AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA | DEPARTMENT OF EDUCATION AND SCIENCE

Scéimeanna Marcála *Scrúduithe Ardteistiméireachta,* 2001

Matamaitic *Ardleibhéal*

Marking Scheme *Leaving Certificate Examination,* 2001

Mathematics *Higher Level*

An Roinn Oideachais agus Eolaíochta

Leaving Certificate Examination 2001

Marking Scheme

MATHEMATICS

Higher Level

Paper 1

General Instructions

Penalties are applied as follows:

numerical slips, misreadings	(-1) each
blunders, major omissions	(-3) each.

Note 1: The lists of slips, blunders and attempts given in the Marking Scheme are not exhaustive.

Note 2: A serious blunder, omission or misreading merits the attempt mark at most.

Note 3: If deductions result in a mark which is less than the attempt mark, the attempt mark is awarded. The attempt mark for a section is the final mark for that section.

Note 4: Particular cases and verifications are, in general, awarded the attempt mark only.

Note 5: Mark all of the candidate's work (including any that is cancelled) and allow the highest scoring solutions.

~~~~~~
~~~~~~

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a)	**10 marks**	**Att 3**

1(a) Find the real numbers a and b such that

$$x^2 + 4x - 6 = (x + a)^2 + b \text{ for all } x \in R.$$

Part (a)	**10 marks**	**Att 3**

1(a)

$$x^2 + 4x - 6 = (x + a)^2 + b$$
$$(x^2 + 4x + 4) - 6 - 4 = (x + a)^2 + b$$
$$(x + 2)^2 - 10 = (x + a)^2 + b$$
$$a = 2, \quad b = -10$$

or

1(a)

$$x^2 + 4x - 6 = (x + a)^2 + b$$
$$x^2 + (4)x + (-6) = x^2 + 2ax + a^2 + b$$

$4 = 2a$ and $-6 = a^2 + b$

$a = 2$ $\qquad -6 = 4 + b$

$\qquad -10 = b$

* Accept solutions based on two values of x.

Blunders (-3)
B1 indices.
B2 expansion of $(x + a)^2$.
B3 completing square.
B4 not like to like.
B5 no 'a' or deduction 'a'.
B6 no 'b' or deduction 'b'.

Slips (-1)
S1 numerical.

Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

1(b) Let $f(x) = 2x^3 + mx^2 + nx + 2$ where m and n are constants.

Given that $x - 1$ and $x + 2$ are factors $f(x)$, find the value of m and the value of n.

$f(1) = 0$	**5 marks**	**Att 2**
$f(-2) = 0$	**5 marks**	**Att 2**
Equations	**5 marks**	**Att 2**
Solving	**5 marks**	**Att 2**

1(b)

$f(x) = 2x^3 + mx^2 + nx + 2$

$(x - 1)$ factor $\Rightarrow f(1) = 0$

$f(1) = 2(1)^3 + m(1)^2 + n(1) + 2 = 0$

$m + n = -4$ ……………………*(i)*

$(x + 2)$ factor $\Rightarrow f(-2) = 0$

$f(-2) = 2(-2)^3 + m(-2)^2 + n(-2) + 2 = 0$

$-16 + 4m - 2n + 2 = 0$

$4m - 2n = 14$

$2m - n = 7$ ……………………………*(ii)*

(i) $m + n = -4$

(ii) $\underline{2m - n = 7}$

$3m = 3$

$m = 1$

(i) $m + n = -4$

$1 + n = -4$

$n = -5$

or

1(b) $(x - 1)(x + 2) = (x^2 + x - 2)$ is a factor.

$(x^2 + x - 2)(ax + b) = 2x^3 + mx^2 + nx + 2$

$\Rightarrow ax^3 = 2x^3$ and $-2b = 2$

$a = 2$ $b = -1$

Factors: $(x^2 + x - 2)(2x - 1) = 2x^3 + 2x^2 - 4x - x^2 - x + 2$

$= 2x^3 + x^2 - 5x + 2$

$m = 1$ and $n = -5$

or

1(b) Since $(x-1)$ and $(x+2)$ factors $\Rightarrow \left(x^2+x-2\right)$ is a factor.

$$\begin{array}{r} 2x+(m-2) \\ x^2+x-2\overline{)2x^3+mx^2+nx+2} \\ \underline{2x^3+2x^2-4x} \\ (m-2)x^2+(n+4)x+2 \\ \underline{(m-2)x^2+(m-2)x-2(m-2)} \\ (n+4)x-(m-2)x+2+2(m-2) \\ (n-m+6)x+(2m-2) \end{array}$$

Since $\left(x^2+x-2\right)$ is a factor, remainder = 0

$\Rightarrow (i): n-m+6=0$ and (ii): $2m-2=0$

$\Rightarrow m=1$

(i): $n-1+6=0$ $\Rightarrow n=-5$

or

1(b)

$(x-1)$ is a factor

$$\begin{array}{r} 2x^2+(m+2)x+(n+m+2) \\ (x-1)\overline{)2x^3+mx^2+nx+2} \\ \underline{2x^3-2x^2} \\ (m+2)x^2+nx \\ \underline{(m+2)x^2-(m+2)x} \\ (n+m+2)x+2 \\ \underline{(n+m+2)x-(n+m+2)} \\ 2+(n+m+2) \end{array}$$

Since $(x-1)$ is a factor, $n+m+4=0$ ……………………*(i)*

$(x+2)$ is a factor

$$\begin{array}{r} 2x^2+(m-4)x+(n-2m+8) \\ x+2\overline{)2x^3+mx^2+nx+2} \\ \underline{2x^3+4x^2} \\ (m-4)x^2+nx \\ \underline{(m-4)x^2+2(m-4)} \\ (n-2m+8)x+2 \\ \underline{(n-2m+8)x+2(n-2m+8)} \\ 2-2(n-2m+8) \end{array}$$

Since $(x-1)$ is a factor, $2-2n+4m-16=0$

$\Rightarrow 4m-2n=14$

$\Rightarrow 2m-n=7$ ……………………*(ii)*

Combining *(i)* and *(ii)*:

(i) $m+n=-4$

(ii) $\underline{2m-n=7}$

$\Rightarrow 3m=3 \Rightarrow m=1$ and $n=-5$

Blunders (-3)
B1 deduction of root from factor.
B2 indices.
B3 2nd value not found (having found 1st).

Slips (-1)
S1 numerical.

Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

1(c)

$x^2 - px + q$ is a factor of $x^3 + 3px^2 + 3qx + r$.

(i) Show that $q = -2p^2$.

(ii) Show that $r = -8p^3$.

(iii) Find the three roots of $x^3 + 3px^2 + 3qx + r = 0$ in terms of *p*.

(i) and (ii)	**10 marks**	**Att 3**
(iii)	**10 marks**	**Att 3**

1(c)(i) and (ii)

$$\begin{array}{r|l} & x + 4p \\ x^2 - px + q & x^3 + 3px^2 + 3q + r \\ & x^3 - px^2 + qx \\ \hline & 4px^2 + 2qx + r \\ & 4px^2 - 4p^2x + 4pq \\ \hline & 2qx + 4p^2x + r - 4pq \\ & (2q + 4p^2)x + (r - 4pq) \end{array}$$

$(x^2 - px + q)$ is a factor $\Rightarrow$ remainder = 0

$$(2q + 4p^2)x + (r - 4pq) = (0)x + 0$$

$$2q + 4p^2 = 0 \quad \text{and} \quad r - 4pq = 0$$

(i)
$$2q + 4p^2 = 0$$
$$2q = -4p^2$$
$$q = -2p^2$$

(ii)
$$r = 4pq$$
$$= 4p(-2p^2)$$
$$= -8p^3$$

1(c)(iii)

$$(x+4p)(x^2-px+q)=0$$

$x+4p=0$ or $x^2-px+q=0$

$x=-4p$ or $x=\dfrac{p\pm\sqrt{p^2-4q}}{2}=\dfrac{p\pm\sqrt{p^2+8p^2}}{2}$

$$=\frac{p\pm\sqrt{9p^2}}{2}$$

$$=\frac{p\pm 3p}{2}$$

$$=\frac{4p}{2}\ or\ \frac{-2p}{2}$$

$$=2p\ \ or\ -p$$

$\Rightarrow$ roots are $-4p$, $-p$ and $2p$

or

1(c)(iii)

$$(x+4p)(x^2-px+q)=0$$

$x+4p=0$ $\quad$ $x^2-px+q=0$

$x=-4p$ $\quad$ $x^2-px-2p^2=0$

$$(x-2p)(x+p)=0$$

$$x=2p\ \ or\ x=\ -p$$

$\Rightarrow$ roots are $-4p$, $-p$ and $2p$

or

1(c)(iii) Let $(x+a)$ be 2nd factor

$$(x^2-px+q)(x+a)=x^3+3px^2+3qx+r$$

$$(x^2-px-2p^2)(x+a)=x^3+3px^2-6p^2x-8p^3$$

Equating terms independent of x:

$$(-2p^2)(a)=-8p^3$$

$$a=\frac{8p^3}{2p^2}=4p$$

$$f(x)=(x+4p)(x^2-px-2p^2)$$

$$=(x+4p)(x-2p)(x+p)$$

$\Rightarrow$ roots are $-4p$, $-p$ and $2p$

Blunders (-3)
B1 indices.
B2 not like to like.
B3 R not shown.
B4 root from factor.
B5 root formula, once only.
B6 factors, once only.

Slips (-1)
S1 numerical.
S2 not changing sign when subtracting in division.

Attempts
A1 remainder $\neq 0$ in division.
A2 any attempt at division.
A3 quadratic not worked on.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

Part (a) **10 marks** **Att 3**

2(a)

Solve the simultaneous equations

$$x - y = 0$$
$$(x+2)^2 + y^2 = 10.$$

Part (a) **10 marks** **Att 3**

2(a)

(i) : $x - y = 0$

(ii) : $(x+2)^2 + y^2 = 10$

(i) : $x = y$

(ii) $(x+2)^2 + x^2 = 10$

$$x^2 + 4x + 4 + x^2 - 10 = 0$$
$$2x^2 + 4x - 6 = 0$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$

$x = -3$ *or* $x = 1$

$y = -3$ *or* $y = 1$

Blunders (-3)
B1 indices.
B2 expansion of $(x+2)^2$.
B3 factors, once only.
B4 deduction of root from factor.
B5 not getting 2^{nd} value (having got 1^{st}).
B6 root formula.

Slips (-1)
S1 numerical.

Attempts
A1 not quadratic.

Worthless
W1 trial and error.

Part (b)(i)	**10 marks**	**Att 3**

2(b)(i) Solve for x

$$|3x+5| < 4.$$

Part (b)(i)	**10 marks**	**Att 3**

2(b)(i)

$$|3x+5| < 4$$

$$\Rightarrow -4 < 3x+5 < 4$$

$$-4 < 3x+5 \qquad\qquad 3x+5 < 4$$

$$-9 < 3x \qquad\qquad 3x < -1$$

$$-3 < x \qquad\qquad x < -\frac{1}{3}$$

$$\Rightarrow \quad -3 < x < -\frac{1}{3}$$

or

2(b)

$$|3x+5| < 4$$

$$(3x+5)^2 < (4)^2$$

$$9x^2 + 30x + 25 - 16 < 0$$

$$9x^2 + 30x + 9 < 0$$

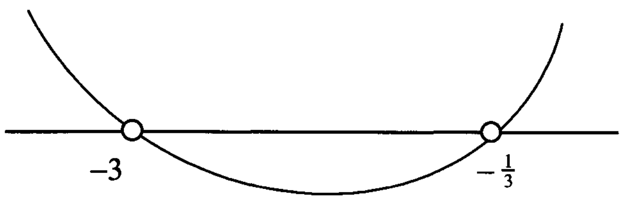
$$3x^2 + 10x + 3 < 0$$

Note: $+x^2 \Rightarrow$ Min Pt

Note: when $3x^2 + 10x + 3 = 0$

$\Rightarrow (3x+1)(x+3) = 0$

$\Rightarrow x = -\frac{1}{3}$ *or* $x = -3$



$$-3 < x < -\frac{1}{3}$$

Blunders (-3)

B1 upper limit.
B2 lower limit.
B3 expansion of $(3x+5)^2$.
B4 inequality sign, e.g. changes to (=), once only.
B5 indices.
B6 factors, once only.
B7 root formula, once only.
B8 deduction of root from factor.
B9 range not shown.
B10 incorrect range.

Attempts
A1 ignores absolute value.

Slips (-1)
S1 numerical.
S2 $\leq$.

Part (b)(ii)	**10 (5, 5) marks**	**Att (2, 2)**

2(b)(ii)	Simplify $\left(x^2+\sqrt{2}+\frac{1}{x^2}\right)\left(x^2-\sqrt{2}+\frac{1}{x^2}\right)$ and express your answer in the form $x^n+\frac{1}{x^n}$ where n is a whole number.

Simplify	**5 marks**	**Att 2**
Express	**5 marks**	**Att 2**

2(b)(ii)

$$\left(x^2+\sqrt{2}+\frac{1}{x^2}\right)\left(x^2-\sqrt{2}+\frac{1}{x^2}\right)$$
$$=\left[\left(x^2+\frac{1}{x^2}\right)+\sqrt{2}\right]\left[\left(x^2+\frac{1}{x^2}\right)-\sqrt{2}\right]$$
$$=\left(x^2+\frac{1}{x^2}\right)^2-\left(\sqrt{2}\right)^2$$
$$=x^4+2\left(x^2\right)\left(\frac{1}{x^2}\right)+\frac{1}{x^4}-2$$
$$=x^4+2+\frac{1}{x^4}-2$$
$$=x^4+\frac{1}{x^4}$$

or

Simplify	**5 marks**	**Att 2**
Express	**5 marks**	**Att 2**

2(b)(ii)

$$\begin{array}{l} x^2+\sqrt{2}+\frac{1}{x^2} \\ x^2-\sqrt{2}+\frac{1}{x^2} \\ \hline x^4+\sqrt{2}x^2+1 \\ \quad -\sqrt{2}x^2-2-\frac{\sqrt{2}}{x^2} \\ \qquad\qquad 1+\frac{\sqrt{2}}{x^2}+\frac{1}{x^4} \\ \hline x^4 \qquad\qquad\qquad +\frac{1}{x^4} \end{array}$$

Blunders (-3)
B1 indices.
B2 expansion of $(a^2+b^2)^2$.
B3 answer not in required form.

Slips (-1)
S1 numerical.
S2 answer not tidied up.

Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

2(c) α and β are real numbers such that $\alpha+\beta=-7$ and $\alpha\beta=11$.

(i) Show that $\alpha^2+\beta^2=27$

(ii) Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ and write your answer in the form $px^2+qx+r=0$ where $p,q,r\in Z$.

(i)	**5 marks**	**Att 2**
(ii)	**15 marks**	**Att 5**

2(c)(i) Given: $\alpha+\beta=-7$ and $\alpha\beta=11$

$$\alpha^2+\beta^2=\alpha^2+2\alpha\beta+\beta^2-2\alpha\beta$$
$$=(\alpha+\beta)^2-2(\alpha\beta)$$
$$=(-7)^2-2(11)$$
$$=49-22$$
$$=27$$

(ii) Roots of new equation : $\frac{\alpha}{\beta}$, $\frac{\beta}{\alpha}$

Sum = $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{27}{11}$

Product = $\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)=1$

Equation : $x^2-(\text{Sum Roots})x+(\text{Product Roots})=0$

$$x^2-\left(\frac{27}{11}\right)x+(1)=0$$
$$11x^2-27x+11=0$$

Blunders (-3)
B1 indices.
B2 expansion of $(\alpha+\beta)^2$.
B3 statement of quadratic equation.
B4 answer not in required form.

Slips (-1)
S1 numerical.

