Coimisiún na Scrúduithe Stáit
STATE EXAMINATIONS COMMISSION

JUNIOR CERTIFICATE EXAMINATION 2003

MATHEMATICS HIGHER LEVEL PAPER 2

MARKING SCHEME

GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,....., S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), it is essential to note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks only.

7 The phrase "and stops" means that no more work is shown by the candidate.

8 The correct answer without work, where the hand symbol is shown, is blunder (-3)
An incorrect answer, without work, is worthless (0)

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 marks**	**Att 3,2,2**
Part (c)	**20 marks**	**Att 3,3**

Part (a) **10 marks** **Att 3**

✍ A solid cone has vertical height 4 cm. The radius of its base is 3 cm.
Find, in terms of π, the volume of the cone.

(a) **10 marks** **Att 3**

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2 4 = 12\pi$$

Blunders (-3)

B1 Error in volume of cone formula
B2 Incorrect substitution into formula
B3 Incorrect squaring
B4 Unfinished

Slips (-1)

S1 Answer not in terms of π
S2 Arithmetic slips

Attempts (3 marks)

A1 Correct formula with some substitution & stops

Worthless (0)

W1 Incorrect formula $\frac{4}{3}\pi r^3$ $2\pi rh$

Part (b) **20 (10,5,5) marks** **Att 3,2,2**

A solid rectangular metal block has length 12 cm and width 5 cm.
The volume of the block is 90 cm^3.

(i) ✍ Find the height of the block in cm.

(ii) ✍ Find the total surface area of the block in cm^2.

(iii) ✍ Each cm^3 of the metal has mass 8·4 g.
The total mass of a number of these metal blocks is 113·4 kg.
How many blocks are there?

(b)(i) **10 marks** **Att 3**

$V = (12)(5)h = 90$ step 1 $\Rightarrow h = \dfrac{90}{(12)(5)}$ step 2 $= 1 \cdot 5$ step 3

Blunders (-3)

B1 Incorrect relevant volume formula
B2 Mathematical Blunder (90 – 60 = 30)
B3 Misplaced decimal point
B4 Uses surface area & finishes
B5 Incorrect substitution e.g. 12.12.h = 90 & continues

Slips (-1)

S1 Arithmetic slips

Attempts (3 marks)

A1 Correct formula, no substitution e.g. l.b.h
A2 12.5.90

(b)(ii) **5 marks** **Att 2**

Surface area $= 2(12 \times 5 + 12 \times 1 \cdot 5 + 5 \times 1 \cdot 5)$
$= 2(60 + 18 + 7 \cdot 5) = 2(85 \cdot 5) = 171 \text{ cm}^2$.

* Allow candidate's h from (b)(i)

Blunders (-3)

B1 Forgets to double surfaces
B2 Omits one surface
B3 Uses volume
B4 Each incorrect substitution

Slips (-1)

S1 Arithmetic slips

Attempts (2 marks)

A1 Any attempt to get a surface area

Worthless (0)

W1 Incorrect answer no work
W2 Uses formula for cone, cylinder or sphere

(b)(iii) **5 marks** **Att 2**

mass of one block = $90 \times 8 \cdot 4$ g
number of blocks = $\dfrac{1000 \times 113.4}{90 \times 8 \cdot 4} = 150$ blocks

Blunders (-3)

B1 Forgets to multiply 8.4 by 90 & continues correctly
B2 Inverts the division
B3 Does not change Kgs to grams

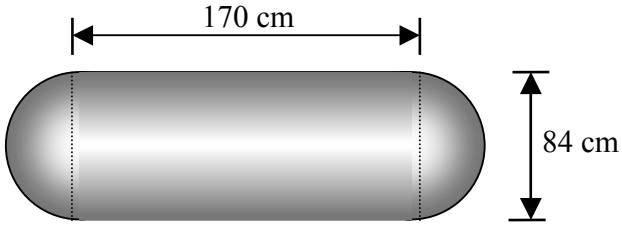
Slips (-1)

S1 Arithmetic slips
MR 84 for 8.4 or similar & continues

Attempts (2 marks)

A1 90 x 8.4 & stops
A2 Correct answer, no work
A3 1000 x 113.4 & stops

Part (c) **20 (10,10)marks** **Att 3,3**



A capsule is made up of a cylindrical section and two hemispherical ends.
The length of the cylindrical section is 170 cm and the diameter is 84 cm.

(i) ✍ Find the surface area of the capsule in cm^2.
Give your answer correct to two significant figures.

(ii) ✍ Find the volume of the capsule in m^3.
Give your answer correct to two decimal places.

(c)(i) **10 marks** **Att 3**

$$\text{Surface area} = 4\pi r^2 + 2\pi rh \qquad \text{step 1}$$
$$= 4\pi(42)^2 + 2\pi(42)(170) \qquad \text{step 2}$$
$$= 7056\pi + 14280\pi$$
$$= 21336\pi = 66995 = 67000\,\text{cm}^2 \qquad \text{step 3}$$

* Initial answer will change depending on the value of π

Blunders (-3)
B1 Sees the diagram as two-dimensional
B2 Each step incorrect or omitted
B3 r = 84 & continues
B4 Mathematical blunders
B5 Each incorrect relevant formula

Slips (-1)
S1 Arithmetic slips
S2 Failure to round off or rounds off incorrectly

Attempts (3 marks)
A1 $4\pi r^2$ or 2πrh with some substitution
A2 r = 42
A3 Two wrong relevant formulae added e.g. $3\pi r^2 + \frac{4}{3}\pi r^3$

