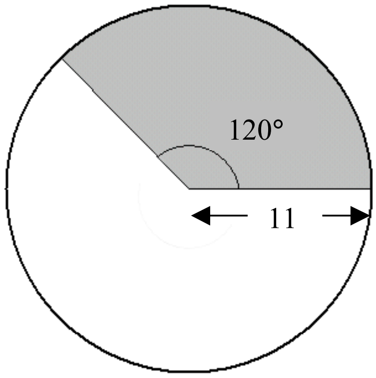
Part (a) **10 (5, 5) marks** **Att (2,2)**

(i) ✍ Find, correct to the nearest cm^2,

the area of a disc of radius 11 cm.

(ii) ✍ Find, correct to the nearest cm^2,

the area of the shaded region in the diagram.



(a) (i) **5 marks** **Att 2**

(i) Area of disc $= 3.14 \times (11)^2$

$= 3.14 \times 121$

$= 379 \cdot 94 \quad = 380$ to nearest cm^2

Blunders (-3)
B1 Incorrect substitution into correct formula
B2 Incorrect squaring
B3 Incorrect relevant area formula with substitution.

Slips (-1)
S1 Answer in terms of π
S2 Fails to round off
S3 Arithmetic slips to a max of -3

Attempts (2 *marks*)
A1 Correct formula with some substitution
A2 $2\pi r$ with r substituted correctly

(a) (ii) **5 marks** **Att 2**

(ii)	Shaded area	=	$379 \cdot 94 \div 3$
		=	$126 \cdot 64 \text{ cm}^2$
		=	127 cm^2

* Accept candidates answer from (a) (i).

Blunders(-3)
B1 Incorrect substitution into correct formula
B2 Incorrect relevant area formula with some substitution
B3 Error in use of 120^0

Slips (-1)
S1 Answer in terms of π
S2 Fails to round off
S3 Arithmetic slips to a max of -3

Attempts (2 *marks*)
A1 Correct formula but no substitution
A2 Indicates division by 3
A3 Indicates $\frac{120}{360}$ or equivalent

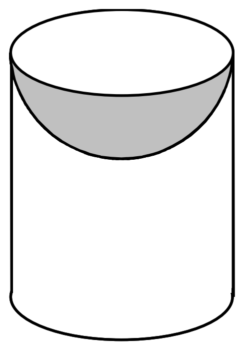
Part (b) **20 (5,15) marks** **Att (2,5)**

(i) A solid metal cylinder has height 20 cm and diameter 14 cm.

✍ Find its curved surface area in terms of π.

(ii) A hemisphere with diameter 14 cm is removed from the top of this cylinder, as shown.

✍ Find the total surface area of the remaining solid in terms of π.



(b) (i) **5 marks** **Att 2**

(i) CSA $= 2\pi rh = 2\pi \times 7 \times 20$ or 280π cm^2.

Blunders (-3)
B1 r = 14
B2 Incorrect relevant formula with some substitution

Slips (-1)
S1 Arithmetic slips to a max of -3
S2 Answer not in terms of π

Attempts (2 *marks*)
A1 Correct formula with some substitution
A2 Volume of cylinder with fully correct substitution

(b) (ii) **15 marks** **Att 5**

(ii) Total Surface Area $= 280\pi + \pi r^2 + 2\pi r^2$ or $280\pi + 3\pi r^2$
$= 280\pi + \pi 7^2 + 2\pi 7^2$ or $280\pi + 3\pi 7^2$
$= 427\pi$ cm^2.

Blunders (-3)
B1 Each part calculated but not added
B2 Omission of cylinder base
B3 Incorrect relevant formula with substitution

Slips (-1)
S1 Arithmetic slips to a max of -3
S2 Answer not in required form (e.g. 1340.78)

Attempts (5 *marks*)
A1 Formula for area of base of cylinder or CSA of hemisphere with some substitution

Worthless (0)
W1 Volume of cylinder and /or hemisphere (with or without substitution)

Part (c) **20 (10,10) marks** **Att (3,3)**

(i)	A cone has radius x and height $3x$.
	✍ Find its volume in term of π and x.
(ii)	A second cone has twice the radius and half the height of the first cone.
	✍ Find the ratio of the volume of the second cone to the volume of the first.

(c) (i) **10 marks** **Att 3**

(i)

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi x^2 3x$$

$$\text{or } \pi x^3$$

Blunders (-3)

B1 Each incorrect substitution into correct formula

B2 Incorrect related formula with substitution

Slips (-1)

S1 Arithmetic slips to a max of -3

Attempts (3 *marks*)

A1 Diagram with x and/or $3x$ shown

A2 Solution with value assigned to x

(c) (ii) **10 marks** **Att 3**

$$\text{Volume} = \frac{1}{3}\pi\,(2x)^2\,\frac{3x}{2}$$

$$\text{or} \quad 2\pi x^3$$

$$\text{Ratio of volume of second to first} = \frac{1}{3}\pi\,(2x)^2\,\frac{3x}{2} : \frac{1}{3}\pi\,x^2\,3x = 2\pi x^3 : \pi x^3 = 2:1$$

* Accept ratio in any order

Blunders (-3)

B1 Each incorrect substitution into correct formula
B2 Incorrect related formula with substitution
B3 Volumes not expressed as a ratio
B4 Ratio not simplified
B5 $(2x)^2 = 2x^2$

Slips (-1)

S1 Arithmetic slips to a max of -3
A1 Correct formula with some substitution

Attempts (3 *marks*)

A2 Diagram with $2x$ and/or $\frac{3x}{2}$ shown
A3 Ratio with value assigned to x

QUESTION 2

Part (a)	**10(5,5) marks**	**Att(2,2)**
Part (b)	**25(5,10,5,5) marks**	**Att (2,3,2,2)**
Part (c)	**15 marks**	**Att 5**

Part (a) **10 (5,5) marks** **Att (2,2)**

a (1 , 4) and *b* (-2 , -1) are two points.