Attempts
A1 not quadratic equation.
A2 incorrect sum or product.

QUESTION 3

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**

Part (a)	**10 (5, 5)marks**	**Att (2, 2)**

3(a) Let $u = \dfrac{1+3i}{3+i}$ where $i^2 = -1$.

(i) Express u in the form $a+ib$ where $a, b \in R$.

(ii) Evaluate $|u|$.

(i)	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**

3(a)(i) $u = \dfrac{1+3i}{3+i} \cdot \dfrac{3-i}{3-i} = \dfrac{3+9i-i-3i^2}{9+1} = \dfrac{6+8i}{10}$

$u = \dfrac{3}{5} + \left(\dfrac{4}{5}\right)i$

(ii) $|u| = \left|\dfrac{3}{5} + \left(\dfrac{4}{5}\right)i\right| = \sqrt{\left(\dfrac{3}{5}\right)^2 + \left(\dfrac{4}{5}\right)^2} = \sqrt{\dfrac{9}{25} + \dfrac{16}{25}} = 1$

Blunders (-3)
B1 *i.*
B2 indices.
B3 modulus.

Slips (-1)
S1 numerical.

Attempts
A1 conjugate.

Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

3(b)(i) Write the simultaneous equations

$$x - \sqrt{3}y = -2$$
$$\sqrt{3}x + y = 2\sqrt{3}$$

in the form $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 2\sqrt{3} \end{pmatrix}$ where A is a 2×2 matrix.

3(b)(ii) Then, find A^{-1} and use it to solve the equations for x and y.

Part b(i)		**5 marks**	**Att 2**
Part b(ii)	A^{-1}	**5 marks**	**Att 2**
	Solve	**10 marks**	**Att 3**

3(b)(i)

$$x-\sqrt{3}y=-2$$
$$\sqrt{3}x+y=2\sqrt{3}$$
$$\begin{pmatrix}1 & -\sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-2\\ 2\sqrt{3}\end{pmatrix}$$

(ii)

$$A=\begin{pmatrix}1 & -\sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}\Rightarrow A^{-1}=\frac{1}{1+3}\begin{pmatrix}1 & \sqrt{3}\\ -\sqrt{3} & 1\end{pmatrix}$$
$$=\frac{1}{4}\begin{pmatrix}1 & \sqrt{3}\\ -\sqrt{3} & 1\end{pmatrix}$$

$$A\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-2\\ 2\sqrt{3}\end{pmatrix}$$
$$A^{-1}.A\begin{pmatrix}x\\ y\end{pmatrix}=A^{-1}\begin{pmatrix}-2\\ 2\sqrt{3}\end{pmatrix}$$
$$\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{4}\begin{pmatrix}1 & \sqrt{3}\\ -\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}-2\\ 2\sqrt{3}\end{pmatrix}$$
$$=\frac{1}{4}\begin{pmatrix}4\\ 4\sqrt{3}\end{pmatrix}$$
$$\Rightarrow\quad\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}1\\ \sqrt{3}\end{pmatrix}$$

Blunders (-3)
B1 incorrect A.
B2 incorrect $\frac{1}{\Delta}$ or no $\frac{1}{\Delta}$.
B3 $A^{-1}\cdot A\neq I$.

Slips (-1)
S1 each incorrect element matrix.

Attempts
A1 uses $A\cdot A^{-1}$ for $A^{-1}\cdot A$.

Part (c)	**20 (5, 10, 5) marks**	**Att (2, 3, 2)**

3(c)

(i) Write $\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} -2 & 3 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ in the form $ax^2 + bxy + cy^2$ where $a, b, c \in Z$.

(ii) Show that $z^2 - 16$ is a factor of $z^3 + (1+i)z^2 - 16z - 16(1+i)$ and hence, find the three roots of $z^3 + (1+i)z^2 - 16z - 16(1+i) = 0$.

(i)		**5 marks**	**Att 2**
(ii)	**Factor**	**10 marks**	**Att 3**
	Roots	**5 marks**	**Att 2**

3(c)(i)

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} -2 & 3 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= \begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} -2x+3y \\ -4x+5y \end{pmatrix}$$
$$= -2x^2 + 3xy - 4xy + 5y^2$$
$$= -2x^2 - xy + 5y^2$$

or

3(c)(i)

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} -2 & 3 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= \begin{pmatrix} -2x-4y & 3x+5y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
$$= -2x^2 - 4xy + 3xy + 5y^2$$
$$= -2x^2 - xy + 5y^2$$

3(c)(ii)

$f(z) = (z^2 - 16)(z + a) = z^3 + (1+i)z^2 - 16z - 16(1+i)$

Equating independent terms: $-16a = -16(1+i)$

$a = (1 + i)$

Check: $(z^2 - 16)(z + (1+i)) = z^3 + (1+i)z^2 - 16z - 16(1+i) \Rightarrow z^2 - 16$ is a factor

$f(z) = (z^2 - 16)[z + (1+i)] = 0$

$z^2 = 16$ or $z = -(1+i)$

$z = \pm 4$

$\Rightarrow$ roots are + 4, – 4 and $-1 - i$.

or

3(c)(ii)	$$\begin{array}{r} z+(1+i) \\ z^2-16\overline{)z^3+(1+i)z^2-16z-16(1+i)} \\ \underline{z^3 \qquad\qquad -16z} \\ (1+i)z^2-16(1+i) \\ \underline{(1+i)z^2-16(1+i)} \end{array}$$ $\Rightarrow f(z)=(z^2-16)[z+(1+i)]=0$ $\Rightarrow \quad z^2=16 \quad$ or $\quad z=-(1+i)$ $\Rightarrow \quad z=\pm 4 \qquad\qquad \Rightarrow \quad$ roots are + 4, – 4 and $-1-i$.

or

3(c)(ii)	$z^2-16=(z-4)(z+4)$ $\Rightarrow z=4 \quad$ or $\quad -4$ Test *f*(-4) and *f*(4): $f(z)=z^3+(1+i)z^2-16z-16(1+i)$ $f(-4)=-64+16(1+i)+64-16(1+i)=0$ $\Rightarrow (z+4)$ is a factor $f(x)=+64+16(1+i)-64-16(1+i)=0$ $\Rightarrow (z-4)$ is a factor $\Rightarrow (z+4)(z-4)=(z^2-16)$ is a factor. Equation: $z^3-[-(1+i)]z^2+(-16)z-[16(1+i)]=0$ $z^3-[(\Sigma\alpha)]z^2+(\Sigma\alpha\beta)z-(\alpha\beta y)=0$ Let roots be 4, – 4 and α $\Sigma\alpha=4+(-4)+\alpha=-(1+i)$ $\alpha=-(1+i)$

Blunders (-3)
B1 indices.
B2 factors.
B3 root from factor.
B4 not like to like.
B5 Factor Theorem.
B6 statement about cubic equation
B7 *i*.
B8 root omitted.
B9 matrix rule.

Slips (-1)
S1 numerical.
S2 not changing sign when subtracting in division.
S3 each incorrect element in matrix.

Attempts
A1 remainder ≠ 0 in division.
A2 any attempt at division.
A3 other factor must be linear.

QUESTION 4

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 15) marks**	**Att (2, 5)**
Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

Part (a) **10 (5, 5)marks** **Att (2, 2)**

4(a) The sum of the first n terms of an arithmetic series is given by $S_n = 3n^2 - 4n$.

Use S_n to find **(i)** the first term, T_1

(ii) the sum of the second term and the third term, $T_2 + T_3$.

Part (a)(i)	**5 marks**	**Att 2**
Part (a)(ii)	**5 marks**	**Att 2**

4(a)(i) $S_n = 3n^2 - 4n$

$n = 1: S_1 = 3(1)^2 - 4(1) = -1$

$\Rightarrow a = -1 = T_1$

(ii) $S_3 = 3(3)^2 - 4(3) = 27 - 12$

$S_3 = T_1 + T_2 + T_3 = 15$

$(-1) + T_2 + T_3 = 15$

$T_2 + T_3 = 16$

or

$S_1 = -1 = T_1$

$S_2 = 3(2)^2 - 4(2) = 4$

$T_1 + T_2 = 4$

$-1 + T_2 = 4$

$T_2 = 5$

$S_3 = 3(3)^2 - 4(3) = 15$

$T_1 + T_2 + T_3 = 15$

$-1 + 5 + T_3 = 15$

$T_3 = 11$

$T_2 + T_3 = 5 + 11 = 16$

Blunders (-3)
B1 indices.
B2 uses incorrect n

Slips (-1)
S1 numerical.

Part (b)	**20 (5, 15) marks**	**Att (2, 5)**

4(b) (i) Show that

$$\frac{1}{(n+2)(n+3)} = \frac{1}{n+2} - \frac{1}{n+3} \text{ for } n \in N.$$

(ii) Hence, find $\sum_{n=1}^{k} \frac{1}{(n+2)(n+3)}$ and evaluate $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$.

(i)	**5 marks**	**Att 2**
(ii)	**15 marks**	**Att 5**

4(b)(i)

$$\frac{1}{n+2} - \frac{1}{n+3} = \frac{(n+3)-(n+2)}{(n+2)(n+3)} = \frac{n+3-n-2}{(n+2)(n+3)} = \frac{1}{(n+2)(n+3)}$$

or

$$\frac{1}{(n+2)(n+3)} = \frac{a}{n+2} + \frac{b}{n+3}$$

$$1 = a(n+3) + b(n+2)$$

$$(0)n + (1) = (a+b)n + (3a+2b)$$

$a + b = 0$ and $3a + 2b = 1$

$a = -b$

$-3b + 2b = 1$

$-b = 1$

$b = -1$

$a = 1$

$$\Rightarrow \quad \frac{1}{(n+2)(n+3)} = \frac{1}{n+2} - \frac{1}{n+3}$$

4(b)(ii)	$\sum_{n=1}^{k}\frac{1}{(n+2)(n+3)}$	$u_1=\frac{1}{3.4}$	$=\frac{1}{3}-\frac{1}{4}$
		$u_2=\frac{1}{4.5}$	$=\frac{1}{4}-\frac{1}{5}$
		$u_3=\frac{1}{5.6}$	$=\frac{1}{5}-\frac{1}{6}$

		$u_{k-2}=\frac{1}{k(k+1)}$	$=\frac{1}{k}-\frac{1}{k+1}$
		$u_{k-1}=\frac{1}{(k+1)(k+2)}$	$=\frac{1}{k+1}-\frac{1}{k+2}$
		$u_k=\frac{1}{(k+2)(k+3)}$	$=\frac{1}{k+2}-\frac{1}{k+3}$
			$\Rightarrow S_k=\frac{1}{3}-\frac{1}{k+3}$
			$\Rightarrow S_\infty=\frac{1}{3}$

Blunders (-3)
B1 indices.
B2 not like to like.
B3 term.
B4 cancellation – must be shown or implied.
B5 sum not to k terms.
B6 no sum to infinity.
B7 n not a natural number.

Slips(-1)
S1 numerical.

Part (c) **20 (5, 15) marks** **Att (2, 5)**

4(c) **(i)** Write $\frac{n^3+8}{n+2}$ in the form an^2+bn+c where $a,b,c\in R$.

(ii) Hence, evaluate $\sum_{n=1}^{30}\frac{n^3+8}{n+2}$.

[Note: $\sum_{n=1}^{k}n=\frac{k}{2}(k+1)$; $\sum_{n=1}^{k}n^2=\frac{k}{6}(k+1)(2k+1)$.]

Part (c)(i)	**5 marks**	**Att 2**
Part (c)(ii)	**15 marks**	**Att 5**

4(c)(i) $$\frac{n^3+8}{n+2}=\frac{(n)^3+(2)^3}{n+2}=\frac{(n+2)(n^2-2n+4)}{(n+2)}=n^2-2n+4$$

or

$$\begin{array}{r l} & n^2-2n+4 \\ n+2 & \overline{)\,n^3+8} \\ & \underline{n^3+2n^2} \\ & -2n^2 \qquad +8 \\ & \underline{-2n^2-4n} \\ & \qquad 4n+8 \\ & \qquad \underline{4n+8} \end{array}$$

$$\Rightarrow \quad f(n)=n^2-2n+4$$

(ii)

$$\sum_{n=1}^{k}\frac{n^3+8}{n+2}=\sum_{n=1}^{k}(n^2-2n+4)$$

$$=\sum_{n=1}^{k}n^2-\sum_{n=1}^{k}2n+4n$$

$$=\frac{k}{6}(k+1)(2k+1)-2\left[\frac{k}{2}(k+1)\right]+4k$$

$$=\frac{k}{6}(k+1)(2k+1)-k(k+1)+4k$$

$k=30:$ $$S_{30}=\frac{30}{6}(31)(61)-30(31)+4(30)$$

$$=8645$$

Blunders (-3)
B1 cubic factors, once only.
B2 indices.
B3 incorrect $\sum n$.
B4 incorrect $\sum n^2$.
B5 $4n$ term wrong.
B6 $n \neq 30$.

Slips (-1)
S1 numerical.

Attempts
A1 remainder $\neq$ 0.
A2 remainder $\neq$ 0 in **(i)** implies maximum of "att 5" in **(ii)**.

QUESTION 5

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

5(a) The second term, T_2, of a geometric sequence is 21.
The third term, T_3, is -63
Find **(i)** the common ratio
(ii) the first term.

(i)	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**

5(a)(i) $T_2 = 21 : T_3 = -63$

$$r = \frac{T_3}{T_2} = \frac{-63}{21} = -3$$

(ii) $T_2 = ar$
$21 = (a)(-3)$
$\Rightarrow r = -7$

* Accept $r = -3$ and $a = -7$ with no work shown.

Blunders (-3)
B1 definition of G.P.
B2 incorrect *a*.
B3 *r*

Slips (-1)
S1 numerical.

Worthless
W1 uses A.P.

Part (b)(i)	10 marks	Att 3

5(b)(i)	Solve $\log_6(x+5) = 2 - \log_6 x$ for $x > 0$

Part (b)(i)	10 marks	Att 3

5(b)(i)

$$\log_6(x+5) = 2 - \log_6 x, \qquad x > 0$$

$$\log_6(x+5) + \log_6 x = 2$$

$$\log_6 x(x+5) = 2$$

$$x^2 + 5x = 6^2 = 36$$

$$x^2 + 5x - 36 = 0$$

$$(x+9)(x-4) = 0$$

$$\Rightarrow x = -9 \text{ or } x = 4 \qquad \text{but} \qquad x > 0 \Rightarrow x = 4$$

or

5(b)(i)

$$\log_6(x+5) = 2 - \log_6 x$$

$$\log_6(x+5) = \log_6(36) - \log_6 x$$

$$\log_6(x+5) = \log_6\left(\frac{36}{x}\right)$$

$$x + 5 = \frac{36}{x}$$

$$x^2 + 5x = 36$$

$$x^2 + 5x - 36 = 0$$

$$(x+9)(x-4) = 0$$

$$\Rightarrow x = -9 \text{ or } x = 4 \qquad \text{but} \qquad x > 0 \Rightarrow x = 4$$

Blunders (-3)
B1 indices.
B2 logs.
B3 factors, once only.
B4 quadratic formula, once only.
B5 $x < 0$ in answer.

Slips (-1)
S1 numerical.

Attempts
A1 not quadratic.

Worthless
W1 drops logs.

Part (b)(ii)	10 marks	Att 3

5(b)(ii) In the binomial expansion of $(1+kx)^6$, the coefficient of x^4 is 240.
Find the two possible real values of k.

