Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2003*
Matamaitic	*Ardleibhéal*
Marking Scheme	*Leaving Certificate Examination, 2003*
Mathematics	*Higher Level*

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MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2003

MATHEMATICS

HIGHER LEVEL

PAPER 1

GENERAL GUIDELINES FOR EXAMINERS - PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

7. A serious blunder, omission or misreading merits the ATTEMPT mark at most.

8. The phrase "and stops" means that no more work is shown by the candidate.

9. Accept the best of two or more attempts – even when attempts have been cancelled.

QUESTION 1

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

1(a)
Express the following as a single fraction in its simplest form:

$$\frac{6y}{x(x+4y)} - \frac{3}{2x}$$

Part (a) **10 marks** **Att 3**

1(a)

$$\frac{6y}{x(x+4y)} - \frac{3}{2x} = \frac{2(6y) - 3(x+4y)}{2x(x+4y)}$$

$$= \frac{12y - 3x - 12y}{2x(x+4y)}$$

$$= \frac{-3x}{2x(x+4y)}$$

$$= \frac{-3}{2(x+4y)}$$

Blunders (-3)
B1 indices.
B2 incorrect cancellation.
B3 answer not in simplest form.

Slips (-1)
S1 numerical.

Worthless
W1 values for *x* and *y*.

Part (b)(i) **10 marks** **Att 3**

1(b)(i) $f(x) = ax^2 + bx + c$ where $a, b, c \in \mathbf{R}$.
Given that k is a real number such that $f(k) = 0$, prove that $x - k$ is a factor of $f(x)$

Part (b)(i) **10 marks** **Att 3**

1(b)(i)

$$f(x) = ax^2 + bx + c$$
$$f(k) = ak^2 + bk + c$$
$$f(x) - f(k) = a(x^2 - k^2) + b(x - k)$$

$f(k) = 0 \Rightarrow$
$$f(x) = a(x-k)(x+k) + b(x-k)$$
$$= (x-k)[a(x+k) + b]$$

$\Rightarrow (x-k)$ is factor

or

1(b)(i)

$$x - k \overline{\smash{)}\, ax^2 + bx + c} \quad \text{quotient: } ax + (ak+b)$$
$$ax^2 - akx$$
$$(ak+b)x + c$$
$$(ak+b)x - k(ak+b)$$
$$c + k(ak+b) = ak^2 + bk + c = f(k)$$

But $f(k) = 0$

$\Rightarrow (x-k)$ is factor

* Possible to divide by $(x - k)$ for full marks.

Blunders (-3)
B1 indices.
B2 factors.
B3 $f(k) = 0$.

Slips (-1)
S1 numerical.
S2 not changing sign when subtracting in division

Attempts
A1 remainder $\neq 0$ in division.
A2 $f(x) = ax^3 + bx^2 + cx + d$ finished correctly.

Part (b)(ii)	**10 (5, 5)marks**	**Att (2, 2)**

1(b)(ii) Show that $2x-\sqrt{3}$ is a factor of $4x^2-2(1+\sqrt{3})x+\sqrt{3}$ and find the other factor.

Show	**5 marks**	**Att 2**
Find	**5 marks**	**Att 2**

1(b)(ii)

By inspection factors are

$(2x-\sqrt{3})$ and $(2x-1)$

$$\Rightarrow \quad (2x-\sqrt{3})(2x-1)=4x^2-2\sqrt{3}x-2x+\sqrt{3}$$

$$=4x^2-2(1+\sqrt{3})x+\sqrt{3}$$

or

1(b)(ii)

$$\begin{array}{r} 2x-1 \\ 2x-\sqrt{3}\,\big)\overline{4x^2-2x-2\sqrt{3}x+\sqrt{3}} \\ \underline{4x^2\qquad\quad -2\sqrt{3}x}\\ -2x+\sqrt{3} \\ \underline{-2x+\sqrt{3}} \end{array}$$

or

1(b)(ii)

If $(2x-\sqrt{3})$ is a factor, then $f\left(\frac{\sqrt{3}}{2}\right)=0$

$f(x): 4x^2-2x-2\sqrt{3}x+\sqrt{3}$

$$f\left(\frac{\sqrt{3}}{2}\right)=4\left(\frac{\sqrt{3}}{2}\right)^2-2\left(\frac{\sqrt{3}}{2}\right)-2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)+\sqrt{3}$$

$$=3-\sqrt{3}-3+\sqrt{3}$$

$$=0$$

$\Rightarrow (2x-\sqrt{3})$ is factor

$\Rightarrow (2x-1)$ is other factor

Blunders (-3)
B1 indices.
B2 factors.
B3 deduction root from factor.

Slips (-1)
S1 numerical.
S2 not changing sign when subtracting in division.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

1(c)
The real roots of $x^2 + 10x + c = 0$ differ by $2p$ where $c, p \in \mathbf{R}$ and $p>0$.

(i) Show that $p^2 = 25 - c$

(ii) Given that one root is greater than zero and the other root is less than zero, find the range of possible values of p.

(i) Show	**10 marks**	**Att 3**
(ii) Range	**10 marks**	**Att 3**

1(c)(i)

$$x^2 + 10x + c = 0$$

$$x = \frac{-10 \pm \sqrt{100 - 4c}}{2(1)}$$

$$= \frac{-10 \pm 2\sqrt{25 - c}}{2}$$

$$\Rightarrow \text{roots are } = -5 \pm \sqrt{25 - c}$$

roots: $-5 + \sqrt{25 - c}$ and $-5 - \sqrt{25 - c}$

$$\left(-5 + \sqrt{25 - c}\right) - \left(-5 - \sqrt{25 - c}\right) = 2p$$

$$2\sqrt{25 - c} = 2p$$

$$p = \sqrt{25 - c}$$

$$p^2 = 25 - c$$

1(c)(ii)

$\alpha = -5+\sqrt{25-c} : \beta = -5-\sqrt{25-c}$ $\qquad$ $\underline{p>0}$

$\alpha > \beta \Rightarrow \alpha > 0$ and $\beta < 0$

$\alpha > 0 \Rightarrow -5+\sqrt{25-c} > 0$ $\qquad$ $\beta < 0 \Rightarrow -5-\sqrt{25-c} < 0$

$-5+p>0$ $\qquad$ $-5-p<0$

$p>5$ $\qquad$ $-5<p$

$\Rightarrow p > 5$ (since $p>0$)

or

1(c)(i)

Let α and β be two roots and let $\alpha > \beta \Rightarrow \alpha > 0$ and $\beta < 0$

$x^2+10x+c=0$

$x^2-(-10)x+c=0$

$x^2-(\alpha+\beta)x+(\alpha\beta)=0$

$\Rightarrow \quad \alpha+\beta=-10$..............(*i*)

$\alpha\beta \quad = c$..................(*ii*)

$\alpha-\beta=2p$.............(*iii*)

(i) $\quad \alpha+\beta=-10$ $\qquad$ *(i)* $\alpha+\beta=-10$

(iii) $\quad \underline{\alpha-\beta=2p}$ $\qquad$ $(p-5)+\beta=-10$

$2\alpha \quad = 2p-10$ $\qquad$ $\beta=-5-p$

$\alpha \quad = p-5$ $\qquad$ $\beta=-p-5$

(iii) $\alpha\beta=c$

$(p-5)(-p-5)=c$

$-p^2+5p-5p+25=c$

$25-p^2=c$

Q1(c)(ii)

$\alpha>0$ $\qquad$ $\beta<0$

$p-5>0$ $\qquad$ $-p-5<0$

$p>5$ $\qquad$ $-5<p$ $\qquad$ ($\Rightarrow p>5$, since $p>0$)

<u>or</u>

$\alpha>0$ and $\beta<0 \Rightarrow \quad \alpha\beta<0$

$c<0$

$25-p^2<0$

$25<p^2$

$\Rightarrow p>5$, since $p>0$

Blunders (-3)

B1 indices.
B2 root formula once only.
B3 mixing up roots once only.
B4 inequality sign.
B5 incorrect deduction from work or no deduction.
B6 statement of quadratic equation once only.
B7 incorrect $(\alpha + \beta)$.
B8 incorrect $\alpha\beta$.
B9 incorrect difference.

Slips (-1)

S1 numerical.

QUESTION 2

Part (a)	**15 (10, 5) marks**	**Att (3, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

Part (a) **15 (10, 5) marks** **Att (3, 2)**

2(a)

Solve the simultaneous equations:

$$3x - y = 8$$
$$x^2 + y^2 = 10$$

Part (a) Quadratic **10 marks** **Att 3**

Finish **5 marks** **Att 2**

2(a)

(i) $3x - y = 8 \Rightarrow y = 3x - 8$

(ii) $x^2 + y^2 = 10$

$$x^2 + (3x-8)^2 = 10$$
$$x^2 + 9x^2 - 48x + 64 - 10 = 0$$
$$10x^2 - 48x + 54 = 0$$
$$5x^2 - 24x + 27 = 0$$
$$(5x-9)(x-3) = 0$$

$\Rightarrow$ $5x - 9 = 0$ or $x - 3 = 0$

$5x = 9$ $\quad x = 3$

$x = \dfrac{9}{5}$

$y = 3x - 8$

$x = \dfrac{9}{5}$: $\quad y = 3\left(\dfrac{9}{5}\right) - 8 = \dfrac{27-40}{5} = \dfrac{-13}{5} \Rightarrow \left(\dfrac{9}{5}, -\dfrac{13}{5}\right)$

$x = 3$ $\quad y = 3(3) - 8 = 9 - 8 = 1 \Rightarrow (3,1)$

Blunders (-3)

B1 indices.

B2 expansion of $(3x-8)^2$ once only.

B3 factors once only.

B4 deduction value from factor.

B5 not getting 2nd value(having got 1st).

B6 root formula once only.

Slips (-1)

S1 numerical.

Attempts

A1 not quadratic.

Worthless

W1 trial and error.

Part (b)(i) **10 marks** **Att 3**

2(b)(i) Solve for x:

$$|4x+7|<1$$

Part (b)(i) **10 marks** **Att 3**

2(b)(i)

$$|4x+7|<1$$

$$\Rightarrow \quad -1<(4x+7)<1$$

$-1<4x+7$

$-8<4x$

$-2<x$

$4x+7<1$

$4x<-6$

$x<-\frac{6}{4}$

$x<-\frac{3}{2}$

Solution: $-2<x<-\frac{3}{2}$

Or

2(b)(i)

$|4x+7|<1$

$(4x+7)^2<(1)^2$

$16x^2+56x+49<1$

$16x^2+56x+48<0$

$2x^2+7x+6<0$

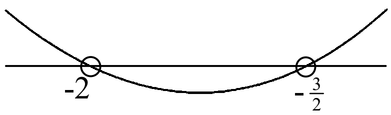
(i) $+2x^2 \quad \Rightarrow$ 

(ii) Solve $2x^2+7x+6=0$

$(2x+3)(x+2)=0$

$2x+3=0$ or $x+2=0$

$x=-\frac{3}{2}$ $\quad x=-2$

(iii) 

$f(x)<0$ when $-2<x<-\frac{3}{2}$

Blunders (-3)
B1 upper limit.
B2 lower limit.
B3 expansion of $(4x+7)^2$ once only.
B4 inequality sign, eg, changes to (=) , once only.
B5 indices.
B6 factors once only.
B7 root formula, once only.
B8 deduction root from factor.
B9 incorrect range.
B10 answer not stated.

Slips (-1)
S1 numerical.
S2 $\leq$.

Attempts
A1 ignore absolute value.

Part (b)(ii) **10 marks** **Att 3**

2 (b)(ii) Given that $x^2 - ax - 3$ is a factor of $x^3 - 5x^2 + bx + 9$ where $a, b \in R$, find the value of a and the value of b.

Part (b)(ii) **10 marks** **Att 3**

2(b)(ii)

$$\begin{array}{r} x + (a-5) \\ x^2 - ax - 3 \overline{\big) x^3 - 5x^2 + bx + 9} \\ \underline{x^3 - ax^2 - 3x} \\ (a-5)x^2 + (b+3)x + 9 \\ \underline{(a-5)x^2 - a(a-5)x - 3(a-5)} \\ (b+3)x + (a^2 - 5a)x + 9 + 3a - 15 = 0 \end{array}$$

$$(b + 3 + a^2 - 5a)x + (3a - 6) = (0)x + (0)$$

$\Rightarrow b + 3 + a^2 - 5a = 0$ ………(i) and $3a - 6 = 0$ …………..(ii)

$3a = 6$

(i) $b + 3 + 4 - 10 = 0$ $a = 2$

$b = 3$

or

2 (b)(ii) Let $(x+p)$ be other factor

$\therefore (x^2-ax-3)(x+p)=x^3-5x+bx+9$

$x^3-(a-p)x^2+(-3-ap)x+(-3p)=x^2-5x^2+bx+9$

Equating like to like

(i) $a-p=5$

(ii) $-3-ap=b$

(iii) $-3p=9$

(iii) $p=-3$

(i) $a-p=5$

$a-(-3)=5$

$a=2$

(ii) $-3-ap=b$

$-3-2(-3)=b$

$-3+6=b \Rightarrow b=3$

or

2(b)(ii) Since (x^2-ax-3) is a factor of $x^3-5x+bx+9$

other factors is $(x-3)$ by inspection.

$\Rightarrow f(3)=0$

$f(x)=x^3-5x^2+bx+9$

$f(3)=(3)^3-5(3)^2+b(3)+9=0$

$27-45+3b+9=0$

$3b=9$

$b=3$

$$\begin{array}{r} x+(a-5) \\ x^2-ax-3\,\overline{)\,x^3-5x^2+3x+9} \\ \underline{x^3-ax^2-3x} \\ ax^2-5x^2+6x+9 \\ (a-5)x^2+6x+9 \\ \underline{(a-5)x^2-a(a-5)x-3(a-5)} \end{array}$$

$[x+(a-5)]=[x+(-3)]$

$\Rightarrow (a-5)=-3 \Rightarrow a=2$

or

2 (b)(ii)

$(x-3)$ other factor by inspection.

$(x-3)(x^2-ax-3)=x^3-5x^2+bx+9$

$x^3-(a+3)x^2+(3a-3)x+9=x^3-5x^2+bx+9$

Equating like to like:

(i) $a+3=5$

$a=2$

(ii) $3a-3=b$

$3(2)-3=b$

$3=b$

Blunders (-3)
B1 indices.
B2 not like to like.
B3 not getting 2nd value, having got 1st .
B4 deduction root from factor.

Slips (-1)
S1 numerical.
S2 not changing sign when subtracting in division.

Attempts
A1 remainder $\neq 0$ in division.
A2 any attempt at division.

Part (c)(i) **10 marks** **Att 3**

2(c)(i) Solve for y :

$$2^{2y+1} - 5(2^y) + 2 = 0$$

Part (c)(i) **10 marks** **Att 3**

2(c)(i)

$$2^{2y+1} - 5(2^y) + 2 = 0$$

Let $x = 2^y$

$$2^{2y+1} = 2^{2y}.2^1 = 2(2^y)^2 = 2x^2$$

$$2x^2 - 5x + 2 = 0$$

$$(2x-1)(x-2) = 0$$

$\Rightarrow \quad 2x - 1 = 0 \quad \text{or} \quad x - 2 = 0$

$\quad x = \frac{1}{2} \quad \text{or} \quad x = 2$

$x = \frac{1}{2}$: $\quad x = 2^y = 2^{-1} \quad \Rightarrow \quad y = -1$

$x = 2$: $\quad x = 2^y = 2^1 \quad \Rightarrow \quad y = 1$

Blunders (-3)
B1 indices.
B2 factors once only.
B3 root formula once only.
B4 if x used for 2^y, not getting y once only.
B5 deduction value from factor.

Slips (-1)
S1 numerical.

Part (c)(ii) **5 marks** **Att 2**

2(c)(ii)
Given that $x = \alpha$ and $x = \beta$ are the solutions of the quadratic equation $2k^2x^2 + 2ktx + t^2 - 3k^2 = 0$ where $k, t \in R$ and $k \neq 0$, show that $\alpha^2 + \beta^2$ is independent of k and t.