(c)(ii) **10 marks** **Att 3**

$$\text{Volume} = \frac{4}{3}\pi r^3 + \pi r^2 h \qquad \text{step1}$$

$$= \frac{4}{3}\pi(0\cdot 42)^3 + \pi(0\cdot 42)^2 1\cdot 7 \qquad \text{step2}$$

$$= 0\cdot 098784\pi + 0\cdot 29988\pi$$

$$= 0\cdot 398664\pi = 1\cdot 2518 = 1\cdot 25\ \text{m}^3 \qquad \text{step 3}$$

* Accept candidate's radius from part (c)(i)

Blunders (-3)
B1 Answer in cm^3
B2 r = 84 (unless in (c)(i))
B3 Each incorrect relevant formula e.g. πrh
B4 Misplaced decimal point (once only)
B5 Mathematical blunders
B6 Each step incorrect or omitted

Slips (-1)
S1 Arithmetic slips
S2 Failure to round off

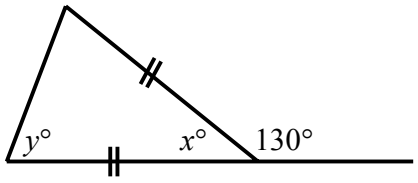
Attempts (3 marks)
A1 $\frac{4}{3}\pi r^3$ or $\pi r^2 h$ with some substitution
A2 r = 42 or .42 & stops

QUESTION 2

Part (a)	**10 marks**	**Att 2,2**
Part (b)	**20 marks**	**Att 3,2,2**
Part (c)	**20 marks**	**Att 2,2,2,2**

Part (a) **10 (5,5) marks** **Att 2,2**

Calculate the value of x
and the value of y in the diagram.



(a) **10 (5,5) marks** **Att 2,2**

$x° + 130° = 180° \Rightarrow x° = 50°$

$2y° = 130° \Rightarrow y° = 65°$

* Accept correct answer without work in each case

Blunders (-3)

B1 x° + 130° = 180° & stops (for x)

B2 x° + x° + y° = 180° giving y = 80°

Slips (-1)

S1 Numerical slips

Attempts (2,2 marks)

A1 shows 180° & stops

A2 States exterior angle theorem & stops

A3 States angles at base of isosceles triangle are equal & stops

Worthless (0)

W1 x = y & stops

Part (b) **20 (10,5,5) marks** **Att 3,2,2**

$a(2, 3)$ and $b(5, -1)$ are two points.

The translation $\overrightarrow{ab}$ maps the point $p(6, 7)$ to the point q.

(i) ✍ Find the co-ordinates of q.

(ii) ✍ Verify that $|ab| = |pq|$.

(b)(i) **10 marks** **Att 3**

$(2, 3) \rightarrow (5, -1)$: x up 3, y down 4
$(6, 7) \rightarrow (9, 3)$.

* (9,3) with no work, Bl (-3) BUT $(2,3) \rightarrow (5,-1)$ and $(6,7) \rightarrow (9,3)$ gets full marks.

- Accept $\vec{ap}$ i.e.$(2,3) \rightarrow (6,7)$ [up 4, up 4] and $(5,-1) \rightarrow (9,3)$

Blunders (-3)
B1 Incorrect sign in Change of x and/or y (once only)
B2 Determines changes between x and y in each point and applies correctly
B3 Change in x applied to y or vice versa

Misreading (-1)
M1 Applies changes to point b instead of point p

Slips (-1)
S1 Incorrect numerical change each time in the correct direction

Attempts (3 marks)
A1 One correct change & stops
A2 Axes showing at least two points
A3 Explanation of a translation & stops

W1 Slopes calculated (apply also to (b)(ii))

(b)(ii) **10 (5,5) marks** **Att 2,2**

$| ab | = \sqrt{(5-2)^2 + (-1-3)^2} = \sqrt{9+16} = 5$ 5 marks

$| pq | = \sqrt{(9-6)^2 + (3-7)^2} = \sqrt{9+16} = 5$ 5 marks

* Allow incorrect point q from part (b)(i)

Blunders (-3)
B1 Incorrect sign(s) in Distance Formula (once only) and continues
B2 Incorrect relevant formula and continues correctly: no square root or no squaring, each time
B3 Incorrect substitution i.e. x and y confused
B4 Mathematical blunders

Slips (-1)
S1 Mixes up x_1 and x_2

Attempts (2 marks)
A1 Correct Distance formula & stops
A2 Any attempt at subtraction of x and/or y coordinates
A3 Axes showing a and b is att 2 BUT showing a,b and p and/or q merits att 2 twice

Part (c) **20 (5,5,5,5) marks** **Att 2,2,2,2**

L is the line $x - 2y - 3 = 0$.

(i) ✍ Find the slope of L.

(ii) ✍ Find the equation of the line K through (-2, 5) which is perpendicular to L.

(iii) ✍ Find the co-ordinates of the point of intersection of L and K.

(iv) ✍ Hence, or otherwise, find the co-ordinates of the image of (-2, 5) under the axial symmetry in L.