Part (b)(ii)	10 marks	Att 3

5(b)(ii) Let $T_{R+1} = 240x^4$

$$T_{R+1} = \binom{6}{R}(kx)^R = 240x^4$$

$$\Rightarrow R = 4$$

$$T_5 = \binom{6}{4}(kx)^4$$

$$= \binom{6}{2}k^4x^4$$

$$= \frac{6.5}{1.2}\cdot k^4 \cdot x^4 = 240x^4$$

$$\Rightarrow \ k^4 = 16 = (\pm 2)^4$$

$$\Rightarrow \ k = \pm 2$$

or

5(b)(ii)

$$(1+kx)^6 = 1 + \binom{6}{1}(kx)^1 + \binom{6}{2}(kx)^2 + \binom{6}{3}(kx)^3 + \binom{6}{4}(kx)^4 \ldots\ldots\ldots\ldots$$

$$T_5 = \binom{6}{4}(kx)^4 = 240x^4$$

$$\frac{6.5}{1.2}\cdot k^4 = 240$$

$$\Rightarrow \ k^4 = 16 = (\pm 2)^4$$

$$\Rightarrow \ k = \pm 2$$

* Accept expansion to 5 terms.

Blunders (-3)
B1 binomial expansion, once only.
B2 $\binom{n}{r}$ or no $\binom{n}{r}$.
B3 indices.
B4 general term.
B5 one value only.

Slips (-1)
S1 numerical.

Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

5(c) Use induction to prove that, for n a positive integer,
$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$
for all $\theta \in R$ and $i^2 = -1$.

P(1)	**5 marks**	**Att 2**
P(*k*)	**5 marks**	**Att 2**
P(*k* + 1) and proof	**10 marks**	**Att 3**

5(c)(i) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta,\ n \in \mathbf{N}_0$.

$n = 1: (\cos\theta + i\sin\theta)^1 = \cos(1)\theta + i\sin(1)\theta = \cos\theta + i\sin\theta$
$\Rightarrow$ True for $n = 1$

Assume True for $n = k$
$(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$

To prove $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$
$$\begin{aligned}(\cos\theta + i\sin\theta)^{k+1} &= (\cos\theta + i\sin\theta)^k \cdot (\cos\theta + i\sin\theta)^1 \\ &= (\cos k\theta + i\sin k\theta).(\cos\theta + i\sin\theta) \\ &= \cos k\theta\cos\theta - \sin k\theta\sin\theta + i\sin k\theta\cos\theta + i\cos k\theta\sin\theta \\ &= \cos(k\theta + \theta) + i\sin(k\theta + \theta) \\ &= \cos(k+1)\theta + i\sin(k+1)\theta\end{aligned}$$
$\Rightarrow$ True for $n = k+1$, if true for $n = k$
But true for $n = 1 \Rightarrow$ true for $n = 2, 3, 4, \ldots\ldots\ldots\ldots.$

Blunders (-3)
B1 indices.
B2 trigonometric error.
B3 *i*.
B4 statement of De Moivre's theorem.

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 15) marks**	**Att (2, 5)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a)	**10 marks**	**Att 3**

6(a) Differentiate $\frac{x}{1+x^2}$ with respect to x.

Part (a)	**10 marks**	**Att 3**

6(a)

$$y=\frac{x}{1+x^2}$$

$$\frac{dy}{dx}=\frac{(1+x^2)(1)-x(2x)}{(1+x^2)^2}$$

$$=\frac{1+x^2-2x^2}{(1+x^2)^2}$$

$$=\frac{1-x^2}{(1+x^2)^2}$$

or

6(a)

$$y=x\cdot(1+x^2)^{-1}$$

$$\frac{dy}{dx}=x\left[-1(1+x^2)^{-2}\cdot(2x)\right]+(1+x^2)^{-1}(1)$$

$$=\frac{-2x^2}{(1+x^2)^2}+\frac{1}{(1+x^2)}$$

$$=\frac{-2x^2+(1+x^2)}{(1+x^2)^2}$$

$$=\frac{1-x^2}{(1+x^2)^2}$$

* Allow full marks if derivatives are not simplified.

Blunders (-3)
B1 differentiation.
B2 indices.

Slips (-1)
S1 numerical.

Attempts
A1 blunder in differentiation formula.

Worthless
W1 integration.

Part (b)	**20 (5, 15) marks**	**Att (2, 5)**

6(b)(i)	Given that $y = \sqrt{x}$, what is $\frac{dy}{dx}$?
6(b)(ii)	Now, find from first principles the derivative of $\sqrt{x}$ with respect to x.

(i)	**5 marks**	**Att 2**
(ii)	**15 marks**	**Att 5**

6(b)(i)

$$y = \sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x)^{-\frac{1}{2}}.(1)$$

$$= \frac{1}{2 \cdot x^{\frac{1}{2}}}$$

$$= \frac{1}{2\sqrt{x}}$$

6(b)(ii)

$$f(x) = x^{\frac{1}{2}}$$

$$f(x+h) = (x+h)^{\frac{1}{2}}$$

$$f(x+h) - f(x) = (x+h)^{\frac{1}{2}} - x^{\frac{1}{2}} \cdot \frac{(x+h)^{\frac{1}{2}} + x^{\frac{1}{2}}}{(x+h)^{\frac{1}{2}} + x^{\frac{1}{2}}}$$

$$= \frac{(x+h)^1 - (x)^1}{(x+h)^{\frac{1}{2}} + (x)^{\frac{1}{2}}}$$

$$= \frac{x+h-x}{(x+h)^{\frac{1}{2}} + x^{\frac{1}{2}}}$$

$$f(x+h) - f(x) = \frac{h}{(x+h)^{\frac{1}{2}} + x^{\frac{1}{2}}}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{(x+h)^{\frac{1}{2}} + x^{\frac{1}{2}}}$$

$$\lim_{h \to o} \frac{f(x+h) - f(x)}{h} = \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{2}}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

or

6(b)(ii)

$$y = x^{\frac{1}{2}}$$

$$y + \Delta y = (x + \Delta x)^{\frac{1}{2}}$$

$$\Delta y = (x + \Delta x)^{\frac{1}{2}} - x^{\frac{1}{2}}$$

$$\Delta y = x^{\frac{1}{2}}\left(1 + \frac{\Delta x}{x}\right)^{\frac{1}{2}} - x^{\frac{1}{2}}$$

$$\Delta y = x^{\frac{1}{2}}\left[1 + \left(\frac{1}{2}\right)\left(\frac{\Delta x}{x}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{\Delta x}{x}\right)^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\right] - x^{-\frac{1}{2}}$$

$$\Delta y = \frac{1}{2}(x)^{-\frac{1}{2}}\Delta x - \frac{1}{8}x^{-\frac{3}{2}}(\Delta x)^2 \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} - \frac{1}{8 \cdot x^{\frac{3}{2}}} \cdot (\Delta x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{1}{2x^{\frac{1}{2}}} - 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

Blunders (-3)
B1 indices.
B2 differentiation.
B3 no limit shown, incorrect limit or no indication of "$\to 0$".
B4 binomial expansion, once only.
B5 must have three terms in expansion.

Attempts
A1 some effort at $f(x+h)$ or $(x+\Delta x)$.
A2 blunder in differentiation formula in **(i)**.
A3 incorrect conjugate in **(ii)**.

Worthless
W1 no 1st principles in **(ii)**.
W2 integration.

Part (c) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

6(c) Let $x = t^2e^t$ and $y = t + 2\ln t$ for $t > 0$.

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in terms of t.

(ii) Hence, show that $\frac{dy}{dx} = \frac{1}{x}$.

$\frac{dx}{dt}$	**5 marks**	**Att 2**
$\frac{dy}{dt}$	**5 marks**	**Att 2**
(ii) Show	**10 marks**	**Att 3**

6(c)(i)

$$x = t^2 e^t \qquad y = t + 2\ln t$$

$$\frac{dx}{dt} = t^2\left(e^t\right) + e^t(2t) \qquad \frac{dy}{dt} = 1 + \frac{2}{t}$$

$$= te^t(t+2)$$

(ii)

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{1+\frac{2}{t}}{te^t(t+2)}$$

$$= \frac{\left(\frac{t+2}{t}\right)}{te^t(t+2)}$$

$$= \frac{1}{t^2e^t}$$

$$= \frac{1}{x}$$

Blunders (-3)
B1 differentiation.
B2 indices.

Attempts
A1 blunder in differentiation formula.
A2 incorrect $\frac{dx}{dt}$ and/or $\frac{dy}{dt}$ used in **(ii)**.
A3 answer not in required form.

Worthless
W1 integration.

QUESTION 7

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

Part (a) **10 marks** **Att 3**

7(a) Taking $x_1 = 1$ as the first approximation to the real root of the equation

$$x^3 + x^2 - 1 = 0,$$

use the Newton-Raphson method to find x_2, the second approximation.

Part (a) **10 marks** **Att 3**

7(a) Newton-Raphson: $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_1 = 1$

$f(x) = x^3 + x^2 - 1$ $\qquad$ $f'(x) = 3x^2 + 2x$

$f(1) = 1 + 1 - 1 = 1$ $\qquad$ $f'(1) = 3 + 2 = 5$

$$x_2 = 1 - \frac{1}{5} = \frac{4}{5} \text{ or } 0.8$$

Blunders (-3)

B1 Newton-Raphson formula, once only.
B2 differentiation.
B3 indices.
B4 $x_1 \neq 1$, once only.

Slips (-1)

S1 numerical.
S2 answer not tidied up.

Part (b)(i) **10 marks** **Att 3**

7(b)(i)	Differentiate $\tan^{-1} 7x$ with respect to x.

Part(b)(i) **10 marks** **Att 3**

7(b)(i)

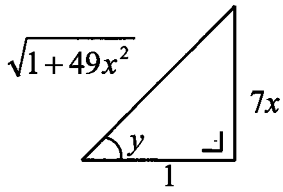
$$y = \tan^{-1}(7x)$$

$$\frac{dy}{dx} = \frac{1}{1+(7x)^2} \cdot 7 = \frac{7}{1+49x^2}$$

or

$$y = \tan^{-1}(7x)$$

$$\tan y = 7x$$



$$\sec^2 y \cdot \frac{dy}{dx} = 7$$

$$\frac{dy}{dx} = \frac{7}{\sec^2 y}$$

$$= 7\left(\frac{1}{\sec^2 y}\right)$$

$$= 7\cos^2 y$$

$$= 7\left(\frac{1}{1+49x^2}\right)$$

$$= \frac{7}{1+49x^2}$$

$$\tan y = \frac{7x}{1} = 7x$$

$$\cos y = \frac{1}{\sqrt{1+49x^2}}$$

$$\cos^2 y = \frac{1}{1+49x^2}$$

Blunders (-3)
B1 differentiation.
B2 indices.
B3 trigonometric formula, e.g. definition of tan, cos or sin.

Attempts
A1 blunder in differentiation formula.

Worthless
W1 integration.

Part (b)(ii)	10 marks	Att 3

7(b)(ii) Given that $y = \sin x \cos x$, find $\frac{dy}{dx}$ and express it in the form $\cos nx$ where $n \in Z$.

Part(b)(ii)	10 marks	Att 3

7(b)(ii)

$$y = \sin x \cdot \cos x = \frac{1}{2}\sin 2x$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot 2 \cdot \cos 2x = \cos 2x$$

or

7(b)(ii)

$$y = (\sin x) \cdot (\cos x)$$

$$\frac{dy}{dx} = (\sin x)(-\sin x) + (\cos x)(\cos x)$$

$$= -\sin^2 x + \cos^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

Blunders (-3)
B1 differentiation.
B2 indices.
B3 trigonometric formula.
B4 not in required form.

Attempts
A1 blunder in differentiation formula.
A2 $\frac{1}{2}\sin 2x$.

Worthless
W1 integration.

Part (c)	20 (5, 15) marks	Att (2, 5)

7(c) Let $g(x) = x^2 + \frac{a}{x^2}$ where a is a real number and $x \in R, x \neq 0$.
Given that *g(x)* has a turning point at $x = 2$,
(i) find the value of a
(ii) prove that *g(x)* has no local maximum points.

Part (c)(i)	**5 marks**	**Att 2**
Part (c)(ii)	**15 marks**	**Att 5**

7(c) $g(x) = x^2 + \frac{a}{x^2} = x^2 + ax^{-2}$

$g'(x) = 2x + a(-2)\cdot x^{-3} = 2x - \frac{2a}{x^3}$; $\quad g''(x) = 2 + a(-2)(-3)\cdot x^{-4} = 2 + \frac{6a}{x^4}$

$g'(x) = 0$ at $x = 2$

(i) $2x - \frac{2a}{x^3} = 0$

$x = \frac{a}{x^3}$

$x^4 = a = (2)^4 = 16$

$\Rightarrow \quad a = 16$

(ii) At any value of x at which there is a turning point, $g'(x) = 0$

$\Rightarrow \quad 2x - \frac{2a}{x^3} = 0$

$\Rightarrow \quad a = x^4$

$g''(x) = 2 + \frac{6a}{x^4}$

$\Rightarrow \quad g''(x) = 2 + \frac{6a}{a}$ for values of x at which a turning point occurs

$= 2 + 6 > 0$ (positive at turning points)

$\Rightarrow \quad g(x)$ has no local maximum points

Blunders (-3)
B1 differentiation.
B2 indices.
B3 deduction.
B4 inequality sign.
B5 turning points found and only one tested.

Slips (-1)
S1 numerical.

Attempts
A1 $g'(x) \neq 0$ in **(i)**.
A2 no test in $g''(x)$ or no $g''(x)$.
A3 blunder in differentiation formula.
A4 no x in $g''(x)$ or $g''(x) = 0$ or $g''(x)$ over-simplified or $g''(2)$ only.

Worthless
W1 no differentiation.
W2 integration.

QUESTION 8

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 15) marks**	**Att (2, 5)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

8(a) Find **(i)** $\int \frac{1}{x^3} dx$ **(ii)** $\int \sin 5x dx$.

Part(i)	**5 marks**	**Att 2**
Part(ii)	**5 marks**	**Att 2**

8(a)(i)

$$\int \frac{dx}{x^3} = \int x^{-3} dx = \frac{x^{-2}}{-2} + c$$

$$= \frac{-1}{2x^2} + c$$

(ii)

$$\int \sin 5x dx = \frac{-\cos 5x}{5} + c$$

* If 'c' shown once $\Rightarrow$ no penalty.

Blunders (-3)
B1 integration.
B2 no 'c', once only. (Penalise 2nd integration.)

Worthless
W1 differentiation instead of integration.

Part 8(b) **20 (10, 10) marks** **Att (3, 3)**

8(b) Evaluate **(i)** $\int_0^3 \frac{12}{x^2+9} dx$ **(ii)** $\int_0^4 \frac{(x+4)}{\sqrt{x^2+8x+1}} dx$.

Part (i)	**10 marks**	**Att 3**
Part(ii)	**10 marks**	**Att 3**

8(b)(i)

$$\int_0^3 \frac{12dx}{x^2+9} = 12\int \frac{dx}{x^2+3^2} = 12\left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3$$
$$= 4\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$
$$= 4\left[\frac{\pi}{4} - 0\right]$$
$$= \pi$$

(ii)

$$\int_0^4 \frac{(x+4)dx}{\sqrt{x^2+8x+1}}$$

$w = x^2 + 8x + 1$

$$= \int \frac{\frac{dw}{2}}{(w)^{\frac{1}{2}}}$$

$\frac{dw}{dx} = 2x + 8 = 2(x+4)$

$$= \frac{1}{2}\int w^{-\frac{1}{2}} \cdot dw$$

$\frac{dw}{2} = (x+4)dx$

$$= \frac{1}{2}\left[\frac{w^{\frac{1}{2}}}{\frac{1}{2}}\right]$$

$$= (x^2+8x+1)^{\frac{1}{2}} \Big]_0^4 = (16+32+1)^{\frac{1}{2}} - (1)^{\frac{1}{2}}$$
$$= (49)^{\frac{1}{2}} - 1$$
$$= 6$$

or

Changing Limits: $x = 4 : w = x^2 + 8x + 1 = 16 + 32 + 1 = 49$

$x = 1 : w = (0)^2 + 8(0) + 1 = 1$

$$\int_0^4 \frac{(x+4)dx}{\sqrt{x^2+8x+1}} = w^{\frac{1}{2}} \Big]_1^{49}$$
$$= (49)^{\frac{1}{2}} - (1)^{\frac{1}{2}} = 7 - 1 = 6$$

* Incorrect substitution and unable to finish yields attempt at most.
* (-3) is maximum deduction when evaluating limits.