Part (c)(ii) **5 marks** **Att 2**

2(c)(ii)

$$2k^2x^2 + 2ktx + t^2 - 3k^2 = 0$$

$$x^2 - \left(-\frac{t}{k}\right)x + \left(\frac{t^2 - 3k^2}{2k^2}\right) = 0$$

$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

$$\Rightarrow \quad \alpha + \beta = -\frac{t}{k} \text{ and } \alpha\beta = \frac{t^2 - 3k^2}{2k^2} = \left(\frac{t^2}{2k^2} - \frac{3}{2}\right)$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2(\alpha\beta)$$

$$= \left(\frac{-t}{k}\right)^2 - 2\left(\frac{t^2}{2k^2} - \frac{3}{2}\right)$$

$$= \frac{t^2}{k^2} - \frac{t^2}{k^2} + 3$$

$$= 3$$

or

2(c)(ii)

$$(2k^2)x^2 + (2kt)x + (t^2 - 3k^2) = 0$$

Roots: $$x = \frac{-2kt \pm \sqrt{4k^2t^2 - 4(2k^2)(t^2 - 3k^2)}}{2(2k^2)}$$

$$= \frac{-2kt \pm \sqrt{4k^2[6k^2 - t^2]}}{4k^2}$$

$$= \frac{-(2k)t \pm (2k)\sqrt{6k^2 - t^2}}{4k^2} = \frac{-t \pm \sqrt{6k^2 - t^2}}{2k}$$

$$\Rightarrow \quad \alpha = \frac{-t + \sqrt{6k^2 - t^2}}{2k} \quad \text{and} \quad \beta = \frac{-t - \sqrt{6k^2 - t^2}}{2k}$$

$$\alpha^2 + \beta^2 = \frac{1}{4k^2}\left[\left(t^2 - 2t\sqrt{6k^2 - t^2} + 6k^2 - t^2\right) + \left(t^2 + 2t\sqrt{6k^2 - t^2} + 6k^2 - t^2\right)\right]$$

$$= \frac{1}{4k^2}\left[2t^2 + 6k^2 - 2t^2 + 6k^2\right]$$

$$= \frac{12k^2}{4k^2}$$

$$= 3$$

Blunders (-3)

B1 indices.

B2 statement of quadratic equation once only.

B3 expansion of $(\alpha + \beta)^2$ once only.

B4 root formula once only.

B5 incorrect $(\alpha + \beta)$

B6 $\alpha\beta$.

Slips (-1)

S1 numerical.

Attempts

A1 *t* and/or *k* in answer.

A2 substitute $x = \alpha$ and/or $x = \beta$

QUESTION 3

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (15, 5) marks**	**Att (5, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

3(a)

Evaluate $\begin{pmatrix} 1 & -2 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Part (a) 1st Multiplication	**5 marks**	**Att 2**
2nd Multiplication	**5 marks**	**Att 2**

3(a)

$$\begin{pmatrix} 1 & -2 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ -7 \end{pmatrix} \qquad \text{or} \qquad \begin{pmatrix} 13 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 17 \end{pmatrix} \qquad\qquad\qquad\qquad = \begin{pmatrix} 17 \end{pmatrix}$$

Slips (-1)
S1 each incorrect element.
S2 numerical.

* Note: cannot get 2nd 5 marks if 1st multiplication not correct "dimension"

Part (b)(i) **10(5, 5) mark** **Att (2, 2)**

3(b)(i)

Given that $z = 2 - i$, calculate $\left|z^2 - z + 3\right|$ where $i^2 = -1$

Part (b)(i) Complex Number	**5 marks**	**Att 2**
Modulus	**5 marks**	**Att 2**

3(b)(i)

$$z = 2 - i$$

$$z^2 = (2-i)^2 = 4 - 4i + i^2 = 3 - 4i$$

$$z^2 - z + 3 = (3 - 4i) - (2 - i) + 3$$

$$= 4 - 3i$$

$$\left|z^2 - z + 3\right| = |4 - 3i| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

Blunders (-3)
B1 i
B2 $(2-i)^2$ once only.
B3 modulus.

Slips (-1)
S1 numerical.

Part (b)(ii) **10 marks** **Att 3**

3(b)(ii) k is a real number such that $\frac{-1+i\sqrt{3}}{-4\sqrt{3}-4i}=ki$.

Find k.

Part (b)(ii) **10 marks** **Att**

3(b)(ii)

$$\frac{-1+i\sqrt{3}}{-4\sqrt{3}-4i}\cdot\frac{-4\sqrt{3}+4i}{-4\sqrt{3}+4i}=\frac{4\sqrt{3}-12i-4i+4\sqrt{3}i^2}{(48)+(16)}$$
$$=\frac{-16i}{64}$$
$$=-\left(\frac{1}{4}\right)i$$

or

3(b)(ii)

$$\frac{-1+i\sqrt{3}}{-4\sqrt{3}-4i}=ki$$
$$\rightarrow \quad -1+i\sqrt{3}=\left(-4k\sqrt{3}\right)i+4k$$
$$\Rightarrow \quad (-1)+\left(\sqrt{3}\right)i=(4k)+\left(-4k\sqrt{3}\right)i$$

Equating like to like:

$-1=4k$ or $-4k\sqrt{3}=\sqrt{3}$

$-\frac{1}{4}=k$ $k=-\frac{1}{4}$

Blunders (-3)
B1 i
B2 indices.
B3 cross multiplication.
B4 not like to like.

Slips (-1)
S1 numerical.

Attempts
A1 not using correct conjugate.

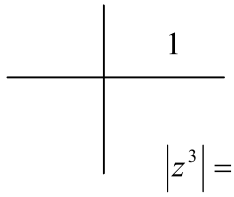
Part (c) **20(15, 5) marks** **Att (5, 2)**

3(c)	$1, \omega, \omega^2$ are the three roots of the equation $z^3 - 1 = 0$.
	(i) Prove that $1 + \omega + \omega^2 = 0$
	(ii) Hence, find the value of $(1 - \omega - \omega^2)^5$

Part (c) **(i) Prove** **15 marks** **Att 5**
(ii) Value **5 marks** **Att 2**

3(c)(i)

$$z^3 - 1 = 0$$



$$z^3 = 1$$

$$z^3 = r(\cos\theta + i\sin\theta)$$

$$= 1(\cos 2n\pi + i\sin 2n\pi)$$

$|z^3| = 1$

$\theta = 2n\pi$

$$z = (\cos 2n\pi + i\sin 2n\pi)^{\frac{1}{3}}$$

$$z = \cos\frac{2n\pi}{3} + i\sin\frac{2n\pi}{3}$$

$$n = 0 : Z_0 = \cos(0) + i\sin(0) = 1$$

$$n = 1: \quad Z_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + i\left(\frac{\sqrt{3}}{2}\right) = \omega$$

$$n = 2: \quad Z_2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - i\left(\frac{\sqrt{3}}{2}\right) = \omega^2$$

$$1 + \omega + \omega^2 = 1 + \left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] + \left[-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$$

$$= \quad 0$$

or

3(c)(i)

$$z^3 - 1 = 0$$

$$z^3 - (0)z^2 + (0)z - 1 = 0$$

$$z^3 - \left(\sum\alpha\right)z^2 + \left(\sum\alpha\beta\right)z - (\alpha\beta y) = 0$$

$$\Rightarrow \sum\alpha = 1 + \omega + \omega^2 = 0$$

or

3(c)(i)

$$z^3 - 1 = 0$$

$$(z-1)(z^2 + z + 1) = 0$$

$$z = 1 \quad \text{or} \quad z = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$1 + \omega + \omega^2 = 1 + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$= 0$$

or

3(c)(i)

$$z^3 - 1 = 0$$

$$(z-1)(z^2 + z + 1) = 0$$

$$z - 1 = 0 \qquad \text{or} \qquad z^2 + z + 1 = 0$$

$$z = 1$$

$$f(z) = z^2 + z + 1$$

Since ω is root,

$$f(\omega) = (\omega)^2 + (\omega) + 1 = 0$$

$$\Rightarrow 1 + \omega + \omega^2 = 0$$

3(c)(ii)

$$1 + \omega + \omega^2 = 0$$

$$1 = -\omega - \omega^2$$

$$1 = -(\omega + \omega^2)$$

$$[1 - \omega - \omega^2]^5 = [1 - (\omega + \omega^2)]^5$$

$$= [1 + 1]^5$$

$$= 32$$

Blunders (-3)

B1 modulus.
B2 argument.
B3 formula De Moivre once only.
B4 application De Moivre.
B5 indices.
B6 polar formula once only.
B7 *i*
B8 factors once only.
B9 roots formula once only.
B10 sum of roots.

Slips (-1)

S1 numerical.
S2 trig value.
S3 $1 = \omega + \omega^2$

Attempts

A1 no general solution when applying De Moivre.

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 marks** **Att 3**

4(a)

Express the recurring decimal 0.252525… in the form $\frac{p}{q}$ where $p, q \in \mathbf{N}$ and $q \neq 0$

Part (a) **10 marks** **Att 3**

4(a)(i)

$$0 \cdot \dot{2}\dot{5} = 0.252525\ldots$$

$$= \frac{25}{100} + \frac{25}{10000} + \frac{25}{1000000} + \ldots\ldots..$$

Geometric Series to ∞ : $a = \frac{25}{100}$ $\quad r = \frac{1}{100}$

$$S_\infty = \frac{a}{1-R} = \frac{\frac{25}{100}}{1-\frac{1}{100}} = \frac{\left(\frac{25}{100}\right)}{\left(\frac{99}{100}\right)} = \frac{25}{99}$$

or

4(a)(i)

Let $x = 0.\dot{2}\dot{5}$

$$x = 0.252525.......$$
$$100x = 25.252525......$$
$$99x = 25$$

$$x = \frac{25}{99}$$

* Accept $\frac{25}{99}$ for full marks and no work shown.

Blunders (-3)
B1 infinity formula once only.
B2 *a*.
B3 *R*.

Slips (-1)
S1 numerical.

Part (b)	**20 (10, 10)**	**Att (3, 3)**

4 (b) In an arithmetic series, the sum of the second term and the fifth term is 18. The sixth term is greater than the third term by 9.

(i) Find the first term and the common difference.

(ii) What is the smallest value of *n* such that $S_n > 600$ where S_n is the sum of the first *n* terms of the series?

Part(b)(i)	**10 marks**	**Att 3**
(ii)	**10 marks**	**Att 3**

4 (b)(i)

Arithmetic series: $u_2 + u_5 = 18 \Rightarrow (a+d)+(a+4d)=18$

$2a+5d = 18 \ldots\ldots\ldots\ldots (i)$

$u_6 - u_3 = 9$

$(a+5d)-(a+2d)=9$

$3d = 9$

$d = 3 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$

(i) $2a+5d = 18$

$2a+15=18$

$a = \frac{3}{2}$ $\qquad$ $a = \frac{3}{2}$

$d = 3$

4(b)(ii)

Let $S_n > 600$

$\frac{n}{2}[2a+(n-1)d] > 600$

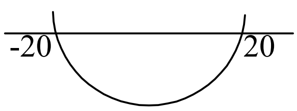
$\frac{n}{2}[3+3n-3] > 600$

$3n^2 > 1200$

$n^2 > 400$

$n > 20$ $\qquad (n > 0)$

$n = 21$



Blunders (-3)

B1 term.
B2 reversing terms in second condition.
B3 formula AP once only.
B4 incorrect *'a'* in formula.
B5 incorrect *'d'* in formula.
B6 inequality sign.
B7 indices.
B8 incorrect deduction or no deduction.
B9 not getting 2nd value (having got 1st value)

Worthless

W1 GP

Part (c)(i)	**10 marks**	**Att 3**

4 (c)

(i) $u_1, u_2, u_3, u_4, u_5, \ldots\ldots..$ is a sequence where $u_1 = 2$ and $u_{n+1} = (-1)^n u_n + 3$

Evaluate u_2, u_3, u_4, u_5 and u_{10} .

Part (c)(i)	**10 marks**	**Att 3**

4(c)(i) $u_{n+1} = (-1)^n u_n + 3 \quad : \quad u_1 = 2$

$$u_2 = (-1)^1(2) + 3 = 1$$

$$u_3 = (-1)^2(1) + 3 = 4$$

$$u_4 = (-1)^3(4) + 3 = -1$$

$$u_5 = (-1)^4(-1) + 3 = 2$$

$\Rightarrow$ 2, 1, 4, -1, 2, 1, 4, -1, 2, 1…………

$\downarrow$ u_{10}

$$u_{10} = 1$$

Blunders (-3)

B1 indices.
B2 term omitted.
B3 deduction u_{10} or no u_{10} .
B4 formula once only.

Slips (-1)

S1 numerical.

Part (c)(ii) **10 marks** **Att 3**

4(c) (ii) *a, b, c, d* are the first, second, third and fourth terms of a geometric sequence, respectively. Prove that $a^2 - b^2 - c^2 + d^2 \geq 0$

Part (c)(ii) **10 marks** **Att 3**

4(c)(ii)

4 terms in G.P. $\Rightarrow a,b,c,d \Rightarrow a, ar, ar^2, ar^3$

$$a^2 - b^2 - c^2 + d^2 = a^2 - (ar)^2 - (ar^2)^2 + (ar^3)^2$$
$$= a^2 - a^2r^2 - a^2r^4 + a^2r^6$$
$$= a^2\left[1 - r^2 - r^4 + r^6\right]$$
$$= a^2\left[(1-r^2) - r^4(1-r^2)\right]$$
$$= a^2(1-r^2)\left[1-r^4\right]$$
$$= a^2(1-r^2)\left[(1-r^2)(1+r^2)\right]$$
$$= a^2(1-r^2)^2(1+r^2)$$
$$\geq 0$$

or

4(c)(ii)

a, b, c, d in GP $\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$

$$\Rightarrow \frac{b}{a} = \frac{d}{c} \Rightarrow d = \frac{bc}{a}$$

$$a^2 - b^2 - c^2 + d^2$$
$$= a^2 - b^2 - c^2 + \frac{b^2c^2}{a^2}$$
$$= \frac{a^4 - a^2b^2 - a^2c^2 + b^2c^2}{a^2}$$
$$= \frac{a^2(a^2 - c^2) - b^2(a^2 - c^2)}{a^2}$$
$$= \frac{(a^2 - c^2)(a^2 - b^2)}{a^2}$$

If GP is increasing $a < c \Rightarrow (a^2 - c^2) < 0$

$|a| < |b| \Rightarrow (a^2 - b^2) < 0$

$\Rightarrow \frac{(a^2 - c^2)(a^2 - b^2)}{a^2} \geq 0$

If GP is decreasing $a > c \Rightarrow (a^2 - c^2) > 0$

$|a| > |b| \Rightarrow (a^2 - b^2) > 0$

$\Rightarrow \frac{(a^2 - c^2)(a^2 - b^2)}{a^2} \geq 0$

Blunders (-3)
B1 definition GP
B2 indices.
B3 factors once only.
B4 incorrect deduction or no deduction.
B5 inequality sign.

Attempts
A1 particular values verified.

Worthless
W1 AP

QUESTION 5

Part (a)	**10 (5, 5)marks**	**Att (2, 2)**
Part (b)	**20(5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20(5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10(5, 5,) marks** **Att 2**

5(a) Solve for x:

$$x = \sqrt{7x-6} + 2$$

Quadratic **5 marks** **Att 2**
Finish **5 marks** **Att 2**

5(a)(i)

$$x - 2 = \sqrt{7x-6}$$
$$(x-2)^2 = 7x - 6$$
$$x^2 - 4x + 4 = 7x - 6$$
$$x^2 - 11x + 10 = 0$$
$$(x-1)(x-10) = 0$$

$\Rightarrow \quad x - 1 = 0 \quad$ or $\quad x - 10 = 0$
$\quad x = 1 \quad$ or $\quad x = 10$

Test $\quad x = 1 \quad$ LHS = -1 $\quad$ RHS $= \sqrt{7x-6} = \sqrt{1} = 1$
LHS $\neq$ RHS

$x = 10 \quad$ LHS = 8 $\quad$ RHS $= \sqrt{70-6} = \sqrt{64} = 8$
LHS = RHS

Solution: $x = 10$

Blunders (-3)
B1 indices.
B2 factors once only.
B3 root formula once only.
B4 deductions values from factors or no values.