(c)(i) **5 marks** **Att 2**

$x - 2y - 3 = 0 \Rightarrow 2y = x - 3 \Rightarrow y = 0{\cdot}5x - 1.5$

Slope is 0·5 Allow any other valid method

Blunders (-3)

B1 Error in manipulation

B2 Error in formula

Attempts (2 mark)

A1 Correct formula & stops

A2 Finds correct point(s) on the line & stops

A3 Mentions x = 0 at y-axis and/or y = 0 at x-axis & stops

(c)(ii) **5 marks** **Att 2**

Slope of K is -2

$y - 5 = -2(x + 2) \Rightarrow 2x + y - 1 = 0$. (not necessary)

* Allow incorrect answer from (c)(i)

* Note y-5 = -2(x + 2) gets full marks

Blunders (-3)

B1 Incorrect signs in equation of line formula & continues

B2 $M_1 \times M_2 = 1$ & continues

B3 Incorrect sign for the slope

B4 Switches x and y when substituting

B5 Substitutes correctly for x and y but no slope

S1 One incorrect sign in formula & continues

Attempts (2 mark)

A1 Correct equation formula & stops

A2 $M_1 \times M_2 = -1$ & stops

A3 Gets the correct slope & stops

A4 2x + y + k =0 & stops

(c)(iii) **5 marks** **Att 2**

$x - 2y = 3 \quad \Rightarrow \quad x - 2y = 3$

$2x + y = 1 \quad \Rightarrow \quad \underline{4x + 2y = 2}$

$5x \quad = 5 \quad \Rightarrow \quad x = 1$

$x - 2y = 3 \quad \Rightarrow \quad 1 - 2y = 3 \quad \Rightarrow \quad y = -1.$ Point of intersection (1, -1).

* Allow candidate's K from part (c)(ii)

Blunders (-3)
B1 Error in manipulation of the equations
B2 Error in signs
B3 Incorrect or no substitution for second value

Attempts (2 marks)
A1 Any correct step & stops
A2 Correct graphical solution

(c)(iv) **5 marks** **Att 2**

$(-2,5) (\in K) \rightarrow (1, -1) \rightarrow (1+3, -1-6) = (4, -7).$

* Allow candidate's point of intersection from (c)(iii)

Blunders (-3)
B1 Incorrect sign in change of x and/or y and continues
B2 Incorrection direction of translation

Slips (-1)
S1 Incorrect numerical change each time

Attempts (2 marks)
A1 Determines one change & stops
A2 Axes showing the correct two points

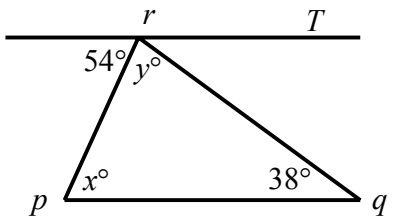
QUESTION 3

Part (a)	**10 marks**	**Att 2,2**
Part (b)	**20 marks**	**Att 3,3**
Part (c)	**20 marks**	**Att 2,2,2,2**

Part (a) **10 (5,5) marks** **Att 2,2**

The line T passes through r and is parallel to pq.

Calculate the value of x and the value of y in the diagram.



(a) **10 (5,5) marks** **Att 2,2**

$x° = 54°$

$54° + y° + 38° = 180° \Rightarrow y° = 180° - 92° = 88°$

S1 Numerical slips

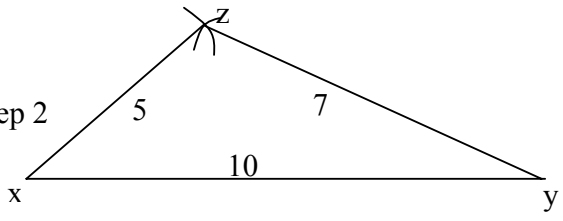
A1 States alternate angles are equal & stops (for x)
A2 States the angle between rq and T is 38° & stops (for y)
A3 States that x° + y° + 38° = 180° & stops (for y
A4 Extends rp (or rq)and gets the exterior angle & stops

Part (b)(i) **10 marks** **Att 3**

Construct a triangle xyz in which $|xy| = 10$ cm, $|yz| = 7$ cm and $|xz| = 5$ cm.

Part (b)(i) **10 marks** **Att 3**

Construction
draw [xy]10 cm in length step 1
with x as centre draw an arc 5cm in length &
with y as centre draw an arc 7cm in length step 2
arcs intersect at z. join zx and zy step 3



* Allow tolerance of ± .2

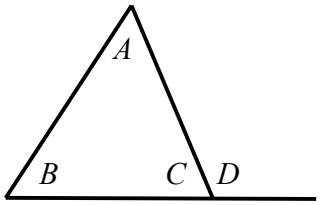
B1 Each step omitted or incorrect
B2 No construction lines visible
B3 Outside of tolerance (once only)
A1 Rough diagram with lengths marked
W1 Triangle with no measurements

Part (b)(ii) **10 marks** **Att 3**

✍ Prove that an exterior angle of a triangle equals the sum of the two interior opposite angles in measure.