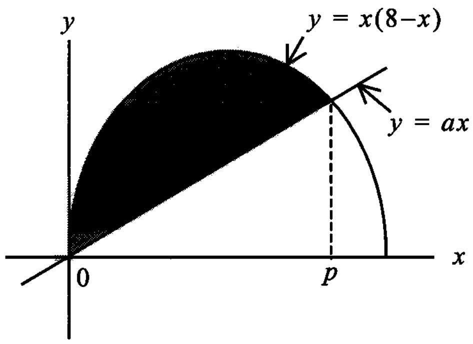
Blunders (-3)
B1 integration.
B2 indices.
B3 limits or no limits.
B4 incorrect order in applying limits.
B5 not calculating substituted limits.
B6 not changing limits.
B7 differentiation.

Slips (-1)
S1 trigonometric value or no trigonometric value.
S2 numerical.

Worthless
W1 differentiation instead of integration except where other work merits attempt.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

8(c) a is a real number such that $0 < a < 8$.
The line $y = ax$ intersects the curve $y = x(8 - x)$ at $x = 0$ and at $x = p$.

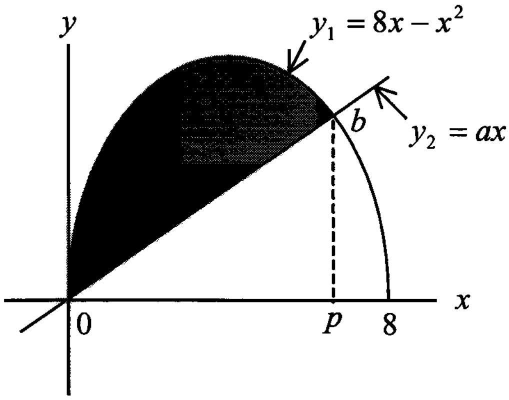


(i) Show that $p = 8 - a$.

(ii) Show that the area between the curve and the line is $\frac{p^3}{6}$ square units.

Part (c)(i)		**5 marks**	**Att 2**
Part (c)(ii)	**1st Area**	**5 marks**	**Att 2**
	2nd Area	**5 marks**	**Att 2**
	Show	**5 marks**	**Att 2**

8(c)(i)



Curve: $y_1 = 8x - x^2$

Line: $y_2 = ax$

Line $\cap$ curve:

$8x - x^2 = ax$

$0 = x^2 + ax - 8x$

$0 = x(x + a - 8)$

$\Rightarrow x = 0$ or $x + a - 8 = 0$

$x = 8 - a$

$\Rightarrow \quad p = 8 - a$

Coordinates of b: $(p,\ 8p - p^2)$

(ii) $A = \int_o^p y_1 \cdot dx - \int_0^p y_2 dx$

$= \int_0^p (8x - x^2)dx - \int_0^p (8 - p)x\,dx$ since $y_2 = ax$ and $p = 8 - a$

$\Rightarrow a = 8 - p$ giving $y_2 = (8 - p)x$

$= 4x^2 - \frac{x^3}{3} - 4x^2 + \frac{px^2}{2} \Big]_0^p$

$= \frac{px^2}{2} - \frac{x^3}{3} \Big]_0^p$

$= \left(\frac{p^3}{2} - \frac{p^3}{3}\right) - 0$

$= \frac{p^3}{6}$ square units

or

8(c)(ii)

$y_1 = 8x - x^2$

At $x = p : y_1 = 8p - p^2$

$$A = \int_0^p y_1\, dx - \text{area}\,\Delta obp$$

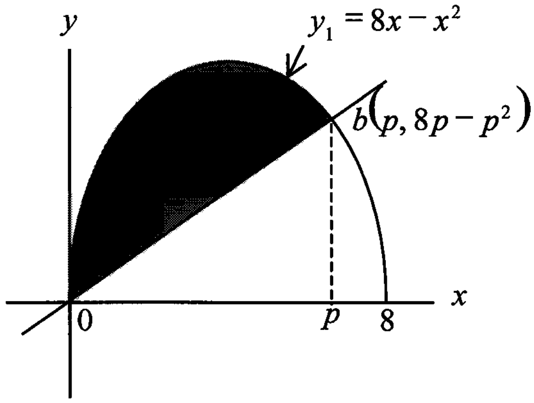
$$A = \int_0^p \left(8x - x^2\right) dx - \frac{1}{2}|op||pb|$$

$$A = \left[4x^2 - \frac{x^3}{3}\right]_0^p - \frac{1}{2}(p)\left(8p - p^2\right)$$

$$A = \left(4p^2 - \frac{p^3}{3}\right) - (0) - 4p^2 + \frac{p^3}{2}$$

$$A = \frac{p^3}{2} - \frac{p^3}{3}$$

$$\Rightarrow A = \frac{p^3}{6} \text{ square units}$$



Blunders (-3)
B1 indices.
B2 factors, once only.
B3 root from factor.
B4 integration.
B5 area formula.
B6 limits or no limits.
B7 not calculating substituted limits.
B8 has *'a'* in solution.

Attempts
A1 some relevant area.
A2 uses volume formula.

Slips (-1)
S1 numerical

Worthless
W1 differentiation instead of integration except where other work merits attempts.
W2 wrong area formula and no work.
W3 graphical methods only.

An Roinn Oideachais agus Eolaíochta

Leaving Certificate Examination 2001

Marking Scheme

MATHEMATICS

Higher Level

Paper 2

General Instructions

Penalties are applied as follows:

numerical slips, misreadings	(–1) each
blunders, major omissions	(–3) each.

Note 1: The lists of slips, blunders and attempts given in the Marking Scheme are not exhaustive.

Note 2: A serious blunder, omission or misreading merits the attempt mark at most.

Note 3: If deductions result in a mark which is less than the attempt mark, the attempt mark is awarded. The attempt mark for a section is the final mark for that section.

Note 4: Particular cases and verifications are, in general, awarded the attempt mark only.

Note 5: Mark all the candidate's work (including any that is cancelled) and allow the highest scoring solutions.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks (5, 5, 5, 5)**	**Att (2, 2, 2, 2)**
Part (c)	**20 marks (5, 5, 5, 5)**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

1(a) A circle with centre $(-3,7)$ passes through the point $(5,-8)$.
Find the equation of the circle.

Circle equation **10 marks** **Att 3**

1(a)

$$r = \sqrt{(5+3)^2+(-8-7)^2} = \sqrt{64+225} = \sqrt{289} = 17.$$

Circle: $(x+3)^2+(y-7)^2 = 289.$

or

Circle: $x^2+y^2+6x-14y+c=0$ as $g=3$ and $f=-7$.

$(5,-8) \in$ circle $\Rightarrow 25+64+30+112+c=0 \Rightarrow c=-231.$

$\therefore$ Circle: $x^2+y^2+6x-14y-231=0.$

Blunders (–3)

B1 incorrect distance formula.

B2 incorrect sign assigned to centre in equation of circle.

Slips (–1)

S1 arithmetic error.

Attempts (3)

A1 correct radius length.

A2 equation of circle without c or radius length evaluated.

Part (b) **20 marks (5, 5, 5, 5)** **Att (2, 2, 2, 2)**

Part (b) (i) **15 marks (5, 5, 5)** **Att (2, 2, 2)**

1(b) The equation of a circle is $(x+1)^2+(y-8)^2=160.$

The line $x-3y+25=0$ intersects the circle at the points p and q.

(i) Find the co-ordinates of p and the co-ordinates of q.

Quadratic equation	**5 marks**	**Att 2**
Solving quadratic	**5 marks**	**Att 2**
Co-ordinates of *p* and *q*	**5 marks**	**Att 2**

1(b)(i)

$x-3y+25=0 \cap (x+1)^2+(y-8)^2=160.$

$x=3y-25 \Rightarrow (3x-24)^2+(y-8)^2=160.$

$9y^2-144y+576+y^2-16y+64-160=0$

$\therefore\ 10y^2-160y+480=0.$

$y^2-16y+48=0 \Rightarrow (y-4)(y-12)=0$

$\therefore$ $(y-4)=0 \Rightarrow y=4$ and hence $x=-13$

$(y-12)=0 \Rightarrow y=12$ and $x=11.$

$\therefore\quad p(-13,\ 4),\ q(11,\ 12).$

Blunders (–3)

B1 incorrect squaring.

B2 incorrect factors.

B3 failure to couple, e.g. y values without corresponding x values.

B4 error in quadratic formula.

Slips (–1)

S1 arithmetic error.

Attempts (2, 2, 2)

A1 x in terms of y.

A2 attempt at solving quadratic equation.

Part (b) (ii) **5 marks** **Att 2**

1(b) (ii) Investigate if [pq] is a diameter of the circle.

Investigate **5 marks** **Att 2**

1(b)(ii)

$p(-13, 4)$, $q(11, 12)$. Mid-point of $[pq] = (-1, 8) =$ centre of circle.

$\therefore [pq]$ diameter.

or

centre of circle $= (-1, 8)$

$(-1, 8) \in x - 3y + 25 = 0$?

$-1 - 24 + 25 = 0$ yes $\therefore [pq]$ diameter.

or

$|\text{radius}| = \sqrt{160}$ and $|pq| = \sqrt{(-13-11)^2 + (4-12)^2} = \sqrt{640}.$

But $\sqrt{640} = 2\sqrt{160} = 2r.$ $\therefore [pq]$ diameter.

Blunders (–3)

B1 incorrect centre.

B2 error in mid-point formula.

B3 error in translation.

B4 error in distance formula.

Slips (–1)

S1 arithmetic error.

Attempts (2)

A1 centre of circle.

A2 mid-point of [pq].

A3 $|pq|$.

Part (c)	**20 marks (5,5, 5, 5)**	**Att (2, 2, 2, 2)**
Part (c) (i)	**15 marks (5, 5, 5)**	**Att (2, 2, 2)**
One equation in *f*, *g* and *c*	**5 marks**	**Att 2**
Three equations	**5 marks**	**Att 2**
Final solution (*g*, *f*, *c*)	**5 marks**	**Att 2**

1(c) The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ passes through the points $(3,3)$ and $(4,1)$. The line $3x - y - 6 = 0$ is a tangent to the circle at $(3,3)$.

(i) Find the real numbers g, f and c.

Values of g, f and c	15 marks (5, 5, 5)	Att (2, 2, 2)

1(c)(i)

$$C: x^2 + y^2 + 2gx + 2fy + c = 0$$

$(3,3) \in C \Rightarrow 9 + 9 + 6g + 6f + c = 0 \Rightarrow 6g + 6f + c = -18$

$(4,1) \in C \Rightarrow 16 + 1 + 8g + 6f + c = 0 \Rightarrow 8g + 2f + c = -17$

$-2g + 4f = -1$

Slope of tangent $3x - y - 6 = 0$ is 3. $\therefore$ Slope of normal $= -\frac{1}{3}$.

Equation of normal at (3, 3): $y - 3 = -\frac{1}{3}(x - 3)$

Normal : $x + 3y = 12$

But $(-g, -f) \in$ normal $\Rightarrow -g - 3f = 12$.

$-2g + 4f = -1 \Rightarrow -2g + 4f = -1$

$-g - 3f = 12 \Rightarrow -2g - 6f = 24$

$10f = -25 \Rightarrow f = -2\frac{1}{2}$ and $g = -4\frac{1}{2}$.

But $6g + 6f + c = -18 \Rightarrow -27 - 15 + c = -18$. $\therefore c = 24$.

Circle: $x^2 + y^2 - 9x - 5y + 24 = 0$.

or

Slope of tangent $3x - y - 6 = 0$ is 3. $\therefore$ Slope of normal $= -\frac{1}{3}$.

Equation of normal at (3, 3) : $y - 3 = -\frac{1}{3}(x - 3)$

Normal : $x + 3y = 12$

Mid-point of chord (M) joining (3, 3) and (4, 1) is $(3\frac{1}{2}, 2)$.

Slope of chord M $= \frac{3-1}{3-4} = -2 \Rightarrow$ slope of mediator of chord $= \frac{1}{2}$.

$\therefore$ Equation of M : $y - 2 = \frac{1}{2}(x - 3\frac{1}{2}) \Rightarrow 2x - 4y = -1$.

$x + 3y = 12 \Rightarrow 2x + 6y = 24$

$2x - 4y = -1 \Rightarrow 2x - 4y = -1$

$10y = 25 \Rightarrow y = 2\frac{1}{2}, x = 4\frac{1}{2}$.

$\therefore$ centre $= (4\frac{1}{2}, 2\frac{1}{2})$. $g = -4\frac{1}{2}, f = -2\frac{1}{2}$.

circle: $x^2 + y^2 - 9x - 5y + c = 0$. But $(3, 3) \in$ circle.

$\therefore 9 + 9 - 27 - 15 + c = 0 \Rightarrow c = 24$.

Blunders (–3)
B1 error in slope formula.
B2 incorrect slope of tangent.
B3 incorrect slope of normal.
B4 incorrect centre.
B5 error in equation of line formula.

Slips (–1)
S1 arithmetic error.

Attempts (2, 2, 2)
A1 slope of tangent.
A2 second equation in *f*, *g* and *c*.

Part (c) (ii)	**5 marks**	**Att 2**

1(c)(ii) Find the co-ordinates of the point on the circle at which the tangent parallel to $3x - y - 6 = 0$ touches the circle.

Find co-ordinates	**5 marks**	**Att 2**

1(c)(ii)

centre $c(4\frac{1}{2}, 2\frac{1}{2})$. Required point $S_c(3, 3) = (6, 2)$.

Blunders (–3)
B1 incorrect centre.
B2 error in mid-point formula.
B3 error in translation.
B4 incorrect squaring.
B5 incorrect factors.

Slips (–1)
S1 arithmetic error.

Attempts (2)
A1 centre of circle.
A2 $S_c(3,3)$.
A3 *x* in terms of *y* for solving.

QUESTION 2

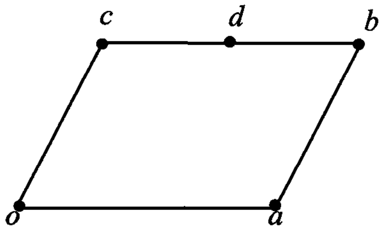
Part (a)	**10 marks (5, 5)**	**Att (2, 2)**
Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (c)	**20 marks (5, 5, 10)**	**Att (2, 2, 3)**

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**
Part (a)(i)	**5 marks**	**Att 2**
Part (a)(ii)	**5 marks**	**Att 2**

2(a) *oabc* is a parallelogram where *o* is the origin.
d is the midpoint of [*cb*].

(i) Express $\vec{b}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.



Express $\vec{b}$	**5 marks**	**Att 2**
Express $\vec{d}$	**5 marks**	**Att 2**

2(a)(i)

$$\vec{b} = \vec{a} + \overrightarrow{ab} = \vec{a} + \vec{c}.$$

2(a)(ii)

$$\vec{d} = \vec{c} + \frac{1}{2}\overrightarrow{cb} = \vec{c} + \frac{1}{2}\vec{a}.$$

or

$$\vec{d} = \frac{1}{2}(\vec{b} + \vec{c}) = \frac{1}{2}(\vec{a} + \vec{c} + \vec{c}) = \frac{1}{2}\vec{a} + \vec{c}.$$

Blunders (–3)

B1 $\overrightarrow{cb} = \vec{c} - \vec{b}$.