Slips (-1)
S1 numerical.
S2 extra value.

Attempts
A1 $x = 10$ and no other work merits 2 marks.

Part (b)	20(5, 5, 10) marks	Att (2, 2, 3)

5(b)

Use induction to prove that 8 is a factor of $7^{2n+1}+1$ for any positive integer *n*.

P(1)	**5 marks**	**Att 2**
Assume	**5 marks**	**Att 2**
P(k +1)	**10 marks**	**Att 3**

5(b)

Let $n = 1$ $\quad 7^{2n+1}+1 = 7^3+1 = 344 \div 8 = 43$

$\Rightarrow$ true for $n=1$

Assume true for $n = k$ $\quad \Rightarrow \left(7^{2k+1}+1\right)$ is divisible by 8.

To prove $\left[7^{2(k+1)+1}+1\right]$ is divisible by 8

i.e. that $\left(7^{2k+3}+1\right)$ is divisible by 8

[using difference between P(k) and P(k+1)]

$$\left(7^{2k+3}+1\right)-\left(7^{2k+1}+1\right)= 7^{2k+1}.7^2-7^{2k+1}$$

$$= 7^{2k+1}\left(49-1\right)$$

$$= 7^{2k}\left(48\right),$$

which is divisible by 8

$\therefore P(k+1)$ true whenever $P(k)$ is true

Since $P(1)$ true, then, by induction, $P(n)$ true for any positive integer *n*.

or

5(b)

Let $n = 1$ $\quad 7^{2n+1}+1 = 7^3+1 = 344 \div 8 = 43$

$\Rightarrow$ true for $n=1$

Assume true for $n = k$: $\quad \Rightarrow \left(7^{2k+1}+1\right)$ is divisible by 8

$\therefore \quad 7^{2k+1}+1 = 8A$, where A is an integer.

$\therefore \quad 7^{2k+1} = 8A-1$

To prove: $\left(7^{2(k+1)+1}+1\right)$ is divisible by 8:

$$7^{2k+3}+1 = 7^2\cdot 7^{2k+1}+1$$
$$= 49\left[8A-1\right]+1$$
$$= 49\left(8A\right)-49+1$$
$$= 49\left(8A\right)-48$$
$$= 8\left[49A-6\right]$$

which is divisible by 8

$\therefore P(k+1)$ true whenever $P(k)$ is true

Since $P(1)$ true, then, by induction, $P(n)$ true for any positive integer *n*.

or

5(b)

Let $n = 1$ $\quad 7^{2n+1}+1 = 7^3+1 = 344 \div 8 = 43$

$\Rightarrow$ true for n =1

$$P(k+1) = \left(7^{2k+3}+1\right) = 7^2.7^{2k+1}+1$$
$$= 49\left(7^{2k+1}\right)+1$$
$$= 48\left(7^{2k+1}\right)+\left(7^{2k+1}+1\right)$$
$$= 48\left(7^{2k+1}\right)+P(k),$$

which is divisible by 8 since *P(k)* divisible by 8

$\therefore P(k+1)$ true whenever $P(k)$ is true

Since $P(1)$ true, then, by induction, $P(n)$ true for any positive integer *n.*

Blunders (-3)

B1 indices.

B2 $n \neq 1$ (accept $n = 0$)

* Note must prove $n = 1$ (not sufficient to state *P(n)* true when $n = 1$).

Part (c) **20(5, 5, 5, 5)marks** **Att (2, 2, 2, 2)**

5(c) Consider the binomial expansion of $\left(ax + \frac{1}{bx}\right)^8$, where a and b are non-zero real numbers.

(i) Write down the general term.

(ii) Given that the coefficient of x^2 is equal to the coefficient of x^4, show that $ab = 2$.

General Term	**5 marks**	**Att 2**
x^2 Term	**5 marks**	**Att 2**
x^4 Term	**5 marks**	**Att 2**
Show	**5 marks**	**Att 2**

5(c) $\left(ax + \frac{1}{bx}\right)^8$

(i) $u_{R+1} = \binom{8}{R}(ax)^{8-R}\left(\frac{1}{bx}\right)^R$

(ii) Let u_{R+1} be term with x^2

$$\binom{8}{R}(ax)^{8-R}(bx)^{-R} = kx^2$$
$$k\left[x^{8-R}\cdot x^{-R}\right] = kx^2$$
$$x^{8-2R} = x^2$$
$$\Rightarrow 8-2R = 2$$
$$6 = 2R$$
$$3 = R$$

$$U_4 = \binom{8}{3}(ax)^5\left(\frac{1}{bx}\right)^3$$

$$= \frac{8.7.6}{1.2.3}\left(a^5x^5\right)\frac{1}{b^3x^3} = \left(\frac{56a^5}{b^3}\right)x^2$$

Now let U_{R+1} be term with x^4

$$x^{8-2R} = x^4$$
$$\Rightarrow 8-2R = 4$$
$$4 = 2R$$
$$2 = R$$

$$U_3 = \binom{8}{2}(ax)^6\left(\frac{1}{bx}\right)^2$$

$$= \frac{8.7}{1.2}.a^6x^6\,\frac{1}{b^2x^2} = \left(\frac{28a^6}{b^2}\right)x^4$$

Equating coefficients: $\frac{56a^5}{b^3} = \frac{28a^6}{b^2}$

$$56a^5b^2 = 28a^6b^3$$
$$ab = 2$$

* Accept multiplication or Binomial expansion simplified.

Blunders (-3)
B1 general term.
B2 error binomial expansion once only.
B3 $\binom{n}{r}$ or no $\binom{n}{r}$.
B4 indices.
B5 not like to like.

Slips (-1)
S1 numerical.

Attempts
A1 correct trial and error.

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 marks** **Att 3**

6(a)

Differentiate $\sqrt{1+4x}$ with respect to x

Part (a) **10 marks** **Att 3**

6(a)

$$y = \sqrt{1+4x} = (1+4x)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(1+4x)^{-\frac{1}{2}}.(4) = \frac{2}{(1+4x)^{\frac{1}{2}}} = \frac{2}{\sqrt{1+4x}}$$

Blunders (-3)
B1 differentiation.
B2 indices.

Attempts
A1 error in differentiation formula.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

6(b) Show that the equation $x^3 - 4x - 2 = 0$ has a root between 2 and 3.
Taking $x_1 = 2$ as the first approximation to this root, use the Newton-Raphson method to find x_3, the third approximation. Give your answer correct to two decimal places.

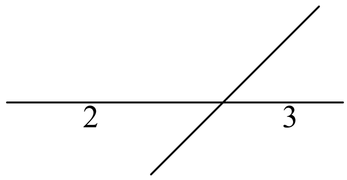
Show	**5 marks**	**Att 2**
***f'*(*x*)**	**5 marks**	**Att 2**
x_2	**5 marks**	**Att 2**
x_3	**5 marks**	**Att 2**

6(b)

$$f(x) = x^3 - 4x - 2$$

$$f(2) = 8 - 8 - 2 < 0$$

$$f(3) = 27 - 12 - 2 > 0$$



$\Rightarrow$ root between 2 and 3 $\Rightarrow$ root = $2 \cdot abc$

$$f(x) = x^3 - 4x - 2$$

$$f'(x) = 3x^2 - 4 \qquad\qquad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

x_n	$f(x) = x^3 - 4x - 2$	$f'(x) = 3x^2 - 4$	x_{n+1}
2	-2	8	$2 - \frac{(-2)}{8} = 2.25$
2.25	0.3906	11.1875	2.25 – 0.0349=2.2151

$$x_3 = 2.2151 = 2.22$$

Blunders (-3)
B1 Newton-Raphson formula once only.
B2 differentiation.
B3 indices.
B4 $x_1 \neq 2$, once only.
B5 inequality sign.
B6 deduction from work or no deduction (root).
B7 incorrect value in table (unless an obvious slip)

Slips (-1)
S1 numerical.
S2 answer not to 2 decimal places.

Worthless
W1 incorrect answer and no work.

* Accept $x_3 = 2.22$ by calculation.

Part (c) **20(5, 5, 10) marks** **Att (2, 2, 3)**

6(c) The function $f(x) = \frac{1}{1-x}$ is defined for $x \in \mathbf{R} \setminus \{1\}$.

(i) Prove that the graph of *f* has no turning points and no points of inflection.

(ii) Write down a reason that justifies the statement: "*f* is increasing at every value of $x \in \mathbf{R} \setminus \{1\}$".

(iii) Given that $y = x + k$ is a tangent to the graph of *f* where *k* is a real number, find the two possible values of *k*.

Turning points & points of inflection	**5 marks**	**Att 2**
Reason	**5 marks**	**Att 2**
Values of *k*	**10 marks**	**Att 3**

6(c)(i) $f(x) = \frac{1}{(1-x)} = (1-x)^{-1}$

$$f'(x) = (-1)(1-x)^{-2}(-1) = (1-x)^{-2}$$

$$f''(x) = -2(1-x)^{-3}(-1) = 2(1-x)^{-2} = \frac{2}{(1-x)^3}$$

$$f'(x) = \frac{1}{(1-x)^2} \neq 0 \Rightarrow \text{ no turning points}$$

$$f''(x) = \frac{2}{(1-x)^3} \neq 0 \Rightarrow \text{ no points of inflection}$$

(ii) $f'(x) = \frac{1}{(1-x)^2} > 0$ for all $x \Rightarrow f(x)$ is an increasing function for all x

(iii) $y = x + k$ is tangent

Slope $m = f'(x) = 1 \quad \Rightarrow \quad \frac{1}{(1-x)^2} = 1$

$$\Rightarrow (1-x)^2 = 1$$

$$1 - 2x + x^2 = 1$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$\Rightarrow x = 0 \quad \text{or} \quad x = 2$$

$x = 0: \quad y = \frac{1}{1-x} = \frac{1}{1-0} = 1 \Rightarrow (0,1)$

$x = 2: y = \frac{1}{1-x} = \frac{1}{1-2} = -1 \Rightarrow (2,-1)$

$(0,\ 1) \in y = x + k$

$1 = 0 + k$

$\Rightarrow k = 1$

or

$(y - y_1) = m(x - x_1)$

$m = 1: \quad (x_1, y_1) = (0,1)$

$(y-1) = 1(x-0)$

$y = x + 1 \Rightarrow k = 1$

$(2,-1) \in y = x + k$

$-1 = 2 + k$

$-3 = k$

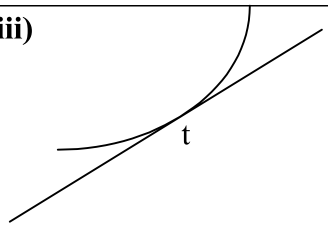
$m = 1: \quad (x_1, y_1) = (2,-1)$

$(y+1) = 1(x-2)$

$y = x - 3 \Rightarrow k = -3$

or

6(c)(iii)



$$T \cap \{curve\} = \{t\}$$

$$\left\{y = \frac{1}{1-x}\right\} \cap \{y = x + k\}$$

$$\Rightarrow \frac{1}{1-x} = x + k$$

$$1 = (x+k)(1-x)$$

$$1 = x - x^2 + k - kx$$

$$x^2 + kx - x + 1 - k = 0$$

$$x^2 + (k-1)x + (1-k) = 0$$

Since T is tangent to curve, quadratic has only one root.

$$\Rightarrow b^2 - 4ac = 0$$

$$(k-1)^2 - 4(1-k) = 0$$

$$k^2 - 2k + 1 - 4 + 4k = 0$$

$$k^2 + 2k - 3 = 0$$

$$(k+3)(k-1) = 0$$

$$k + 3 = 0 \quad \text{or} \quad k - 1 = 0$$

$$k = -3 \quad \text{or} \quad k = 1$$

Blunders (-3)
B1 differentiation.
B2 indices.
B3 $f'(x) \neq 0$
B4 $f''(x) \neq 0$
B5 deduction.
B6 factors once only.
B7 root formula once only.
B8 no y values.
B9 equation straight line
B10 $b^2 - 4ac \neq 0$.
B11 deduction roots from factors.
B12 expansion of $(k-1)^2$ once only.

Slips (-1)
S1 numerical.

Attempts
A1 error in differentiation formula.
A2 graphical methods.

Worthless
W1 integration.

QUESTION 7

Part (a)	**10 (5, 10)marks**	**Att (2, 3)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

Part (a) **15 (5, 10) marks** **Att (2, 3)**

7(a)

Differentiate each of the following with respect to x :

(i) $\cos^4 x$ **(ii)** $\sin^{-1}\frac{x}{5}$

(i)	**5 marks**	**Att 2**
(ii)	**10 marks**	**Att 3**

7(a)(i)

$$y = \cos^4 x = (\cos x)^4$$

$$\frac{dy}{dx} = 4(\cos x)^3(-\sin x) = -4\cos^3 x \sin x$$

(ii)

$$y = \sin^{-1}\left(\frac{x}{5}\right) \qquad a = 5$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}} = \frac{1}{\sqrt{25 - x^2}}$$

Blunders (-3)
B1 differentiation.
B2 indices.

Attempts
A1 error in differentiation formula.

Part (b)(i) **10 marks** **Att 3**

7(b)(i)	The parametric equations of a curve are:
	$x = \cos t + t \sin t$
	$y = \sin t - t\cos t$ where $0 < t < \dfrac{\pi}{2}$.
	Find $\dfrac{dy}{dx}$ and write your answer in its simplest form.

Part(b)(i) **10 marks** **Att**

7(b)(i)

$$x = \cos t + t\sin t$$

$$\frac{dx}{dt} = (-\sin t) + [t(\cos t) + \sin t(1)]$$

$$= -\sin t + t\cos t + \sin t$$

$$= t\cos t$$

$$y = \sin t - t\cos t$$

$$\frac{dy}{dt} = \cos t - [t(-\sin t) + (\cos t)(1)]$$

$$= \cos t + t\sin t - \cos t$$

$$= t\sin t$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{t\sin t}{t\cos t} = \tan t$$

Blunders (-3)
B1 differentiation.
B2 indices.
B3 $\dfrac{dy}{dx}$

Slips (-1)
S1 not in simplest form.

Attempts
A1 error in differentiation formula.

Part (b)(ii) **10 marks** **Att 3**

7(b)(ii)	Given that $\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$, find the value of $\frac{dy}{dx}$ at the point (2, -3)

Part(b)(ii) **10 marks** **Att 3**

7(b)(ii)

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$$

$$\frac{1}{y}=\frac{1}{6}-\frac{1}{x}$$

$$\frac{1}{y}=\frac{x-6}{6x}$$

$$y=\frac{6x}{x-6}$$

$$\frac{dy}{dx}=\frac{(x-6)(6)-(6x)(1)}{(x-6)^2}=\frac{6x-36-6x}{(x-6)^2}=\frac{-36}{(x-6)^2}$$

At (2, -3) : $x = 2$: $\frac{dy}{dx}=\frac{-36}{(x-6)^2}=\frac{-36}{(2-6)^2}=\frac{-36}{16}=\frac{-9}{4}$

Or

7(b)(ii)

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$$

$$x^{-1}+y^{-1}=\frac{1}{6}$$

$$-1.x^{-2}+(-1).y^{-2}.\frac{dy}{dx}=0$$

$$-\frac{1}{x^2}-\frac{1}{y^2}\cdot\frac{dy}{dx}=0$$

$$-\frac{1}{x^2}=\frac{1}{y^2}\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{-y^2}{x^2}$$

At (2, -3) $x = 2$, $y = -3$ $\frac{dy}{dx}=\frac{-(-3)^2}{(2)^2}=\frac{-9}{4}$

Blunders (-3)
B1 algebra.
B2 differentiation.
B3 substitution of (2, -3) or no substitution.

Slips (-1)
S1 numerical.

Attempts
A1 error in differentiation formula.

Part (c)(i) **10 marks** **Att 3**

7(c)(i) Given that $y = \ln\frac{1+x^2}{1-x^2}$ for $0 < x < 1$, find $\frac{dy}{dx}$ and write your answer in the form $\frac{kx}{1-x^k}$ where $k \in \mathbf{N}$.