(b)(ii) **10 marks** **Att 3**

Given triangle with angles A, B, C, D, as shown



To Prove: $|\angle D| = |\angle A| + |\angle B|$ step 1

Proof: $|\angle A| + |\angle B| + |\angle C| = 180°$

$|\angle C| + |\angle D| = 180°$ step 2

Hence, $|\angle C| + |\angle D| = |\angle A| + |\angle B| + |\angle C|$

Thus, $|\angle D| = |\angle A| + |\angle B|$ step 3

* Memorised proof, no diagram, full marks if all steps are given

Blunders (-3)

B1 Each step omitted or incorrect

B2 Each incomplete step

Attempts (3 marks)

A1 Labelled diagram & stops

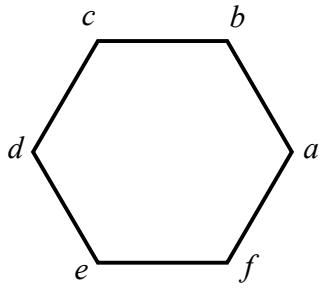
A2 Proves 3 angles total 180° correctly

Worthless (0)

W1 Draws a triangle & stops

Part (c) **20 (5,5,5,5)marks** **Att 2,2,2,2**

The diagram shows a regular hexagon.
(A regular hexagon has six equal sides and six equal angles.)



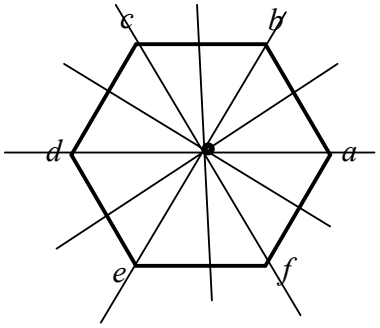
(i) How many axes of symmetry has the hexagon?
(ii) Copy the diagram into your answerbook and draw in the axes of symmetry.
(iii) $[ad]$ and $[cf]$ intersect at o.
What is the measure of the angle of the rotation, about o, which maps a onto c?
(iv) Describe one transformation which maps $[af]$ to $[cd]$.

(c) **20 (5,5,5,5)marks** **Att 2,2,2,2**

(i) 6 axes **(ii)** →

(iii) 120° or 240°

(iv) axial symmetry in *be*
translation *a* to *c or f to d*
or central symmetry in *o*
or R_{180°



(i) S1 12
A1 Any number between 1 and 5 inclusive
W1 any other number

(ii) S1 Each axis missing, to max of 3
A1 One axis drawn & stops
W1 Copies diagram & stops

(iii) B1 $\frac{360}{12}$ or $\frac{360}{12} = 30^\circ$ & stops
A1 Any indication of 30° or 60° or 90°
Note: Accept correct answer without work.

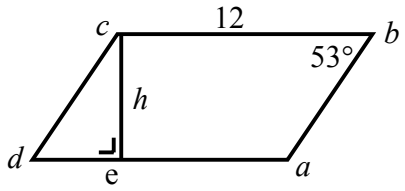
(iv) B1 Central symmetry in eb
B2 Axial symmetry in o
A1 Mentions eb e.g. $\vec{eb}$
W1 Central symmetry or axial symmetry or translation or rotation & stops

QUESTION 4

Part (a)	**10 marks**	**Att 2,2**
Part (b)	**20 marks**	**Att 6**
Part (c)	**20 (5,5,5,5)marks**	**Att 2,2,2,2**

Part (a) **10 (5,5) marks** **Att 2,2**

In the parallelogram *abcd*,
$|\angle abc| = 53°$ and $|bc| = 12$ cm.



(i) Find $|\angle bcd|$.

(ii) ✍ Find the perpendicular height, *h*,
given that the area of *abcd* is 90 cm^2.

(a)(i) **5 marks** **Att 2**

$|\angle bcd| = 180° - 53° = 127°$

Blunders (-3)

B1 Assumes angles in Parm total 180° and gets answer 37°

B2 Gets $|\angle dce|$ & stops

B3 Gets $|\angle bcd| = 254°$ $(360° - 2(53°))$

Slips (-1)

S1 Numerical slips

Attempts (2 marks)

A1 States that the sum of the angles in a Parm is 360°

A2 Gets $|\angle cda| = 53°$ & stops

A3 $|\angle bce| = 90°$

(a)(ii) **5 marks** **Att 2**

$(12)(h) = 90 \Rightarrow h = 7\frac{1}{2}$

Blunders (-3)

B1 Gets the area of triangle cda or cba & stops

B2 Error in transposing

B3 Uses $\frac{1}{2}(12).h = 90$

Attempts (2 marks)

A1 12 x h = 90 & stops

A2 Finds |cd| & stops (9.39) or |ab| & stops

Part (b) **20 marks** **Att 6**

✍ Prove that if two sides of a triangle are equal in measure, then the angles opposite these sides are equal in measure.

(b) **20 marks** **Att 6**

abc is a triangle with $|ab| = |ac|$

To Prove: $|\angle abc| = |\angle acb|$ Step1

Construction: Join *a* to *d*, the midpoint of [*bc*] step 2

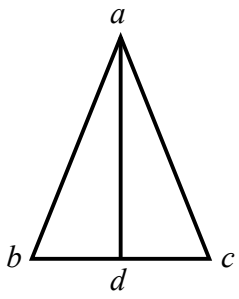
Proof: $|ab| = |ac|$ (given) step 3

$|ad| = |ad|$ step 4

$|bd| = |dc|$ (construction) step 5

Thus triangles *abd* and *adc* are congruent (SSS)

Thus $|\angle abc| = |\angle acb|$ step 6



* For construction candidate may use ad ⊥ bc but 6 steps still apply
* Step 3 may be implied in the construction in both proofs
* Steps 2,3 and 4 may be indicated on diagram
* Memorised proof, no diagram, full marks if all steps are given
* In step 2 omits "mid-point of [bc)" or similar is a blunder

Blunders (-3)

B1 Each step incorrect or omitted

Attempt (6 marks)

A1 ad shown in a diagram (even without letters)

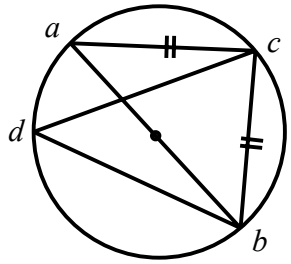
Worthless (0)

W1 A triangle and nothing else

W2 Wrong theorem

Part (c) **20 (5,5,5,5) marks** **Att 2,2,2,2**

a, d, b, c are points on a circle, as shown.
$[ab]$ is a diameter of the circle.
$|ab| = 12$ cm and $|ac| = |cb|$.