Slips (–1)

S1 arithmetic error.

Attempts (2)

A1 $\vec{b} = \vec{a} + \overrightarrow{ab}$.

A2 $\vec{d} = \vec{c} + 1/2\overrightarrow{cb}$.

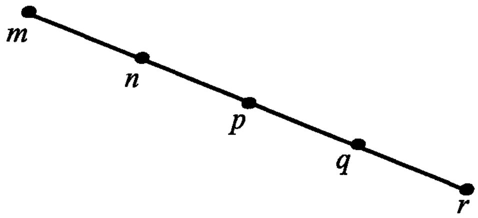
A3 $\vec{d} = 1/2(\vec{b} + \vec{c})$.

Worthless (0)

W1 $\vec{d} = 1/2\vec{a}$.

Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (b)(i)	**10 marks**	**Att 3**
Part (b)(ii)	**10 marks**	**Att 3**

2(b)



[mr] is divided into four line segments of equal length by the points n, p and q.

Given that $\vec{m} = -2\vec{i} + 3\vec{j}$ and $\vec{q} = 7\vec{i} - 9\vec{j}$, express

(i) $\vec{p}$ in terms of $\vec{i}$ and $\vec{j}$

(ii) $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

Express $\vec{p}$	**10 marks**	**Att 3**
Express $\vec{r}$	**10 marks**	**Att 3**

2(b)(i)

$$\vec{mp} = \frac{2}{3}(\vec{mq}) \Rightarrow 3(\vec{p} - \vec{m}) = 2(\vec{q} - m).$$

$$3\vec{p} = 2\vec{q} + \vec{m} \Rightarrow 3\vec{p} = 14\vec{i} - 18j - 2\vec{i} + 3\vec{j}$$

$$3\vec{p} = 12\vec{i} - 15\vec{j} \Rightarrow \vec{p} = 4\vec{i} - 5\vec{j}.$$

or

$$\vec{p} = \frac{\vec{m} + 2\vec{q}}{1+2} = \frac{-2\vec{i} + 3\vec{j} + 14\vec{i} - 18\vec{j}}{3} = 4\vec{i} - 5\vec{j}.$$

2(b)(ii)

$$\vec{r} = \frac{2\vec{q} - \vec{p}}{2-1} = 14\vec{i} - 18\vec{j} - 4\vec{i} + 5\vec{j} = 10\vec{i} - 13\vec{j}.$$

or

$$\vec{q} = \frac{1}{2}(\vec{p} + \vec{r}) \Rightarrow \vec{r} = 2\vec{q} - \vec{p} = 10\vec{i} - 13\vec{j}.$$

Blunders (–3)

B1 error in ratio formula.

B2 $\vec{mq} = \vec{m} - \vec{q}$, once only in part **(i)** and once only in part **(ii)**.

Slips (–1)

S1 arithmetic error.

Attempts (3, 3)

A1 $\vec{mp} = 2/3\vec{mq}$ and stops.

A2 correct ratio formula with some substitution.

A3 $\vec{pq} = \vec{qr}$ and stops.

Part (c)	**20 marks (5, 5, 10)**	**Att (2, 2, 3)**

Part (c)(i)	**5 marks**	**Att 2**

2(c) *rst* is a triangle where $\vec{r} = -\vec{i} + 2\vec{j}$, $\vec{s} = -4\vec{i} - 2\vec{j}$ and $\vec{t} = 3\vec{i} - \vec{j}$.

(i) Express $\overrightarrow{rs}$, $\overrightarrow{st}$ and $\overrightarrow{tr}$ in terms of $\vec{i}$ and $\vec{j}$.

Express $\overrightarrow{rs}$, $\overrightarrow{st}$ and $\overrightarrow{tr}$	**5 marks**	**Att 2**

2(c)(i)

$$\overrightarrow{rs} = \vec{s} - \vec{r} = -4\vec{i} - 2\vec{j} + \vec{i} - 2\vec{j} = -3\vec{i} - 4\vec{j}.$$
$$\overrightarrow{st} = \vec{t} - \vec{s} = 3\vec{i} - \vec{j} + 4\vec{i} + 2\vec{j} = 7\vec{i} + \vec{j}.$$
$$\vec{tr} = \vec{r} - \vec{t} = -\vec{i} + 2\vec{j} - 3\vec{i} + \vec{j} = -4\vec{i} + 3\vec{j}.$$

Blunders (–3)

B1 $\overrightarrow{rs} = \vec{r} - \vec{s}$ or similar error.

B2 two correct vectors found.

Slips (–1)

S1 arithmetic error.

Attempts (2)

A1 one correct vector found.

Part (c)(ii)	**5 marks**	**Att 2**

2(c)(ii) Show that the triangle *rst* is right-angled at *r*.

Show Δrst right-angled at *r*	**5 marks**	**Att 2**

2(c)(ii)

$$\overrightarrow{rs}.\overrightarrow{rt} = (-3\vec{i} - 4\vec{j})(4\vec{i} - 3\vec{j}) = -12 + 12 = 0. \quad \therefore \overrightarrow{rs} \perp \overrightarrow{rt}.$$

or

$$\left|\overrightarrow{st}\right|^2 = 49 + 1 = 50 \ ; \ \left|\overrightarrow{rs}\right|^2 + \left|\overrightarrow{rt}\right|^2 = 25 + 25 = 50 = \left|\overrightarrow{st}\right|^2.$$

$\therefore$ triangle *rst* right-angled at *r*.

Blunders (–3)
B1 incorrect vector multiplication.
B2 error in distance formula.
B3 incorrect formula for norm of vector.

Slips (–1)
S1 arithmetic error.

Attempts (2)
A1 $\vec{sr}.\vec{tr}$ and stops.
A2 length of a side correct.

Worthless (0)
W1 diagram.

Part (c)(iii)	**10 marks**	**Att 3**

> **2(c) (iii)** Find the measure of $\angle rst$.

Measure of $\angle rst$	**10 marks**	**Att 3**

2(c)(iii)

$$\cos\angle rst = \frac{\vec{sr}.\vec{st}}{\left|\vec{sr}\right|.\left|\vec{st}\right|} = \frac{(3\vec{i}+4\vec{j})(7\vec{i}+\vec{j})}{\left|3\vec{i}+4\vec{j}\right|.\left|7\vec{i}+\vec{j}\right|} = \frac{21+4}{5\sqrt{50}} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

$$\cos\angle rst = \frac{1}{\sqrt{2}} \Rightarrow \angle rst = 45^0.$$

or

$|rs| = \sqrt{9+16} = 5$ and $|tr| = \sqrt{16+9} = 5.$
As $|rs| = |tr|$ and $\angle rst = 90^0$ [by part (c) (ii)] $\therefore \angle rst = 45^0$.

Blunders (–3)
B1 incorrect vector multiplication.
B2 error in distance formula.
B3 incorrect formula for norm of vector.
B4 incorrect direction of vector.
B5 $\cos^{-1}\angle rst$ incorrect.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 scalar product with substitution.
A2 $|rs|$ or $|rt|$ correct.

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (c)	**20 marks (5, 5, 10)**	**Att (2, 2, 3)**

Part (a)	**10 marks**	**Att 3**

3(a) The line B contains the points $(6,-2)$ and $(-4,10)$.
The line A with equation $ax+6y+21=0$ is perpendicular to B.

Find the value of the real number a.

Find value of *a*	**10 marks**	**Att 3**

3(a)

$$\text{Slope B} = \frac{10+2}{-4-6} = \frac{12}{-10} = -\frac{6}{5}. \quad \text{A} \perp \text{B} \Rightarrow \text{slope A} = \frac{5}{6}.$$

$$\text{A: } ax+6y+21=0 \Rightarrow y = -\frac{ax}{6} - \frac{21}{6}. \quad \therefore \text{slope A} = -\frac{a}{6}.$$

$$-\frac{a}{6} = \frac{5}{6} \Rightarrow a = -5.$$

Blunders (–3)
B1 error in slope formula.
B2 incorrect perpendicular slope.
B3 incorrect slope of A, other than slip.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 slope of line A or line B.
A2 equation of line B.

Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (b)(i)	**10 marks**	**Att 3**

3(b) f is the transformation $(x, y) \to (x', y')$ where

$$x' = -5x - 6y$$
$$y' = 4x + 3y.$$

L is the line $x - 9y = 2$.

(i) Find the equation of $f(L)$, the image of L under f.

Find equation of $f(L)$	**10 marks**	**Att 3**

3(b)(i)

$$\begin{aligned} x' &= -5x - 6y \\ 2y' &= 8x + 6y \end{aligned} \quad \Rightarrow \quad x' + 2y' = 3x. \qquad \therefore\ x = \frac{1}{3}(x' + 2y').$$

But $y' = 4x + 3y \;\Rightarrow\; y' = \frac{4}{3}(x' + 2y') + 3y$

$$9y = 3y' - 4x' - 8y' \qquad \therefore\ y = \frac{1}{9}(-4x' - 5y').$$

$$\therefore\ f(L):\ \frac{1}{3}(x' + 2y') - (-4x' - 5y') = 2$$

$$x' + 2y' + 12x' + 15y' = 6$$

$$\therefore\ f(L):\ 13x' + 17y' = 6.$$

Blunders (–3)
B1 incorrect matrix or matrix multiplication.
B2 $f(L)$ not simplified to $ax' + by' + c = 0$.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 attempt at expressing x or y in terms of primes.
A2 correct matrix for f.

Part (b)(ii) **10 marks** **Att 3**

3(b) *M* is a line containing the point $(1, k)$ where $k \in \mathbf{Z}$ **(ii)** Given that $f(M)$ is $5x' - 2y' + 3k = 0$, find the value of *k*.

Find value of *k* **10 marks** **Att 3**

3(b)(ii)

$$f(M)\text{: } 5x' - 2y' + 3k = 0.$$

$$\therefore\ M: 5(-5x - 6y) - 2(4x + 3y) + 3k = 0$$

$$-25x - 30y - 8x - 6y + 3k = 0$$

$$-33x + 36y + 3k = 0 \quad \Rightarrow \quad 11x + 12y - k = 0.$$

But $(1, k) \in 11x + 12y - k = 0 \quad \Rightarrow \quad 11 + 12k - k = 0. \quad \therefore\ k = -1.$

or

$$f(1, k) = (-5 - 6k,\ 4 + 3k) \in f(M).$$

$$\therefore\ 5(-5 - 6k) - 2(4 + 3k) + 3k = 0$$

$$-33 - 33k = 0 \quad \Rightarrow \quad k = -1.$$

Blunders (–3)

B1 correct *M* but *k* not evaluated.

B2 incorrect $f(1, k)$.

Slips (–1)

S1 arithmetic error.

Attempts (3)

A1 correct $f(1, k)$.

A2 substitutes for x' and y' and stops.

Worthless (0)

W1 substitutes point $(1, k)$ into $f(M)$.

Part (c) **20 marks (5, 5, 10)** **Att (2, 2, 3)**

Part (c)(i) **5 marks** **Att 2**

3(c) N is the line $tx+(t-2)y+4=0$ where $t\in\mathbf{R}$.
(i) Write down the slope of N in terms of t.

Slope of ***N*** **5 marks** **Att 2**

3(c)(i)

$$tx+(t-2)y+4=0 \;\Rightarrow\; (t-2)y=-tx-4$$

$$y=\left(\frac{t}{2-t}\right)x-4. \qquad \therefore \text{ Slope } N = \frac{t}{2-t}.$$

Blunders (–3)

B1 expresses line in the form $x=my+c$.

B2 slope $=\dfrac{2-t}{t}$.

Slips (–1)

S1 arithmetic error.

Attempts (2)

A1 $(t–2)y=–tx–4$ and stops.

Part (c)(ii) **15 marks (5,10)** **Att (2, 3)**

3(c) (ii) Given that the angle between N and the line $x-3y+1=0$ is $45°$, find the two possible values of t.

Tan 45^0 = formula (filled in)*	**5 marks**	**Att 2**
Two values of *t*	**10 marks**	**Att 3**

3(c)(ii)

$$N: x - 3y + 1 = 0 \;\Rightarrow\; y = \frac{1}{3}x + \frac{1}{3}. \qquad \text{Slope } N = m_1 = \frac{1}{3}.$$

$$\tan 45^0 = \pm\frac{m_1 - m_2}{1 + m_1 m_2}, \quad \text{where } m_2 = \frac{t}{t-2}.$$

$$1 = \pm\frac{\frac{1}{3} - \frac{t}{2-t}}{1 + \frac{t}{3(2-t)}} = \pm\frac{2 - t - 3t}{6 - 3t + t} = \pm\frac{2 - 4t}{6 - 2t}$$

$$1 = \pm\left(\frac{1-2t}{3-t}\right) \;\Rightarrow\; 3 - t = \pm(1 - 2t).$$

$$3 - t = 1 - 2t \quad \text{or} \quad 3 - t = -1 + 2t$$

$$\therefore\; t = -2 \quad \text{or} \quad t = \frac{4}{3}.$$

* With or without ±, ± can be inserted later in final 10 mark part.

Blunders (–3)
- B1 sign error in formula for angle between two lines.
- B2 tan45° incorrect.
- B3 error in dealing with common denominator, other than slip.

Slips (–1)
- S1 arithmetic error.

Attempts (2, 3)
- A1 slope of line $x - 3y + 1 = 0$.
- A2 no ± used in formula resulting in one value for *t*.

QUESTION 4

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**
Part (b)	**20 marks (5, 5, 5, 5)**	**Att (2, 2, 2, 2)**
Part (c)	**20 marks (10, 10)**	**Att (3, 3)**

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**

Part (a)(i)	**5 marks**	**Att 2**
Part (a)(ii)	**5 marks**	**Att 2**

4(a) The length of an arc of a circle is 10 cm. The radius of the circle is 4 cm. The measure of the angle at the centre of the circle subtended by the arc is θ.

(i) Find θ in radians.

(ii) Find θ in degrees, correct to the nearest degree.

Find θ in radians	**5 marks**	**Att 2**
Find θ in degrees	**5 marks**	**Att 2**

4(a)(i)

$$r\theta = 10 \Rightarrow 4\theta = 10 \Rightarrow \theta = 2.5 \text{ radians.}$$

4(a)(ii) 1 radian = 57.296^0. $\therefore$ 2.5 radians = 143^0.

or

$$\theta = \frac{2.5 \times 180^0}{\pi} = 143^0.$$

Blunders (–3)

B1 error in arc length formula.

B2 angle given in degrees for part **(i)**.

B3 error in converting to degrees.

Slips (–1)

S1 arithmetic error.

Attempts (2, 2)

A1 correct substitution into arc length formula.

Part (b)	20 marks (5, 5, 5, 5)	Att (2, 2, 2, 2)

Part (b)(i)	5 marks	Att 2

4(b) (i) Write $\cos 2x$ in terms of $\sin x$.

Write $\cos 2x$ in terms of $\sin x$	5 marks	Att 2

4(b)(i)

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x.$$

Blunders (–3)
B1 error in $\cos^2 x$ conversion to $1 - \sin^2 x$.

Slips (–1)
S1 arithmetic error.

Attempts (2)
A1 $\cos 2x = \cos^2 x - \sin^2 x$ and stops.
A2 $\cos^2 x = 1 - \sin^2 x$.