Part (c)(i) **10 marks** **Att 3**

7(c)(i)

$$y = \ln\left(\frac{1+x^2}{1-x^2}\right)$$

$$y = \ln(1+x^2) - \ln(1-x^2)$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}(2x) - \frac{1}{1-x^2}(-2x)$$

$$= \frac{2x}{(1+x^2)} + \frac{2x}{(1-x^2)}$$

$$= \frac{2x(1-x^2) + 2x(1+x^2)}{(1+x^2)(1-x^2)}$$

$$= \frac{2x - 2x^3 + 2x + 2x^3}{1-x^4} = \frac{4x}{1-x^4}$$

or

7(c)(i)

$$y = \ln\left(\frac{1+x^2}{1-x^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{\left[\frac{1+x^2}{1-x^2}\right]} \cdot \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2}$$

$$= \frac{(1-x^2)}{(1+x^2)} \cdot \frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2}$$

$$= \frac{4x}{(1+x^2)(1-x^2)}$$

$$= \frac{4x}{1-x^4}$$

Blunders (-3)
B1 differentiation.
B2 logs.
B3 indices.
B4 not in required form.

Slips (-1)
S1 numerical.

Attempts
A1 error in differentiation formula.

Part (c)(ii) **5 marks** **Att 2**

(c)(ii) Given that $f(\theta)=\sin(\theta+\pi)\cos(\theta-\pi)$, find the derivative of $f(\theta)$ and express it in the form $\cos n\theta$, where $n \in Z$

Part (c)(ii) **5 marks** **Att 2**

7(c)(ii)

$$f(\theta)=\sin(\theta+\pi).\cos(\theta-\pi)$$
$$f'(\theta)=\sin(\theta+\pi)[-\sin(\theta-\pi)]+\cos(\theta-\pi)[\cos(\theta+\pi)]$$
$$=\cos(\theta+\pi)\cos(\theta-\pi)-\sin(\theta+\pi)\sin(\theta-\pi)$$
$$=\cos A.\cos B-\sin A.\sin B$$
$$=\cos[(\theta+\pi)+(\theta-\pi)]$$
$$=\cos 2\theta$$

or

7(c)(ii)

$$f(\theta)=\sin(\theta+\pi)\cos(\theta-\pi)$$
$$=\frac{1}{2}[2\sin(\theta+\pi)\cos(\theta-\pi)]$$
$$=\frac{1}{2}\langle\sin[(\theta+\pi)+(\theta-\pi)]+\sin[(\theta+\pi)-(\theta-\pi)]\rangle$$
$$f(\theta)=\frac{1}{2}[\sin 2\theta+\sin 2\pi]$$
$$f'(\theta)=\frac{1}{2}[2\cos 2\theta]=\cos 2\theta$$

or

7(c)(ii)

$$f(\theta)=\sin(\theta+\pi)\cos(\theta-\pi)$$

$\cos\pi=-1$
$\sin\pi=0$

$$\sin(\theta+\pi)=\sin\theta\cos\pi+\cos\theta\sin\pi$$
$$=-\sin\theta+0$$
$$=-\sin\theta$$
$$\cos(\theta-\pi)=\cos\theta\cos\pi+\sin\theta\sin\pi$$
$$=-\cos\theta+0$$
$$=-\cos\theta$$

$$f(\theta)=\sin(\theta+\pi)\cos(\theta-\pi)=(-\sin\theta)(-\cos\theta)$$
$$f(\theta)=\sin\theta\cos\theta=\frac{1}{2}[\sin 2\theta]$$
$$f'(\theta)=\frac{1}{2}[2\cos 2\theta]=\cos 2\theta$$

Blunders (-3)
B1 trig formula.
B2 differentiation.
B3 not in required form.

Slips (-1)
S1 numerical.
S2 trig value.

Attempts
A1 error in differentiation formula.

QUESTION 8

Part (a)	**10 (5, 5)marks**	**Att (2, 2)**
Part (b)	**20 (10, 10)**	**Att (3, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a)	**10 (5, 5)marks**	**Att (2, 2)**

(a)

Find **(i)** $\int (x^3 + 2)dx$ **(ii)** $\int e^{7x} dx$

Part(a) (i)	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**

8(a) **(i)** $\int (x^3 + 2)dx = \frac{x^4}{4} + 2x + c$

(ii) $\int e^{7x} dx = \frac{e^{7x}}{7} + c$

* If c shown once $\Rightarrow$ no penalty

Blunders (-3)

B1 integration.

B2 no 'c' once only. (Penalise 1st integration)

B3 indices.

Attempts

A1 only c correct.

Worthless

W1 differentiation instead of integration.

Part (b)	20 (10, 10)	Att (3, 3)

8(b)(i) Evaluate $\int_0^1 \frac{2x}{\sqrt{1+x^2}}dx$

(ii) By letting $u = \sin x$, evaluate $\int_0^{\frac{\pi}{2}} \cos x \sin^6 x dx$

Part (i)	10 marks	Att 3
Part(ii)	10 marks	Att 3

8(b)(i) $\int_0^1 \frac{2x}{\sqrt{1+x^2}}dx$

$= \int \frac{dw}{w^{\frac{1}{2}}}$

$= \int w^{\frac{-1}{2}} \cdot dw$

$= \frac{w^{\frac{1}{2}}}{\frac{1}{2}}$

$= 2.w^{\frac{1}{2}} \Big]_1^2$

$= 2\left[\sqrt{2} - \sqrt{1}\right] = 2\left(\sqrt{2} - 1\right)$

Let $w = 1 + x^2$

$\frac{dw}{dx} = 2x$

$dw = 2x.dx$

$x = 1 \ : \ w = 1 + x^2 = 1 + 1 = 2$

$x = 0 \ : \ w = 1 + x^2 = 1 + 0 = 1$

or

8(b)(i) $2w^{\frac{1}{2}} = 2\left(1 + x^2\right)^{\frac{1}{2}} \Big]_0^1$

$= 2\left[(2)^{\frac{1}{2}} - (1)^{\frac{1}{2}}\right]$

$= 2\left[\sqrt{2} - 1\right]$

8(b)(ii) $\int_0^{\frac{\pi}{2}} \sin^6 x.(\cos x dx)$

$u = \sin x$

$= \int u^6.du$

$= \frac{u^7}{7} \Big]_0^1$

$= \frac{1}{7}[1 \quad - 0]$

$= \frac{1}{7}$

$\frac{du}{dx} = \cos x$

$du = \cos x \cdot dx$

$x = \frac{\pi}{2} : \ u = \sin\frac{\pi}{2} = 1$

$x = 0 \ : \ u = \sin 0 = 0$

or

8(b)(ii) $\ldots\ \frac{1}{7}\left[u^7\right] = \frac{1}{7}\left[(\sin x)^7\right]_0^{\frac{\pi}{2}}$

$$= \frac{1}{7}\left[\left(\sin\frac{\pi}{2}\right)^7 - (\sin 0)^7\right]$$

$$= \frac{1}{7}\left[(1)^7 - (0)^7\right]$$

$$= \frac{1}{7}(1) = \frac{1}{7}$$

Blunders (-3)

B1 integration.
B2 indices.
B3 limits.
B4 no limits.
B5 incorrect order in applying limits.
B6 not calculating substituted limits.
B7 not changing limits.
B8 differentiation.

Slips (-1)

S1 numerical.
S2 trig value.

Worthless:

W1 differentiation instead of integration except where other work merits attempt.

Note: Incorrect substitution and unable to finish yields attempt at most.
Note: (-3) is maximum deduction when evaluating limits

Part (c)(i) **10 marks** **Att 3**

8(c)(i) Show that $\int_a^{2a} \sin 2x\,dx = \sin 3a \sin a$.

Part (c)(i) **10 marks** **Att 3**

8(c)(i) $\int_a^{2a} \sin 2x\,dx = \frac{-\cos 2x}{2}\Big]_a^{2a}$

$$= -\frac{1}{2}\left[\cos 2(2a) - \cos 2(a)\right]$$

$$= -\frac{1}{2}\left[\cos 4a - \cos 2a\right]$$

$$= -\frac{1}{2}\left[-2\sin 3a \sin a\right]$$

$$= \sin 3a \sin a$$

or

8(c)(i)

$$\int_a^{2a} \sin 2x dx = \int 2\sin x(\cos x dx) \qquad u = \sin x$$

$$= \int 2.u.du \qquad \frac{du}{dx} = \cos x$$

$$= u^2 \qquad du = \cos x.dx$$

$$= \sin^2 x$$

$$= \frac{1}{2}[1 - \cos 2x]_a^{2a} = \frac{1}{2}[\cos 2a - \cos 4a] = \sin 3a.\sin a$$

Blunders (-3)
B1 integration.
B2 limits.
B3 no limits.
B4 incorrect order in applying limits.
B5 trig formula.
B6 not calculating substituted limits.

Slips (-1)
S1 numerical

Worthless
W1 differentiation instead of integration except where other work merits attempts.

Note: (-3) is maximum deduction in evaluation of limits.
Note: Incorrect substitution and unable to finish merits attempt at most.

Part (c)(ii) **10 marks** **Att 3**

(c)(ii) Use integration methods to show that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$

Part (c)(ii) **10 marks** **Att 3**

8(c)(ii)

$$V = \int_{-r}^{r} \pi y^2 dx$$

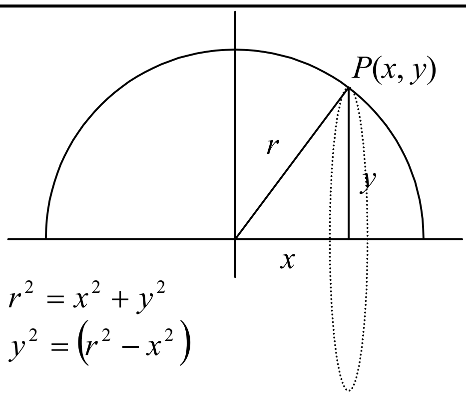
$$= \pi(2)\int_0^r y^2 dx$$

$$= 2\pi \int (r^2 - x^2)dx$$

$$= 2\pi \left[r^2 x - \frac{x^3}{3} \right]_0^r$$

$$= 2\pi \left[r^3 - \frac{r^3}{3} \right]$$

$$= 2\pi \left(\frac{2r^3}{3} \right) = \frac{4\pi r^3}{3}$$



$$r^2 = x^2 + y^2$$
$$y^2 = (r^2 - x^2)$$

Blunders (-3)
B1 equation circle.
B2 volume formula.(must be quadratic).
B3 limits.
B4 no limits.
B5 incorrect order in applying limits.
B6 indices.

Slips (-1)
S1 numerical

Attempts
A1 uses $v = \pi y$

Worthless
W1 differentiation instead of integration.
W2 $\int 4\pi R^2 dR$

Note: (-3) is maximum deduction in evaluation of limits.

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2003

MATHEMATICS

HIGHER LEVEL

PAPER 2

GENERAL GUIDELINES FOR EXAMINERS - PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled as B1, B2, B3,......, S1, S2, S3,...., M1, M2, etc. Note that these lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any correct relevant step in a part of a question merits *at least* the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,....etc.

4. The *same* error in the *same* section of a question is penalised *once* only.

5. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

6. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

7. A serious blunder, omission or misreading merits the ATTEMPT mark at most.

8. The phrase “and stops” means that no more work is shown by the candidate.

9. Accept the best of two or more attempts – even when attempts have been cancelled.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, -)**
Part (c)	**20 (10,10) marks**	**Att (3, 3)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Substitutes into x^2 and y^2		
or Cartesian point on circle found		
or t^2(t) in terms of x and in terms of y	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1(a)

For all values of $t \in \mathbf{R}$, the point $\left(\frac{3-3t^2}{1+t^2}, \frac{6t}{1+t^2} \right)$ lies on the circle $x^2 + y^2 = r^2$.

Find *r*, the radius of the circle.

Radius of circle **10 (5, 5) marks** **Att (2, 2)**

1 (a)

$$t = 0 \Rightarrow (3, 0) \in x^2 + y^2 = r^2 \Rightarrow 9 = r^2. \quad \therefore r = 3.$$

or

$$\left(\frac{3-3t^2}{1+t^2} \right)^2 + \left(\frac{6t}{1+t^2} \right)^2 = \frac{9-18t^2+9t^4+36t^2}{(1+t^2)^2} = \frac{9t^4+18t^2+9}{(1+t^2)^2}$$

$$= \frac{9(t^4+2t^2+1)}{(t^2+1)^2} = \frac{9(t^2+1)^2}{(t^2+1)^2} = 9.$$

$$\therefore r^2 = 9 \Rightarrow r = 3.$$

or

$$x = \frac{3-3t^2}{1+t^2} \Rightarrow x + xt^2 = 3 - 3t^2 \Rightarrow t^2(x+3) = 3 - x$$

$$\therefore t^2 = \frac{3-x}{3+x}.$$

$$\text{But } y^2 = \frac{36t^2}{(1+t^2)^2} = \frac{36\left(\frac{3-x}{3+x}\right)}{\left(1+\frac{3-x}{3+x}\right)^2} = \frac{36(3-x)(3+x)}{36}.$$

$$\therefore y^2 = 9 - x^2 \Rightarrow x^2 + y^2 = 9. \quad \therefore r = 3.$$

∗ Accept $r^2 = 9$.

Blunders (-3)

B1 incorrect squaring (apply once if same type of error).
B2 incorrect factorisation.

Slips (-1)

S1 arithmetic error.

Attempts (2, 2 marks)

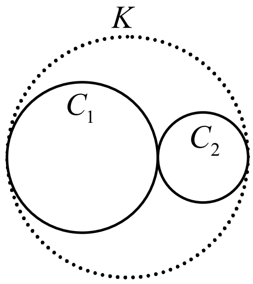
A1 substitutes for x only or substitutes for y only.
A2 fails to eliminate t.
A3 assigns value for t.
A4 attempt at simplification

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, -)**
Part (b) (i)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**
Centre & radius C_1	**5 marks**	**Att 2**
Centre & radius C_2	**5 marks**	**Att 2**
Final conclusion	**5 marks**	**Att 2**

1 (b) (i)

$C_1: x^2 + y^2 + 2x - 2y - 23 = 0$ and
$C_2: x^2 + y^2 - 14x - 2y + 41 = 0$ are two circles.

(i) Prove that C_1 and C_2 touch externally.



C_1 and C_2 touch externally	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

1 (b) (i)

Centre $C_1 = c_1(-1, 1)$, radius $= \sqrt{1+1+23} = 5$. $\therefore r_1 = 5$.
Centre $C_2 = c_2(7, 1)$, radius $= \sqrt{49+1-41} = 3$. $\therefore r_2 = 3$.
$|c_1c_2| = \sqrt{64+0} = 8$.
As $|c_1c_2| = r_1 + r_2$, the circles touch externally.

Blunders (-3)

B1 incorrect sign in centre point or incorrect g, f.
B2 error in radius length formula.
B3 error in distance formula.
B4 fails to conclude.

Slips (-1)

S1 arithmetic error.

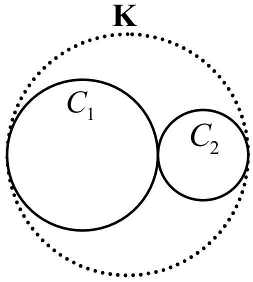
Attempts (2, 2, 2 marks)

A1 a correct centre
A2 a correct radius.
A3 sum of radii.

Part (b) (ii) **5 marks** **Hit/Miss**

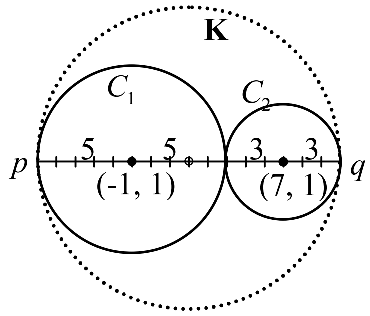
1 (b) (ii)

K is a third circle.
Both C_1 and C_2 touch *K* internally.
The centres of the three circles lie in a straight line.
Find the equation of *K*.