(i) ✍ Write down $|\angle bca|$, giving a reason for your answer.

(ii) ✍ Find $|\angle cdb|$.

(iii) ✍ Find $|bc|$.

(iv) ✍ Find the area of Δabc.

(c) **20 (5,5,5,5) marks** **Att 2,2,2,2**

(i) $|\angle bca| = 90°$, angle in semicircle

(ii) $|ac| = |bc| \Rightarrow |\angle bac| = 45°$
$|\angle cdb| = |\angle bac| \Rightarrow |\angle cdb| = 45°$

(iii) $|ac|^2 + |bc|^2 = 12^2 \Rightarrow 2|bc|^2 = 144 \Rightarrow |bc|^2 = 72 \Rightarrow |bc| = \sqrt{72}$ (accept) $= 6\sqrt{2}$

(iv) Area $= \frac{1}{2}|bc| \times |ac| = \frac{1}{2} \times 6\sqrt{2} \times 6\sqrt{2} = 36$

(i) B1 Reason not given or incorrect or measured using a protractor

(ii) W1 Reproduction of original diagram & stops and also for (iii) and (iv)

(iii) May use the Sine rule $\frac{|bc|}{\sin 45°} = \frac{12}{\sin 90°}$ or $\text{Sin } 45° = \frac{|\text{bc}|}{12} = \frac{1}{\sqrt{2}}$ etc.

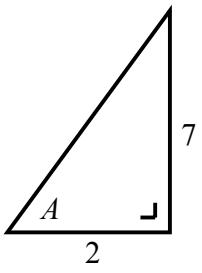
(iv) May use trig area formula $\frac{1}{2}|ca||cb|\sin 90°$ etc.

QUESTION 5

Part (a)	**10 marks**	**Att 2,2**
Part (b)	**20 marks**	**Att 3,3**
Part (c)	**20 marks**	**Att 3,3**

Part (a) **10 (5,5) marks** **Att 2,2**

✍ Use the information given in the diagram to find sin A and cos A.
Give your answers in surd form.



(a) **10 (5,5) marks** **Att 2,2**

$7^2 + 2^2 = 49 + 4 = 53$ 5 marks

$\sin A = \frac{7}{\sqrt{53}}$ $\cos A = \frac{2}{\sqrt{53}}$ 5 marks

* Note that this part is divided in to two parts as per scheme

Blunders (-3)

B1 Incorrect use of Pythagoras' Theorem
B2 Incorrect squaring
B3 Incorrect trigonometric ratios
B4 Omits either sine or cosine
B5 Uses 53 for $\sqrt{53}$

Slips (-1)

S1 Numerical slips
S2 Answer not in surd form (0.9615 and 0.2747) – once only

Attempts (2,2 marks)

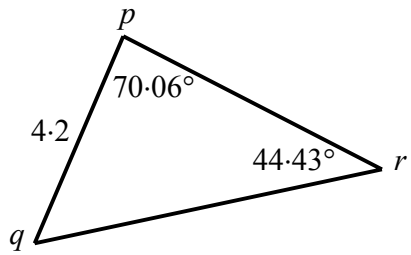
A1 Any reference to Pythagoras
A2 States correct trigonometric ratio for sine or cosine

Part (b) **20 (10,10) marks** **Att 3,3**

In the triangle pqr,
$|\,pq\,| = 4{\cdot}2$ cm, $|\,\angle rpq\,| = 70\cdot 06°$
and $|\,\angle qrp\,| = 44\cdot 43°$.

(i) ✍ Find $|\,qr\,|$, giving your answer correct to two decimal places.

(ii) ✍ Hence, or otherwise, find the area of $\Delta\, pqr$.
Give your answer correct to two decimal places.



Part (b)(i) **10 marks** **Att 3**

$$\frac{|\,qr\,|}{\sin 70\cdot 06°} = \frac{4\cdot 2}{\sin 44\cdot 43} \qquad \text{step 1}$$

$$\frac{|qr|}{.9401} = \frac{4.2}{.7001} \qquad \text{step 2}$$

$$|qr| = \frac{4.2 \text{ x } .9401}{.7001} = 5.64 \qquad \text{step 3}$$

To be applied to parts (b) and (c)

Blunders (-3)
B1 Each step incorrect or omitted
B2 Incorrect trigonometric ratio
B3 Incorrect ratio in Sine rule
B4 Error in transposition
B5 Takes 1° = 100'
B6 Decimal error
B7 Reading wrong page of tables or calculator in the wrong mode
B8 Failure to calculate
B9 Early rounding off which affects the accuracy of the answer

Slips (-1)
S1 Numerical slips
S2 Slips reading tables e.g. wrong column
S3 Fails to round off
MR1 Fails to distinguish between degrees & minutes and decimal degrees e.g. 70.06° & 70°6'
i.e. apply once only throughout
Attempts (3 marks)
A1 Partly filled in Sine Rule & stops