(i) ✍ Find the slope of *ab*.

(ii) Find the equation of *ab*.

(a) (i) **5 marks** **Att 2**

(i) Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{-2-1} = \frac{-5}{-3}$ or $\frac{5}{3}$

Blunders (-3)
B1 Incorrect slope formula and continues
B2 Mixes both *x* and *y* in substitution
B3 Substitutes correctly but slope not found

Slips (-1)
S1 Incorrect sign after substitution
S2 Arithmetic slips to a max of -3

Attempt (2 *marks*)
A1 Writes slope formula with or without some substitution
A2 Some attempt at difference of *y*'s or difference of *x*'s

(a) (ii) **5 marks** **Att 2**

Equation $y - 4 = \frac{5}{3}(x-1)$ or $y - 4 = \frac{-5}{-3}(x\text{-}1)$

$y\text{- -}1 = \frac{5}{3}(x \text{ - - } 2)$ or $y\text{- -}1 = \frac{-5}{-3}(x \text{ - - } 2)$

Or $y = mx + c$

$y = \frac{5}{3}x + c$

$4 = \frac{5}{3}.1 + c$

$c = 4 - \frac{5}{3}$ or $\frac{7}{3}$

* May find another point on *ab* (e.g. midpoint and continues)

Blunders (-3)
B1 Incorrect relevant formula and continues
B2 Switches both *x* and *y* in substitution
B3 Substitutes correctly for *x* and *y* but no slope

Slips (-1)
S1 Incorrect sign after substitution

Attempts (2 *marks*)
A1 Correct line formula and stops

Part (b) **25 (5,10,5,5) marks** **Att (2,3,2,2)**

L is the line $3x - 4y + 7 = 0$ and contains the point $p\,(-1, h)$.

M is the line $4x + 3y - 24 = 0$ and contains the point $q\,(k, 0)$.

(i) ✍ Find the values of h and k.

(ii) L and M intersect at the point r.

✍ Find the coordinates of r.

(iii) Show p, q, r, L and M on a coordinate diagram on graph paper.

(iv) ✍ Prove that $\angle prq$ is a right angle.

(b) (i) **5 marks** **Att 2**

(i)

$3(-1) - 4h + 7 = 0$ $\qquad$ $4k + 3(0) - 24 = 0$

$h = 1$ $\qquad$ $k = 6$

Blunders (-3)

B1 Mixes x and y in substitution

B2 Transposition error

Slips (-1)

S1 Arithmetic slips to a max of -3 e.g. $3(0) \neq 0$

Attempts (2 *marks*)

A1 Some attempt at substitution

(b) (ii) **10 marks** **Att 3**

(ii)

$3x - 4y + 7 = 0 \Rightarrow 9x - 12y + 21 = 0$

$4x + 3y - 24 = 0 \Rightarrow \underline{16x + 12y - 96 = 0}$

$25x \quad -75 = 0 \qquad \Rightarrow x = 3$

$3x - 4y + 7 = 0 \Rightarrow 9 - 4y + 7 = 0 \Rightarrow y = 4$

* (3,4) without work $\Rightarrow$ Attempt mark

* Accept $(3, 4) \in L$ and $(3,4) \in M$ shown in each case.

Blunders (-3)

B1 Error in manipulation of both equations

B2 Transposition error

B3 No substitution for second value

Slips (-1)

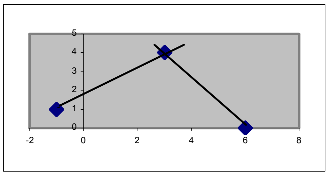
S1 Arithmetic slips to a max of -3

Attempts (3 *marks*)

A1 Any correct step and stops

A2 Graphical solution correct

(b) (iii) **5 marks** **Att 2**



Plot p (-1,1), r (3,4), q (6,0), $p \in L$, $q \in M$, $r \in L \cap M$

Slips (-1)

S1 Each element missing

Attempts (2 *marks*)

A1 One point only plotted

A2 Axes only drawn

(b) (iv) **5 marks** **Att 2**

(iv)

Slope $pr = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-4}{-1-3} = \dfrac{3}{4}$ or L: $y = \dfrac{3}{4}x + \dfrac{7}{4} \Rightarrow$ slope $= \dfrac{3}{4}$

Slope $qr = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-4}{6-3} = \dfrac{-4}{3}$ $M: y = \dfrac{-4}{3}x + 8 \Rightarrow$ slope $= \dfrac{-4}{3}$

$\dfrac{3}{4} \cdot \dfrac{-4}{3} = -1 \Rightarrow |\angle prq|$ right angle

* If product = -1 no need for conclusion

Blunders (-3)

B1 Incorrect relevant formula

B2 Mixes both x and y in substitution

B3 Substitutes correctly but slope not found

B4 Errors in transposition

Slips (-1)

S1 Incorrect conclusion for product $\neq$ -1

S2 Slopes found and stops

S3 Lengths of sides of triangle prq calculated but relationship not established

Attempts (2 *marks*)

A1 Correct formula and stops

Part (c) **15 marks** **Att 5**

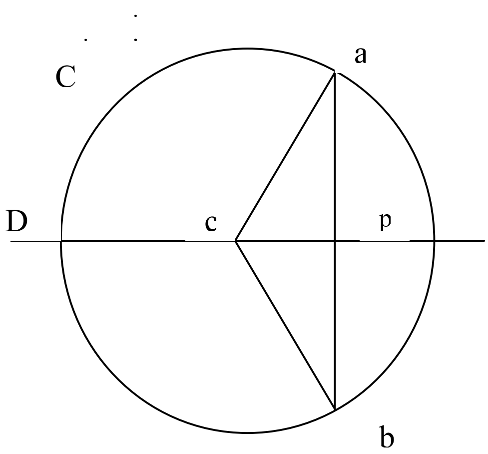
✍ Prove that a line through the centre of a circle perpendicular to a chord bisects the chord.