Equation of circle *K* **5 marks** **Hit/Miss**

1 (b) (ii)



Diameter of $K = 10 + 6 = 16 \Rightarrow$ radius = 8.
p is 5 units to the left of (-1, 1) $\Rightarrow$ p (-6, 1)
q is 3 units to the right of (7, 1) $\Rightarrow$ q (10, 1)

Midpoint of $[pq]$ = centre of circle $K = (2, 1)$.

$\therefore$ Equation of circle $K : (x-2)^2 + (y-1)^2 = 64$.

* Award full marks to candidates who indicate some understanding of the criteria for touching circles. For example:
(i) states $d = R - r$, $d = R + r$ or similar
(ii) finds any one of the relevant quantities, e.g. centre and/or radius of one or more of the three circles involved.

* **Note:** work presented in part (b)(i) may be of merit for part (b)(ii).

Part (c)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c) (i)	**10 marks**	**Att 3**

1 (c) (i)

The line $ax + by = 0$ is a tangent to the circle $x^2 + y^2 - 12x + 6y + 9 = 0$

where $a, b \in \mathbf{R}$ and $b \neq 0$.

(i) Show that $\dfrac{a}{b} = -\dfrac{3}{4}$.

Show	**10 marks**	**Att 3**

1 (c) (i)

Centre of circle $= (6, -3)$ and radius $= \sqrt{36 + 9 - 9} = 6$.

$\perp$ distance from $(6, -3)$ to tangent $ax + by = 0$ equals radius 6.

$$\therefore \left| \frac{6a - 3b}{\sqrt{a^2 + b^2}} \right| = 6 \Rightarrow \left| \frac{2a - b}{\sqrt{a^2 + b^2}} \right| = 2.$$

$$(2a - b)^2 = 4a^2 + 4b^2$$

$$4a^2 - 4ab + b^2 = 4a^2 + 4b^2 \Rightarrow 4ab = -3b^2 \Rightarrow 4a = -3b$$

$$\therefore \frac{a}{b} = -\frac{3}{4}.$$

or

$$y = -\frac{ax}{b} \Rightarrow x^2 + \frac{a^2x^2}{b^2} - 12x - \frac{6ax}{b} + 9 = 0$$

$$\therefore x^2\left(1 + \frac{a^2}{b^2}\right) + x\left(-12 - \frac{6a}{b}\right) + 9 = 0.$$

Line a tangent, so perfect square $\Rightarrow \left(-12 - \dfrac{6a}{b}\right)^2 - 4\left(1 + \dfrac{a^2}{b^2}\right)(9) = 0$

$$144b^2 + 144ab + 36a^2 - 36b^2 - 36a^2 = 0$$

$$108b^2 + 144ab = 0 \Rightarrow 3b + 4a = 0. \quad \therefore \frac{a}{b} = -\frac{3}{4}.$$

Blunders (-3)
B1 error in perpendicular distance formula.
B2 error in centre point.
B3 error in radius distance formula.
B4 error in squaring.
B5 error in quadratic formula.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 uses perpendicular distance and fails to finish. .
A2 quadratic in x or y and fails to finish

Part (c) (ii) **10 marks** **Att 3**

1 (c) (ii)

(ii) Hence, or otherwise, find the co-ordinates of the point of contact.

Point of contact **10 marks** **Att 3**

1(c) (ii)

$$ax + by = 0 \quad \Rightarrow \quad y = \frac{3}{4}x \text{ as } \frac{a}{b} = -\frac{3}{4}.$$

$$y = \frac{3}{4}x \;\cap\; x^2 + y^2 - 12x + 6y + 9 = 0$$

$$\text{gives } x^2 + \frac{9x^2}{16} - 12x + \frac{9}{2}x + 9 = 0$$

$$25x^2 - 120x + 144 = 0$$

$$(5x - 12)(5x - 12) = 0 \Rightarrow 5x - 12 = 0$$

$$\therefore\; x = \frac{12}{5} \text{ and hence } y = \frac{9}{5}. \quad \text{Point of contact is } \left(\frac{12}{5}, \frac{9}{5}\right).$$

Blunders (-3)
B1 error in factors or quadratic formula..
B2 finds x value but fails to give y value.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 quadratic in x or y and fails to finish.
A2 slope or equation of normal.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10,10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 marks** **Att 3**

2 (a)

oabc is a parallelogram where *o* is the origin, $\vec{a} = 3\vec{i} - \vec{j}$ and $\vec{b} = 4\vec{i} + 3\vec{j}$.

Express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

Express $\vec{c}$ **10 marks** **Att 3**

2 (a)

$$\vec{c} = \vec{ab} = \vec{b} - \vec{a} = 4\vec{i} + 3\vec{j} - 3\vec{i} + \vec{j} \;\Rightarrow\; \vec{c} = \vec{i} + 4\vec{j}.$$

or

$$\vec{b} = \vec{a} + \vec{c} \;\Rightarrow\; \vec{c} = \vec{b} - \vec{a} = \vec{i} + 4\vec{j}.$$

Blunders (-3)

B1 $\vec{ab} = \vec{a} - \vec{b}$.
B2 incorrect translation.
B3 error in translation.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 diagram with correct order of points oabc.
A2 $\vec{oc} = \vec{ab}$ and stops.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

Part (b) (i) **10 marks** **Att 3**

2 (b) (i)

$\vec{p} = 2\vec{i} + \vec{j}$, $\vec{q} = 3\vec{i} + k\vec{j}$, $\vec{r} = 3\vec{i} + t\vec{j}$ where $k, t \in \mathbf{R}$ and *o* is the origin.

(i) Given that $\vec{p} \perp \vec{q}$, calculate the value of *k*.

Calculate value of *k* **10 marks** **Att 3**

2 (b) (i)

$$\vec{p} \perp \vec{q} \;\Rightarrow\; \vec{p}.\vec{q} = 0.$$

$$\therefore\; (2\vec{i} + \vec{j})(3\vec{i} + k\vec{j}) = 0 \;\Rightarrow\; 6 + k = 0. \quad k = -6.$$

Blunders (-3)

B1 $\vec{p}.\vec{q} = -1$.

B2 incorrect vector multiplication.

B3 incorrect formula.

Slips (-1)

S1 arithmetic error.

Attempts (3 marks)

A1 $\vec{p}.\vec{q} = 0$.

A2 a vector perpendicular to $\vec{p}$ or $\vec{q}$.

Part (b) (ii) **10 marks** **Att 3**

2 (b) (ii)
(ii) Given that $\lvert \angle por \rvert = 45°$, calculate the two possible values of *t*.

Values of *t* **10 marks** **Att 3**

2 (b) (ii)

$$\cos 45^0 = \frac{\vec{p}.\vec{q}}{|\vec{p}|.|\vec{q}|} \Rightarrow \frac{1}{\sqrt{2}} = \frac{(2\vec{i}+\vec{j})(3\vec{i}+t\vec{j})}{|2\vec{i}+\vec{j}|.|3\vec{i}+t\vec{j}|}$$

$$\frac{1}{\sqrt{2}} = \frac{6+t}{\sqrt{5}\sqrt{9+t^2}} \Rightarrow 5(9+t^2) = 2(6+t)^2$$

$$\therefore\ 45+5t^2 = 72+24t+2t^2$$

$$3t^2-24t-27=0 \Rightarrow t^2-8t-9=0 \Rightarrow (t-9)(t+1)=0$$

$$\therefore\ t=9 \text{ or } t=-1.$$

Blunders (-3)

B1 incorrect formula for norm of vector.

B2 incorrect vector multiplication.

B3 incorrect squaring.

B4 incorrect factors.

B5 error in slope formula.

B6 sign error in $\tan\theta$ formula.

B7 only one value of *t* found.

B8 $\vec{op}.\vec{ro}$.

Slips (-1)

S1 arithmetic error.

Attempts (3 marks)

A1 scalar product with substitution.

A2 norm of vector found.

A3 $\vec{p}.\vec{r}$ correct.

A4 one correct slope.

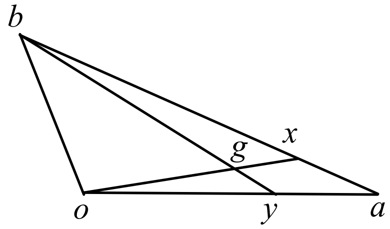
Part (c) **20 (10, 5, 5)** **Att (3, 2, 2)**

Part (c) (i) **10 marks** **Att 3**

2 (c) (i)

oab is a triangle where *o* is the origin.

(i) *x* is a point on [*ab*] such that | *ax* | : | *xb* | = 1 : 3.
Express $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$.



Express $\vec{x}$ **10 marks** **Att 3**

2 (c) (i)

$$\vec{x} = \frac{3}{4}\vec{a} + \frac{1}{4}\vec{b} \quad \textbf{or} \quad \vec{x} = \vec{a} + \frac{1}{4}\overrightarrow{ab} = \vec{a} + \frac{1}{4}\left(\vec{b} - \vec{a}\right) \Rightarrow \vec{x} = \frac{3}{4}\vec{a} + \frac{1}{4}\vec{b}.$$

Blunders (-3)

B1 error in ratio formula.

B2 $\overrightarrow{ab} = \vec{a} - \vec{b}$.

Slips (-1)

S1 arithmetic error.

Attempts (3 marks)

A1 $\vec{x} = \vec{a} + \frac{1}{4}\overrightarrow{ab}$.

Part (c) (ii) **5 marks** **Att 2**

2 (c) (ii)

(ii) *y* is a point on [*oa*] such that | *oy* |: | *ya* | = 2 : 1.
Express $\overrightarrow{by}$ in terms of $\vec{a}$ and $\vec{b}$.

Express $\overrightarrow{by}$ **5 marks** **Att 2**

2 (c) (ii)

$$\overrightarrow{by} = \vec{y} - \vec{b} = \frac{2}{3}\vec{a} - \vec{b} \quad \textbf{or} \quad \overrightarrow{by} = \overrightarrow{bo} + \overrightarrow{oy} = -\vec{b} + \frac{2}{3}\vec{a}.$$

Blunders (-3)

B1 error in ratio formula.

B2 $\overrightarrow{by} = \vec{b} - \vec{y}$.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 $\overrightarrow{by} = \overrightarrow{ba} + \overrightarrow{ay}$.

Part (c) (iii) **5 marks** **Att 2**

2 (c) (iii)   **(iii)** [*ox*] and [*by*] intersect at *g*.   Given that $\vec{g} = m\vec{x}$ and $\overrightarrow{bg} = n\,\overrightarrow{by}$ where $m, n \in \mathbf{R}$,   find the value of *m* and the value of *n*.

Find value of *m* and of *n* **5 marks** **Att 2**

2 (c) (iii)

$$\vec{g} = m\vec{x} \;\Rightarrow\; \vec{g} = \frac{3m}{4}\vec{a} + \frac{m}{4}\vec{b}.$$

$$\overrightarrow{bg} = n\,\overrightarrow{by} \;\Rightarrow\; \vec{g} - \vec{b} = \frac{2n}{3}\vec{a} - n\vec{b}$$

$$\therefore\; \vec{g} = \frac{2n}{3}\vec{a} + \vec{b}(1-n).$$

$$\therefore\; \frac{3\text{m}}{4} = \frac{2n}{3} \text{ and } \frac{m}{4} = 1-n.$$

$$\frac{2n}{3} = 3(1-n) \quad \therefore\; n = \frac{9}{11}. \quad \text{But } m = 4(1-n) \Rightarrow m = \frac{8}{11}.$$

Blunders (-3)
B1 error in solving other than slip.
B2 finds *m* but fails to give *n* value.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 $\vec{g}$ in terms of $\vec{a}$ and $\vec{b}$ and scalar *m* or $\vec{g}$ in terms of $\vec{a}$ and $\vec{b}$ and scalar *n*.
A2 equates coefficients.
A3 two equations in *m* and *n* and stops.

QUESTION 3

Part (a)	**20 marks**	**Att 6**
Part (b)	**30 (5, 5, 5, 5, 5, 5) marks**	**Att (2, 2, 2, 2, 2, 2)**

Part (a) **20 marks** **Att 6**

3 (a) f is the transformation $(x, y) \to (x', y')$ where $x' = x + y$ and $y' = x - y$.
L is the line $4x - 2y - 1 = 0$.
Find the equation of $f(L)$, the image of L under f.

Find equation of *f* (*L*) **20 marks** **Att 6**

3 (a)

$$x' = x + y$$
$$y' = x - y \;\Rightarrow\; x' + y' = 2x \quad \therefore\; x = \tfrac{1}{2}(x' + y').$$
But $y = x' - x = x' - \tfrac{1}{2}(x' + y') \;\Rightarrow\; y = \tfrac{1}{2}(x' - y')$.
$$f(L):\; 2(x' + y') - (x' - y') = 1$$
$$f(L):\; x' + 3y' - 1 = 0.$$

or

$(0, -\tfrac{1}{2}),\ (\tfrac{1}{4}, 0) \in L$; $f(0, -\tfrac{1}{2}) = (-\tfrac{1}{2}, \tfrac{1}{2})$ and $f(\tfrac{1}{4}, 0) = (\tfrac{1}{4}, \tfrac{1}{4})$.
$$\text{Slope of } f(L) = \frac{\tfrac{1}{2} - \tfrac{1}{4}}{-\tfrac{1}{2} - \tfrac{1}{4}} = -\frac{1}{3}.$$
$$f(L):\; y' - \tfrac{1}{2} = \tfrac{-1}{3}(x' + \tfrac{1}{2}) \Rightarrow 3y' - \tfrac{3}{2} = -x' - \tfrac{1}{2}$$
$$\therefore\; f(L): x' + 3y' - 1 = 0.$$

Blunders (-3)
B1 incorrect matrix.
B2 error in slope formula.
B3 image line not in the form of $ax' + by' + c = 0$.
B4 incorrect matrix multiplication.

Slips (-1)
S1 arithmetic error.

Attempts (6 marks)
A1 expressing x or y in terms of primes.
A2 correct matrix for f.
A3 a correct image point on f(L).

Part (b) 30 (5, 5, 5, 5, 5, 5) marks Att (2, 2, 2, 2, 2, 2)
Part (b) (i) 10 (5, 5) marks Att (2, 2)
Point on *M* 5 marks Att 2
Equation of *M* 5 marks Att 2

3 (b) (i)

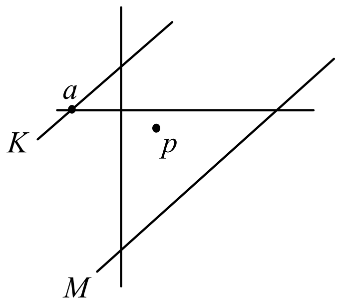
K is the line $3x - 4y + 9 = 0$.

The point $a(-3, 0)$ is on *K*.

The line *M* is parallel to *K*.

The point $p(2, -1)$ is midway between *K* and *M*.

(i) Find the equation of *M*.



Equation of *M* 10 (5, 5) marks Att (2, 2)

3 (b) (i)

p mid-point of [aq] where $q \in M$.

$a(-3, 0) \rightarrow p(2, -1) \rightarrow q(7, -2)$ and $q \in M$.

Equation of $M : 3x - 4y = k$ as M parallel to K.

$(7, -2) \in M \Rightarrow 21 + 8 = k. \therefore k = 29.$

Equation of line $M : 3x - 4y = 29$.

Blunders (-3)
B1 error in translation or central symmetry.
B2 error in mid-point formula.
B3 error in slope of *M* other than slip.
B4 error in equation line formula.

Slips (-1)
S1 arithmetic error.

Attempts (2, 2 marks)
A1 one component of point on *M* correct.
A2 slope of *M*.

Part (b) (ii) 5 marks Att 2

3 (b) (ii) Calculate the distance between *K* and *M*.

Distance between *K* and *M* 5 marks Att 2

3 (b) (ii)

$\perp$ distance from point $a(-3, 0)$ to the line $3x - 4y - 29 = 0$

$$\left|\frac{-9 + 0 - 29}{\sqrt{9 + 16}}\right| = \left|\frac{-38}{5}\right| = \frac{38}{5}.$$

Blunders (-3)
B1 error in perpendicular distance formula.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 perpendicular distance formula with some substitution.