Part (b)(ii)	15 marks (5, 5, 5)	Att (2, 2, 2)

4(b) (ii) Hence, find all the solutions of the equation

$$\cos 2x - \sin x = 1 \qquad \text{in the domain } 0° \le x \le 360°.$$

Quadratic in sin*x*	5 marks	Att 2
Solve for sin*x*	5 marks	Att 2
Final solution for *x*	5 marks	Att 2

4(b)(ii)

$$\cos 2x - \sin x = 1$$
$$1 - 2\sin^2 x - \sin x - 1 = 0$$
$$2\sin^2 x + \sin x = 0$$
$$\sin x(2\sin x + 1) = 0$$
$$\therefore \quad \sin x = 0 \quad \text{or} \quad \sin x = -\frac{1}{2}.$$
$$x = 0^0,\ 180^0,\ 360^0 \quad \text{or} \quad x = 210^0,\ 330^0.$$

$$\text{Solution} = \{\ 0^0,\ 180^0,\ 210^0,\ 330^0,\ 360^0\ \}$$

Blunders (–3)
B1 error in factors.
B2 missing solution or incorrect 'solution'.

Slips (–1)
S1 arithmetic error.

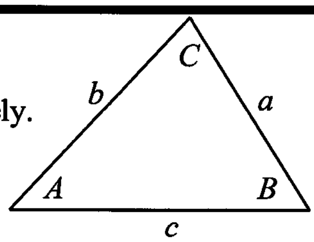
Attempts (2, 2, 2)
A1 $\cos 2x$ replaced by $1-2\sin^2 x$.
A2 correct factors.
A3 one correct solution for x.

Part (c) **20 marks (10, 10)** **Att (3, 3)**

Part (c)(i) **10 marks** **Att 3**

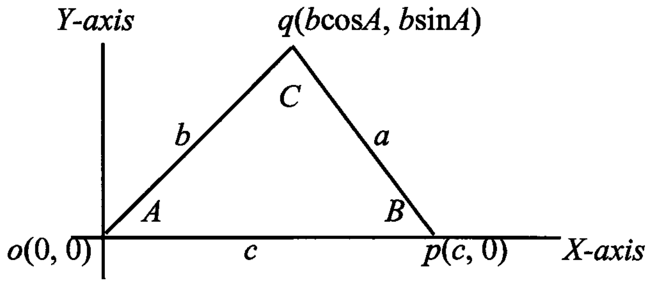
4(c) A triangle has sides a, b and c.
The angles opposite a, b and c are A, B and C, respectively.

(i) Prove that $a^2 = b^2 + c^2 - 2bc\cos A$.



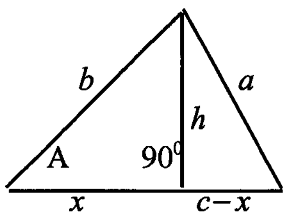
Cosine rule **10 marks** **Att 3**

4(c)(i)



$$|pq|^2 = a^2 = (b\cos A - c)^2 + (b\sin A - 0)^2$$
$$\therefore\ a^2 = b^2\cos^2 \text{A} - 2bc\cos A + c^2 + b^2\sin^2 A$$
$$a^2 = b^2(\cos^2 A + \sin^2 \text{A}) + c^2 - 2bc\cos A$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$

or



$$h^2 = b^2 - x^2.$$
$$h^2 = a^2 - (c-x)^2.$$

$$a^2 - c^2 + 2cx - x^2 = b^2 - x^2$$
$$a^2 = b^2 + c^2 - 2cx$$
$$\text{But } \cos A = \frac{x}{b} \Rightarrow x = b\cos A$$
$$\therefore a^2 = b^2 + c^2 - 2bc\cos A.$$

Blunders (–3)
B1 error in distance formula.
B2 error in Pythagoras.
B3 error in squaring.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 diagram of value.
A2 finds length of pq.
A3 $h^2 = b^2 - x^2$.

Part (c)(ii) **10 marks** **Att 3**

4(c) (ii) Show that $c(b\cos A - a\cos B) = b^2 - a^2$.

Show that **10 marks** **Att 3**

4(c)(ii)

$$bc\cos A - ac\cos B = bc\left(\frac{b^2+c^2-a^2}{2bc}\right) - ac\left(\frac{a^2+c^2-b^2}{2ac}\right)$$
$$= \frac{b^2+c^2-a^2}{2} - \frac{a^2+c^2-b^2}{2}$$
$$= \frac{2b^2-2a^2}{2}$$
$$= b^2 - a^2.$$

Blunders (–3)
B1 any error in substitution.
B2 incorrect substitution for cos$A$.
B3 incorrect substitution for cos$B$.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 substitution for $bc\cos A$ or $\cos A$.
A2 relevant substitution but fails to finish.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks (10, 5, 5)**	**Att (3, 2, 2)**
Part (c)	**20 marks (5, 5, 10)**	**Att (2, 2, 3)**

Part (a)	**10 marks**	**Att 3**

5(a) Evaluate $\lim\limits_{\theta \to 0} \dfrac{\sin 7\theta}{\sin 2\theta}$.

Evaluate	**10 marks**	**Att 3**

5(a) $$\lim_{\theta \to 0} \frac{\sin 7\theta}{\sin 2\theta} = \lim_{\theta \to 0} \frac{\frac{\sin 7\theta}{7\theta}}{\frac{\sin 2\theta}{2\theta}} \times \frac{7}{2} = \frac{7}{2}.$$

or

$$f(\theta) = \sin 7\theta \Rightarrow f'(\theta) = 7\cos 7\theta$$
$$g(\theta) = \sin 2\theta \Rightarrow g'(\theta) = 2\cos 2\theta$$
$$\lim_{\theta \to 0} \frac{f(\theta)}{g(\theta)} = \frac{f'(0)}{g'(0)} = \frac{7\cos 0}{2\cos 0} = \frac{7}{2}.$$

* accept correct answer without work.

Blunders (–3)

B1 $\sin 7\theta = 7\sin\theta$ or $\sin 2\theta = 2\sin\theta$

B2 differentiation error.

B3 $\cos 0 \neq 1$.

Slips (–1)

S1 arithmetic error.

Attempts (3)

A1 $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$.

Part (b)	20 marks (10, 5, 5)	Att (3, 2, 2)

Part (b)(i)	10 marks	Att 3

5(b) xyz is a triangle where $|xy| = 8$ cm and $|yz| = 6$ cm.

Given that the area of triangle xyz is 12 cm^2, find

(i) the two possible values of $|\angle xyz|$

Evaluate $|\angle xyz|$	10 marks	Att 3

5(b)(i)

$$\text{area } \Delta\, xyz = 12$$

$$\frac{1}{2}(8)(6)\sin \angle xyz = 12$$

$$\sin\angle xyz = \frac{1}{2} \Rightarrow \angle xyz = 30^0 \text{ or } \angle xyz = 150^0.$$

Blunders (–3)
B1 error in triangle area formula.
B2 only one angle given.
B3 incorrect angle given.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 triangle area formula with some substitution.

Part (b)(ii)	10 marks (5, 5)	Att (2, 2)

5(b) (ii) the two possible values of $|xz|$, correct to one decimal place.

Find $|xz|$	10 marks (5, 5)	Att (2, 2)

5(b)(ii)

$$|xz|^2 = 64 + 36 - 2(8)(6)\cos\angle xyz$$

$$|xz|^2 = 100 - 96\cos\angle xyz.$$

For $\angle xyz = 30^0$, $|xz|^2 = 100 - 96\cos 30^0 \Rightarrow |xz| = 4.1$ cm.

For $\angle xyz = 150^0$, $|xz|^2 = 100 - 96\cos 150^0 \Rightarrow |xz| = 13.5$ cm.

Blunders (–3)
B1 error in cosine rule formula (apply once)
B2 incorrect evaluation of $\cos 30^0$ or $\cos 150^0$.
B3 incorrect evaluation other, than slip.

Slips (–1)
S1 arithmetic error.

Attempts (2, 2)
A1 cosine rule with some substitution.

Part (c)	**20 marks (5, 5, 10)**	**Att (2, 2, 3)**
Part (c)(i)	**10 marks(5, 5)**	**Att (2, 2)**

5(c) A is an obtuse angle such that

$$\sin\left(A+\frac{\pi}{6}\right)+\sin\left(A-\frac{\pi}{6}\right)=\frac{4\sqrt{3}}{5}.$$

(i) Find $\sin A$ and $\tan A$.

Find Sin A	**5 marks**	**Att 2**
Find Sin A	**5 marks**	**Att 2**

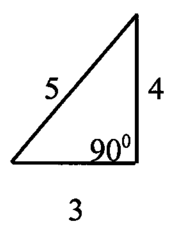
5(c)(i)

$$\sin\left(A+\frac{\pi}{6}\right)+\sin\left(A-\frac{\pi}{6}\right)=\frac{4\sqrt{3}}{5}.$$

$$2\sin A\cos\frac{\pi}{6} = \frac{4\sqrt{3}}{5}$$

$$2.\frac{\sqrt{3}}{2}\sin A = \frac{4\sqrt{3}}{5} \Rightarrow \sin A = \frac{4}{5}.$$

$$\therefore \tan A = -\frac{4}{3} \text{ as A is obtuse.}$$



Blunders (–3)
B1 error in $\sin A + \sin B$ formula.
B2 error in $\sin(A+B)$ or $\sin(A-B)$ formula.
B3 $\cos\frac{\pi}{6}$ incorrect.
B4 $\sin A$ incorrect.
B5 $\tan A$ incorrect.

Slips (–1)
S1 arithmetic error.

Attempts (2, 2)
A1 use of sin$A$ + sin$B$ formula.
A2 expansion of $\sin(A+\frac{\pi}{6})$ or $\sin(A-\frac{\pi}{6})$.

Part (c) (ii) **10 marks** **Att 3**

5(c)(ii) Given that $\tan(A+B)=\frac{1}{2}$, find $\tan B$ and express your answer in the form $\frac{p}{q}$ where p,q **Z** and $q \neq 0$.

Find tan$B$ **10 marks** **Att 3**

5(c)(ii)

$$\tan(A+B) = \frac{1}{2}$$

$$\frac{\tan A + \text{Tan}B}{1 - \tan A \tan B} = \frac{1}{2} \Rightarrow \frac{-\frac{4}{3} + \tan B}{1 + \frac{4}{3}\tan B} = \frac{1}{2}$$

$$\frac{-4 + 3\tan B}{3+4\tan B} = \frac{1}{2}$$

$$-8 + 6\tan B = 3 + 4\tan B$$
$$2\tan B = 11$$
$$\therefore \tan B = \frac{11}{2}.$$

Blunders (–3)
B1 error in expansion of tan(A+B).
B2 error in common denominator.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 tan(A+B) expansion with some substitution.

QUESTION 6

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**
Part (b)	**20 marks (5, 5, 10)**	**Att (2, 2, 3)**
Part (c)	**20 marks (5, 10, 5)**	**Att (2, 3, 2)**

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**

Part (a)(i)	**5 marks**	**Att 2**
Part (a)(ii)	**5 marks**	**Att 2**

6(a)(i) How many different sets of three books or of four books can be selected from six different books?

(ii) How many of the above sets contain one particular book?

How many different sets	**5 marks**	**Att 2**
Sets with one particular book	**5 marks**	**Att 2**

6(a) (i)

$$^6C_3 + {}^6C_4 = 20+15 = 35.$$

(ii)

$$^5C_2 + {}^5C_3 = 10+10 = 20.$$

Blunders (–3)
B1 $^6C_3 \times {}^6C_4$.
B2 $^5C_2 \times {}^5C_3$ (second part).

Slips (–1)
S1 arithmetic error.

Attempts (2, 2)
A1 $^6C_3$ or $^6C_4$.
A2 $^6P_3 + {}^6P_4$.
A3 $^5C_2$ or $^5C_3$ (second part).
A4 $^5P_2 + {}^5P_3$ (second part).

Part (b)	**20 marks (5, 5, 10)**	**Att (2, 2, 3)**

6(b) Solve the difference equation

$$u_{n+2} - 8u_{n+1} + 11u_n = 0, \qquad \text{where } n \geq 0,$$

given that $u_0 = 0$ and $u_1 = 2\sqrt{15}$.

Characteristic equation	**5 marks**	**Att 2**
Characteristic roots	**5 marks**	**Att 2**
Final solution	**10 marks**	**Att 3**

6(b)

$$u_0 = 0$$

$$x^2 - 8x + 11 = 0$$

$$x = \frac{8 \pm \sqrt{64-44}}{2} = \frac{8 \pm \sqrt{20}}{2} = \frac{8 \pm 2\sqrt{5}}{2}$$

$$\therefore \; x = 4 \pm \sqrt{5}.$$

$$u_n = k(4+\sqrt{5})^n + l(4-\sqrt{5})^n.$$

$$u_0 = 0 \;\Rightarrow\; k + l = 0 \;\Rightarrow\; l = -k.$$

$$u_1 = 2\sqrt{15} \;\Rightarrow\; k(4+\sqrt{5}) + l(4-\sqrt{5}) = 2\sqrt{15}$$

$$k(4+\sqrt{5}) - k(4-\sqrt{5}) = 2\sqrt{15}$$

$$4k + k\sqrt{5} - 4k + k\sqrt{5} = 2\sqrt{15}$$

$$2k\sqrt{5} = 2\sqrt{15}$$

$$k = \sqrt{3} \;\Rightarrow\; l = -\sqrt{3}.$$

$$\therefore \; u_n = \sqrt{3}(4+\sqrt{5})^n - \sqrt{3}(4-\sqrt{5})^n.$$

Blunders(–3)
B1 error in characteristic equation.
B2 error in quadratic formula.
B3 incorrect use of initial conditions.
B4 roots in decimal form (apply in final solution).

Slips (–1)
S1 arithmetic error.

Attempts (2, 2, 3)
A1 error in characteristic equation.
A2 quadratic formula with some substitution.
A3 correct form of u_n and stops.
A4 an equation in *k* and *l*.

Part (c)	**20 marks (5, 10, 5)**	**Att (2, 3, 2)**
Part (c)(i)	**5 marks**	**Att 2**

6(c) A box contains four silver coins, two gold coins and x copper coins.
Two coins are picked at random, and without replacement, from the box.

(i) Write down an expression in x for the probability that the two coins are copper.

Expression in x	**5 marks**	**Att 2**

6(c)(i)

4 silver, 2 gold and x copper coins.
Total number of coins = $x + 6$.

$$\text{P(two copper coins)} = \frac{x(x-1)}{(x+6)(x+5)}.$$

Blunders (–3)
B1 incorrect number of possible outcomes.

Slips (–1)
S1 arithmetic error.

Attempts (2)
A1 correct number of possible outcomes.
A2 favourable outcomes correct.
A3 probabilities added.
A4 probability of one copper coin correct.

Part (c) (ii)	10 marks	Att 3

> **6(c)** If it is known that the probability of picking two copper coins is $\frac{4}{13}$,
>
> **(ii)** how many coins are in the box?

Number of coins in the box	10 marks	Att 3

6(c)(ii)

$$\frac{x(x-1)}{(x+6)(x+5)} = \frac{4}{13}$$

$$13x(x-1) = 4(x+6)(x+5)$$

$$13x^2 - 13x = 4x^2 + 44x + 120$$

$$9x^2 - 57x - 120 = 0$$

$$3x^2 - 19x - 40 = 0 \Rightarrow (x-8)(3x+5) = 0$$

$$\therefore\ x = 8 \quad \text{as} \quad x \neq -\frac{5}{3}.$$

$\therefore$ 8 copper coins.

$\therefore$ 14 coins are in the box

Blunders(–3)
B1 incorrect factors or error in quadratic formula.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 stops at equation in x^2.
A2 due to previous error has two values that are not natural numbers.

Part (c)(iii) **5 marks** **Att 2**

6(c)(iii)	what is the probability that neither of the two coins picked is copper?

P(neither coin is copper) **5 marks** **Att 2**

6(c)(iii)

4 silver, 2 gold, 8 copper.
Total = 14 coins.