(c) **15 marks** **Att 5**

Given: Circle C, centre c on D, with chord $ab \perp$ D, and $ab \cap$ D = {p}
Construction: Join ca and cb step 1
To Prove : $|ap| = |bp|$
Proof : $|ca| = |cb|$ step 2
$|\angle cpa| = |\angle cpb|$ (right angles) step3
$|cp| = |cp|$
$\Rightarrow$RHS $\Rightarrow \Delta cap$ and Δcpb congruent step 4
$\Rightarrow |ap| = |bp|$ step5

or $|ca| = |cb|$
$\Rightarrow |\angle cap| = |\angle cbp|$ step2
$|\angle cpa| = |\angle cpb|$ given
$\Rightarrow |\angle acp| = |\angle \text{bcp}|$ step 3
$\Rightarrow$ASA $\Rightarrow \Delta cap$ and Δcpb congruent step 4
$\Rightarrow |ap| = |bp|$ step5

or $|ca| = |cb|$
$\Rightarrow |\angle cap| = |\angle cbp|$ step 2
$|\angle cpa| = |\angle cpb|$ (right angles)
$\Rightarrow |\angle acp| = |\angle \text{bcp}|$ step3
$\Rightarrow |cp| = |cp|$
$\Rightarrow$ SAS $\Rightarrow \Delta cap$ and Δcpb congruent step 4
$\Rightarrow |ap| = |bp|$ step5



* Some steps may be indicated on diagram
* Accept any other valid proofs

Blunders (-3)
B1 Each step incorrect or omitted
B2 Each step incomplete

Attempts (5 *marks*)
A1 Diagram with circle drawn, and diameter or chord indicated

Worthless (0)
W1 Wrong Theorem
W2 Circle and nothing else

QUESTION 3

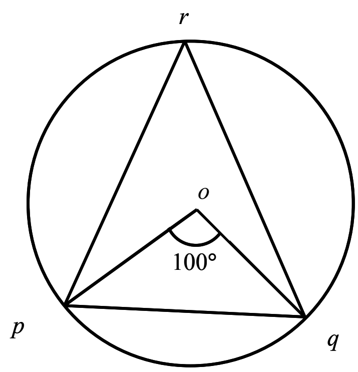
Part (a)	**10(5,5) marks**	**Att (2,2)**
Part (b)	**20 marks**	**Att 7**
Part (c)	**20 (10,10) marks**	**Att (3,3)**

Part (a) **10 (5, 5) marks** **Att(2,2)**

o is the centre of the circle, as shown.

(i) Find $|\angle prq|$.

(ii) ✍ Find $|\angle opq|$



(a) (i) **5 marks** **Att 2**

(i) $|\angle prq| = \frac{1}{2}(100°) = 50°$

* Accept correct answer without work

Blunders (-3)

B1 Finds $\frac{1}{2}$ reflex angle

Slips (-1)

S1 Arithmetic slips to a max of -3

Attempts (2 *marks*)

A1 Reflex angle and stops

A2 $|\angle prq| = 200°$

(a) (ii) **5 marks** **Att 2**

(ii) $|op| = |oq| \Rightarrow |\angle oqp| = |\angle opq|$

$|\angle oqp| + |\angle opq| = 80°$

$|\angle opq| = 40°$

* Accept correct answer on diagram with indication of isosceles triangle

Blunders(-3)

B1 Isosceles triangle not implied or indicated

B2 $|\angle opq| = 80°$

Slips (-1)

S1 Arithmetic slips to a max of -3

Attempts (2 *marks*)

A1 Indicates sum of angles of triangle =180°

Part (b) **20 marks** **Att 7**

Prove that the measure of the angle at a centre of the circle is twice the measure of the angle at the circumference, standing on the same arc.

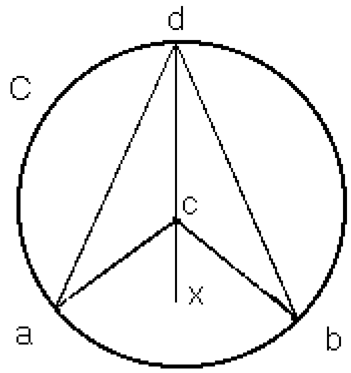
(b) **20 marks** **Att 7**

Given: Circle C, centre *c*, with points *a,b,d* on arc

Construction: Join *ac, bc, ad, bd*

Join *dc* and produce to *x* — Step1

To prove: $|\angle acb| = 2|\angle adb|$



Proof:

$|ac| = |cd|$.