Part (b) (iii)	**5 marks**	**Att 2**

3 (b) (iii)

(iii) Calculate the measure of the acute angle between *ap* and *K*.
Give your answer correct to the nearest degree.

Measure of acute angle	**5 marks**	**Att 2**

3 (b) (iii)

$a(-3,0),\ \ p(2,-1).\quad \text{Slope } ap = \dfrac{0+1}{-3-2} = -\dfrac{1}{5}.$

$K: 3x - 4y + 9 = 0.\ \ \text{Slope } K = \dfrac{3}{4}.$

$\tan\theta = \pm\dfrac{\frac{3}{4}+\frac{1}{5}}{1-\frac{3}{20}} = \pm\dfrac{19}{17}.\ \ \text{For acute angle, } \theta = \tan^{-1}\left(\dfrac{19}{17}\right) = 48^0.$

Blunders (-3)
B1 error in slope formula.
B2 sign error in tan θ formula.
B3 error in $\tan^{-1}\left(\frac{19}{17}\right)$ calculation.

Slips (-1)
S1 arithmetic error.
S2 angle not correct to nearest degree.

Attempts (2 marks)
A1 one correct slope.
A2 tan θ formula with some substitution.

Part (b) (iv)	**10 (5, 5) marks**	**Att (2, 2)**
$\vert ab\vert$ or $\vert ab\vert^2$ in terms of x and y	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

3 (b) (iv)

(iv) $b(x, y)$ is a point on K such that $|ab| = 15$ and $x > 0$.
Find the value of x and the value of y.

Value of x and value of y	**10 (5, 5) marks**	**Att (2, 2)**

3 (b) (iv)

$a(-3, 0),\ b(x, y)$ and $|ab| = 15.$

$(x+3)^2 + (y-0)^2 = 225.$

$(x, y) \in 3x - 4y + 9 = 0 \ \Rightarrow\ y = \frac{1}{4}(3x+9).$

$\therefore\ (x+3)^2 + \frac{1}{16}(3x+9)^2 = 225$

$16x^2 + 96x + 144 + 9x^2 + 54x + 81 = 225$

$25x^2 + 150x - 3375 = 0 \ \Rightarrow\ x^2 + 6x - 135 = 0$

$(x-9)(x+15) = 0 \Rightarrow x = 9. \quad x \neq -15$ as $x > 0$.

$\therefore\ x = 9$ and $y = 9.\ \therefore\ b(9, 9).$

Blunders (-3)
B1 incorrect squaring.
B2 incorrect factors.
B3 x value found and stops.

Slips (-1)
S1 arithmetic error.

Attempts (2, 2 marks)
A1 expression for $|ab| = 15$.
A2 K: expressed y in terms of x or x in terms of y.
A3 quadratic in x or in y.

QUESTION 4

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 (10, 15) marks**	**Att (3, 5)**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

Part (a)	**10 marks**	**Att 3**

4 (a) The circumference of a circle is 30π cm.
The area of a sector of the circle is 75 cm^2.

Find, in radians, the angle in this sector.

Measure of angle in sector	**10 marks**	**Att 3**

4 (a)

Circumference $= 2\pi r = 30\pi \Rightarrow r = 15\text{cm}.$

Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}(15)^2\theta = 75.$

$\therefore \ \theta = \frac{2}{3}$ radians.

Blunders (-3)
B1 error in circumference formula.
B2 error in area sector formula.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 correct value for *r*.

Part (b)	**25 (10, 15) marks**	**Att (3, 5)**
Express as product	**10 marks**	**Att 3**
Solutions for *x*	**15 marks**	**Att 5**

4 (b) Find all the solutions of the equation

$$\sin 2x + \sin x = 0$$

in the domain $0° \le x \le 360°$.

Find solutions	**25 (10, 15)**	**Att (3, 5)**

4 (b)

$\sin 2x + \sin x = 0$

$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0 \ \Rightarrow \ \sin\frac{3x}{2} = 0 \ \text{ or } \ \cos\frac{x}{2} = 0.$

$\frac{3x}{2} = 0^0, 180^0, 360^0, 540^0 \quad \text{or} \quad \frac{x}{2} = 90^0.$

$x = 0^0, 120^0, 240^0, 360^0 \quad \text{or} \quad x = 180^0.$

Solution $= \{0^0, 120^0, 180^0, \ 240^0, 360^0\}$ in the domain $0^0 \le x \le 360^0$.

or

$$\sin 2x + \sin x = 0$$
$$2\sin x \cos x + \sin x = 0$$
$$\sin x(2\cos x + 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}.$$
$$x = 0^0, 180^0, 360^0 \quad \text{or} \quad x = 120^0, 240^0$$
$$x = \{0^0, 120^0,\ 240^0, 180^0, 360^0\}.$$

* ignore solutions outside of range, correct or otherwise.

Blunders (-3)
B1 error in sin A + sin B formula.
B2 error in sin 2A formula.
B3 missing solution or incorrect solution.

Slips (-1)
S1 arithmetic error.

Attempts (3, 5 marks)
A1 expresses sin $2x = 2\sin x \cos x$ and stops.
A2 one correct solution.
A3 solutions for x not given.

Part (c) **15 (10, 5) marks** **Att (3, 2)**

Part (c) (i) **10 marks** **Att 3**

4 (c) (i)

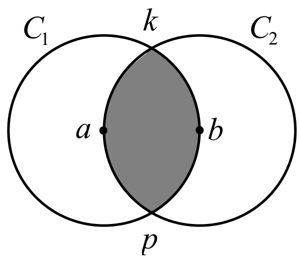
C_1 is a circle with centre a and radius r.

C_2 is a circle with centre b and radius r.

C_1 and C_2 intersect at k and p.

$a \in C_2$.

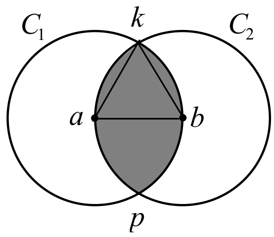
$b \in C_1$.



(i) Find, in radians, the measure of angle *kap*.

Measure of angle *kap* **10 marks** **Att 3**

4 (c) (i)



Triangle *kab* is equilateral with sides of length *r*.

$$\therefore |\angle kab| = \frac{\pi}{3} \Rightarrow |\angle kap| = \frac{2\pi}{3}$$

* accept angle in degrees. Penalise in part (c) (ii).

Blunders (-3)
B1 an incorrect length in one triangle side used.
B2 error in cosine rule formula.
B3 $|\angle kab| = 60^0$ and $|\angle kap|$ not given.

Slips (-1)
S1 arithmetic error.

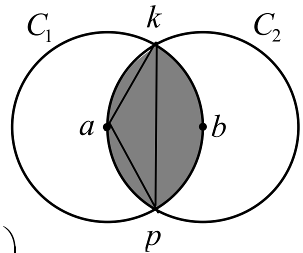
Attempts (3 marks)
A1 draws triangle showing length of side.

Part (c) (ii) **5 marks** **Att 2**

4 (c) (ii) Calculate the area of the shaded region.
Give your answer in terms of *r* and *π*.

Area of shaded region 5 marks **Att 2**

4 (c) (ii)



Area triangle *kap* $= \frac{1}{2}r^2 \sin\frac{2\pi}{3} = \frac{1}{2}r^2\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2.$

area of sector *apk* in $C_1 = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\frac{2\pi}{3} = \frac{\pi}{3}r^2.$

Area of shaded region *kpb* in $C_1 = \frac{\pi}{3}r^2 - \frac{\sqrt{3}}{4}r^2 = r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

$\therefore$ Area of required shaded region $= 2r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right).$

Blunders (-3)
B1 error in triangle area formula.
B2 error in sector area formula.
B3 finds area of triangle and of sector but fails to finish.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 triangle area formula with some substitution.
A2 sector area formula with some substitution.

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 (15, 10) marks**	**Att (5, 3)**
Part (c)	**15 (10, 5) marks**	**Att (3, 2)**

Part (a) **10 marks** **Att 3**

5 (a) Find the value of $\sin 15°$ in surd form.

Value of Sin 15^0 **10 marks** **Att 3**

5 (a)

$$\sin 15^0 = \sin(45^0 - 30^0)$$
$$= \sin 45^0 \cos 30^0 - \cos 45^0 \sin 30^0$$
$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

Blunders (-3)
B1 error in sin (A – B) formula.
B2 sin 45^0 etc incorrect.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 sin 15^0 = sin (45^0 – 30^0) or sin 15^0 = sin (60^0 – 45^0).
A2 sin 45^0 etc not in surd form.

Worthless (0)
W1 sin 15^0 evaluated from calculator.

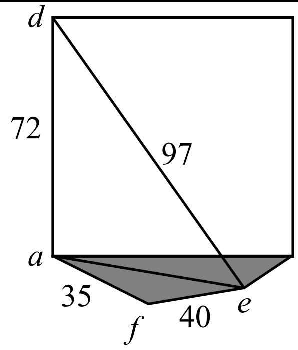
Part (b) **25 (15, 10) marks** **Att (5, 3)**

Part (b) (i) **15 marks** **Att 5**

5 (b) (i) a, f and e are points on horizontal ground.
d is a point on a vertical wall directly above a.

$|ad| = 72$ m, $|de| = 97$ m,
$|af| = 35$ m and $|fe| = 40$ m.

(i) Calculate $|ae|$.



Calculate | *ae* | **15 marks** **Att 5**

5 (b) (i)

$$|ae|^2 = 97^2 - 72^2 = 4225$$
$$|ae| = \sqrt{4225} = 65.$$

Blunders (-3)
B1 incorrect squaring.
B2 incorrect application of Pythagoras.
B3 error in square root.

Slips (-1)
S1 arithmetic error.

Attempts (5 marks)
A1 application of Pythagoras.

Part (b) (ii) **10 marks** **Att 3**

5 (b) (ii) Hence, calculate $|\angle afe|$.

Calculate $|\angle afe|$ **10 marks** **Att 3**

5 (b) (ii)

$$\cos\angle afe = \frac{35^2 + 40^2 - 65^2}{2(35)(40)} = \frac{-1400}{2800} = -\frac{1}{2}$$
$$|\angle afe| = 120^0.$$

Blunders (-3)
B1 error in cosine rule formula.
B2 error in substitution into cosine rule formula.
B3 incorrect squaring.
B4 incorrect evaluation of angle.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 cosine formula with some substitution.

Part (c) **15 (10, 5) marks** **Att (3, 2)**

Part (c) (i) **10 marks** **Att 3**

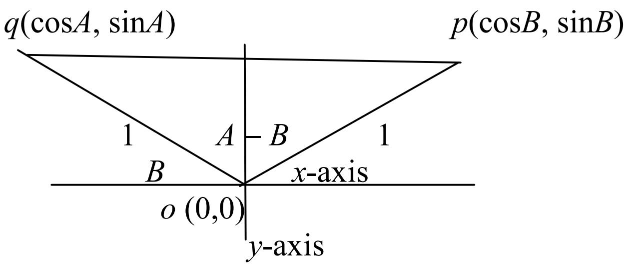
5 (c) (i)	Using the identity $\cos(A - B) = \cos A \cos B + \sin A \sin B$, or otherwise, prove: $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

Prove sin(*A*+*B*) **10 marks** **Att 3**

5 (c) (i)

$$\sin(A+B) = \cos\left(90^0 - (A+B)\right)$$
$$= \cos\left((90^0 - A) - B\right)$$
$$= \cos(90^0 - A)\cos B + \sin(90^0 - A)\sin B$$
$$= \sin A \cos B + \cos A \sin B.$$

or



Area triangle *opq* = $\frac{1}{2}(1)(1)\text{Sin}(A-B)$
Area triangle *opq* = $\frac{1}{2}\left|\sin A\cos B - \cos A\sin B\right|$.

$\therefore\ \sin(A-B) = \sin A\cos B - \cos A\sin B$
Replacing B by $-B \Rightarrow \sin(A+B) = \sin A\cos(-B) - \cos A\sin(-B)$
$= \sin A\cos B + \cos A\sin B$.

Blunders (-3)
B1 incorrect expansion of $\cos[(90^0 - A) - B)]$

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 $\sin(A + B) = \cos[90^0 - (A + B)]$
A2 area of triangle *opq*.

Part (c) (ii) **5 marks** **Att 2**

5 (c) (ii)	Prove: $\sin(A+B)\sin(A-B) = (\sin A + \sin B)(\sin A - \sin B)$.

Prove **5 marks** **Att 2**

5 (c) (ii)

$$\sin(A+B)\sin(A-B) = [\sin A\cos B + \cos A\sin B][\sin A\cos B - \cos A\sin B]$$
$$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$$
$$= \sin^2 A\left(1-\sin^2 B\right) - \left(1-\sin^2 A\right)\sin^2 B$$
$$= \sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$$
$$= \sin^2 A - \sin^2 B$$
$$= (\sin A + \sin B)(\sin A - \sin B).$$

or

$$\sin(A+B)\sin(A-B) = \frac{1}{2}\left(\cos 2B - \cos 2A\right) = \frac{1}{2}\left(1 - 2\sin^2 B - 1 + 2\sin^2 A\right)$$
$$= \sin^2 A - \sin^2 B = (\sin A + \sin B)(\sin A - \sin B).$$

Blunders (-3)

B1 error in expansion of sin (A + B) or sin (A–B).

B2 error in sin (A + B). sin (A–B) formula.

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 correct expansion of sin (A–B).

A2 correct application of sin (A + B).sin (A–B) formula.

QUESTION 6

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Part (a) (i) **5 marks** **Att 2**

6 (a) (i)

Eight people, including Kieran and Anne, are available to form a committee. Five people must be chosen for the committee.

(i) In how many ways can the committee be formed if both Kieran and Anne must be chosen?

Part (a) (i) **5 marks** **Att 2**

6 (a) (i) Kieran and Anne chosen. $\therefore$ Select 3 from 6.

Solution = ${}^6C_3 = 20$.

Blunders (-3)
B1 ${}^6C_5$.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 ${}^8C_3$.

Part (a) (ii) **5 marks** **Att 2**

6 (a) (ii) In how many ways can the committee be formed if neither Kieran nor Anne can be chosen?

Part (a) (ii) **5 marks** **Att 2**

6 (a) (ii)

Both Kieran and Anne not chosen. $\therefore$ Select 5 from 6.

Solution = ${}^6C_5 = 6$.

Blunders (-3)
B1 ${}^6C_3$.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 $8C_5$.

Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (b) (i)	15 (5, 5, 5) marks	Att (2, 2, 2)

Characteristic roots	5 marks	Att 2
Simultaneous equations	5 marks	Att 2
Final solution	5 marks	Att 2

6 (b) (i) Solve the difference equation $u_{n+2} - 4u_{n+1} + 3u_n = 0$, where $n \geq 0$, given that $u_0 = -2$ and $u_1 = 4$.

Solve	15 (5, 5, 5) marks	Att (2, 2, 2)

6 (b) (i)

$$u_{n+2} - 4u_{n+1} + 3u_n = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0 \quad \Rightarrow \quad x - 3 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 3 \quad \text{or} \quad x = 1.$$
$$u_n = l(\alpha)^n + k(\beta)^n = l(3)^n + k(1)^n.$$
$$u_n = l(3)^n + k$$
$$u_0 = -2 \quad \therefore \; l + k = -2$$
$$u_1 = 4 \quad \therefore 3l + k = 4 \quad \Rightarrow \quad -2l = -6$$
$$\therefore \; l = 3, \; k = -5.$$
$$u_n = 3(3)^n - 5.$$

Blunders (-3)
B1 error in characteristic equation.
B2 error in factors or in quadratic formula.
B3 incorrect use of initial conditions.

Slips (-1)
S1 arithmetic error.

Attempts (2, 2, 2 marks)
A1 correct characteristic equation.
A2 an equation in *k* and *l*.
A3 correct value for *k* or *l*.

Part (b) (ii) **5 marks** **Att 2**

6 (b) (ii)	Verify that the solution you have obtained in **(i)** satisfies the difference equation.