Required Probability = P(first not copper).P(second not copper)

$$= \frac{6}{14} \times \frac{5}{13} = \frac{30}{182} = \frac{15}{91}.$$

or

Probability = P(both gold) + P(both silver) + P(one gold and one silver)

$$= \frac{2}{14}.\frac{1}{13} + \frac{4}{14}.\frac{3}{13} + \frac{2}{14}.\frac{4}{13} \times 2$$

$$= \frac{2+12+16}{182} = \frac{30}{182}$$

$$= \frac{15}{91}.$$

or

P = 1 − [P(both copper) + P(one copper and one gold) + P(one copper and one silver)]

$$= 1 - \left[\frac{8}{14}.\frac{7}{13} + \frac{8}{14}.\frac{2}{13} \times 2 + \frac{8}{14}.\frac{4}{13} \times 2 \right] = 1 - \frac{152}{182}$$

$$= \frac{30}{182} = \frac{15}{91}.$$

Blunders (–3)

B1 incorrect number of possible outcomes.
B2 incorrect probability for any two coins-e.g. P(both gold) incorrect.

Slips (–1)
S1 arithmetic error.

Attempts (2)
A1 correct number of possible outcomes.
A2 favourable outcomes correct.

QUESTION 7

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**
Part (b)	**20 marks (5, 5, 5, 5)**	**Att (2, 2, 2, 2)**
Part (c)	**20 marks (5, 15)**	**Att (2,5)**

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**

Part (a)(i)	**5 marks**	**Att 2**
Part (a)(ii)	**5 marks**	**Att 2**

7(a) (i) How many different four-letter arrangements can be made from the letters of the word FRIDAY if each letter is used no more than once in each arrangement?

(ii) How many of the above arrangements begin with the letter D and also end with a vowel?

No. of arrangements	**5 marks**	**Att 2**
Begin with letter D, end in vowel	**5 marks**	**Att 2**

7(a)(i)

$$^{6}P_4 = 360.$$

(ii)

$$1 \times 4 \times 3 \times 2 = 24 \quad \textbf{or} \quad \frac{1}{6}(360) \times \frac{2}{5} = 24$$

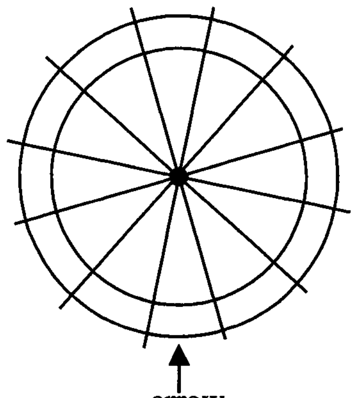
Blunders (–3)
B1 $^{6}C_4$.

Slips (–1)
S1 arithmetic error.

Attempts (2, 2)
A1 1×4×3×1 (second part).

Part (b)	**20 marks (5, 5, 5, 5)**	**Att (2, 2, 2, 2)**
Part (b)(i)	**5 marks**	**Att 2**
Part (b)(ii)	**5 marks**	**Att 2**
Part (b)(iii)	**5 marks**	**Att 2**
Part (b)(iv)	**5 marks**	**Att 2**

7(b) To play a game a player spins a wheel.



The wheel is fixed to a wall. It spins freely around its centre point.
Its rim is divided equally into twelve regions. Three of the regions are coloured red. Four are coloured blue. Five are coloured green.

When the wheel stops an arrow fixed to the wall points to one of the regions. All the regions are equally likely to stop at the arrow. The colour of the region to which the arrow points is the outcome of the game.

When the game is played twice, calculate the probability that

(i) both outcomes are green

(ii) both outcomes are the same colour

(iii) the first outcome is red and the second outcome is green

(iv) one outcome is green and the other outcome is blue.

Both green	**5 marks**	**Att 2**
Both same colour	**5 marks**	**Att 2**
First red, second green	**5 marks**	**Att 2**
One green, one blue	**5 marks**	**Att 2**

7(b)(i)

P(both outcomes are green) $= \frac{5}{12}.\frac{5}{12} = \frac{25}{144}.$

(ii) P(both outcomes same colour)
= P(both green) + P(both red) + P(both blue)

$$= \frac{5}{12}.\frac{5}{12} + \frac{3}{12}.\frac{3}{12} + \frac{4}{12}.\frac{4}{12} = \frac{25+9+16}{144} = \frac{50}{144}$$

$$= \frac{25}{72}.$$

(iii)

P(first red and second green) $= \frac{3}{12}.\frac{5}{12} = \frac{15}{144} = \frac{5}{48}.$

(iv)

P(one green and one blue) $= \frac{5}{12}.\frac{4}{12} \times 2 = \frac{40}{144} = \frac{5}{18}.$

Blunders (–3)
B1 incorrect number of possible outcomes.

Slips (–1)
S1 arithmetic error.

Attempts (2, 2, 2, 2)
A1 correct number of possible outcomes.
A2 correct number of favourable outcomes.

Part (c) **20 marks (5, 15)** **Att (2, 5)**

Part (c)(i) **5 marks** **Att 2**

7(c) Consider the numbers $1, \quad k, \quad 3k-2, \quad 9$ where $k \in \mathbf{Z}$. The mean of these numbers is $\bar{x}$. The standard deviation is σ. **(i)** Express $\bar{x}$ in terms of k.

Express $\bar{x}$ **5 marks** **Att 2**

7(c)(i)

$$\bar{x} = \frac{1+k+3k-2+9}{4} = \frac{8+4k}{4}$$

$$= 2 + k.$$

Blunders (–3)

B1 incorrect form of mean.

Slips (–1)

S1 arithmetic error.

Attempts (2)

A1 $4k+8$.

Part (c)(ii)	**15 marks**	**Att 5**

7(c) (ii) Given that $\sigma = \sqrt{20}$, find the value of k.

Find value of k	**15 marks**	**Att 5**

7(c)(ii)

x	$\lvert d \rvert$	$\lvert d \rvert^2$
1	$\lvert -1-k \rvert$	k^2+2k+1
k	$\lvert -2 \rvert$	4
$3k-2$	$\lvert 2k-4 \rvert$	$4k^2-16k+16$
9	$\lvert 7-k \rvert$	$k^2-14k+49$

$$\Sigma d^2 = 6k^2 - 28k + 70$$

$$\sigma^2 = \frac{6k^2 - 28k + 70}{4} = 20$$

$$\therefore 6k^2 - 28k + 70 = 80 \Rightarrow 6k^2 - 28k - 10 = 0$$

$$3k^2 - 14k - 5 = 0 \Rightarrow (k-5)(3k+1) = 0$$

$$\therefore k = 5 \text{ as } k \neq -\frac{1}{3} \notin Z.$$

Blunders (–3)
B1 incorrect deviation.
B2 incorrect d^2.
B3 any incorrect step in calculating σ.
B4 incorrect factors or error in user of quadratic formula.

Slips (–1)
S1 arithmetic error.

Attempts (5)
A1 any correct deviation.
A2 quadratic equation.
A3 fails to finish to k value.

QUESTION 8

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (c)	**20 marks (5, 5, 5, 5)**	**Att (2, 2, 2, 2)**

Part (a)	**10 marks**	**Att 3**

8(a) Use integration by parts to find $\int x\cos x\,dx$.

Integration	**10 marks**	**Att 3**

8(a)

$$\int x\cos x dx = u.v - \int v.du$$

$$u = x \Rightarrow du = dx \;\; ; \;\; dv = \cos x dx \Rightarrow v = \int \cos x dx = \sin x.$$

$$\therefore \int x\cos x = x\sin x - \int \sin x dx$$

$$= x\sin x + \cos x + \text{constant.}$$

Blunders (–3)
B1 incorrect differentiation or integration.
B2 constant of integration omitted.
B3 incorrect ‘parts’ formula.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 correct assigning to parts formula.
A2 correct differentiation or integration.

Part (b) **20 marks (10, 10)** **Att (3, 3)**

Part (b)(i)	**10 marks**	**Att 3**
Part (b)(ii)	**10 marks**	**Att 3**

8(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series for $f(x)$.

(i) Derive the Maclaurin series for $f(x) = \sin x$ up to and including the term containing x^7.

(ii) Write down the general term and use the Ratio Test to show that the series converges for all x **R**.

Maclaurin series	**10 marks**	**Att 3**
General term & ratio test	**10 marks**	**Att 3**

8(b)(i)

$$\begin{aligned}
f(x) &= \sin x &\Rightarrow\ f(0) &= \sin 0 &= 0\\
f'(x) &= \cos x &\Rightarrow\ f'(0) &= \cos 0 &= 1\\
f''(x) &= -\sin x &\Rightarrow\ f''(0) &= -\sin 0 &= 0\\
f'''(x) &= -\cos x &\Rightarrow\ f'''(0) &= -\cos 0 &= -1\\
f^{iv}(x) &= \sin x &\Rightarrow\ f^{iv}(0) &= \sin 0 &= 0\\
f^{v}(x) &= \cos x &\Rightarrow\ f^{v}(0) &= \cos 0 &= 1\\
f^{vi}(x) &= -\sin x &\Rightarrow\ f^{vi}(0) &= -\sin 0 &= 0\\
f^{vii}(x) &= -\cos x &\Rightarrow\ f^{vii}(0) &= -\cos 0 &= -1
\end{aligned}$$

$$\therefore\quad f(x) = \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots..$$

(ii)

$$u_n = \frac{x^{2n-1}}{(2n-1)!}(-1)^{n-1} \Rightarrow u_{n+1} = \frac{x^{2n+1}}{(2n+1)!}(-1)^{n}.$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+1}(-1)^n}{(2n+1)!}\cdot\frac{(2n-1)!}{x^{2n-1}!(-1)^{n-1}}\right| = \lim_{x\to\infty}\left|\frac{-x^2}{4n^2+2n}\right|$$

$$= \lim_{x\to\infty}\left|\frac{\frac{x^2}{n^2}}{4+\frac{2}{n}}\right| = \left|\frac{0}{4}\right| = 0 < 1. \quad \therefore \text{ converges for all } x \in \mathbf{R}.$$

Blunders (–3)

B1 incorrect differentiation - apply more than once.
B2 incorrect evaluation of sin 0^0 or cos 0^0.
B3 each term not derived.

B4 (–1) omitted from general term.
B5 an incorrect power in general term.
B6 factorial omitted or term with factorial incorrect in general term.
B7 error in evaluating limit other than slip.
B8 evaluates limit as 0 and stops.

Slips (–1)
S1 arithmetic error.

Attempts (3, 3)
A1 $f(0)$ correct.
A2 a correct differentiation.
A3 any one correct term.
A4 any part of U_n correct.
A5 U_{n+1} correct.
A6 ratio test with some correct substitution.

Worthless (0)
W1 general term with no x component.

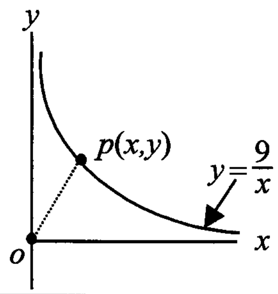
Part (c)	**20 marks (5, 5, 5, 5)**	**Att(2, 2, 2, 2)**
Part (i)	**5 marks**	**Att 2**

8(c) o is the origin, $(0,0)$.

$p(x, y)$ is a point on the curve $y = \frac{9}{x}$, where $x > 0$.

$|op|$ is the distance from the origin to p.

(i) Express $|op|$ in terms of x.



Express | *op* | in terms of *x*	**5 marks**	**Att 2**

8(c)(i)

$$|op| = \sqrt{x^2 + y^2} = \sqrt{x^2 + \frac{81}{x^2}}.$$

Blunders (–3)
B1 error in distance formula.
B2 $|op|$ not expressed in terms of x only or y only.

Slips (–1)
S1 arithmetic error.

Attempts (2)
A1 $|op| = \sqrt{x^2 + y^2}$.

Part (ii)	**10 marks (5, 5)**	**Att (2, 2)**
Part (iii)	**5 marks**	**Att 2**

8(c) (ii) Given that there is one value of x for which $|op|$ is a minimum, find this value of x.

(iii) Hence, find the minimum value of $|op|$.

Correct differentiation	**5 marks**	**Att 2**
Minimum x value	**5 marks**	**Att 2**
Minimum value of $\mid op \mid$	**5 marks**	**Att 2**

8(c)(ii)

$$l = |op| = \sqrt{x^2 + \frac{81}{x^2}}$$

$$l = (x^2 + 81x^{-2})^{\frac{1}{2}}$$

$$\frac{dl}{dx} = \frac{1}{2}\left(x^2 + 81x^{-2}\right)^{-\frac{1}{2}}.(2x - 162x^{-3}) = \frac{x - \frac{81}{x^3}}{\left(x^2 + 81x^{-2}\right)^{\frac{1}{2}}}$$

$$\frac{dl}{dx} = 0 \Rightarrow x - \frac{81}{x^3} = 0$$

$$\therefore \quad x^4 = 81 \quad \Rightarrow \quad x = 3 \quad \text{as } x > 0.$$

(iii)

minimum value of $|op| = \sqrt{x^2 + \frac{81}{x^2}}$ for $x = 3$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}.$

Blunders (–3)
B1 error in differentiation.

Slips (–1)
S1 arithmetic error.

Attempts (2, 2, 2)
A1 $\frac{dl}{dx} = 0$ and stops.

QUESTION 9

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**
Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (c)	**20 marks (5, 5, !0)**	**Att (2, 2, 3)**

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**

Part (a)(i)	**5 marks**	**Att 2**
Part (a)(ii)	**5 marks**	**Att 2**

9(a) Two fair dice are thrown.

(i) What is the probability of getting a four on both dice?

(ii) What is the probability of getting a four on at least one die?

P(a four on both dice)	**5 marks**	**Att 2**
P(at least one four)	**5 marks**	**Att 2**

9(a)(i)

P(of getting a four on both dice) $= \frac{1}{6}.\frac{1}{6} = \frac{1}{36}.$

(ii)

P(of getting at least one four both dice)
1 − P(of getting no four on either dice)

$= 1 - \frac{5}{6}.\frac{5}{6} = \frac{11}{36}.$

Blunders (–3)
B1 incorrect number of possible outcomes.
B2 probabilities added instead of multiplied or converse.
B3 any theory error.

Slips (–1)
S1 arithmetic error.

Attempts (2, 2)
A1 correct number of possible outcomes.
A2 correct number of favourable outcomes.

Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (b) (i)	**10 marks**	**Att 3**
Part (b) (ii)	**10 marks**	**Att 3**

9(b) The probability of passing a driving test is $\frac{2}{3}$. Six students take the test.
Use a binomial distribution to find

(i) the probability that none of the students passes the test

(ii) the probability that half of the students pass the test.

Probability no students pass test	**10 marks**	**Att 3**
Probability half pass test	**10 marks**	**Att 3**

9(b)(i)

$$p = \frac{2}{3},\ q = \frac{1}{3},\ n = 6.$$

$$\binom{6}{0}p^0q^6 = \binom{6}{0}.\left(\frac{2}{3}\right)^0.\left(\frac{1}{3}\right)^6 = \frac{1}{729}.$$

(ii)

$$\binom{6}{3}p^3q^3 = \binom{6}{3}.\left(\frac{2}{3}\right)^3.\left(\frac{1}{3}\right)^3 = \frac{160}{729}.$$

Blunders (–3)
B1 error in binomial.
B2 incorrect q.

Slips (–1)
S1 arithmetic error.

Attempts (3, 3)
A1 use of binomial.
A2 $\left(\frac{2}{3}\right)^3$.

Part (c) **20 marks (5, 5, 10)** **Att (2, 2, 3)**

9(c)

A particular drug gives relief from pain. The period of pain relief reported by people who are treated with the drug is normally distributed with mean 50 hours and standard deviation 16 hours.

In a random sample of 64 people who have been treated with the drug, what is the probability that the mean period of pain relief reported is between 48 hours and 53 hours?