$\Rightarrow |\angle cad| = |\angle adc|$ — step 2

But $|\angle acx| = |\angle cad| + |\angle adc|$ — step 3

$\Rightarrow |\angle acx| = 2|\angle adc|$ — step 4

Similarly $|\angle bcx| = 2|\angle bdc|$

$\Rightarrow |\angle acx| + |\angle bcx| = 2|\angle adc| + 2|\angle bdc|$ — step 5

$\Rightarrow |\angle acb| = 2|\angle adb|$ — step 6

* Steps 1 and 2 may be indicated on diagram

Blunders (-3)

B1 Each step incorrect or omitted

B2 Each step incomplete

Attempts (7 *marks*)

A1 Diagram with angle at centre and or angle at arc indicated

A2 Theorem proven for angle in a semi-circle

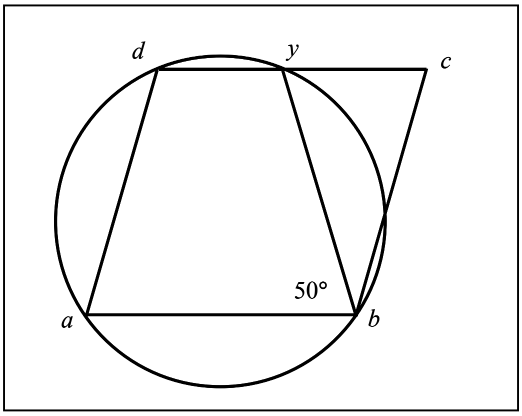
Part (c) **20 (10,10) marks** **Att (3,3)**

abcd is a parallelogram and *a*, *b*, *y* and *d* are points on the circle.

$|\angle aby| = 50°$.

(i) ✍ Find $|\angle ady|$.

(ii) ✍ Prove $|by| = |bc|$.



(c) (i) **10 marks** **Att 3**

(i)	$|\angle ady| = 180° - 50° = 130°$

* Accept correct answer given on diagram

Blunders (-3)

B1 Uses 360° instead of 180°

Slips (-1)

S1 Arithmetic errors to a max of -3

Attempts (3 *marks*)

A1 Indicates sum of the opposite angles in cyclic quadrilateral = 180°

A2 Diagram drawn with a correct modification

Worthless (0)

W1 $|\angle ady| = 50°$

(c) (ii) **10 marks** **Att 3**

(ii) $\lvert\angle abc\rvert = 130°$ $\Rightarrow \lvert\angle ybc\rvert = 80°$ $\lvert\angle aby\rvert = \lvert\angle byc\rvert$ alternates $\Rightarrow \lvert\angle byc\rvert = 50°$ $\Rightarrow \lvert\angle bcy\rvert = 180° - (80° + 50°) = 50°$ $\Rightarrow \lvert\angle byc\rvert = \lvert\angle bcy\rvert$ $\Rightarrow \lvert by\rvert = \lvert bc\rvert$	or $\lvert\angle dab\rvert = \lvert\angle bcy\rvert$ $\lvert\angle dab\rvert + \lvert\angle byd\rvert = \lvert byd\rvert + \lvert\angle byc\rvert$ $\Rightarrow \lvert\angle dab\rvert = \lvert\angle byc\rvert$ Then $\lvert\angle bcy\rvert = \lvert\angle byc\rvert$ $\Rightarrow \lvert by\rvert = \lvert bc\rvert$

Blunders (-3)
B1 Angles proven equal but no conclusion indicated

Slips (-1)
S1 Arithmetic slips to a max of -3

Attempts (3 *marks*)
A1 Indicates $\lvert\angle ybc\rvert = 80°$
A2 Indicates $\lvert\angle byc\rvert = 50°$

Worthless (0)
W1 Takes Δ *bcy* as right-angled
W2 $\lvert\angle abc\rvert$ right-angled

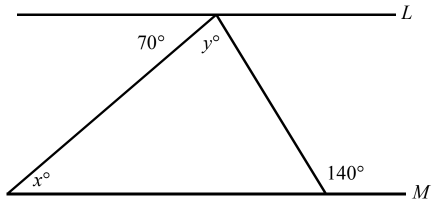
QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20(10,10)marks**	**Att (3,3)**
Part (c)	**20 (5,5,10) marks**	**Att (2,2,3)**

Part (a) **10 marks** **Att 3**

The line L is parallel to the line M.

✍ Calculate the value of x and the value of y, in the diagram.



(a) **10 marks** **Att 3**

$x = 70°$ or $x + y = 140° \Rightarrow 70° + y = 140 \Rightarrow y = 70°$

$40° + x + y = 180°$.

$40° + 70° + y = 180° \Rightarrow y = 70°$

* Accept answers indicated on diagram

Blunders (-3)

B1 One value found

Slips (-1)

S1 Arithmetic slip

Attempts (3 *marks*)

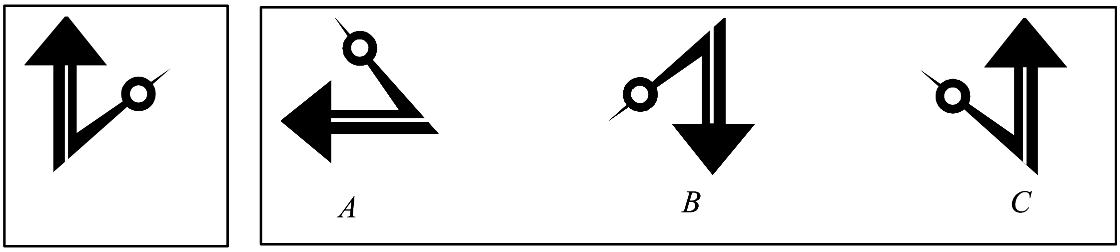
A1 $x + y = 140°$

A2 Sum of angles of triangle equals 180°

Part (b) **20 (10,10) marks** **Att (3,3)**

(i) ✍ Show how to divide a line segment into three equal parts. All construction lines must be clearly shown.