Verify **5 marks** **Att 2**

6 (b (ii)

$$u_{n+2} - 4u_{n+1} + 3u_n$$
$$= 3(3)^{n+2} - 5 - 12(3)^{n+1} + 20 + 9(3)^n - 15$$
$$= 27(3)^n - 36(3)^n + 9(3)^n$$
$$= 0.$$

Blunders (-3)
B1 u_{n+2} or u_{n+1} incorrect.
B2 error in indices.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 correct substitution into difference equation.
A2 u_{n+2} or u_{n+1} correct.

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**
Part (c) (i) **5 marks** **Att 2**

6 (c) (c) Ten discs, each marked with a different whole number from 1 to 10, are placed in a box.
Three of the discs are drawn at random (without replacement) from the box.

(i) What is the probability that the disc with the number 7 is drawn?

Probability disc with 7 is drawn **5 marks** **Att 2**

6 (c) (i)

Number of possible outcomes = ${}^{10}C_3 = 120$.
Number of favourable outcomes (one 7 and any two others) = ${}^{9}C_2 = 36$

Probability $= \frac{36}{120} = \frac{3}{10}$.

or

Probability $= \frac{1}{10}.\frac{9}{9}.\frac{8}{8} \times 3 = \frac{3}{10}$. (×3 as the 7 can be drawn on the first or second or third disc).

Blunders (-3)
B1 favourable and possible outcomes given, but probability not expressed.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 correct number of possible outcomes.
A2 correct number of favourable outcomes.
A3 failure to allow for order, i.e. number 7 can be drawn first, second or third.

Part (c) (ii) 5 marks Att 2

6 (c) (ii)	What is the probability that the three numbers on the discs drawn are odd?

Probability of three odd numbers 5 marks Att 2

6 (c) (ii)

$$\text{Number of possible outcomes } = {}^{10}C_3 = 120.$$

$$\text{Number of favourable outcomes } = {}^{5}C_3 = 10$$

$$\text{Probability} = \frac{10}{120} = \frac{1}{12}.$$

or

$$P(\text{three odd}) = P(\text{first odd}).P(\text{second odd}).P(\text{third odd}) = \frac{5}{10}.\frac{4}{9}.\frac{3}{8} = \frac{60}{720} = \frac{1}{12}.$$

Blunders (-3)
B1 favourable and possible outcomes given, but probability not expressed.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 correct number of possible outcomes.
A2 correct number of favourable outcomes.

Part (c) (iii) 5 marks Att 2

6 (c) (iii)	What is the probability that the product of the three numbers on the discs drawn is even?

Probability product even 5 marks Att 2

6 (c) (iii)

For product even as least one of the numbered discs must be even.
At least one even = all possible outcomes − outcome of all three odd.
Possible outcomes = ${}^{10}C_3 = 120$.
Favourable outcomes = ${}^{10}C_3 - {}^{5}C_3 = 120 - 10 = 110$.
$\therefore$ Probability (three odd) = $\frac{110}{120} = \frac{11}{12}$.

or

P(Three odd) = P(E, E, E) + P(E, E, O) + P(E, O, O)

$$= \frac{5}{10}.\frac{4}{9}.\frac{3}{8} + \frac{5}{10}.\frac{4}{9}.\frac{5}{8} \times 3 + \frac{5}{10}.\frac{5}{9}.\frac{4}{8} \times 3$$

$$= \frac{60 + 300 + 300}{720} = \frac{660}{720} = \frac{11}{12}.$$

Blunders (-3)
B1 favourable and possible outcomes given, but probability not expressed.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 correct number of possible outcomes.
A2 correct number of favourable outcomes.
A3 P(even, even, even) or P(even, even, odd) or P(even, odd, odd) calculated.
A4 solution = 1 – P(three odd) and stops.

Part (c) (iv) **5 marks** **Att 2**

6 (c) (iv) What is the probability that the smallest number on the discs drawn is 4?

Probability smallest number is 4 **5 marks** **Att 2**

6 (c) (iv)

Choose 4 and any two discs numbered 5, 6, 7, 8, 9, 10.

Favourable outcomes $= {}^{6}C_{2} = 15$.

Possible outcomes = ${}^{10}C_{3} = 120$.

Probability $= \frac{15}{120} = \frac{1}{8}$.

or

Probability $\frac{1}{10}.\frac{6}{9}.\frac{5}{8} \times 3 = \frac{90}{720} = \frac{1}{8}$.

Blunders (-3)
B1 favourable and possible outcomes given, but probability not expressed.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 correct number of possible outcomes.
A2 correct number of favourable outcomes.
A3 failure to allow for order, i.e. number 4 can be drawn first, second or third.

QUESTION 7

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

Part (a) (i) **5 marks** **Att 2**

7 (a) Five cars enter a car park. There are exactly five vacant spaces in the car park.

(i) In how many different ways can the five cars park in the vacant spaces?

Part (a) (i) **5 marks** **Att 2**

7 (a) (i)

Solution = $5\times4\times3\times2\times1 = 5! = 120$.

Blunders (-3)
B1 5^5.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 two cars parked in 5×4 ways.

Part (a) (ii) **5 marks** **Att 2**

7 (a) (ii) Two of the cars leave the car park without parking.
In how many different ways can the remaining three cars park in the five vacant spaces?

Part (a) (ii) **5 marks** **Att 2**

7 (a) (ii)

Solution = $5\times4\times3 = 60$.

Blunders (-3)
B1 5^3.

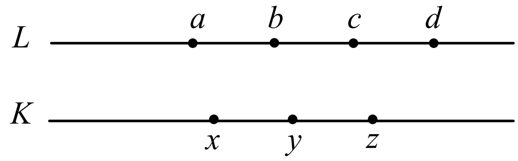
Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 two cars parked in 5×4 ways.

Part (b) **20 (5, 5, 5, 5)** **Att (2, 2, 2, 2)**

Part (b) (i) **5 marks** **Att 2**

7 (b) (i)



L and K are distinct parallel lines.

a, b, c and d are points on L such that $|ab| = |bc| = |cd| = 1$ cm.

x, y and z are points on K such that $|xy| = |yz| = 1$ cm.

(i) How many different triangles can be constructed using three of the named points as vertices?

Number of triangles **5 marks** **Att 2**

7 (b) (i)

Form triangles with one point from K and two points from L or one point from L and two points from K.

Solution = ${}^3C_1 \times {}^4C_2 \ + \ {}^4C_1 \times {}^3C_2 = 18 + 12 = 30.$

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 ${}^3C_1 \times {}^4C_2$ or ${}^4C_1 \times {}^3C_2$

Part (b) (ii) **5 marks** **Att 2**

7 (b) (ii) How many different quadrilaterals can be constructed using four of the named points as vertices?

Quadrilaterals **5 marks** **Att 2**

7 (b) (ii)

Form quadrilateral with two points from L and two points from K.

Solution = ${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18.$

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 ${}^4C_2$ or ${}^3C_2$ or ${}^4C_1 + {}^3C_2$.

Part (b) (iii) **5 marks** **Att 2**

7 (b) (iii)	How many different parallelograms can be constructed using four of the named points as vertices?

Parallelograms **5 marks** **Att 2**

7 (b) (iii)

$[xy]$ with $[ab]$ or $[bc]$ or $[cd] = 1 \times 3 = 3$

$[yz]$ with $[ab]$ or $[bc]$ or $[cd] = 1 \times 3 = 3$

$[xz]$ with $[ac]$ or $[bd] = 1 \times 2 = 2$

$\therefore$ Total = 8.

Blunders (-3)

B1 one set of parallelograms omitted.

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 one set of possible parallelograms correctly counted.

Part (b) (iv) **5 marks** **Att 2**

7 (b) (iv)	If one quadrilateral is constructed at random, what is the probability that it is *not* a parallelogram?

Probability **5 marks** **Att 2**

7 (b) (iv)

Number of quadrilaterals = 18. Solution from part (b) (ii).

Number of parallelograms = 8 Solution from part (b) (iii).

$\therefore$ Number of quadrilaterals not parallelograms = 18−10 = 10.

Probability $= \frac{10}{18} = \frac{5}{9}$.

Blunders (-3)

B1 possible and favourable outcomes found but probability not given.

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 correct number of favourable outcomes.

A2 correct number of possible outcomes.

Part (c) **20 marks (5, 5, 10)** **Att (2, 2, 3)**
Part (c) (i) **5 marks** **Att 2**

7 (c)	The mean of the real numbers a and b is $\bar{x}$. The standard deviation is σ. **(i)** Express σ in terms of a, b and $\bar{x}$.

Express σ **5 marks** **Att 2**

7 (c) (i)

$$\sigma = \sqrt{\frac{(a-\bar{x})^2 + (b-\bar{x})^2}{2}}.$$

Blunders (-3)
B1 error in standard deviation formula.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 any correct deviation.

Part (c) (ii) **5 marks** **Att 2**

7 (c) (ii) Hence, express σ in terms of a and b only.

Express σ **5 marks** **Att 2**

7 (c) (ii)

$$\bar{x} = \frac{a+b}{2}.$$

$$\sigma = \sqrt{\frac{\left(a - \frac{a+b}{2}\right)^2 + \left(b - \frac{a+b}{2}\right)^2}{2}} = \sqrt{\frac{(a-b)^2 + (b-a)^2}{8}}$$

$$= \sqrt{\frac{2a^2 - 4ab + 2b^2}{8}} = \sqrt{\frac{a^2 - 2ab + b^2}{4}} = \sqrt{\frac{(a-b)^2}{4}}$$

$$\sigma = \frac{a-b}{2}.$$

Blunders (-3)
B1 error in mean.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 correct mean.

Part (c) (iii) **10 marks** **Att 3**

7 (c) (iii)	Show that $\bar{x}^2 - \sigma^2 = ab$.

Show **10 marks** **Att 3**

7 (c) (iii)

$$\bar{x}^2 - \sigma^2 = \left(\frac{a+b}{4}\right)^2 - \left(\frac{a-b}{4}\right)^2 = \frac{a^2 + 2ab + b^2 - a^2 + 2ab - b^2}{4}$$

$$\bar{x}^2 - \sigma^2 = \frac{4ab}{4} = ab.$$

Blunders (-3)

B1 incorrect squaring.

Slips (-1)

S1 arithmetic error.

Attempts (3 marks)

A1 $\bar{x}^2 - \sigma^2$ expressed in terms of *a* and of *b*.

QUESTION 8

Part (a)	**10 marks**	**Att 3**
Part (b)	**25 (15, 5, 5) marks**	**Att (5, 2, 2)**
Part (c)	**15 (5, 5, 5) marks**	**Att (2, 2, 2)**

Part (a) **10 marks** **Att 3**

8 (a) Use integration by parts to find $\int xe^{-5x}\,dx$.

Integration **10 marks** **Att 3**

8 (a)

$$\int xe^{-5x}\,dx = \int u\,dv = uv - \int v\,du.$$

$$u = x \Rightarrow du = dx. \quad dv = e^{-5x}dx \Rightarrow v = \int e^{-5x}dx = -\frac{1}{5}e^{-5x}.$$

$$\therefore \int xe^{-5x}\,dx = -\frac{1}{5}xe^{-5x} + \frac{1}{5}\int e^{-5x}dx = -\frac{1}{5}xe^{-5x} - \frac{1}{25}e^{-5x} + \text{constant.}$$

Blunders (-3)
B1 incorrect differentiation or integration.
B2 constant of integration omitted.
B3 incorrect 'parts' formula.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 correct assigning to parts formula.
A2 correct differentiation or integration.

Part (b) **25 marks (15, 5, 5)** **Att (5, 2, 2)**
Part (b) (i) **15 marks** **Att 5**

8 (b) (i)

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$ is the Maclaurin series.

(i) Derive the Maclaurin series for $f(x) = \log_e(1+x)$ up to and including the term containing x^4.

(i) Maclaurin series **15 marks** **Att 5**

8 (b) (i)

$$f(x) = \log_e(1+x) \quad \Rightarrow \quad f(0) = 0$$
$$f'(x) = \frac{1}{1+x} = (1+x)^{-1} \quad \Rightarrow \quad f'(0) = 1$$
$$f''(x) = -1(1+x)^{-2} \quad \Rightarrow \quad f''(0) = -1$$
$$f'''(x) = 2(1+x)^{-3} \quad \Rightarrow \quad f'''(0) = 2$$
$$f''''(x) = -6(1+x)^{-4} \quad \Rightarrow \quad f''''(0) = -6.$$

$$\therefore f(x) = 0 + x - \frac{x^2}{2!} + \frac{2x^2}{3!} - \frac{6x^4}{4!} + \ldots$$
$$\therefore f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

Blunders (-3)
B1 incorrect differentiation.
B2 incorrect evaluation of $f^n(0)$.
B3 each term not derived.

Slips (-1)
S1 arithmetic error.

Attempts (5 marks)
A1 $f(0)$ correct.
A2 a correct differentiation.
A3 any one correct term.

Part (b) (ii) **10 marks (5, 5)** **Att (2, 2)**
General term **5 marks** **Att 2**
Show **5 marks** **Att 2**

8 (b) (ii) Write down the general term and use the Ratio Test to show that the series converges for $-1 < x < 1$.

Part (b) (ii) **10 marks (5, 5)** **Att (2, 2)**

8 (b) (ii)

$$u_n = \frac{(-1)^{n+1}x^n}{n} \quad \Rightarrow \quad u_{n+1} = \frac{(-1)^{n+2}x^{n+1}}{n+1}.$$

$$\underset{n\to\infty}{\text{Limit}}\left|\frac{u_{n+1}}{u_n}\right| = \underset{n\to\infty}{\text{Limit}}\left|\frac{(-1)^{n+2}x^{n+1}}{n+1} \times \frac{n}{(-1)^n x^n}\right|$$

$$\underset{n\to\infty}{\text{Limit}}\left|\frac{(-1)xn}{n+1}\right| = \underset{n\to\infty}{\text{Limit}}\left|\frac{x}{1+\frac{1}{n}}\right| = |\,x\,|.$$

Convergent for $|\,x\,| < 1 \;\Rightarrow\; -1 < x < 1.$

Blunders (-3)

B1 (−1) omitted from general term.
B2 an incorrect power in general term.
B3 error in u_{n+1}.
B4 error in evaluating limit other than slip.
B5 evaluates limit as $|x|$ and stops.

Slips (-1)

S1 arithmetic error.

Attempts (2, 2 marks)

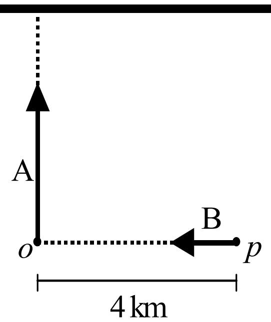
A1 any part of u_n correct.
A2 u_{n+1} correct.
A3 states ratio test correctly.

Part (c) **15 (5, 5, 5) marks** **Att (2, 2, 2)**
Part (c) (i) **5 marks** **Att 2**

8 (c) The point p is 4 km due east of the point o.

At noon, A leaves o and travels north at a steady speed of 12 km/h. At the same time, B leaves p and travels towards o at a steady speed of 6 km/h.

(i) Write down expressions in x for the distances that A and B will each have travelled at x minutes after noon.



(i) Expressions in *x* **5 marks** **Att 2**

8 (c) (i)

A travels at 12km/h and *B* travels at 12km/h.

In x minutes, *A* travels a distance of $x \times \frac{12}{60} = \frac{x}{5}$ km.

In x minutes, *B* travels a distance of $x \times \frac{6}{60} = \frac{x}{10}$ km.

Blunders (-3)

B1 distance travelled by A given as $12x$ and/or distance travelled by B given as $6x$.

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 correct distance travelled by A given or correct distance travelled by B given.

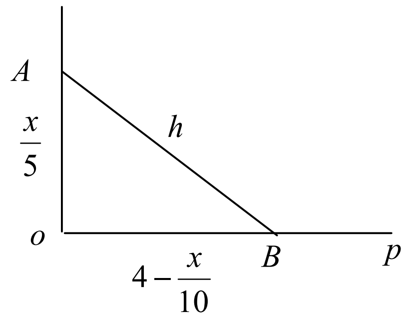
Part (c) (ii) **5 marks** **Att 2**

8 (c) (ii)	Find an expression in x for the distance that B will be from A at x minutes after noon.