Worthless
W1 Treats triangle pqr as a right-angled triangle

(b)(ii) **10 marks** **Att 3**

$|\angle pqr| = 180° - (70\cdot06° + 44\cdot43°) = 65\cdot51°$ step 1

$0\cdot5 \times 4\cdot2 \times 5\cdot64 \times \sin 65\cdot51°$ step 2

$\text{Area} = 10\cdot778 = 10\cdot78$ step 3

* Allow the candidate's $|\text{qr}|$ from (b)(i)

B10 Uses only one side, $\frac{1}{2}|\text{pq}|\sin 65.51°$

B11 Halves the 65.51 in the sin 65.51° & continues

B12 Incorrect formula

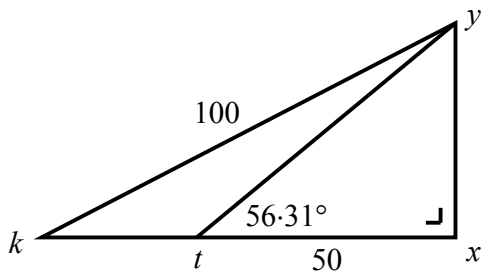
A2 Area formula from the tables with some substitution & stops

Part (c) **20 (10,10)marks** **Att 3,3**

A vertical mast $[xy]$ stands on level ground.
A straight wire joins y, the top of the mast, to t, a point on the ground. t is 50 m from x, the bottom of the mast.

(i) ✍ If $|\angle ytx| = 56\cdot31°$, find $|xy|$, the height of the mast.

(ii) ✍ A second straight wire joins y to k, another point on the ground. If the length of this wire is 100 m, find $|\angle ykt|$, correct to the nearest degree.



(c)(i) **10 marks** **Att 3**

1 $\tan 56\cdot31° = \frac{|xy|}{50}$ step 1 $\Rightarrow |xy| = 50(1\cdot5000)$ step 2 $= 75$ step 3

2 $\tan 33.69° = \frac{50}{|\text{xy}|}$ step 1 $\Rightarrow .6666 = \frac{50}{|\text{xy}|}$ step 2 $\Rightarrow |\text{xy}| = \frac{50}{.6666} = 75$ step 3

3 $\frac{50}{\text{Sin}33.69°} = \frac{|\text{xy}|}{\text{Sin}56.31°}$ step 1 $\Rightarrow \frac{50}{.5547} = \frac{|\text{xy}|}{.8321}$ step 2 $\Rightarrow |\text{xy}| = \frac{.8321 \text{ x } 50}{.5547} = 75$ step 3

MR1 Gets $|\text{yt}|$ for $|\text{xy}|$ (90.14)

A3 Gets 33.69° & stops

(c)(ii) **10 marks** **Att 3**

1 $\sin \angle ykx = \frac{75}{100}$ step 1 $= 0.75$ step 2 $\Rightarrow |\angle ykx| = 48 \cdot 59° = 49°$ step 3

2
$\text{Cos}\angle kyx = \frac{75}{100} = 0.75$ step 1

$\Rightarrow |\angle kyx| = 41.24°$ step 2

$\Rightarrow |\angle ykx| = 90° - 41.24 = 49°$ step 3

* There are other methods, e.g. Pythagoras, Sine Rule on triangle kty

* Accept candidate's $|xy|$ from (c)(i)

A4 Gets 123.69° & stops

Two other methods

3 Δykx $\frac{100}{\text{Sin } 90°} = \frac{75}{\sin \angle ykx}$ step 1 $\Rightarrow$ $\frac{100}{1} = \frac{75}{\sin \angle ykx}$ step 2

$\Rightarrow$ $100 \text{ Sin } \angle yks = 75$ $\Rightarrow \text{Sin } \angle ykx = \frac{75}{100} = 0.75$ $\Rightarrow |\angle ykx| = 49°$ step 3

4
Δxyt $|yt| = 90.14$ step 1

Δykt $\frac{90.14}{\text{Sin } \angle ykx} = \frac{100}{\text{Sin } 123.69°}$ step 2

$\Rightarrow 100 \text{ Sin } \angle ykx = 90.14 \text{ Sin } 123.69°$ $\Rightarrow |\angle ykx| = 49°$ step 3

QUESTION 6

Part (a)	**10 marks**	**Att 2,2**
Part (b)	**20 marks**	**Att 3,2,2**
Part (c)	**20 marks**	**Att 2,2,2,2**

Part (a) **10 (5,5) marks** **Att 2,2**

(i) ✍ Show that 13 is the mean of the numbers 6, 11, 15, 16, 17.

(ii) ✍ 14 is the mean of the numbers 6, 11, 15, 16, 17, x.
Find the value of x.

(a) **10 (5,5) marks** **Att 2,2**

(i) $\dfrac{6+11+15+16+17}{5} = \dfrac{65}{5} = 13$

(ii) $\dfrac{6+11+15+16+17+x}{6} = \dfrac{65+x}{6} = 14 \Rightarrow 65+x = 84 \Rightarrow x = 19$

(i) B1 Incorrect denominator

S1 Numerical slips, same for (ii)

A1 Adds some or all of the numbers & stops
A2 Division by 5

W1 Multiplies the 5 numbers instead of adding, same for (ii)

(ii) B1 Error in transposing
B2 Incorrect denominator

A1 Adds some or all of the numbers
A2 6 x 14 = 84 & stops
A3 States that the mean is $\dfrac{6+11+15+16+17+x}{6}$ & stops

Part (b) **20 (10 + 5 + 5)marks** **Att 3,2,2**

The duration of each log-on to the internet in a public library was recorded over a certain period.
The results are summarised in the following table:

Duration (minutes)	0 – 3	3 – 6	6 – 9	9 – 15	15 – 21	21 – 30
Number of log-ons	3	5	9	20	21	12

[Note: 3 – 6 means 3 minutes or more but less than 6 minutes etc.]