(ii) Each of the three figures labelled *A*, *B* and *C* shown below in the box on the right is the image of the figure shown in the box on the left under a transformation. For each of *A*, *B* and *C*, state what the transformation is (translation, central symmetry, axial symmetry or rotation) and in the case of a rotation, state the angle.

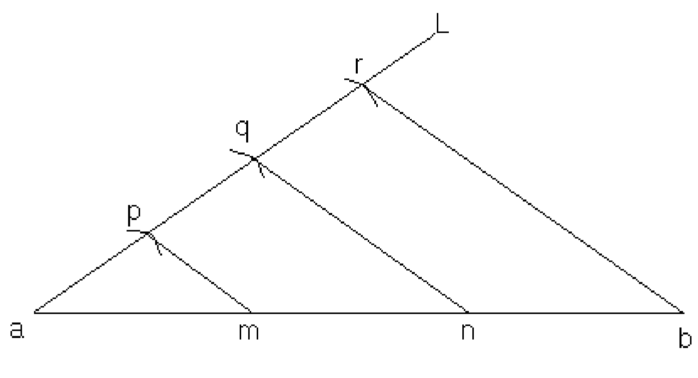


(b) (i) **10 marks** **Att 3**

(i) To divide a line segment [*ab*] into three equal parts

Construction: Draw a line L at an angle to [*ab*]
With a compass mark points *p,q,r* on L
such that $|ap| = |pq| = |qr|$.
Join *rb*
Draw lines through *p* and *q* parallel to *rb.* These lines meet [*ab*] at *m* and *n*.

m and *n* are the points which divide [*ab*] into three equal parts.



* No need for written explanation

* Allow a tolerance of 2 mm for points on L, and for points on [*ab*]

Blunders (-3)

B1 For each parallel not shown in construction

B2 Third arc not joined to the point *b*

Attempts (3 *marks*)

A1 Line L drawn

A2 Straight line divided into three equal parts

(b) (ii) **10 marks** **Att 3**

(ii)	*A*	Rotation 90°(anti-clockwise) or 270°(clockwise)
	B	Central Symmetry or Rotation 180°
	C	Axial Symmetry

* Accept angle of rotation without reference to clockwise or anticlockwise
* One correct transformation 4 marks
* Two correct transformations 7 marks
* Three correct transformations 10 marks

Slips (-1)
S1 No angle or incorrect angle of rotation

Attempts (3 *marks*)
A1 Any attempt at drawing the original figure under one of the given transformations

Part (c) **20 (5,5,10) marks** **Att (2,2,3)**

$[om]$ is parallel to $[pq]$.

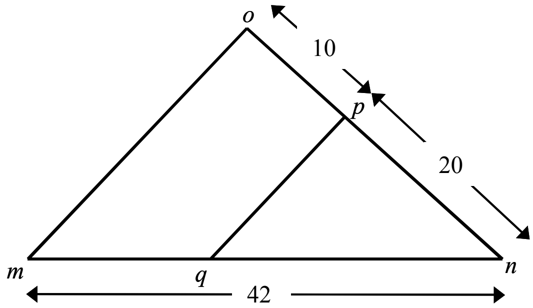
$|op| = 10$ cm, $|pn| = 20$ cm

and $|mn| = 42$ cm.

(i) ✍ Find $|qm|$.

(ii) ✍ If $|qm| = |pq|$,

find $|om|$.



(iii) ✍ Find $\dfrac{\text{area } \Delta\ pqn}{\text{area } \Delta\ omn}$ as a fraction in its simplest form.

[Hint area of $\Delta = \frac{1}{2}ab\sin C$].

(c) (i) **5 marks** **Att 2**

(i) $\frac{30}{10} = \frac{42}{|qm|} \Rightarrow |qm| = \frac{42.10}{30} = 14$ or $\frac{20}{10} = \frac{42-|qm|}{|qm|}$

$\Rightarrow 20\,|qm| = 10(42- \,|qm|)$

$\Rightarrow 30\,|qm| = 420$

$\Rightarrow |qm| = 14$

or $\frac{20}{10} = \frac{2}{1} \Rightarrow |qm| = \frac{1}{3}(42) = 14$

Blunders (-3)

B1 $\frac{30}{10} = \frac{|qm|}{42}$ or equivalent

B2 Transposition error

Slips (-1)

S1 Arithmetic slip

S2 $|qn|$ correct, but $|qm|$ not found

Attempts (2 *marks*)

A1 One correct relevant ratio

(c) (ii) **5 marks** **Att 2**

(ii)	$\lvert qm \rvert = \lvert pq \rvert = 14 \quad \Rightarrow \dfrac{20}{14} = \dfrac{30}{\lvert om \rvert} \Rightarrow \lvert om \rvert = 21$

Blunders (-3)

B1 $\dfrac{14}{20} = \dfrac{30}{\lvert om \rvert}$ or equivalent

B2 Transposition error

Slips (-1)

S1 Arithmetic slips to a max of -3

Attempts (2 *marks*)

A1 One correct relevant ratio

(c) (iii) **10 marks** **Att 3**

(iii) $\dfrac{area\Delta\, pqn}{area\Delta\, omn} = \dfrac{\frac{1}{2}20.28.Sin\lvert\angle qnp\rvert}{\frac{1}{2}30.42Sin\lvert\angle mno\rvert}$ $= \dfrac{4}{9}$.	or $\lvert\angle qpn\rvert = \lvert\angle mon\rvert$ since $qp \,//\, mo$ $\dfrac{area\Delta\, pqn}{area\Delta\, omn} = \dfrac{\frac{1}{2}.20.14.Sin\lvert\angle qpn\rvert}{\frac{1}{2}.30.21.Sin\lvert\angle mon\rvert} = \dfrac{4}{9}$.