Evaluating $\sigma_{\bar{x}}$	**5 marks**	**Att 2**
Standard Units	**5 marks**	**Att 2**
Final solution	**10 marks**	**Att 3**

9(c)

$\mu_{\bar{x}} = 50, \ \sigma = 16, n = 64.$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{8} = 2.$$

$P(48 \le x \le 53) = P(Z_1 \le Z \le Z_2).$

$$Z_1 = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{48-50}{2} = -1$$

$$Z_2 = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{53-50}{2} = 1.5$$

$P(-1 \le Z \le 1.5)$

$= P(Z \le 1.5) - P(Z \le -1) = P(Z \le 1.5) - P(Z \ge 1)$

$= P(Z \le 1.5) - [1 - P(Z \le 1)]$

$= 0.9332 - [1 - 0.8413] = 0.9332 - 0.1587$

$= 0.7745.$

Blunders (–3)

B1 error in $\sigma_{\bar{x}}$.

B2 error in standard units other than slip.

B3 incorrect reading from tables.

B4 error in determining correct area required.

Slips (–1)

S1 arithmetic error.

Attempts (2, 2, 3)

A1 correct expression for $\sigma_{\bar{x}}$.

A2 correct expression for standard unit.

A3 a correct step in outlining correct required area.

QUESTION 10

Part (a)	**10 marks**	**Att 3**
Part (b)	**40 marks (10, 10, 5, 5, 5, 5)**	**Att (3, 3, 2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

10(a) A binary operation $\circ$ is defined by $a \circ b = \dfrac{a+b}{2}$ where a, b **R**.
Investigate if $(a \circ b) \circ c = a \circ (b \circ c)$.

* accept counter example.

Investigate **10 marks** **Att 3**

10(a)

$$a \circ b = \frac{a+b}{2}$$

Is $(a \circ b) \circ c = a \circ (b \circ c)$?

$$\frac{a+b}{2} \circ c = a \circ \frac{b+c}{2} \ ?$$

$$\frac{a+b+2c}{4} \neq \frac{2a+b+c}{4}$$ $\therefore$ not associative.

Blunders (–3)
B1 error in applying operation.

Slips (–1)
S1 arithmetic error.

Attempts (3)
A1 one correct operation other than $a \circ b$.

Part (b) **40 marks (10, 10, 5, 5, 5, 5)** **Att (3, 3, 2, 2, 2, 2)**

Part (b) (i) **10 marks** **Att 3**

10(b) The group G, * is defined by the following Cayley table:

*	*e*	*a*	*b*	*c*	*d*	*f*	*g*	*h*
e	*e*	*a*	*b*	*c*	*d*	*f*	*g*	*h*
a	*a*	*b*	*d*	*h*	*e*	*c*	*f*	*g*
b	*b*	*d*	*e*	*g*	*a*	*h*	*c*	*f*
c	*c*	*h*	*g*	*b*	*f*	*a*	*e*	*d*
d	*d*	*e*	*a*	*f*	*b*	*g*	*h*	*c*
f	*f*	*c*	*h*	*a*	*g*	*e*	*d*	*b*
g	*g*	*f*	*c*	*e*	*h*	*d*	*b*	*a*
h	*h*	*g*	*f*	*d*	*c*	*b*	*a*	*e*

(i) Find the order of each element.

Find order of elements **10 marks** **Att 3**

10(b)(i)

order(e) = 1, order(a) = 4, order(b) = 2, order(c) = 4,
order(d) = 4, order(f) = 2, order(g) = 4, order(h) = 2.

or

By Lagrange the order of an element is 1, 2, 4 or 8.

$e^1 = e \Rightarrow$ order(e) = 1. $a^2 = b \Rightarrow a^4 = b*b = e$ $\therefore$ order(a) = 4.

$b^2 = e \Rightarrow$ order(b) = 2. $c^2 = b \Rightarrow c^4 = b^2 = e$ $\therefore$ order(c) = 4.

$d^2 = b \Rightarrow d^4 = b^2 = e$ $\therefore$ order(d) = 4. $f^2 = e$ $\therefore$ order(f) = 2.

$g^2 = b \Rightarrow g^4 = b^2 = e$ $\therefore$ order(g) = 4. $h^2 = e$ $\therefore$ order(h) = 2.

Blunders (–3)

B1 order of element incorrect.

B2 incorrect operation each time.

Slips (–1)

S1 arithmetic error.

Attempts (3)

A1 order of one element correct.

A2 any valid work towards determining order of an element.

Part (b)(ii)	10 marks	Att 3

10(b)(ii) Write down three subgroups of order two.

Three subgroups of order two	10 marks	Att 3

10(b)(ii)

A subgroup of order two has to contain e and an element of order two.

$\therefore$ Subgroups are $\{e, b\}$, $\{e, f\}$, $\{e, h\}$.

Blunders (–3)

B1 one incorrect subgroup.

Slips (–1)

S1

Attempts (3)

A1 one correct subgroup of order two.

A2 two incorrect subgroups.

Part (b)(iii)	10 marks (5, 5)	Att (2, 2)

10(b)(iii) H = $\{e, c, x, y\}$ is a subgroup of G. What elements of G do x and y represent?

x **value**	**5 marks**	**Att 2**
y **value**	**5 marks**	**Att 2**

10(b)(iii)

$$c * c = b \quad \therefore x = b.$$

$$c * c = g \quad \therefore y = g.$$

$$\therefore \text{H} = \{e, c, b, g\}.$$

Blunders (–3)

B1 $c * c$ incorrect.

B2 $b * c$ incorrect.

Slips (–1)

S1 arithmetic error.

Attempts (2, 2)

A1 operating c with x or y and fails to finish.

A2 operating b with x or y and fails to finish.

Part (b)(iv)	**10 marks(5, 5)**	**Att (2, 2)**

10(b)(iv) Show that K $= \{e, b, f, h\}$ is a subgroup of G and explain why H and K are not isomorphic.

Subgroup	**5 marks**	**Att 2**
Not isomorphic	**5 marks**	**Att 2**

10(b)(iv)

$$K = \{e, b, f, h\}.$$

$*$	e	b	f	h
e	e	b	f	h
b	b	e	h	f
f	f	h	e	b
h	h	f	b	e

$\therefore$ Closed : No new element.

$e^{-1} = e$

$b^{-1} = b$

$f^{-1} = f$

$h^{-1} = h$

K is closed and each element has an unique inverse.

$\therefore K = \{e, b, f, h\}$ is a subgroup G.

Elements of H	e	c	b	g
Order of element of H	1	4	2	4
Elements of K	e	b	f	h
Order of elements of K	1	2	2	2

Different order of elements. $\therefore H$ and K are not isomorphic.

Blunders (–3)

B1 failure to show closure for K.

B2 failure to state inverses in K.

B3 order of element incorrect.

Slips (–1)

S1 arithmetic error.

Attempts (2, 2)

A1 Cayley table for K.

A2 states condition for isomorphism.

A3 incomplete work with $\phi : f(a \circ b) = f(a) * f(b)$.

QUESTION 11

Part (a)	**10 marks (5, 5)**	**Att (2, 2)**
Part (b)	**20 marks (10, 10)**	**Att (3, 3)**
Part (c)	**20 marks (5, 15)**	**Att (2, 5)**

Part (a)	**10 marks (5, 5)**	**Att (2,2)**

Part (a)(i)	**5 marks**	**Att 2**
Part (a)(ii)	**5 marks**	**Att 2**

11(a) h is the transformation $(x,y) \to (x',y')$ where $x' = 5x$ and $y' = 3y$.

(i) Find the image of the circle $x^2 + y^2 = 4$ under h.

(ii) Show that the image is an ellipse and find its eccentricity.

Find image	**5 marks**	**Att 2**
Eccentricity	**5 marks**	**Att 2**

11(a)(i)

$$x' = 5x, \quad y' = 3y. \qquad \text{C: } x^2 + y^2 = 4.$$

$$x = \frac{x'}{5} \text{ and } y = \frac{y'}{3}.$$

$$f(C)\text{: } \left(\frac{x'}{5}\right)^2 + \left(\frac{y'}{3}\right)^2 = 4 \Rightarrow \frac{x'^2}{100} + \frac{y'^2}{36} = 1.$$

(ii)

$$\frac{x'^2}{100} + \frac{y'^2}{36} = 1 \text{ is an ellipse with } a^2 = 100 \text{ and } b^2 = 36.$$

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{100-36}}{10} = \frac{8}{10}. \qquad \therefore e = \frac{4}{5}.$$

Blunders (–3)

B1 substitutes $5x$, $3y$ into circle equation.

B2 error in eccentricity formula.

Slips (–1)

S1 arithmetic error.

Attempts (2, 2)

A1 x in terms of x' or y in terms of y'.

A2 correct value for a or b.

A3 $e = \dfrac{\sqrt{a^2 - b^2}}{a}$ and stops.

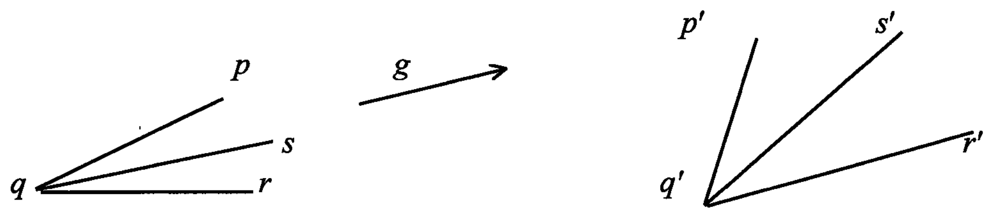
Part (b) **10 marks (10, 10)** **Att (3, 3)**

Part (b) (i) **10 marks** **Att 3**

11(b)	Let g be a similarity transformation.
(i)	The angle $\angle pqr$ is mapped to the angle $\angle p'q'r'$ under g. Given that the line qs bisects $\angle pqr$, show that $q's'$ bisects $\angle p'q'r'$.

Show that $q's'$ bisects $\angle p'q'r'$ **10 marks** **Att 3**

11(b)(i)



$$|\angle pqs| = |\angle sqr| \Rightarrow |\angle p'q's'| = |\angle s'q'r'|$$

as g is a similarity transformation.

$\therefore$ $q's'$ bisects $\angle p'q'r'$.

Blunders (–3)

B1 reason not given why $\angle p'q's' = \angle s'q'r'$.

Slips (–1)

S1 arithmetic error.

Attempts (3)

A1 mapping of angular bisector.

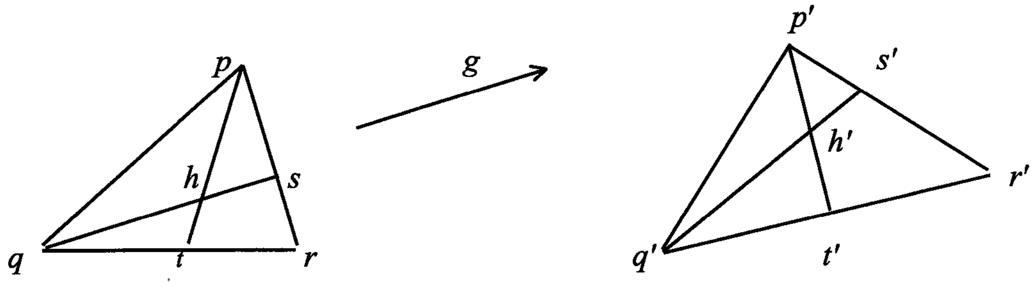
A2 diagram showing mappings.

Part (b)(ii) **10 marks** **Att 3**

11(b)(ii) Hence, prove that if h is the incentre of the triangle pqr, $g(h)$ is the incentre of the triangle $p'q'r'$.

Prove $g(h)$ incentre **10 marks** **Att 3**

11(b)(ii)



qs bisects $\angle pqr$ and pt bisects $\angle qpr$

$\therefore$ h is incentre of triangle pqr.

By part (i) $q's'$ bisects $\angle p'q'r'$ and $p't'$ bisects $\angle q'p'r'$

$\therefore$ h' is incentre of triangle $p'q'r'$.

But $g(qs \cap pt) = h'$

$\Rightarrow$ $g(h) = h'$.

$\therefore$ $g(h)$ is incentre of triangle $p'q'r'$.

Blunders (–3)

B1 reason not given why h' is incentre.

B2 reason why $g(h)$ incentre not given.

Slips (–1)

S1 arithmetic error.

Attempts (3)

A1 diagram of merit.

A2 mapping of angular bisectors.

Part (c) **20 marks (5, 15)** **Att (2, 5)**

Part (c)(i) **5 marks** **Att 2**

11(c)	f is the transformation $(x,y) \to (x',y')$ where $x' = ax$ and $y' = by$ for $a > b > 0$.
(i)	Given that $f(C)$ is the ellipse $\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1$, show that C is the circle $x^2 + y^2 = 1$.

Show *C* is the circle **5 marks** **Att 2**

11(c)(i)

$$f(C) = \frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1. \qquad x' = ax \text{ and } y' = by.$$

$$\therefore C = \frac{a^2x^2}{a^2} + \frac{b^2y^2}{b^2} = 1 \Rightarrow C\colon x^2 + y^2 = 1.$$

Blunders (–3)

B1 substitutes $\frac{x'}{a}, \frac{y'}{b}$ into f(C).

Slips (–1)

S1 arithmetic error.

Attempts (2)

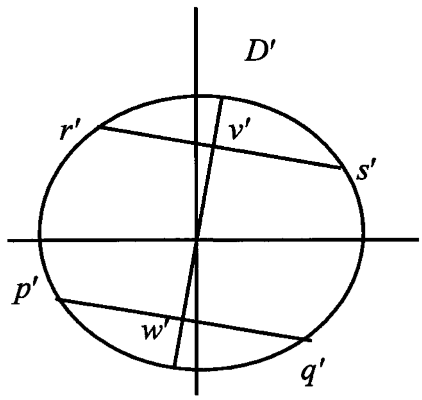
A1 fails to finish to $x^2 + y^2 = 1$.

Part (c) (ii) **15 marks** **Att 5**

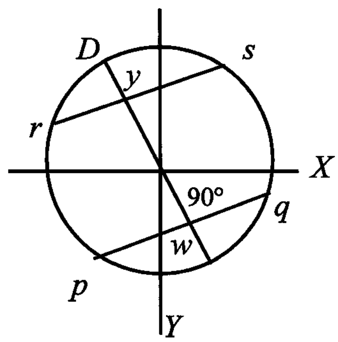
11(c)(ii)	Hence, show that the locus of midpoints of parallel chords of the ellipse $f(C)$ is a diameter (less its endpoints) of $f(C)$.

Show that $f(C)$ is diameter **15 marks** **Att 5**

11(c)(ii)



f^{-1}



$C: x^2 + y^2 = 1.$

$f(C)$: $\dfrac{x'^2}{a^2} + \dfrac{y'^2}{b^2} = 1.$

$|r's'|$ is parallel to $|p'q'|$.

v' and w' are midpoints of $[r's']$ and $[p'q']$ respectively.

Join of the points v' and w' is the line D'.

To prove : D' is a diameter of ellipse $f(C)$.

By f^{-1}, $[p'q']$ maps to $[pq]$ and $[r's']$ maps to $[rs]$.

But $[p'q']$ is parallel to $[r's']$

$[pq]$ is parallel to $[rs]$ as parallelism is an invariant map.

Also v is midpoint of $[rs]$ and w is midpoint of $[pq]$

as the mapping of midpoints is invariant.

$v, w \in D \Rightarrow D$ is a diameter of C.

By f, D maps to D'. $[f(0,0) = (0,0)]$

D' is a diameter of $f(C)$ (less its endpoints).

Blunders (–3)

B1 reason not given why $[rs]$ is parallel to $[pq]$.

B2 reason not given why v and w are mid-points of chords.

Slips (–1)

S1 arithmetic error.

Attempts (5)

A1 diagram of value.

A2 mapping of chords.