(i) Draw a histogram to illustrate the data in the table.

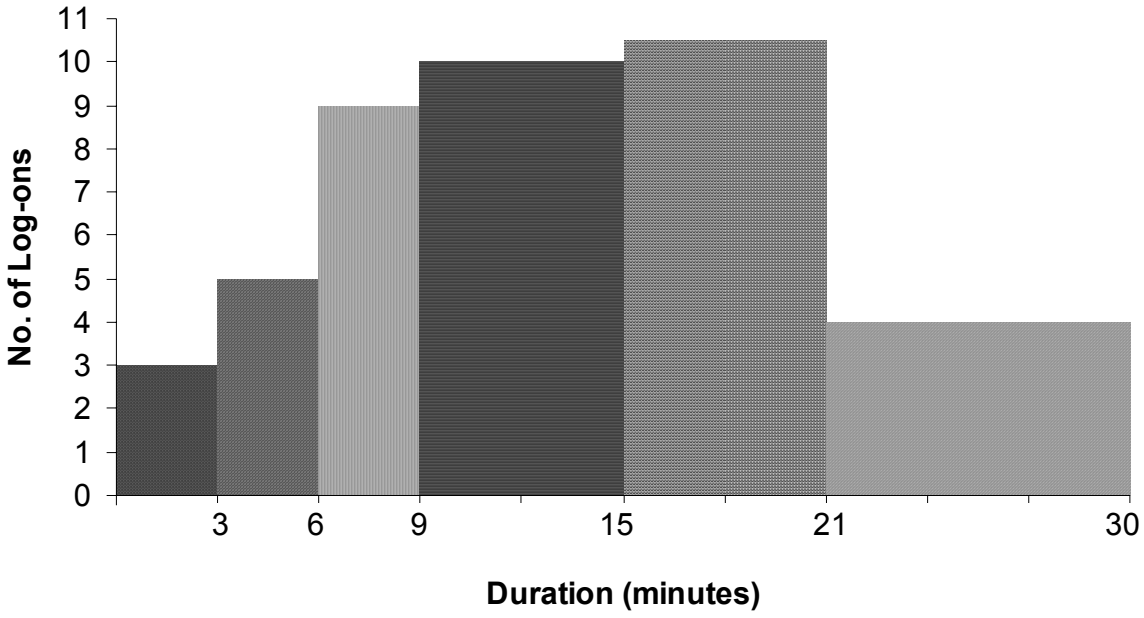
(ii) ✍ What was the total number of log-ons made?

(iii) ✍ In which class interval does the median lie?

(b)(i) **10 marks** **Att 3**

Histogram

Duration	0-3	3-6	6-9	9-15	15-21	21-30
Log-ons	3	5	9	20	21	12
Base	3	3	3	6	6	9
Height	3	5	9	10	10.5	4



Blunders (-3)
B1 Scale not uniform
B2 Each width incorrect and inconsistent with candidate's scale
B3 Each height out of proportion, but if error is consistent apply once only
B4 Bars with correct width and height but separated
B5 'Number of log-ons' on the horizontal axis
B6 Use other type of graph
B7 Any other error apart from above
B8 Each missing bar to a max of 3

Attempts (3 marks)
A1 Axes scaled or partly scaled & stops
A2 Calculates the heights only
A3 Frequency polygon / curve

Worthless (0)
W1 Pie-chart

(b)(ii) **5 marks** **Att 2**

70

* Remember answer 70 needs work for full marks

Blunders (-3)
B1 Adds heights instead of log-ons
B2 Adds 3+6+9+15 … to get 84

Slips (-1)
S1 Incorrect addition or omits a number

Attempts (2 marks)
A1 Adds the mid-interval values (69)
A2 Any effort to add any two adjacent values

Worthless (0)
W1 Finds the mid-interval values & stops
W2 looks up "log"of numbers

(b)(iii) **5 marks** **Att 2**

9 – 15 over 35

Blunders (-3)
B1 Shows adding, stops at 37. Fails to list class interval 9 – 15
B2 Adds heights (41.5) and concludes class interval is 15 – 21

Slips (-1)
S1 Incorrect addition

Attempts (2 marks)
A1 70/ 2 = 35 & stops
A2 Some effort at adding 3,5,9,etc.
A3 Graphical. May state class interval after (c)(iii) or may return and state answer in part (c)

Part (c) **20 (5,5,5,5,5)marks** **Att 2,2,2,2**

(i) Copy the following cumulative frequency table into your answerbook and use the table in part **(b)** to complete it:

Duration (minutes)	< 3	< 6	< 9	< 15	< 21	< 30
Number of log-ons						

(ii) On graph paper construct the ogive.

Use your graph to estimate:

(iii) ✍ the median

(iv) ✍ the number of log-ons lasting at least 10 minutes.