Blunders (-3)

B1 Incorrect relevant formula

B2 Substitutes incorrectly into correct formula

B3 No indication of equal angles (method 2)

B4 Ratio not indicated

B5 Ratio not simplified

B6 Transposition error

Slips (-1)

S1 Arithmetic slips to a max of -3

S2 Fraction not in simplest form

Attempts (3 *marks*)

A1 Area of triangle with some substitution

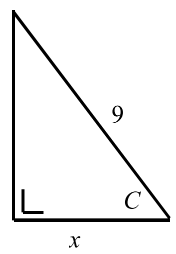
QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10,10) marks**	**Att (3,3)**
Part (c)	**20(5,5,10) marks**	**Att (2,2,3)**

Part (a) **10 marks** **Att 3**

✍ Given that $\cos C = \frac{2}{3}$,

find the value of x.



(a) **10 marks** **Att 3**

$$\text{Cos } C = \frac{x}{9} \quad \Rightarrow \frac{2}{3} = \frac{x}{9} \Rightarrow x = 6$$

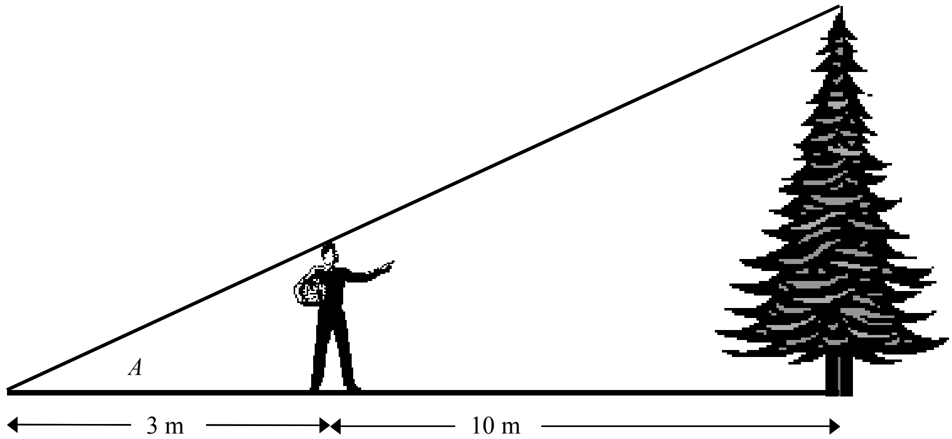
Blunders (-3)
- B1 Incorrect ratio for Cos C
- B2 Error in cross multiplication
- B3 Incorrect ratio in use of Sin function or Sine Rule
- B4 Reads wrong page of tables or uses calculator in incorrect mode
- B5 $1^0 \neq 60^1$
- B6 $\frac{2}{3} = \frac{x}{9}$ and stops

Slips (-1)
- S1 Arithmetic slips to a max of -3
- S2 Slip reading tables (e.g. wrong column)
- S3 Fails to distinguish between degrees and minutes and degrees in decimal format

Attempts (3 *marks*)
- A1 Indicates use of x and 9 in a ratio
- A2 Finds measure of angle C and stops
- A3 Finds value of third angle and stops
- A4 Writes down Sin, Cos, or Tan ratio and stops

Part (b) **20 (10,10) marks** **Att (3,3)**

Some students wish to estimate the height of a tree standing on level ground. One of them stands so that the end of his shadow coincides with the end of the shadow of the tree, as shown in the diagram. This student is $1 \cdot 6$ m tall.
His friend then measures the distances shown in the diagram.
A is the angle of elevation of the sun.



(i) ✍ Find *A*, correct to the nearest degree.

(ii) ✍ Find the height of the tree correct to one decimal place.

(b) (i) **10 marks** **Att 3**

(i) $\text{Tan } A = \frac{1.6}{3} = .5333 \Rightarrow A = 28 \cdot 07° = 28°$

Blunders (-3)
B1 Incorrect ratio for Tan A
B2 Error in cross multiplication
B3 Reads wrong page of tables or uses calculator in incorrect mode or finds Tan(.5333)
B4 Error in Theorem of Pythagoras.
B5 Incorrect ratio in use of Sin function or Cos function
B6 Incorrect ratio in use of Sine Rule
B7 $1^0 \neq 60^1$
B8 Tan A = .5333 and stops
B9 Early rounding off which affects final answer

Slips (-1)
S1 Arithmetic slips to a max of -3
S2 Finds value of other acute angle
S3 Slip reading tables (e.g. wrong column)
S4 Fails to distinguish between degrees and minutes and degrees in decimal format
S5 Fails to round off
Attempts (3 *marks*)
A1 Indicates use of 1.6 and 3 in a ratio
A2 Finds value of hypotenuse and stops

(b) (ii) **10 marks** **Att 3**

(ii) $\text{Tan } A = \frac{h}{13} \Rightarrow h = 13\,.(\,.5333) = 6\cdot 9329 = 6.9$ or $\frac{3}{1.6} = \frac{13}{h} \Rightarrow h = 6.933 = 6.9$

or $\frac{Sin28^0}{h} = \frac{Sin62^0}{13} \Rightarrow h = \frac{13\,Sin28^0}{Sin62^0} = 6.933 = 6.9$