Distance from *A* to *B* **5 marks** **Att 2**

8 (c) (ii)

$$h = \sqrt{\left(\frac{x}{5}\right)^2 + \left(4 - \frac{x}{10}\right)^2}$$



Blunders (-3)
B1 error in distance from B to A.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 correct distance from o to B.

Part (c) (iii) **5 marks** **Att 2**

8 (c) (iii)	At how many minutes after noon will B be closest to A?

How many minutes **5 marks** **Att 2**

8 (c) (iii)

$$h = \left(\frac{x^2}{25} + (4 - \frac{x}{10})^2\right)^{\frac{1}{2}}$$

$$\frac{dh}{dx} = \frac{1}{2}\left(\frac{x^2}{25} + (4 - \frac{x}{10})^2\right)^{-\frac{1}{2}} . \left(\frac{2x}{25} + 2(4 - \frac{x}{10})(-\frac{1}{10})\right).$$

For minimum $\frac{dh}{dx} = 0$

$$\therefore \frac{2x}{25} - \frac{1}{5}\left(4 - \frac{x}{10}\right) = 0 \;\Rightarrow\; 2x - 20 + \frac{x}{2} = 0$$

$5x = 40 \;\Rightarrow\; x = 8$ minutes.

Note: $\frac{d^2h}{dx^2} > 0$ for $x = 8 \Rightarrow$ minimum.

or

$$\frac{x^2}{25}+16-\frac{4x}{5}+\frac{x^2}{100} = \frac{x^2}{20}-\frac{4x}{5}+16$$

$$=\frac{1}{20}\left(x^2-16x+320\right)=\frac{1}{20}\left(x^2-16x+64+256\right)$$

$$=\frac{1}{20}\left((x-8)^2+256.\right)$$

Minimum value for $x - 8 = 0$.

Minimum at $x = 8$ minutes.

* $\frac{d^2x}{dh^2} > 0$ for $x = 8$ not required.

Blunders (-3)

B1 error in differentiation.

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 equates $dh/dx = 0$ and stops.

A2 some correct differentiation and stops.

A3 if previous error does not allow solution.

QUESTION 9

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

9 (a) z is a random variable with standard normal distribution.
Calculate $P(-2.13 < z \le 1.46)$.

Calculate **10 marks** **Att 3**

9 (a)

$$P(-2.13 < z \le 1.46)$$
$$= P(z \le 1.46) - P(z \le -2.13)$$
$$= P(z \le 1.46) - [1 - P(z < 2.13)]$$
$$= 0.9279 - (1 - 0.9834)$$
$$= 0.9113.$$

Blunders (-3)
B1 incorrect area defined.
B2 incorrect reading from tables.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 $P(z \le 1.46)$ found.
A2 $P(z < 2.13)$ found.

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**
Part (b) (i) **5 marks** **Att 2**

9 (b) Whenever Anne's mobile phone rings, the probability that she answers the call is $\frac{3}{4}$.
A friend phones Anne six times.

(i) What is the probability that she misses all the calls?

P(misses all calls) **5 marks** **Att 2**

9 (b) (i)

$$P(\text{Anne answers}) = \frac{3}{4}. \quad P(\text{Anne does not answer}) = \frac{1}{4}.$$

$$P(\text{Anne misses all calls}) = \left(\frac{1}{4}\right)^6 = \frac{1}{2096}.$$

Blunders (-3)
B1 incorrect value for q.
B2 error in binomial.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 $q = ¼$.

Part (b)(ii) **5 marks** **Att 2**

9 (b) (ii) What is the probability that she misses the first two calls and answers the others?

Probability **5 marks** **Att 2**

9 (b) (ii)

$$\text{Probability} = \left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4 = \frac{81}{4096}.$$

Blunders (-3)
B1 powers of probability interchanged.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 $(¼)^2$ or $(¾)^4$ in solution

Part (b) (iii) **5 marks** **Att 2**

9 (b) (iii) What is the probability that she answers exactly one of the calls?

Probability **5 marks** **Att 2**

9 (b) (iii)

$$\text{Probability} = {}^6C_1\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^1 = \frac{9}{2048}.$$

Blunders (-3)
B1 binomial coefficient omitted.
B2 p and q interchanged in solution.

Slips (-1)
S1 arithmetic error.

Attempts (2 marks)
A1 a correct substitution into binomial.

Part (b) (iv) **5 marks** **Att 2**

9 (b) (iv)	What is the probability that she answers at least two of the calls?

Probability **5 marks** **Att 2**

9 (b) (iv)

P(At least two calls answered) = 1 − [P(one answered) + P(none answered)].

$$= 1 - [\ {}^6C_1 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^1 + \left(\frac{1}{4}\right)^6\ ].$$

$$= 1 - \frac{18}{4096} - \frac{1}{4096}$$

$$= \frac{4077}{4096}.$$

Blunders (-3)

B1 omits one essential probability.

Slips (-1)

S1 arithmetic error.

Attempts (2 marks)

A1 P = 1 − [P(one answered) + P(none answered)].

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

Find $\bar{x}$	**5 marks**	**Att 2**
Find σ	**5 marks**	**Att 2**
Correct confidence interval	**5 marks**	**Att 2**
Final solution	**5 marks**	**Att 2**

9 (c) In a newspaper advertisement, a driving school claimed that 80% of its clients passed their driving test on their first attempt.

1000 people who had attended the school and who had taken the test for the first time were randomly selected.

Find, at the 5% level of significance, the interval in which the number who passed should lie in order that the claim made in the advertisement be accepted.

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

9 (c)

H_0 : Claim is true.

$$p = \frac{80}{100} = 0.8 \quad \Rightarrow \quad q = 0.2. \qquad \text{n} = 1000.$$

$$\bar{x} = np = 1000(0.8) = 800.$$

$$\sigma = \sqrt{npq} = \sqrt{160} = 12.65.$$

$$\begin{aligned} 5\% \text{ confidence interval} &= \bar{x} \pm 1.96\sigma \\ &= 800 \pm 1.96(12.65) \\ &= 800 \pm 24.79 \\ &= [776, 824]. \end{aligned}$$

$\therefore$ Accept claim if number lies between 776 and 824 inclusive.

Blunders (-3)
B1 incorrect value of p or q
B2 incorrect formula for mean.
B3 incorrect formula for σ.
B4 error in confidence given.
B5 final answer not simplified.

Slips (-1)
S1 arithmetic error.

Attempts (2, 2, 2, 2 marks)
A1 correct value of p or of q.
A2 correct formula for mean.
A3 correct formula for σ.
A4 error in confidence interval.
A5 substitutes into confidence and stops.

QUESTION 10

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Show $A^2 = B$	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

10 (a)

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},\ C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},\ D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$M = \{A, B, C, D\}$ is a cyclic group under matrix multiplication.

Verify that A is a generator of M.

Verify A is a generator of M	**10 (5, 5) marks**	**Att (2, 2)**

10 (a)

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \therefore\ A^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = B.$$

$$A^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = C.$$

$$A^4 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = D.$$

$\therefore\ A$ is a generator.

Blunders (-3)

B1 incorrect matrix multiplication.

B2 A^3 or A^4 not evaluated.

Slips (-1)

S1 arithmetic error.

Attempts (2, 2 marks)

A1 attempt at evaluating A^2.

A2 attempt at evaluating A^3 or A^4.

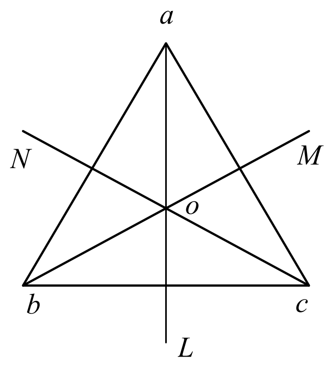
Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

Part (b) (i) **10 (5, 5) marks** **Att (2, 2)**
Find one composition **5 marks** **Att 2**
Find other composition and finish **5 marks** **Att 2**

10 (b) (i)

abc is an equilateral triangle.
L, M and N, the perpendicular bisectors of the sides, intersect at o.

$D_3 = \{I_\pi, R_{120°}, R_{240°}, S_L, S_M, S_N\}$, under composition, is the symmetry group of *abc*.



(i) Investigate whether $S_L \circ S_M = S_M \circ S_L$.

Investigate **10 (5, 5) marks** **Att (2, 2)**

10 (b) (i)

$S_L \circ S_M = R_{120^0}$ and $S_M \circ S_L = R_{240^0}$
$\therefore\ S_L \circ S_M \neq S_M \circ S_L$.

Blunders (-3)
B1 $S_L \circ S_M$ evaluated as incorrect rotation.
B2 $S_M \circ S_L$ evaluated as incorrect rotation.

Slips (-1)
S1 arithmetic error.

Attempts (2, 2 marks)
A1 $S_L \circ S_M = R_{2\alpha}$.
A2 $S_M \circ S_L = R_{2\beta}$.

Part (b) (ii)	10 marks	Att 3

10 (b) (ii) Write down the centralizer of $R_{120°}$.

Centralizer of $R_{120°}$	10 marks	Att 3

10 (b) (ii)

Elements that commute with $R_{120°}$

$C\left(R_{120^0}\right) = \{I_\pi, R_{120^0}, R_{240^0}\}.$

Blunders (-3)

B1 gives exactly two elements, both correct.

Slips (-1)

S1 arithmetic error.

Attempts (3 marks)

A1 given exactly one element, which is correct.

Part (c)	20 (10, 10)	Att (3, 3)
$f\left(R_{120°}{}^{-1}\right) = f(R_{240^0})$ or $[f(R_{120^0})]^{-1}$	10 marks	Att 3
Final conclusion	10 marks	Att 3

10 (c)

$$P = \left\{\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix},\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix},\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix},\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\right\}$$

P is a group under composition of permutations.

D_3 is the symmetry group of the equilateral triangle *abc*, as described in **(b)**.

$f: D_3 \to P$ is an isomorphism where $f\left(R_{120°}\right) = \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$.

Find $f\left(R_{120°}{}^{-1}\right)$, justifying your answer.

Find $f\left(R_{120°}{}^{-1}\right)$	20 marks (10, 10)	Att (3, 3)

10 (c)

$$f\left(R_{120^0}{}^{-1}\right) = f\left(R_{240^0}\right) = \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}.$$

or

$$f\left(R_{120^0}{}^{-1}\right) = \left[f\left(R_{120^0}\right)\right]^{-1} = \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}^{-1} = \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}.$$

Blunders (-3)

B1 incorrect inverse permutation, but with one column correct and two columns incorrect (given that only 1, 2 and 3 are used).

Slips (-1)

S1 arithmetic error.

Attempts (3, 3 marks)

A1 $R_{120^0}{}^{-1} = R_{240^0}$.

A2 incorrect inverse permutation with all columns incorrect but uses 1, 2 and 3 in 'solution'.

QUESTION 11

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 marks** **Att 3**

11 (a)

The polar of a point *p* with respect to the circle $x^2 + y^2 = 9$ is $2x - 5y = 27$.

Given that the polar of the point (x_1, y_1) with respect to the circle $x^2 + y^2 = r^2$ is $xx_1 + yy_1 = r^2$, find the coordinates of *p*.

Find coordinates of *p* **10 marks** **Att 3**

11 (a)

$x^2 + y^2 = 9$. The polar of p is $2x - 5y = 27$.

$2x - 5y = 27 \Rightarrow \frac{2}{3}x - \frac{5}{3}y = 9$. Line is in the form of $xx_1 + yy_1 = r^2$.

$\therefore p$ is $\left(\frac{2}{3}, -\frac{5}{3}\right)$.

Blunders (-3)

B1 error in *dy/dx*.

Slips (-1)

S1 arithmetic error.

Attempts (3 marks)

A1 line expressed in the form of $xx' + yy' = r^2$.

A2 finds *dy/dx*.

A3 solves between line and circle.

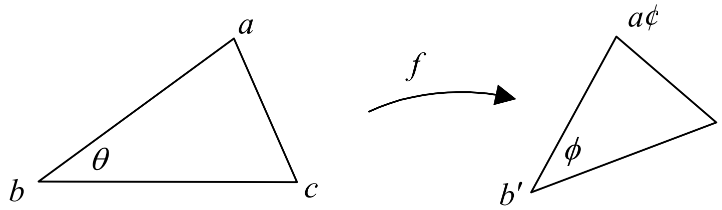
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Cos θ	**5 marks**	**Att 2**
Cos ϕ	**5 marks**	**Att 2**
$\lvert p'q' \rvert = k \lvert pq \rvert$	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

11 (b) f is a similarity transformation.
f maps the angle θ onto the angle ϕ.

Prove that θ and ϕ are equal in measure.

Prove	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

11 (b)



$$\cos\theta = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab|.|bc|}$$

$$\cos\phi = \frac{|a'b'|^2 + |b'c'|^2 - |a'c'|^2}{2|a'b'|.|b'c'|}.$$

f is a similarity transformation.

$\therefore\ |p'q'| = k|pq|$ where k is a scalar.

$$\therefore\ \cos\phi = \frac{k^2|ab|^2 + k^2|bc|^2 - k^2|ac|^2}{2k^2|ab|.|bc|}$$

$$\cos\phi = \frac{|ab|^2 + |bc|^2 - |ac|^2}{2|ab|.|bc|} = \cos\theta.$$

$\cos\theta = \cos\phi \ \Rightarrow\ |\angle\theta| = |\angle\phi|$ as $0^0 \le \theta \le 180^0$.

Blunders (-3)
B1 error in cosine formula.
B2 cos ϕ not simplified fully.
B3 error in definition of similarity transformation.

Slips (-1)
S1 arithmetic error.

Attempts (2, 2, 2, 2 marks)
A1 diagram of value.
A2 fails to square k.
A3 k^2 incorrectly cancelled.

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

Part (c) (i) **10 marks** **Att 3**

11 (c) (i) [*cd*] is a diameter of the ellipse $\frac{x^2}{100}+\frac{y^2}{25}=1$ where *c* is the point $\left(5\sqrt{2},\ \frac{5}{2}\sqrt{2}\right)$. Find the equation of the tangent to the ellipse at *c*.

Equation of tangent **10 marks** **Att 3**

11 (c) (i)

$$\frac{x^2}{100}+\frac{y^2}{25}=1, \quad \text{point } c\left(5\sqrt{2},\ \frac{5}{2}\sqrt{2}\right).$$

Tangent at (x_1, y_1) is $\frac{xx_1}{100}+\frac{yy_1}{25}=1$

$$\therefore \text{ tangent at c} \left(5\sqrt{2}, \frac{5}{2}\sqrt{2}\right) \text{ is } \frac{5\sqrt{2}x}{100}+\frac{\frac{5\sqrt{2}}{2}y}{25}=1$$

$$\therefore \frac{10x}{100}+\frac{10y}{50}=\sqrt{2} \Rightarrow \frac{x}{10}+\frac{y}{5}=\sqrt{2}.$$

Blunders (-3)
B1 error in tangent formula.

Slips (-1)
S1 arithmetic error.

Attempts (3 marks)
A1 formula for equation of tangent.
A2 finds dy/dx.

Part (c) (ii) **10 (5, 5) marks** **Att (2, 2)**
Equation of *st* **5 marks** **Att 2**
Coordinates of *s* and *t* **5 marks** **Att 2**

11 (c) (ii) The diameter [*st*] is conjugate to the diameter [*cd*].
Find the equation of *st* and the coordinates of the points *s* and *t*.

Part (c) (ii) **10 (5, 5) marks** **Att (2, 2)**

11 (c) (ii)

[*st*] and [*cd*] are conjugate diameters of ellipse *E*.

$\therefore$ Tangent at *c* is parallel to *st*.

Equation of *st* is $\frac{x}{10} + \frac{y}{5} = k$.

Blunders (-3)

B1 error in slope.
B2 error in line equation formula.
B3 incorrect squaring.
B4 finds the coordinates of one point.

Slips (-1)

S1 arithmetic error.

Attempts (2, 2 marks)

A1 recognises that tangent at *c* is parallel to *st*.
A2 correct slope of *st*.
A3 equation in x^2 or y^2 from solving.