(c)(i) **5 marks** **Att 2**

Duration (minutes)	< 3	< 6	< 9	< 15	< 21	< 30
Number of log-ons	3	8	17	37	58	70

Blunders (-3)
B1 Omits any number or puts numbers in the wrong places

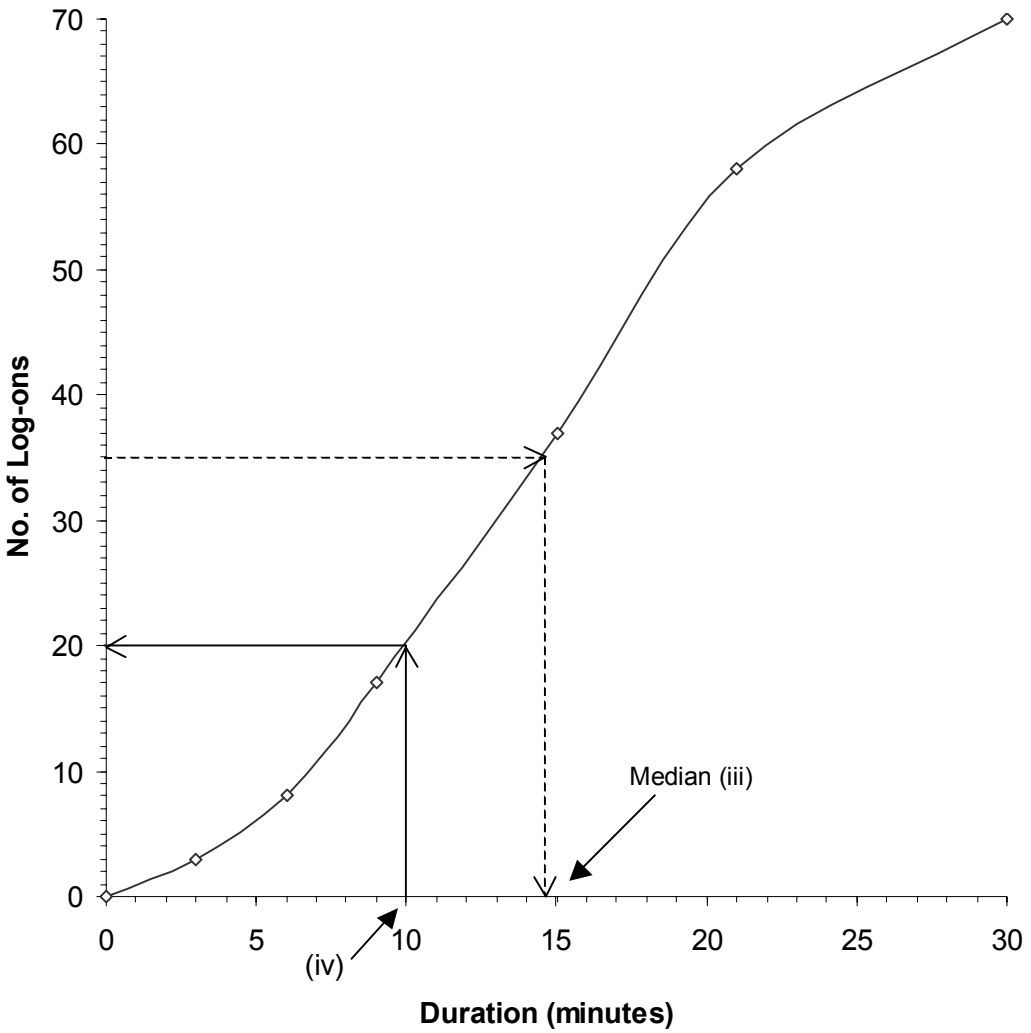
Slips (-1)
S1 Incorrect addition

Attempts (2 mark)
A1 Any one value filled in correctly into table A2 Any indication of addition
Worthless (0)
W1 Copies table & stops

(c)(ii) **5 marks** **Att 2**

Ogive

* Accept ogive consistent with student's cumulative frequency table



Blunders (-3)

B1 Scale not uniform
B2 Points not joined or joined by line segments
B3 Each point omitted or plotted incorrectly (if not consistent or slip)
B4 Interchanges axes
B5 Draws histogram

Slips (-1)

S1 Slips in plotting points (to a max of 3)

Attempts (2 marks)

A1 Axes scaled or partly scaled & stops
A2 Frequency polygon or curve
A3 Couples named e.g. (3,3) & stops
A4 Bar chart

(c)(iii) **5 marks** **Att 2**

Median = 14.4

* Accept answer consistent with candidate's graph (with tolerance ± 3)

Blunders (-3)

B1 Median read from wrong starting point of correct axis
B2 Mid-value of wrong axis taken as starting point (& continues)
B3 Mathematical error in finding median of a histogram
B4 Stops dead in mid-air before reading corresponding value

Slips (-1)

S1 Median indicated but value not written
S2 Value just outside tolerance

Attempts (2 marks)

A1 Some attempt to find median on graph
A2 Attempt to find mode or mean

(c)(iv) **5 marks** **Att 2**

Log-ons lasting at least 10 minutes	50 (70 – 20)

* Accept answer consistent with candidate's graph (within tolerance ± 3)

Blunders (-3)

B1 Line drawn from wrong starting point of correct axis
B2 No subtraction
B3 Line drawn from 10 on the vertical axis
B4 Starting point correct but reading far outside of tolerance

Slips (-1)

S1 Starting point correct but reading just outside tolerance. Continues to subtract from 70
S2 70 + 20