* Accept 28.07^0 or 28^0 from above

Blunders (-3)

B1 Incorrect ratio for Tan function
B2 Error in cross multiplication
B3 Reads wrong page of tables or uses calculator in incorrect mode
B4 Incorrect ratio in use of Sine Rule
B5 Incorrect ratio in use of Sin function or Cos function
B6 Takes adjacent as 10 instead of 13
B7 $1^0 \neq 60^1$
B8 $\frac{3}{1.6} = \frac{h}{13}$ or equivalent
B9 Takes 10 instead of 13 in ratio method

Slips (-1)

S1 Arithmetic slips to a max of -3
S2 Slip reading tables (e.g. wrong column)
S3 Fails to distinguish between degrees and minutes and degrees in decimal format
S4 Fails to round off or rounds off incorrectly

Attempt (3 *marks*)

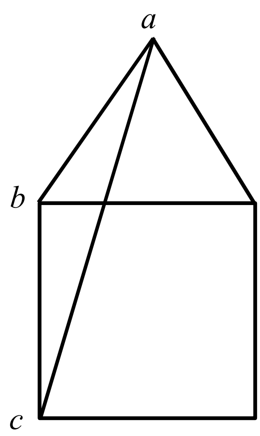
A1 Indicates use of h in a ratio
A2 Indicates use of 3 and 1.6 in a ratio
A3 Indicates use of 13
A4 Sine Rule with some substitution
A5 Calculates hypotenuse (14.7) and stops
A6 Finds angle 62^0 and stops

Part (c) **20 (5, 5,10) marks** **Att (2,2,3)**

The diagram shows an equilateral triangle and a square, each of side 6.
a is joined to c.

(i) ✍ ` Find $|\angle abc|$ and $|\angle bac|$.

(ii) ✍ Find $|ac|$, correct to one decimal place.



(c) (i) **10(5, 5) marks** **Att (2,2)**

(i)	$|\angle abc| = 60° + 90° = 150°$	$|\angle bac| = |\angle bca|$ since $|bc| = |ba|$
		$\Rightarrow |\angle bac| = \frac{1}{2}(180° - 150°) = 15°$

* Accept answer given on diagram
* Accept candidates answer for $|\angle abc|$ for further work

Blunders (-3)
B1 Fails to divide by 2

Slips (-1)
S1 Arithmetic slip
S2 60° and 90° indicated but not added

Attempts (2 *marks*)
A1 Indicates 60° angle(s) in equilateral triangle
A2 Indicates angle(s) in square 90°
A3 Indicates 180° is sum of angles in triangle
A4 Identifies that triangle is isosceles

(c) (ii) **10 marks** **Att 3**

(ii)	$\frac{Sin150^0}{\lvert ac \rvert} = \frac{Sin15^0}{6} \Rightarrow \lvert ac \rvert = \frac{6Sin150^0}{Sin15^0} = \frac{6(.5)}{(0.2588)} = 11\cdot 59 = 11\cdot 6$

* $|\angle abc|$ treated as 60° or 90° gives rise to special cases, apply A1 at most.

Blunders (-3)
B1 Incorrect ratio in use of Sine Rule
B2 Error in cross multiplication
B3 Reads wrong page of tables or uses calculator in incorrect mode
B4 $1^0 \neq 60^1$
B5 Early rounding off which effects answer

Slips (-1)
S1 Arithmetic slips to a max of -3
S2 Slip reading tables (e.g. wrong column)
S3 Fails to distinguish between degrees and minutes and degrees in decimal format
S4 Fails to round off

Attempts (2 *marks*)
A1 Sine Rule with some substitution

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10,10) marks**	**Att (3,3)**
Part (c)	**20(5,10,5) marks**	**Att(2,3,2)**

Part (a) **10 marks** **Att 3**

✍ 6 is the mean of the numbers 3, 1, 9, x, 5.

Find the value of x.

(a) **10 marks** **Att 3**

$$\frac{3+1+9+x+5}{5} = 6 \Rightarrow \frac{18+x}{5} = 6 \Rightarrow 18 + x = 30 \Rightarrow x = 12$$

Blunders (-3)
B1 Incorrect denominator
B2 Error in transposition
B3 $18x$ in numerator

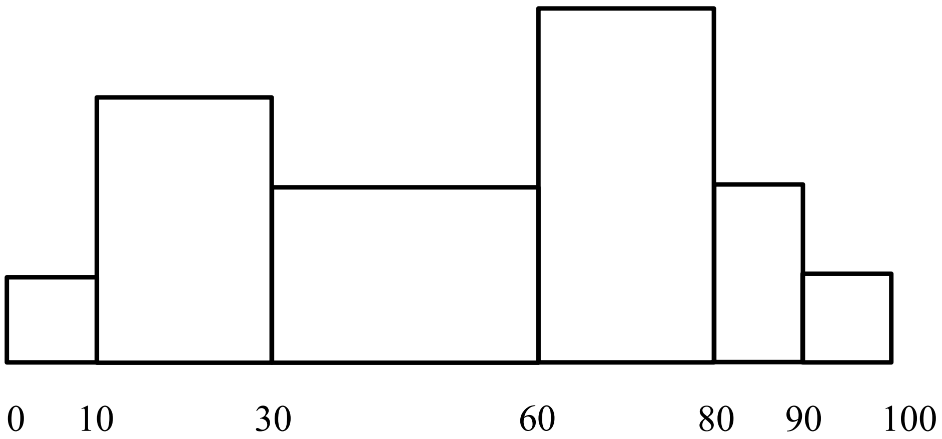
Slips (-1)
S1 Arithmetic slips to a max of -3

Attempts (3 *marks*)
A1 Adds some or all of the numbers
A2 Indication of division by 5
A3 $\frac{3+1+9+x+5}{5} = 6$ and stops

Part (b) **20 (10,10) marks** **Att (3,3)**

The times taken by a number of athletes to finish a race after the winner crossed the finish line were recorded.

The results are shown in the following histogram.



(i) Given that there are 6 athletes in the 10 – 30 time interval, complete the following frequency table.

Number of seconds after winner	0 – 10	10 – 30	30 – 60	60 – 80	80 – 90	90–100
Number of athletes		**6**				

[Note 10 – 30 means 10 or more but less than 30, etc.]

(ii) ✍ Taking mid-interval values, calculate the mean time taken to finish the race after the winner, correct to the nearest second.

(b) (i) **10 marks** **Att 3**

Number of seconds after winner	0 – 10	10 – 30	30 – 60	60 – 80	80 – 90	90–100
Number of athletes	1	**6**	6	8	2	1

Blunders (-3)
B1 Heights taken as frequency
B2 Correct ratios but incorrect values
B3 Mishandling of base

Slips (-1)
S1 Arithmetic slips to a max of -3

Attempts (3 *marks*)
A1 Bases taken as frequency

(b) (ii) **10 marks** **Att 3**

(ii)	Mean = $\dfrac{1(5)+6(20)+6(45)+8(70)+2(85)+1(95)}{1+6+6+8+2+1} = \dfrac{1220}{24}$ $= 50\cdot 83$ =51 sec

* Accept candidate's work from (b) (i)

Blunders (-3)
B1 Division by 6
B2 Division by sum of mid interval values
B3 Use of value other than mid interval values
B4 Consistently adds mid interval value to frequency instead of multiplying

Slips (-1)
S1 Arithmetic slips to a max of -3
S2 Fails to round off

Attempts (3 *marks*)
A1 Some or all mid intervals identified
A2 One correct multiplication in numerator
A3 Indicates division by 24
A4 Sum of frequencies divided by 6 or sum of mid interval values divided by 6

Part (c) **20 (5,10, 5) marks** **Att (2,3,3)**

The number of people voting in a polling station on election day was recorded every two hours. The following are the results.

Time	800 – 1000	1000 – 1200	1200 – 1400	1400 – 1600	1600 – 1800	1800 – 2000	2000 – 2200
Number of people	200	300	250	350	800	550	350

[Note 1000 – 1200 means 1000 or later but before 1200, etc.]

(i) Draw up a cumulative frequency table.

(ii) On graph paper construct the ogive.

(iii) ✍ Use your graph to estimate the number of people who cast their vote between 1700 and 1900.

(c) (i) **5 marks** **Att 2**

Time	800 – 1000	800 – 1200	800 – 1400	800 – 1600	800 – 1800	800 – 2000	800 – 2200
Number of people	200	500	750	1100	1900	2450	2800

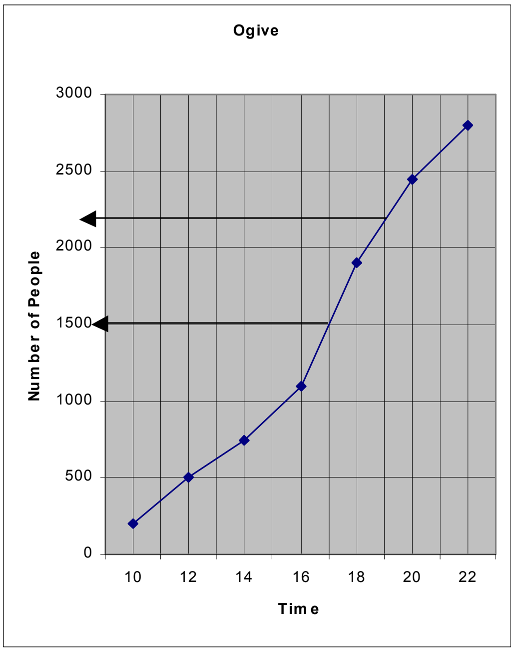
Blunders (-3)
B1 Omits any number or puts numbers in wrong place

Slips (-1)
S1 Arithmetic slips to a max of -3

Attempts (2 *marks*)
A1 Any one value filled in correctly into table
A2 Any indication of addition of frequencies

Worthless (0)
W1 Copies table and stops

(c) (ii) **10 marks** **Att 3**



Blunders (-3)
B1 Incorrect scales
B2 Plots points but does not join them
B3 Draws a 'cumulative' histogram
B4 Points joined with straight lines
B5 Draws trend graph from original table

Slips (-1)
S1 Each incorrect plot
S2 Each point omitted

Attempts (3 *marks*)
A1 Draws axes and stops

(c) (iii) **5 marks** **Att 2**

(iii)	2175 – 1500	=	675

* Accept answer consistent with candidate's ogive with a tolerance of ± 200
* Trend graph or cumulative histogram in (c) (i) attracts attempt mark at most in (c) (ii).

Blunders (-3)
B1 Line drawn from incorrect starting point of correct axis (once only)
B2 No subtraction of values indicated

Slips (-1)
S1 Work correct but outside tolerance
S2 Adds both values

Attempts (2 *marks*)
A1 Graphical indication, but number not stated